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Series Expansions and Polynomial Approximations of Monomolecular Growth Model for Some Populations of *Eucalyptus camaldulensis Dehn.* From Eastern Mediterranean Forest Research Manager

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ABSTRACT

In this study, firstly the series expansions of monomolecular growth model from first degree polynomial to (n-1)th degree polynomial, were given with respect to (t-r) where t is time, n is the number of data, r is integer number: $t_0 \le r \le t_{n-1}$, t_0 and t_{n-1} are initial and final values of time, respectively. Secondly, monomolecular growth model's series expansions having m-th degree polynomials, studied on the data taken for Eucalyptus camaldulensis Dehn. from Eastern Mediterranean Forest Research Manager were given with R^2 with respect to (t-k), respectively where t is time; n is the number of data points; m, k are integer numbers $1 < m \le n-1$, $0 \le k \le 9$. Finally, for each data set, polynomial approximations having m-th degree were given with R^2. For each purpose, the tables and the graphs were used for analyzing the differences.

Keywords:
Series expansions; polynomial approximation; monomolecular growth model; growth models; Eucalyptus camaldulensis Dehn.

INTRODUCTION

In mathematics, a series expansion is a method for calculating a function that cannot be expressed by only elementary operations such as addition, subtraction, multiplication and division. The resulting so called series often can be limited to a finite number of terms, thus yielding an approximation of the function. The fewer terms of the sequence are used, the simpler this approximation will be. Actually, there are some kinds of series such as divergent series, Taylor series and power series. A Taylor series is a representation of a function as an infinite sum of terms that are calculated from the values of the function's derivatives at a single point. If the Taylor series is centered at zero, then that series is also called a Maclaurin series [1]. The general idea behind Taylor series is that if a function satisfies certain criteria, then the function can be expressed as an infinite series of polynomials. In its most general terms, the value of a function, f(x), in the vicinity of the point x0, is given by:

$$f(x) = \sum_{r=0}^{\infty} a_r (x - x_0)^r \qquad (1)$$

where x_0 is the initial point of the series, a_r are the coefficient of the series.

A polynomial function is a function such as quadratic, cubic, and so on, involving only non-negative integer powers of x. The degree of a polynomial is the highest power of x in its expression. Actually polynomial is the special condition of a Maclaurin series when n is finite. In its most general terms, the value of a polynomial, p(x) is given by:

$$p(x) = \sum_{r=0}^{n} a_r x^r \qquad (2)$$

where a_r are the coefficient of the polynomial, n is the degree of the polynomial. A central problem of mathematical analysis is the approximation to more general functions by polynomials and the estimation of how small the discrepancy can be made.

Growth models have generally had sigmoidal shape. These models have one inflection point. For these growth models, growth rate increased continually until

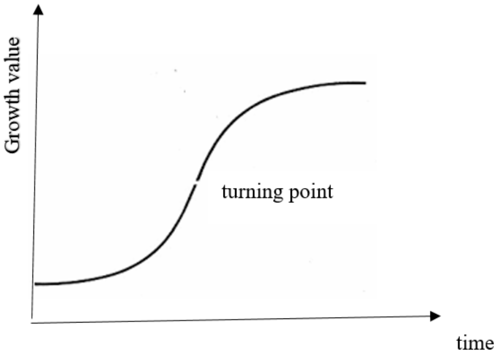

Figure 1. Sigmoidal function

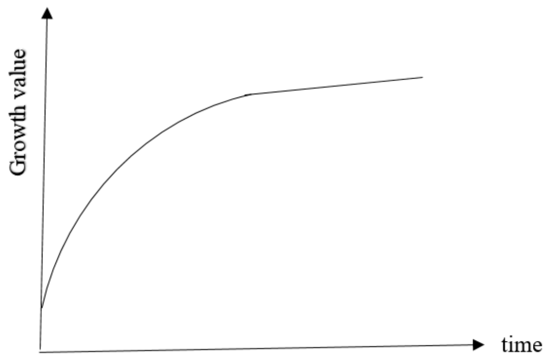

Figure 2. Increasing function by decreasing rate

the inflection point and the highest growth rate occurs at inflection point. After that point the growth rate decreased continually (Table 1).

The monomolecular or Brody function [2] is of decaying exponential type with no inflection point. Fabens [3] described a similar function based on the work of Bertalanffy. For these functions the highest growth rate occurs at birth and decreased continually (Table 2) [4].

In this study the series expansions of only one of the growth models were presented to investigate the series expansions. For that reason, series expansions of Monomolecular growth model w.r.t. (t-r) were given below where t is time, r is integer number $t_0 \le r \le t_{n-1}$, t_0 and t_{n-1} is initial and final values of time, respectively and n is the number of data points.

Monomolecular Growth Model (M.G.M.):

Series expansions of Monomolecular growth model, $y = a(1 - b\exp(-ct))$, with respect to (w.r.t.) (t-0), (t-1) and (t-2) are given in Tables 1-3, respectively.

Table 1. Series expansions of Monomolecular growth model w.r.t. (t-0)

Degrees of series expansion of M.G.M. w.r.t.(t-o)	$y = a(1 - b\exp(-ct))$
1	$a(1-\mathrm{b}) + \mathrm{abct}$
2	$a(1-\mathrm{b}) + \mathrm{abct} - \frac{abc^2t^2}{2}$
3	$a(1-\mathrm{b}) + \mathrm{abct} - \frac{abc^2t^2}{2} + \frac{abc^3t^3}{6}$
4	$a(1-\mathrm{b}) + \mathrm{abct} - \frac{abc^2t^2}{2} + \frac{abc^3t^3}{6} - \frac{abc^4t^4}{24}$
5	$a(1-\mathrm{b}) + \mathrm{abct} - \frac{abc^2t^2}{2} + \frac{abc^3t^3}{6} - \frac{abc^4t^4}{24} + \frac{abc^5t^5}{120}$
6	$a(1-\mathrm{b}) + \mathrm{abct} - \frac{abc^2t^2}{2} + \frac{abc^3t^3}{6} - \frac{abc^4t^4}{24} + \frac{abc^5t^5}{120} - \frac{abc^6t^6}{720}$
7	$a(1-\mathrm{b}) + \mathrm{abct} - \frac{abc^2t^2}{2} + \frac{abc^3t^3}{6} - \frac{abc^4t^4}{24} + \frac{abc^5t^5}{120} - \frac{abc^6t^6}{720} + \frac{abc^7t^7}{5040}$
8	$a(1-\mathrm{b}) + \mathrm{abct} - \frac{abc^2t^2}{2} + \frac{abc^3t^3}{6} - \frac{abc^4t^4}{24} + \frac{abc^5t^5}{120} - \frac{abc^6t^6}{720} + \frac{abc^7t^7}{5040} - \frac{abc^8t^8}{40320}$
9	$a(1-\mathrm{b}) + \mathrm{abct} - \frac{abc^2t^2}{2} + \frac{abc^3t^3}{6} - \frac{abc^4t^4}{24} + \frac{abc^5t^5}{120} - \frac{abc^6t^6}{720} + \frac{abc^7t^7}{5040} - \frac{abc^8t^8}{40320} + \frac{abc^9t^9}{362880}$

Table 2. Series expansions of Monomolecular growth model w.r.t. (t-1)

Degrees of series expansion of M.G.M. w.r.t.(t-1)	$y = a(1 - b\exp(-ct))$
1	$a(1-\mathrm{be}^{(-c)}) + \mathrm{abe}^{(-c)}\mathrm{c}(\mathrm{t}-1)$

Table 2. Series expansions of Monomolecular growth model w.r.t. (t-1)(continued)

