THE NEW SET DIFFERENCE OPERATION BETWEEN INTERVAL VALUED INTUITIONISTIC FUZZY SETS

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Abstract. In this paper, the new set difference is defined between interval-ued intuitionistic fuzzy sets (IVIFSs).A different perspective is reduced to the formal set difference between IVIFSs. This new set difference $(-.)$ between IVIFSs is introduced by using . operation. Instead of comparing this operator with all other set difference operations, the set difference operation $(-_\cap)$ which is defined by using $\cap$ is used.

Some fundamental properties which are provided and not provided on $-_\cap$ are examined whether satisfy or not satisfy on $-.$ operation. By the help of these examinations, it is seen that $-_\cap$ and $-.$ operations have different properties. The new properties about $-.$ are studied.

1. Introduction

The set diffence between fuzzy sets which are defined by Zadeh in 1965, is given by using the concept of infimum[6]. However, there are many set difference conceptions between IFSs which are defined by Atanassov in 1983 because all implications and negations on IFSs theory generate different set differences between these sets[1].

Atanassov defined some fundamental operations on IFSs in [1],[2],[4] like as complement, $\cap$, $\cup$, $+$, $.$,@, $\&$, $\#$, $*$ New set differences can be defined between IVIFSs by the help of diversity of set differences between IFSs.

2. On Intuitionistic Fuzzy Sets

Some conceptions which are used in this paper, are given below,

The concept of fuzzy sets which is the extension of crisp sets by expanding the truth value set to unit interval [0.1], was introduced by Zadeh in 1965[6]. Let X be a universal set. Then the function $\mu : X \to [0,1]$ is a fuzzy set over $X.\forall x \in X$, $\mu(x)$ is the membership degree of x and $1 - \mu(x)$ is the non-membership degree of x. All fuzzy sets over X are denoted by $FS(X)$.

Another extension of fuzzy sets is the concept of interval valued fuzzy sets that was defined by Sambuc in 1975[5]. Let X be a set and A be interval valued fuzzy set. Then the function $M_A : X \to \wp([0,1]$ generates interval valued fuzzy set,where

$\wp(x)$ is the power set of X. This means that for $x \in X$, $M_A(x)$ is a closed interval within $[0, 1]$. All interval valued fuzzy sets over X are denoted by $IVFS(X)$.

Intuitionistic fuzzy sets which are the one of the extensions of fuzzy sets were defined by Atanassov in 1983[1]. Let X be a universal set and A be an IFSs. Then $\mu_A(x)$ and $\upsilon_A(x)$ functions such that $0 \leq \mu_A(x) + \upsilon_A(x) \leq 1$ where $\mu_A : X \to [0.1]$ and $\upsilon_A : X \to [0, 1]$ respectively are called membership and non-membership degree of each $x \in X$.It is important to point that the sum of membership and non-membership degree of x over fuzzy set equals to 1 while this sum is less than 1 or equals to 1 over IFSs. All IFSs over X are denoted by $IFS(X)$.

The concept of interval valued intuitionistic fuzzy sets which is the extension both IFSs and interval valued fuzzy sets was defined by Atanassov and Gargov in 1989[3]. Let X be a set and A be interval valued intuitionistic fuzzy set. Then the functions $M_A : X \to \wp([0, 1]$ and $N_A : X \to \wp([0, 1]$ generate interval valued intuitionistic fuzzy set, where $M_A(x) \subseteq [0, 1]$ and $N_A(x) \subseteq [0, 1]$ are closed interval such that $\sup M_A(x) + \sup N_A(x) \leq 1$. All interval valued intuitionistic fuzzy sets over X are denoted by $IVIFS(X)$.

In this paper, an interval valued fuzzy set A over universe set X is shown by $IVFS\ A$ and an interval valued intuitionistic fuzzy set A over universe set X is shown by $IVIFS\ A$.

Definition 1. [1]*An intuitionistic fuzzy set (shortly, IFS) on a set X is has the form*

$$A = \{\langle x, \mu_A(x), \upsilon_A(x) \mid x \in X \rangle\}$$

where, $\mu_A(x)$ ($\mu_A : X \to [0.1]$) is called membership degree of x in A and $\upsilon_A(x)$ ($\upsilon_A : X \to [0, 1]$) is called non-membership degree of x in A. Also, μ_A and υ_A functions have inequality: $\forall x \in X, 0 \leq \mu_A(x) + \upsilon_A(x) \leq 1$

3. On Interval Valued Intuitionistic Fuzzy Sets

In this section, the definition of IVIFSs which is introduced by Atanassov and Gargov and the operations on IVIFSs are given.

Definition 2. [5]*X is a set. $IVFS\ A$ is*

$$A = \{\langle x, M_A(x) \rangle \mid x \in X\}$$

the function; $M_A : X \to \wp[0, 1]$ where $\wp(X)$ is the power set of X. $\forall x \in X$, $M_A(x)$ is a closed interval within $[0, 1]$.

Definition 3. [3]$M_A(x) \subset [0, 1]$ *and* $N_A(x) \subset [0, 1]$ *are closed interval and*

$\forall x \in X, M_A : X \to \wp[0, 1]$ and $N_A : X \to \wp[0, 1]$ satisfy that, $\sup M_A(x) + \sup N_A(x) \leq 1$.

$$A = \{\langle x, M_A(x), N_A(x) \rangle \mid x \in X\}$$

is called interval valued intuitionistic fuzzy set over X. It is shortly denoted IVIFSs A

Some operations betwwen IVIFSs are given below.