Degrees of series expansion of M.G.M. w.r.t.(t-1)	$y = a(1-b\exp(-ct))$
2	$a(1-\mathrm{be}^{(-c)})+\mathrm{abe}^{(-c)}\mathrm{c}(\mathrm{t}-1)-\frac{1}{2}\mathrm{abe}^{(-c)}\mathrm{c}^2(\mathrm{t}-1)^2$
3	$a(1-\mathrm{be}^{(-c)})+\mathrm{abe}^{(-c)}\mathrm{c}(\mathrm{t}-1)-\frac{1}{2}\mathrm{abe}^{(-c)}\mathrm{c}^2(\mathrm{t}-1)^2+\frac{1}{6}\mathrm{abe}^{(-c)}\mathrm{c}^3(\mathrm{t}-1)^3$
4	$a(1-\mathrm{be}^{(-c)})+\mathrm{abe}^{(-c)}\mathrm{c}(\mathrm{t}-1)-\frac{1}{2}\mathrm{abe}^{(-c)}\mathrm{c}^2(\mathrm{t}-1)^2+\frac{1}{6}\mathrm{abe}^{(-c)}\mathrm{c}^3(\mathrm{t}-1)^3-\frac{1}{24}\mathrm{abe}^{(-c)}\mathrm{c}^4(\mathrm{t}-1)^4$
5	$a(1-\mathrm{be}^{(-c)})+\mathrm{abe}^{(-c)}\mathrm{c}(\mathrm{t}-1)-\frac{1}{2}\mathrm{abe}^{(-c)}\mathrm{c}^2(\mathrm{t}-1)^2+\frac{1}{6}\mathrm{abe}^{(-c)}\mathrm{c}^3(\mathrm{t}-1)^3-\frac{1}{24}\mathrm{abe}^{(-c)}\mathrm{c}^4(\mathrm{t}-1)^4+\frac{1}{120}\mathrm{abe}^{(-c)}\mathrm{c}^5(\mathrm{t}-1)^5$
6	$a(1-\mathrm{be}^{(-c)})+\mathrm{abe}^{(-c)}\mathrm{c}(\mathrm{t}-1)-\frac{1}{2}\mathrm{abe}^{(-c)}\mathrm{c}^2(\mathrm{t}-1)^2+\frac{1}{6}\mathrm{abe}^{(-c)}\mathrm{c}^3(\mathrm{t}-1)^3-\frac{1}{24}\mathrm{abe}^{(-c)}\mathrm{c}^4(\mathrm{t}-1)^4+\frac{1}{120}\mathrm{abe}^{(-c)}\mathrm{c}^5(\mathrm{t}-1)^5$ $-\frac{1}{720}\mathrm{abe}^{(-c)}\mathrm{c}^6(\mathrm{t}-1)^6$
7	$a(1-\mathrm{be}^{(-c)})+\mathrm{abe}^{(-c)}\mathrm{c}(\mathrm{t}-1)-\frac{1}{2}\mathrm{abe}^{(-c)}\mathrm{c}^2(\mathrm{t}-1)^2+\frac{1}{6}\mathrm{abe}^{(-c)}\mathrm{c}^3(\mathrm{t}-1)^3-\frac{1}{24}\mathrm{abe}^{(-c)}\mathrm{c}^4(\mathrm{t}-1)^4+\frac{1}{120}\mathrm{abe}^{(-c)}\mathrm{c}^5(\mathrm{t}-1)^5$ $-\frac{1}{720}\mathrm{abe}^{(-c)}\mathrm{c}^6(\mathrm{t}-1)^6+\frac{1}{5040}\mathrm{abe}^{(-c)}\mathrm{c}^7(\mathrm{t}-1)^7$
8	$a(1-\mathrm{be}^{(-c)})+\mathrm{abe}^{(-c)}\mathrm{c}(\mathrm{t}-1)-\frac{1}{2}\mathrm{abe}^{(-c)}\mathrm{c}^2(\mathrm{t}-1)^2+\frac{1}{6}\mathrm{abe}^{(-c)}\mathrm{c}^3(\mathrm{t}-1)^3-\frac{1}{24}\mathrm{abe}^{(-c)}\mathrm{c}^4(\mathrm{t}-1)^4+\frac{1}{120}\mathrm{abe}^{(-c)}\mathrm{c}^5(\mathrm{t}-1)^5$ $-\frac{1}{720}\mathrm{abe}^{(-c)}\mathrm{c}^6(\mathrm{t}-1)^6+\frac{1}{5040}\mathrm{abe}^{(-c)}\mathrm{c}^7(\mathrm{t}-1)^7-\frac{1}{40320}\mathrm{abe}^{(-c)}\mathrm{c}^8(\mathrm{t}-1)^8$
9	$a(1-\mathrm{be}^{(-c)})+\mathrm{abe}^{(-c)}\mathrm{c}(\mathrm{t}-1)-\frac{1}{2}\mathrm{abe}^{(-c)}\mathrm{c}^2(\mathrm{t}-1)^2+\frac{1}{6}\mathrm{abe}^{(-c)}\mathrm{c}^3(\mathrm{t}-1)^3-\frac{1}{24}\mathrm{abe}^{(-c)}\mathrm{c}^4(\mathrm{t}-1)^4+\frac{1}{120}\mathrm{abe}^{(-c)}\mathrm{c}^5(\mathrm{t}-1)^5$ $-\frac{1}{720}\mathrm{abe}^{(-c)}\mathrm{c}^6(\mathrm{t}-1)^6+\frac{1}{5040}\mathrm{abe}^{(-c)}\mathrm{c}^7(\mathrm{t}-1)^7-\frac{1}{40320}\mathrm{abe}^{(-c)}\mathrm{c}^8(\mathrm{t}-1)^8+\frac{1}{362880}\mathrm{abe}^{(-c)}\mathrm{c}^9(\mathrm{t}-1)^9$

Table 3. Series expansions of Monomolecular growth model w.r.t. (t-2)

Degrees of series expansion of M.G.M.	$y = a(1-b\exp(-ct))$
1	$a(1-\mathrm{be}^{(-2c)})+\mathrm{abe}^{(-2c)}\mathrm{c}(\mathrm{t}-2)$
2	$a(1-\mathrm{be}^{(-2c)})+\mathrm{abe}^{(-2c)}\mathrm{c}(\mathrm{t}-2)-\frac{1}{2}\mathrm{abe}^{(-2c)}\mathrm{c}^2(\mathrm{t}-2)^2$
3	$a(1-\mathrm{be}^{(-2c)})+\mathrm{abe}^{(-2c)}\mathrm{c}(\mathrm{t}-2)-\frac{1}{2}\mathrm{abe}^{(-2c)}\mathrm{c}^2(\mathrm{t}-2)^2+\frac{1}{6}\mathrm{abe}^{(-2c)}\mathrm{c}^3(\mathrm{t}-2)^3$
4	$a(1-\mathrm{be}^{(-2c)})+\mathrm{abe}^{(-2c)}\mathrm{c}(\mathrm{t}-2)-\frac{1}{2}\mathrm{abe}^{(-2c)}\mathrm{c}^2(\mathrm{t}-2)^2+\frac{1}{6}\mathrm{abe}^{(-2c)}\mathrm{c}^3(\mathrm{t}-2)^3-\frac{1}{24}\mathrm{abe}^{(-2c)}\mathrm{c}^4(\mathrm{t}-2)^4$
5	$a(1-\mathrm{be}^{(-2c)})+\mathrm{abe}^{(-2c)}\mathrm{c}(\mathrm{t}-2)-\frac{1}{2}\mathrm{abe}^{(-2c)}\mathrm{c}^2(\mathrm{t}-2)^2+\frac{1}{6}\mathrm{abe}^{(-2c)}\mathrm{c}^3(\mathrm{t}-2)^3-\frac{1}{24}\mathrm{abe}^{(-2c)}\mathrm{c}^4(\mathrm{t}-2)^4+\frac{1}{120}\mathrm{abe}^{(-2c)}\mathrm{c}^5(\mathrm{t}-2)^5$
6	$a(1-\mathrm{be}^{(-2c)})+\mathrm{abe}^{(-2c)}\mathrm{c}(\mathrm{t}-2)-\frac{1}{2}\mathrm{abe}^{(-2c)}\mathrm{c}^2(\mathrm{t}-2)^2+\frac{1}{6}\mathrm{abe}^{(-2c)}\mathrm{c}^3(\mathrm{t}-2)^3-\frac{1}{24}\mathrm{abe}^{(-2c)}\mathrm{c}^4(\mathrm{t}-2)^4+\frac{1}{120}\mathrm{abe}^{(-2c)}\mathrm{c}^5(\mathrm{t}-2)^5$ $-\frac{1}{720}\mathrm{abe}^{(-2c)}\mathrm{c}^6(\mathrm{t}-2)^6$
7	$a(1-\mathrm{be}^{(-2c)})+\mathrm{abe}^{(-2c)}\mathrm{c}(\mathrm{t}-2)-\frac{1}{2}\mathrm{abe}^{(-2c)}\mathrm{c}^2(\mathrm{t}-2)^2+\frac{1}{6}\mathrm{abe}^{(-2c)}\mathrm{c}^3(\mathrm{t}-2)^3-\frac{1}{24}\mathrm{abe}^{(-2c)}\mathrm{c}^4(\mathrm{t}-2)^4+\frac{1}{120}\mathrm{abe}^{(-2c)}\mathrm{c}^5(\mathrm{t}-2)^5$ $-\frac{1}{720}\mathrm{abe}^{(-2c)}\mathrm{c}^6(\mathrm{t}-2)^6+\frac{1}{5040}\mathrm{abe}^{(-2c)}\mathrm{c}^7(\mathrm{t}-2)^7$
8	$a(1-\mathrm{be}^{(-2c)})+\mathrm{abe}^{(-2c)}\mathrm{c}(\mathrm{t}-2)-\frac{1}{2}\mathrm{abe}^{(-2c)}\mathrm{c}^2(\mathrm{t}-2)^2+\frac{1}{6}\mathrm{abe}^{(-2c)}\mathrm{c}^3(\mathrm{t}-2)^3-\frac{1}{24}\mathrm{abe}^{(-2c)}\mathrm{c}^4(\mathrm{t}-2)^4+\frac{1}{120}\mathrm{abe}^{(-2c)}\mathrm{c}^5(\mathrm{t}-2)^5$ $-\frac{1}{720}\mathrm{abe}^{(-2c)}\mathrm{c}^6(\mathrm{t}-2)^6+\frac{1}{5040}\mathrm{abe}^{(-2c)}\mathrm{c}^7(\mathrm{t}-2)^7-\frac{1}{40320}\mathrm{abe}^{(-2c)}\mathrm{c}^8(\mathrm{t}-2)^8$

Table 3. Series expansions of Monomolecular growth model w.r.t. (t-2)(continued)

Degrees of series expansion of M.G.M.	$y = a(1 - b\exp(-ct))$
9	$a(1-\mathrm{be}^{(-2c)}) + \mathrm{abe}^{(-2c)}c(t-2) - \frac{1}{2}\mathrm{abe}^{(-2c)}c^2(t-2)^2 + \frac{1}{6}\mathrm{abe}^{(-2c)}c^3(t-2)^3 - \frac{1}{24}\mathrm{abe}^{(-2c)}c^4(t-2)^4 + \frac{1}{120}\mathrm{abe}^{(-2c)}c^5(t-2)^5 - \frac{1}{720}\mathrm{abe}^{(-2c)}c^6(t-2)^6 + \frac{1}{5040}\mathrm{abe}^{(-2c)}c^7(t-2)^7 - \frac{1}{40320}\mathrm{abe}^{(-2c)}c^8(t-2)^8 + \frac{1}{362880}\mathrm{abe}^{(-2c)}c^9(t-2)^9$

Similarly, the remainder series expansions of Monomolecular growth model w.r.t. (t-r) could easily be shown in a similar manner where t is time, r is integer number $3 \le r \le t_{n-1}$, and n is the number of data points.

MATERIAL AND METHODS

In this study, monomolecular growth model was studied on the data taken for Eucalyptus camaldulensis Dehn. in Table 3.1. [5]. For the presentation of the models, the measurements of the mean tree lengths (m) in the age-structured of Eucalyptus camaldulensis Dehn. from Eastern Mediterranean Forest Research Manager were used [6] in this study in Table 4.

Table 4. Average heights of samples of trees, Eucalyptus camaldulensis Dehn. according to each age class

Ages (year)	Planting Age (0)	1	2	3	4	5	6	7	8	9
Average heights of the trees (m)	0.41	3.23	7.45	11.41	14.83	18.11	18.95	19.69	21.50	23.40

While the degrees of Taylor series expansions in the neighborhood of (t-k) were increasing, these expansions did not show uniform convergence and also continuous decrease of Sum of Squared Errors (SSE) and continuous increase of coefficient of determination (R^2) were not found. However, R^2 of the series expansion having m-th degree polynomial with respect to (t-k) generally increased uniformly or kept on the same value while k is increasing where t is time, m and k are integer numbers $1 < m \le n-1$, $0 \le k \le 9$.

RESULTS AND DISCUSSIONS

By using Table 4, the series expansions of Monomolecular growth model were given in the following tables. Since this monomolecular growth model is a nonlinear model, we have started to fit the monomolecular growth model by using second degree polynomial and then for fitting the model we have found third, fourth, fifth, sixth seventh, eighth and ninth degree polynomials respectively.

For each degree polynomial, we got the series expansions w.r.t. (t-k) where k is an integer, $0 \le k \le 9$, respectively. While k was increasing, R^2 of the series expansions generally increased uniformly or kept on the same value (Table 5-12).

Since we got the same Sum of Squared Errors (SSE) and R^2 of all series expansions for 10 and higher degrees, we did not make a table for them.

Table 5. Second degree series expansions of Monomolecular growth model with respect to (t-k) where t is time and k is integer number: $0 \le k \le 9$ and their the values of R^2

Series expansion of M.G.M. w.r.t.(t-k)	$y = a(1 - b\exp(-ct))$	R^2
t-0	-0.229272729041378 + 4.52265151414627 t - 0.218409090681348 t^2	0.992
t-1	-0.0108636352328890 + 4.08583332987219 t - 0.218409090163881 $(t-1)^2$	0.992
t-2	0.644363620911388 + 3.64901516282561 t - 0.218409093453883 $(t-2)^2$	0.992
t-3	1.73640908901081 + 3.21219697132384 t - 0.218409089883235 $(t-3)^2$	0.992
t-4	3.26527273991154 + 2.77537878811066 t - 0.218409092757985 $(t-4)^2$	0.992
t-5	5.23095453353882 + 2.33856060811264 t - 0.218409090505056 $(t-5)^2$	0.992
t-6	7.63345457478377 + 1.90174242086438 t - 0.218409092246574 $(t-6)^2$	0.992
t-7	10.4727727007942 + 1.46492424543531 t - 0.218409090291847 $(t-7)^2$	0.992
t-8	13.7489090224325 + 1.02810607045345 t - 0.218409089669624 $(t-8)^2$	0.992
t-9	17.4618636837107 + 0.591287872286244 t - 0.218409091611633 $(t-9)^2$	0.992

Table 6. Third degree series expansions of Monomolecular growth model with respect to (t-k) where t is time and k is integer number: $0 \le k \le 9$ and their the values of R^2

Series expansion of M.G.M. w.r.t.(t-k)	$y = a(1 - b\exp(-ct))$	R^2
t-0	$0.680675653955140 + 4.03930304476615\,t - 0.243612355615930\,t^2 + 0.00979492074928451\,t^3$	0.981
t-1	$0.584963946591832 + 3.84007601577501\,t - 0.278675898534364\,(t-1)^2 + 0.0134824165120530\,(t-1)^3$	0.985
t-2	$1.16594322063417 + 3.49770679765588\,t - 0.297738072649563\,(t-2)^2 + 0.0168963962263288\,(t-2)^3$	0.988
t-3	$2.44639528844028 + 3.03058995617843\,t - 0.220092625499482\,(t-3)^2 + 0.0160474863319337\,(t-3)^3$	0.990
t-4	$4.05447741988629 + 2.58111809971917\,t - 0.228191132177259\,(t-4)^2 + 0.0134492600474202\,(t-4)^3$	0.991
t-5	$5.85739358168628 + 2.18639715508841\,t - 0.194574411773702\,(t-5)^2 + 0.0115438623548554\,(t-5)^3$	0.991
t-6	$7.81113723933999 + 1.83610476659678\,t - 0.167778046315177\,(t-6)^2 + 0.0102207213653038\,(t-6)^3$	0.991
t-7	$9.87660717922102 + 1.52160347167066\,t - 0.145386300126691\,(t-7)^2 + 0.00926092185340971\,(t-7)^3$	0.992
t-8	$12.0145428710101 + 1.23768798475280\,t - 0.125887808125803\,(t-8)^2 + 0.00853620644833953\,(t-8)^3$	0.992
t-9	$14.1839446628479 + 0.981364682263551\,t - 0.108335472603878\,(t-9)^2 + 0.00797296175854134\,(t-9)^3$	0.992

Table 7. Fourth degree series expansions of Monomolecular growth model with respect to (t-k) where t is time and k is integer number: $0 \le k \le 9$ and their the values of R^2

Series expansion of M.G.M. w.r.t.(t-k)	$y = a(1 - b\exp(-ct))$	R^2
t-0	$-0.360744304989450 + 4.87762730457168\,t - 0.360902568792858\,t^2 + 0.0178024622804074\,t^3 - 0.000658614728703753\,t^4$	0.991
t-1	$-0.0438660693301349 + 4.23718011942472\,t - 0.329511408421765\,(t-1)^2 + 0.0170833376946988\,(t-1)^3$ $- 0.000664257183512074\,(t-1)^4$	0.991
t-2	$0.890733909398483 + 3.62671056678083\,t - 0.291729601195510\,(t-2)^2 + 0.0156443253735637\,(t-2)^3$ $- 0.000629208302973788\,(t-2)^4$	0.991
t-3	$2.26927816549750 + 3.07908820492831\,t - 0.252588727915214\,(t-3)^2 + 0.0138138438445744\,(t-3)^3$ $- 0.000566599754878032\,(t-3)^4$	0.991
t-4	$3.92458563383874 + 2.60640074179350\,t - 0.216329159127621\,(t-4)^2 + 0.0119700971377770\,(t-4)^3$ $- 0.000496754203405687\,(t-4)^4$	0.991
t-5	$5.73867075761085 + 2.20328073831089\,t - 0.184587731331811\,(t-5)^2 + 0.0103096653294535\,(t-5)^3$ $- 0.000431864560167892\,(t-5)^4$	0.991
t-6	$7.63416937574968 + 1.85894948599934\,t - 0.157378864957577\,(t-6)^2 + 0.00888247450934917\,(t-6)^3$ $- 0.000375995627321871\,(t-6)^4$	0.991
t-7	$9.55867222085055 + 1.56310018181112\,t - 0.134151138918962\,(t-7)^2 + 0.00767557033245839\,(t-7)^3$ $- 0.000329373163004440\,(t-7)^4$	0.991
t-8	$11.4745852571453 + 1.30732060953373\,t - 0.114255413382824\,(t-8)^2 + 0.00665702526326516\,(t-8)^3$ $- 0.000290900781264948\,(t-8)^4$	0.991
t-9	$13.3530729890336 + 1.08508386831010\,t - 0.0971025039789758\,(t-9)^2 + 0.00579303686677019\,(t-9)^3$ $- 0.000259205026373660\,(t-9)^4$	0.991

Table 8. Fifth degree series expansions of Monomolecular growth model with respect to (t-k) where t is time and k is integer number: $0 \le k \le 9$ and their the values of R^2

Series expansion of M.G.M. w.r.t.(t-k)	$y = a(1 - b\exp(-ct))$	R^2
t-0	$-0.398733365062543 + 5.06605109275128\,t - 0.431023810813641\,t^2 + 0.0244479078591920\,t^3$ $- 0.00104002409558871\,t^4 + 0.0000353944435862534\,t^5$	0.990
t-1	$-0.0519088215751733 + 4.29129401652697\,t - 0.362192522099715\,(t-1)^2 + 0.0203797770648711\,(t-1)^3$ $- 0.000860043943217098\,(t-1)^4 + 0.0000290356693072724\,(t-1)^5$	0.991
t-2	$0.924442518467425 + 3.62833242423506\,t - 0.303655112677479\,(t-2)^2 + 0.0169419293629748\,(t-2)^3$ $- 0.000708934969040578\,(t-2)^4 + 0.0000237323048425372\,(t-2)^5$	0.991
t-3	$2.30476251761820 + 3.07162584755017\,t - 0.256007762241889\,(t-3)^2 + 0.0142248172099415\,(t-3)^3$ $- 0.000592790887132543\,(t-3)^4 + 0.0000197627023634046\,(t-3)^5$	0.991

Table 8. Fifth degree series expansions of Monomolecular growth model with respect to (t-k) where t is time and k is integer number: $0 \le k \le 9$ and their the values of R^2(continued)

Series expansion of M.G.M. w.r.t.(t-k)	$y = a(1 - b\exp(-ct))$	R^2
t-4	$3.94333421828184 + 2.60142194209407\,t - 0.216525639783785\,(t-4)^2 + 0.0120148015783598\,(t-4)^3 - 0.000500017424419706\,(t-4)^4 + 0.0000166472944620995\,(t-4)^5$	0.991
t-5	$5.73254173978960 + 2.20283829673356\,t - 0.183425184277512\,(t-5)^2 + 0.0101822569173987\,(t-5)^3 - 0.000423926339533021\,(t-5)^4 + 0.0000141197412564034\,(t-5)^5$	0.991
t-6	$7.59228787448331 + 1.86419135505680\,t - 0.155514792387977\,(t-6)^2 + 0.00864891564050795\,(t-6)^3 - 0.000360755433332042\,(t-6)^4 + 0.0000120379930236846\,(t-6)^5$	0.991
t-7	$9.46427391817074 + 1.57581977438594\,t - 0.131901861644774\,(t-7)^2 + 0.00736044455004877\,(t-7)^3 - 0.000308048025055050\,(t-7)^4 + 0.0000103138972212963\,(t-7)^5$	0.991
t-8	$11.3069711424540 + 1.32963294973438\,t - 0.111868341452826\,(t-8)^2 + 0.00627467694344072\,(t-8)^3 - 0.000263959201280032\,(t-8)^4 + 0.888325701143577\,10^{-5}\,(t-8)^5$	0.991
t-9	$13.0912078624012 + 1.11889304425699\,t - 0.0948209241970095\,(t-9)^2 + 0.00535708496996129\,(t-9)^3 - 0.000226993880452155\,(t-9)^4 + 0.769466559543478\,10^{-5}\,(t-9)^5$	0.991

Table 9. Sixth degree series expansions of Monomolecular growth model with respect to (t-k) where t is time and k is integer number: $0 \le k \le 9$ and their the values of R^2

Series expansion of M.G.M. w.r.t.(t-k)	$y = a(1 - b\exp(-ct))$	R^2
t-0	$-0.429522115482260 + 5.04575821101274\,t - 0.415950307240173\,t^2 + 0.0228594198476887\,t^3 - 0.000942215452201562t^4 + 0.0000310688535153583\,t^5 - 0.853726940852854\,10^{-6}\,t^6$	0.991
t-1	$-0.0564049542016347 + 4.28180099690690\,t - 0.354716647774369\,(t-1)^2 + 0.0195904947940020\,(t-1)^3 - 0.000811466325337110\,(t-1)^4 + 0.0000268896770226637\,(t-1)^5 - 0.742539264833108\,10^{-6}\,(t-1)^6$	0.991
t-2	$0.917252604051328 + 3.62802249505953\,t - 0.301198922338968\,(t-2)^2 + 0.0166703837405814\,(t-2)^3 - 0.000691988765246638\,(t-2)^4 + 0.0000229796006863249\,(t-2)^5 - 0.635923194332095\,10^{-6}\,(t-2)^6$	0.991
t-3	$2.29893582306801 + 3.07263401789004\,t - 0.255309485257502\,(t-3)^2 + 0.0141426829842476\,(t-3)^3 - 0.000587567717444554\,(t-3)^4 + 0.0000195287314559782\,(t-3)^5 - 0.540889927743661\,10^{-6}\,(t-3)^6$	0.991
t-4	$3.94000249171290 + 2.60198312158441\,t - 0.216276386757099\,(t-4)^2 + 0.0119845705584294\,(t-4)^3 - 0.000498077715364870\,(t-4)^4 + 0.0000165600534006362\,(t-4)^5 - 0.458822924238559\,10^{-6}\,(t-4)^6$	0.991
t-5	$5.72855228163803 + 2.20335108905297\,t - 0.183185744072797\,(t-5)^2 + 0.0101533272048586\,(t-5)^3 - 0.000422071817804583\,(t-5)^4 + 0.0000140363540573868\,(t-5)^5 - 0.388992320961550\,10^{-6}\,(t-5)^6$	0.991
t-6	$7.58181706814705 + 1.86560662707831\,t - 0.155166918568407\,(t-6)^2 + 0.00860373323741911\,(t-6)^3 - 0.000357796428051299\,(t-6)^4 + 0.0000119035102919737\,(t-6)^5 - 0.330014374474623\,10^{-6}\,(t-6)^6$	0.991
t-7	$9.43970404519211 + 1.57921656112010\,t - 0.131446193031418\,(t-7)^2 + 0.00729395494273774\,(t-7)^3 - 0.000303556406692425\,(t-7)^4 + 0.0000101066148905417\,(t-7)^5 - 0.280408248938763\,10^{-6}\,(t-7)^6$	0.991
t-8	$11.2595567001189 + 1.33609057445753\,t - 0.111354547940796\,(t-8)^2 + 0.00618712313079704\,(t-8)^3 - 0.000257828440846414\,(t-8)^4 + 0.859533628201506\,10^{-5}\,(t-8)^5 - 0.238788648110308\,10^{-6}\,(t-8)^6$	0.991
t-9	$13.0119301184415 + 1.12940133849068\,t - 0.0943207770723575\,(t-9)^2 + 0.00525140100590662\,(t-9)^3 - 0.000219282644138534\,(t-9)^4 + 0.732526470041843\,10^{-5}\,(t-9)^5 - 0.203920618607707\,10^{-6}\,(t-9)^6$	0.991

Table 10. Seventh degree series expansions of Monomolecular growth model with respect to (t-k) where t is time and k is integer number: $0 \le k \le 9$ and their the values of R^2

Series expansion of M.G.M. w.r.t.(t-k)	$y = a(1 - b\exp(-ct))$	R^2
t-0	$-0.434828123120450 + 5.06217611621347\,t - 0.421486259768881\,t^2 + 0.0233958233370519\,t^3 - 0.000973990023871748\,t^4 + 0.0000324384930740797\,t^5 - 0.900296552605892\,10^{-6}\,t^6 + 10^{-7}\,t^7$	0.991
t-1	$-0.0573655320578679 + 4.28498718704676\,t - 0.356403272656266\,(t-1)^2 + 0.0197625317129037\,(t-1)^3 - 0.000821873050139327\,(t-1)^4 + 0.0000273436752075354\,(t-1)^5 - 0.758102253675752\,10^{-6}\,(t-1)^6 + 0.180157314312161\,10^{-7}\,(t-1)^7$	0.991
t-2	$0.918211722789252 + 3.62827592183363\,t - 0.301656184232409\,(t-2)^2 + 0.0167198701616699\,(t-2)^3 - 0.000695048053468011\,(t-2)^4 + 0.0000231146195255585\,(t-2)^5 - 0.640585967576221\,10^{-6}\,(t-2)^6 + 0.152167288907535\,10^{-7}\,(t-2)^7$	0.991

Table 10. Seventh degree series expansions of Monomolecular growth model with respect to (t-k) where t is time and k is integer number: $0 \le k \le 9$ and their the values of R^2(continued)

Series expansion of M.G.M. w.r.t.(t-k)	$y = a(1 - b\exp(-ct))$	R^2
t-3	$2.29978449123599 + 3.07250276125247\, t - 0.255412094951390\, (t-3)^2 + 0.0141546579051981\, (t-3)^3 - 0.000588326701358795\, (t-3)^4 + 0.0000195626519467975\, (t-3)^5 - 0.542070347291260\; 10^{-6}\, (t-3)^6 + 0.128746900929205\; 10^{-7}\, (t-3)^7$	0.991
t-4	$3.94030372900206 + 2.60191113903930\, t - 0.216284326063926\, (t-4)^2 + 0.0119857820402478\, (t-4)^3 - 0.000498160131610271\, (t-4)^4 + 0.0000165638598060696\, (t-4)^5 - 0.458957930502304\; 10^{-6}\, (t-4)^6 + 0.109002741650714\; 10^{-7}\, (t-4)^7$	0.991
t-5	$5.72815540513387 + 2.20338912024644\, t - 0.183158896998366\, (t-5)^2 + 0.0101501761510335\, (t-5)^3 - 0.000421871709150150\, (t-5)^4 + 0.0000140274010092443\, (t-5)^5 - 0.388680521444407\; 10^{-6}\, (t-5)^6 + 0.923127207450638\; 10^{-8}\, (t-5)^7$	0.991
t-6	$7.57989286794269 + 1.86587022056233\, t - 0.155111243519455\, (t-6)^2 + 0.00859634559110287\, (t-6)^3 - 0.000357310449479465\, (t-6)^4 + 0.0000118813982945828\, (t-6)^5 - 0.329236628913246\; 10^{-6}\, (t-6)^6 + 0.781991348136531\; 10^{-8}\, (t-6)^7$	0.991
t-7	$9.43437496048621 + 1.57996492941454\, t - 0.131362230938031\, (t-7)^2 + 0.00728118934615032\, (t-7)^3 - 0.000302688135219226\, (t-7)^4 + 0.0000100664990673188\, (t-7)^5 - 0.278984626093062\; 10^{-6}\, (t-7)^6 + 0.662727963110582\; 10^{-8}\, (t-7)^7$	0.991
t-8	$11.2479524265758 + 1.33769725429912\, t - 0.111251118362609\, (t-8)^2 + 0.00616821745834955\, (t-8)^3 - 0.000256493421190692\, (t-8)^4 + 0.853262720497008\; 10^{-5}\, (t-8)^5 - 0.236541892189077\; 10^{-6}\, (t-8)^6 + 0.562065038281674\; 10^{-8}\, (t-8)^7$	0.991
t-9	$12.9905117588151 + 1.13229795488828\, t - 0.0942165079072939\, (t-9)^2 + 0.00522639253147285\, (t-9)^3 - 0.000217439434179989\, (t-9)^4 + 0.723710012239584\; 10^{-5}\, (t-9)^5 - 0.200728762237868\; 10^{-6}\, (t-9)^6 + 0.477208139504774\; 10^{-8}\, (t-9)^7$	0.991

Table 11. Eighth degree series expansions of Monomolecular growth model with respect to (t-k) where t is time and k is integer number: $0 \le k \le 9$ and their the values of R^2

Series expansion of M.G.M. w.r.t.(t-k)	$y = a(1 - b\exp(-ct))$	R^2
t-0	$-0.433608228852283 + 5.05878273346574\, t - 0.420376461012291\, t^2 + 0.0232883914680598\, t^3 - 0.000967613367070306\, t^4 + 0.0000321628268544778\, t^5 - 0.890892535589199\; 10^{-6}\, t^6 + 0.211519123408160\; 10^{-7}\, t^7 - 0.439422214886404\; 10^{-9}\, t^8$	0.991
t-1	$-0.0570835327271100 + 4.28434713167022\, t - 0.356095349204152\, (t-1)^2 + 0.0197313451077891\, (t-1)^3 - 0.000819989605240820\, (t-1)^4 + 0.0000272615150778572\, (t-1)^5 - 0.755284172293431\; 10^{-6}\, (t-1)^6 + 0.179359336817422\; 10^{-7}\, (t-1)^7 - 0.372688204959517\; 10^{-9}\, (t-1)^8$	0.991
t-2	$0.918092664360992 + 3.62821699634121\, t - 0.301582499855520\, (t-2)^2 + 0.0167119743776485\, (t-2)^3 - 0.000694561407905749\, (t-2)^4 + 0.0000230931684527820\, (t-2)^5 - 0.639845547514368\; 10^{-6}\, (t-2)^6 + 0.151956748165362\; 10^{-7}\, (t-2)^7 - 0.315771465900738\; 10^{-9}\, (t-2)^8$	0.991
t-3	$2.29967652480385 + 3.07251590966884\, t - 0.255397423275344\, (t-3)^2 + 0.0141529712095596\, (t-3)^3 - 0.000588220286709800\, (t-3)^4 + 0.0000195579062840601\, (t-3)^5 - 0.541905397884822\; 10^{-6}\, (t-3)^6 + 0.128699779731700\; 10^{-7}\, (t-3)^7 - 0.267448510324584\; 10^{-9}\, (t-3)^8$	0.991
t-4	$3.94025714066446 + 2.60191851167342\, t - 0.216281109727798\, (t-4)^2 + 0.0119853916037267\, (t-4)^3 - 0.000498135084735814\, (t-4)^4 + 0.0000165627337578257\, (t-4)^5 - 0.458918604508833\; 10^{-6}\, (t-4)^6 + 0.108991472048245\; 10^{-7}\, (t-4)^7 - 0.226494377319102\; 10^{-9}\, (t-4)^8$	0.991
t-5	$5.72808264413668 + 2.20339840980785\, t - 0.183155223840188\, (t-5)^2 + 0.0101497262503318\, (t-5)^3 - 0.000421842771380205\, (t-5)^4 + 0.0000140260983893956\, (t-5)^5 - 0.388634995026958\; 10^{-6}\, (t-5)^6 + 0.922996678863383\; 10^{-8}\, (t-5)^7 - 0.191807871159936\; 10^{-9}\, (t-5)^8$	0.991
t-6	$7.57958865400335 + 1.86591267002906\, t - 0.155103330790887\, (t-6)^2 + 0.00859527301173300\, (t-6)^3 - 0.000357239514632815\, (t-6)^4 + 0.0000118781633245046\, (t-6)^5 - 0.329122708492054\; 10^{-6}\, (t-6)^6 + 0.781663106991428\; 10^{-8}\, (t-6)^7 - 0.162438673307304\; 10^{-9}\, (t-6)^8$	0.991
t-7	$9.43340041614564 + 1.58010329740721\, t - 0.131348514134999\, (t-7)^2 + 0.00727903135353300\, (t-7)^3 - 0.000302540332077490\, (t-7)^4 + 0.0000100596519606013\, (t-7)^5 - 0.278741341343180\; 10^{-6}\, (t-7)^6 + 0.662022920969791\; 10^{-8}\, (t-7)^7 - 0.137579180955098\; 10^{-9}\, (t-7)^8$	0.991
t-8	$11.2455245506238 + 1.33803742398329\, t - 0.111232356224621\, (t-8)^2 + 0.00616456951527712\, (t-8)^3 - 0.000256233338471745\, (t-8)^4 + 0.852037094647629\; 10^{-5}\, (t-8)^5 - 0.236102249283624\; 10^{-6}\, (t-8)^6 + 0.560783151011653\; 10^{-8}\, (t-8)^7 - 0.116546125878152\; 10^{-9}\, (t-8)^8$	0.991
t-9	$12.9854825929341 + 1.13298835527921\, t - 0.0941966567280208\, (t-9)^2 + 0.00522100696351228\, (t-9)^3 - 0.000217037270694356\, (t-9)^4 + 0.721779184738210\; 10^{-5}\, (t-9)^5 - 0.200029081738918\; 10^{-6}\, (t-9)^6 + 0.475154820445459\; 10^{-8}\, (t-9)^7 - 0.987609345356764\; 10^{-10}\, (t-9)^8$	0.991

Table 12. Ninth degree series expansions of Monomolecular growth model with respect to (t-k) where t is time and k is integer number: $0 \le k \le 9$ and their the values of R^2

Series expansion of M.G.M. w.r.t.(t-k)	$y = a(1 - b\exp(-ct))$	R^2
t-0	$-0.433914558778196 + 5.059455884447531\,t - 0.420582327983030\,t^2 + 0.0233081051468270\,t^3$ $- 0.000968777804111872\,t^4 + 0.0000322130152692428\,t^5 - 0.892600921432618\,10^{-6}\,t^6$ $+ 0.212000308823644\,10^{-7}\,t^7 - 0.440578915273099\,10^{-9}\,t^8 + 0.813876301136041\,10^{-11}\,t^9$	*0.991*
t-1	$-0.0571499574470016 + 4.28446268530283\,t - 0.356145918363578\,(t-1)^2 + 0.0197364172956297\,(t-1)^3$ $- 0.000820294746301659\,(t-1)^4 + 0.0000272747970710612\,(t-1)^5 - 0.755739078972881\,10^{-6}\,(t-1)^6$ $+ 0.179488009911674\,10^{-7}\,(t-1)^7 - 0.372998476241402\,10^{-9}\,(t-1)^8 + 0.689010992232219\,10^{-11}\,(t-1)^9$	*0.991*
t-2	$0.918105241106358 + 3.62822766715208\,t - 0.301593044792331\,(t-2)^2 + 0.0167130939238083\,(t-2)^3$ $- 0.000694630181288771\,(t-2)^4 + 0.0000230961946821788\,(t-2)^5 - 0.639949888883564\,10^{-6}\,(t-2)^6$ $+ 0.151986395254758\,10^{-7}\,(t-2)^7 - 0.315843187893670\,10^{-9}\,(t-2)^8 + 0.583426016792124\,10^{-11}\,(t-2)^9$	*0.991*
t-3	$2.29968888992291 + 3.07251464656294\,t - 0.255399178727401\,(t-3)^2 + 0.0141531715870146\,(t-3)^3$ $- 0.000588232899679598\,(t-3)^4 + 0.0000195584681292460\,(t-3)^5 - 0.541924912976777\,10^{-6}\,(t-3)^6$ $+ 0.128705352016109\,10^{-7}\,(t-3)^7 - 0.267462038297415\,10^{-9}\,(t-3)^8 + 0.494054854300104\,10^{-11}\,(t-3)^9$	*0.991*
t-4	$3.94026043483711 + 2.60191783595405\,t - 0.216281244612934\,(t-4)^2 + 0.0119854096658842\,(t-4)^3$ $- 0.000498136275464446\,(t-4)^4 + 0.0000165627879797823\,(t-4)^5 - 0.458920512275896\,10^{-6}\,(t-4)^6$ $+ 0.108992021414677\,10^{-7}\,(t-4)^7 - 0.226495719029905\,10^{-9}\,(t-4)^8 + 0.418381782633787\,10^{-11}\,(t-4)^9$	*0.991*
t-5	$5.72807619305896 + 2.20339915670248\,t - 0.183154886929434\,(t-5)^2 + 0.0101496854694080\,(t-5)^3$ $- 0.000421840157482132\,(t-5)^4 + 0.0000140259809233224\,(t-5)^5 - 0.388630893661299\,10^{-6}\,(t-5)^6$ $+ 0.922984927554837\,10^{-8}\,(t-5)^7 - 0.191805011281830\,10^{-9}\,(t-5)^8 + 0.354301318154633\,10^{-11}\,(t-5)^9$	*0.991*
t-6	$7.57954821613993 + 1.86591836038502\,t - 0.155102348424211\,(t-6)^2 + 0.00859513792151890\,(t-6)^3$ $- 0.000357230547959905\,(t-6)^4 + 0.0000118777537310939\,(t-6)^5 - 0.329108271260228\,10^{-6}\,(t-6)^6$ $+ 0.781621484504528\,10^{-8}\,(t-6)^7 - 0.162428499551898\,10^{-9}\,(t-6)^8 + 0.300037441989715\,10^{-11}\,(t-6)^9$	*0.991*
t-7	$9.43324763988232 + 1.58012515736214\,t - 0.131346529575979\,(t-7)^2 + 0.00727871069844188\,(t-7)^3$ $- 0.000302518248498711\,(t-7)^4 + 0.0000100586265305825\,(t-7)^5 - 0.278704861012072\,10^{-6}\,(t-7)^6$ $+ 0.661917120015674\,10^{-8}\,(t-7)^7 - 0.137553212440026\,10^{-9}\,(t-7)^8 + 0.254088692515500\,10^{-11}\,(t-7)^9$	*0.991*
t-8	$11.2450867730620 + 1.33809928927531\,t - 0.111229311596016\,(t-8)^2 + 0.00616394705399585\,(t-8)^3$ $- 0.000256188607611311\,(t-8)^4 + 0.851825651257341\,10^{-5}\,(t-8)^5 - 0.236026283820578\,10^{-6}\,(t-8)^6$ $+ 0.560561457335014\,10^{-8}\,(t-8)^7 - 0.116491476952360\,10^{-9}\,(t-8)^8 + 0.215185273819447\,10^{-11}\,(t-8)^9$	*0.991*
t-9	$12.9844573832616 + 1.13313065903825\,t - 0.0941932390184142\,(t-9)^2 + 0.00521997247537813\,(t-9)^3$ $- 0.000216959143732002\,(t-9)^4 + 0.721402578591395\,10^{-5}\,(t-9)^5 - 0.199892351253455\,10^{-6}\,(t-9)^6$ $+ 0.474753169633848\,10^{-8}\,(t-9)^7 - 0.986614792067005\,10^{-10}\,(t-9)^8 + 0.182253068693670\,10^{-11}\,(t-9)^9$	*0.991*

The highest value of R^2 (0.992) was found at the second degree expansions of monomolecular growth model. The values of R^2 of third, fourth, ... and ninth degree expansions of monomolecular growth model were generally found as 0.991. Furthermore, the values of R^2 of tenth and higher degree expansions in the neighborhood of (t-k), where t is time and k is integer number, $0 \le k \le 9$ are the same with those of third, fourth,... and ninth degree expansions of monomolecular growth model: R^2=0.991.

For all degree series expansions, the values of R^2 are generally increasing or keeping the same level w.r.t. (t-k) while k is increasing where t is time and k is the value of age. Even so the best approaches according to R^2 were found at the second degree expansions of Monomolecular growth model.

The research for the second degree series expansions of monomolecular growth model in the neighborhood of (t-k), where t is time and k is integer number, $0 \le k \le 9$ was done and for each one the same R^2 was found (0.992). Moreover, the research for the third, fourth, ... and ninth degree series expansions of monomolecular growth model in the neighborhood of (t-k) was done and for each one the same R^2 was generally found (0.991).

Since the number of data points is 10 and the only ninth degree polynomial for monomolecular model is unique, for all series expansions of ninth degree polynomial are actually the same function. For that reason, R^2 is the same for all series expansions of ninth degree polynomial.

Here the following question comes to mind. I wonder if it can be directly fitted (n-1)th degree polynomial instead of using the series expansions of Monomolecular growth model. I wonder how it results. After this, this investigation will be done.

For each degree of polynomial approximations, we got the approach equations. While the degree of polynomial was increasing, R^2 of the polynomial approximations generally increased uniformly (Table 13).

For ninth degree polynomial, SSE was found as zero, but actually since the degree of freedom is zero, values for the items of ninth degree polynomial are not available. We can see that in the plot in Fig. 3.

Table 13. Polynomial approximations and their values of R^2

Polynomial Degree	*Polynomial Approximations*	R^2
1	$2.39163636363637 + 2.55696969696970\, t$	0.9475
2	$- 0.229272727272729 + 4.52265151515151\, t - 0.218409090909091\, t^2$	0.9918
3	$- 0.269902097902094 + 4.59697746697747\, t - 0.240174825174825\, t^2 + 0.00161227661227660\, t^3$	0.9918
4	$0.404405594405547 + 1.78736208236217\, t + 1.33633158508155\, t^2 - 0.279349261849256\, t^3 + 0.0156089743589741\, t^4$	0.9988
5	$0.427328671328617 + 1.55532960372979\, t + 1.55123543123531\, t^2 - 0.346844988344958\, t^3 + 0.0242051282051249\, t^4 - 0.000382051282051152\, t^5$	0.9989
6	$0.379328671328654 + 2.97852960372992\, t - 0.399697902098311\, t^2 + 0.586155011655198\, t^3 - 0.176128205128243\, t^4 + 0.0194179487179522\, t^5 - 0.000733333333333456\, t^6$	0.9992
7	$0.410025915261181 - 0.709987883685088\, t + 6.36290493624040\, t^2 - 3.86973469708383\, t^3 + 1.22379402337848\, t^4 - 0.207414787581678\, t^5 + 0.0175571078431354\, t^6 - 0.000580648926237101\, t^7$	0.9998
8	$0.411627313865612 - 1.98339717437018\, t + 9.21625186286223\, t^2 - 6.26102315856714\, t^3 + 2.22612775732909\, t^4 - 0.439277287576093\, t^5 + 0.0476133578424037\, t^6 - 0.00262529178332981\, t^7 + 0.0000567956349192339\, t^8$	0.9998
9	$0.409999999958488 + 16.8924801581856\, t - 39.2281795615017\, t^2 + 42.2230666864486\, t^3 - 23.0818489570843\, t^4 + 7.22164814776001\, t^5 - 1.34619791659509\, t^6 + 0.147586970891698\, t^7 - 0.00877356150747743\, t^8 + 0.000218033509688805\, t^9$	1

As it was seen in Table 13, as the degree of the polynomial approximation increases, a better approach is provided. Even significantly better results than series expansions of monomolecular growth model were found.

Since degree of freedom of SSE is n-2 where n is the number of data points, (n-2)th degree polynomial has better approximation especially when n is large. It shows that it can be directly used (n-2)th degree polynomial approximation instead of using series expansions of any model. For example, in the following figure (Fig. 4), it can be seen that how ninth degree polynomial deviates from the data points while eighth degree polynomial is perfect fitting the data set.

Fig. 5 showing Monomolecular growth model and its ninth degree series expansion w.r.t. (t-0) with eighth and ninth degree polynomial approximations is presented below. In this Fig. 5, the graphics of monomolecular growth model and its ninth degree series expansion overlap. It also seems that eighth degree polynomial shows better approach.

But as the number of data points increases, it can be met with some approach problems. For example, if we add the points (10,25) and (11,27), it can be seen that the graphics of tenth and eleventh degree polynomial approximations exclusively deviate from the data endpoints. This situation is presented in the following figure (Fig. 6).

In addition to the points (10,25) and (11,27) if we also add the points (12,28), (13,31), (14,34), (15,36), (16,39), (17,42),

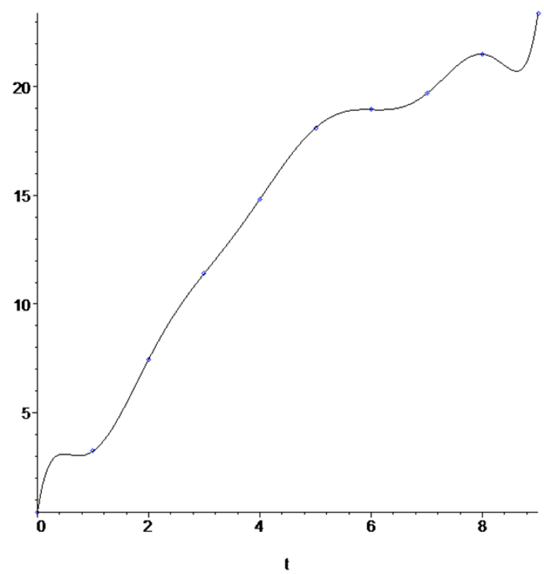

Figure 3. Ninth degree polynomial approximation

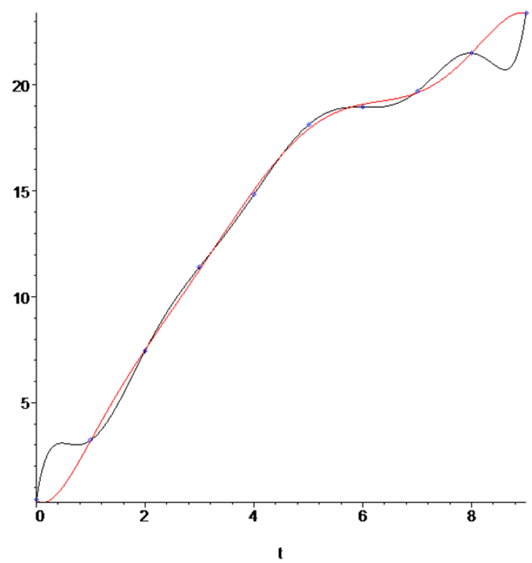

Figure 4. Eighth degree (red) and ninth degree (black) polynomial approximation

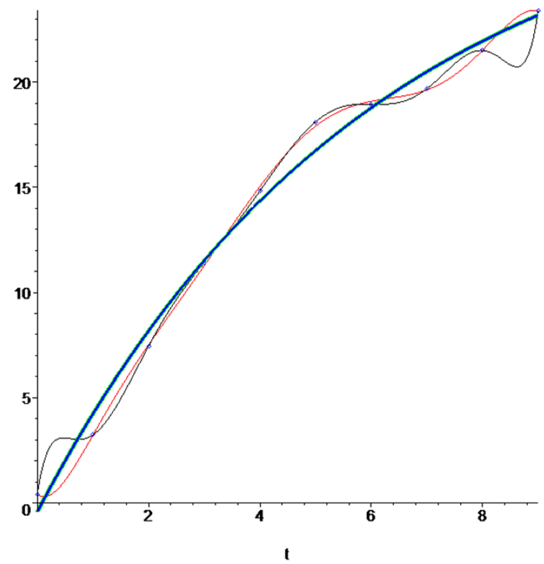

Figure 5. Monomolecular growth model (blue) and its ninth degree series expansion w.r.t. (t-0) (green) with eighth degree (red) and ninth degree (black) polynomial approximations

(18,46) and (19,52), it can be seen that although the graphic of tenth degree polynomial approximation exclusively deviates from the data endpoints, the graphics of eighteenth and nineteenth degree polynomial approximations have huge irreparable deviations from the data points. This situation is also presented in the following figure (Fig. 7).

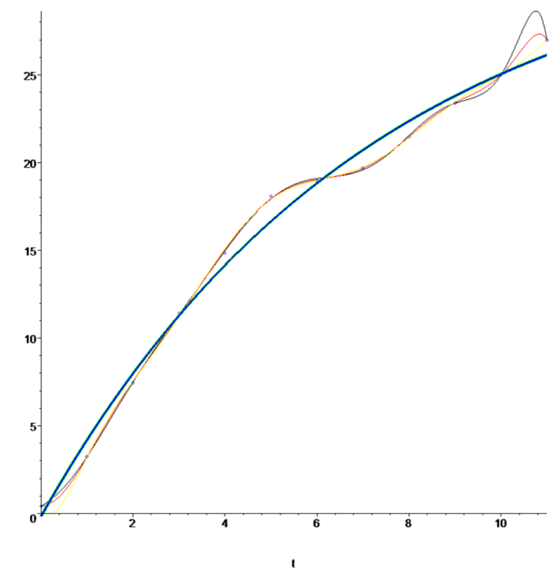

Figure 6. Monomolecular growth model (blue) and its eleventh degree series expansion w.r.t.(t-0) (green) with ninth degree (yellow), tenth degree (red) and eleventh degree (black) polynomial approximations

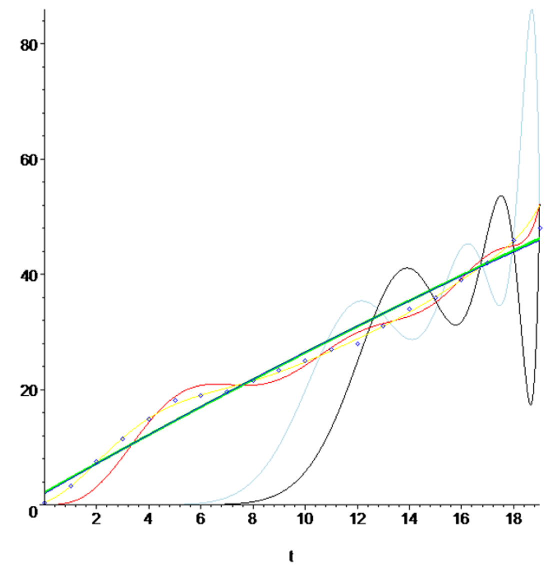

Figure 7. Monomolecular growth model (blue) and its nineteenth degree series expansion w.r.t. (t-0) (green) with ninth degree (yellow), tenth degree (red), eighteenth degree (light blue) and nineteenth degree (black) polynomial approximations

CONCLUSION

It can be said that if there are too many data points especially much more than 10, polynomial approximation can be much more problematic. However, high degree series expansions of Monomolecular growth model have not any problem. For that reason, polynomial approximation should be used especially when the number of data points is 10 or fewer. Nevertheless, if researcher decides to do polynomial approximation of any model, he can do (n-1)th degree polynomial approximation where n is the number of data points. Although R^2 of (n-1)th degree polynomial is closer or equal to one, in order to see whether there is any deviation particularly at endpoints or not he must draw the graph of the polynomial function. If there is any deviation for (n-1)th degree polynomial approximation, (n-2)th degree series expansion and its R^2 should be used.

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