Definition 4. [3]*For any two IVIFSs A and B, following relations are defined:*

(1) $A \subset_{\square,\inf} B :\Leftrightarrow \forall x \in X, \inf M_A(x) \leq \inf M_B(x)$

(2) $A \subset_{\square,\sup} B :\Leftrightarrow \forall x \in X \sup M_A(x) \leq \sup M_B(x)$
(3) $A \subset_{\Diamond,\inf} B :\Leftrightarrow \forall x \in X, \inf N_A(x) \geq N_B(x)$
(4) $A \subset_{\Diamond,\sup} B :\Leftrightarrow \forall x \in X, \inf N_A(x) \geq N_B(x)$
(5) $A \subset_{\square} B :\Leftrightarrow A \subset_{\square,\inf}$ and $A \subset_{\square,\sup} B$
(6) $A \subset_{\Diamond} B :\Leftrightarrow A \subset_{\Diamond,\inf}$ and $A \subset_{\Diamond,\sup} B$
(7) $A \subset B :\Leftrightarrow A \subset_{\square} B$ and $A \subset_{\Diamond} B$
(8) $A = B :\Leftrightarrow A \subset B$ and $B \subset A$

In this study, some operations between IVIFSs which are used to give the new set difference operation.

Definition 5. [3]*For any two IVIFSs A and B the following operations are defined:*

(1) $A^c = \{\langle x, N_A(x), M_A(x) \rangle \mid x \in X\}$

(2) $A \cap B = \left\{ \left\langle \begin{array}{l} x, [\min \{\inf M_A(x), \inf M_B(x)\}, \min \{\sup M_A(x), \sup M_B(x)\}], \\ [\max \{\inf N_A(x), \inf N_B(x)\}, \max \{\sup N_A(x), \sup N_B(x)\}] \end{array} \right\rangle \mid x \in X \right\}$

(3) $A \cup B = \left\{ \left\langle \begin{array}{l} x, [\max \{\inf M_A(x), \inf M_B(x)\}, \max \{\sup M_A(x), \sup M_B(x)\}], \\ [\min \{\inf N_A(x), \inf N_B(x)\}, \min \{\sup N_A(x), \sup N_B(x)\}] \end{array} \right\rangle \mid x \in X \right\}$

(4) $A -_{\cap} B = \left\{ \left\langle \begin{array}{l} x, [\min \{\inf M_A(x), \inf N_B(x)\}, \min \{\sup M_A(x), \sup N_B(x)\}], \\ [\max \{\inf N_A(x), \inf M_B(x)\}, \max \{\sup N_A(x), \sup M_B(x)\}] \end{array} \right\rangle \mid x \in X \right\}$

(5) $A.B = \left\{ \left\langle \begin{array}{l} x, [\inf M_A(x). \inf M_B(x), \sup M_A(x). \sup M_B(x)], \\ \left[\begin{array}{l} \inf N_A(x) + \inf N_B(x) - \inf N_A(x). \inf N_B(x), \\ \sup N_A(x) + \sup N_B(x) - \sup N_A(x). \sup N_B(x) \end{array} \right] \end{array} \right\rangle \mid x \in X \right\}$

Theorem 1. [3]*For every three IVIFS A, B and C,*

(1) $A \cap B = B \cap A$
(2) $A \cup B = B \cup A$
(3) $A.B = B.A$
(4) $(A \cap B) \cap C = A \cap (B \cap C)$
(5) $(A \cup B) \cup C = A \cup (B \cup C)$
(6) $(A.B).C = A.(B.C)$
(7) $(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$
(8) $(A \cap B).C = (A.C) \cap (B.C)$
(9) $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$
(10) $(A \cup B).C = (A.C) \cup (B.C)$

The theorem that is given below is related to some properties of the concept of complement. Also, we examine these properties by using the new set difference.

Theorem 2. [3]*For every IVIFS A and B,*

(1) $(A^c \cap B^c)^c = A \cup B$
(2) $(A^c \cup B^c)^c = A \cap B$

Definition 6. [3]*There are some special sets on uncertain set theories. These special sets are null set and universel set on crisp set theory. Special sets on IVIFSs set theory are given below,*

(1) $0^* = \{\langle x, [0,0], [1,1] \rangle \mid x \in X\}$
(2) $U^* = \{\langle x, [0,0], [0,0] \rangle \mid x \in X\}$
(3) $X^* = \{\langle x, [1,1], [0,0] \rangle \mid x \in X\}$

Remark 1. [3]*It is clearly seen that:*

$$0^* \subset U^* \subset X^*$$

Remark 2. [3]$\forall$ *IVIFS A:*

$$A \cap 0^* \;=\; A.0^* = 0^*$$
$$A \cup 0^* \;=\; A + 0^* = 0^*$$

4. The New Set Differences Operations Between IVIFSs

In this section, we define new set difference between IVIFSs by using $\cdot$ operation. Accroding to these, the definition of $A -_{.} B$ is given below.

Definition 7. *For any two IVIFSs A and B, following operation are given:*

$$A-_{.}B = \left\{ \left\langle \begin{array}{c} x, [\inf M_A(x).\inf N_B(x), \sup M_A(x).\sup N_B(x)], \\ \left[\begin{array}{c} \inf N_A(x) + \inf M_B(x) - \inf N_A(x).\inf M_B(x), \\ \sup N_A(x) + \sup M_B(x) - \sup N_A(x).\sup M_B(x) \end{array} \right] \end{array} \right\rangle \mid x \in X \right\}$$

Proposition 1. *For every IVIFSs A and B,*

$$A -_{.} B \in IVIFS(X)$$

Proof. Arbitrary IVIFSs A and B are given,

$$A-_{.}B = \left\{ \left\langle \begin{array}{c} x, [\inf M_A(x).\inf N_B(x), \sup M_A(x).\sup N_B(x)], \\ \left[\begin{array}{c} \inf N_A(x) + \inf M_B(x) - \inf N_A(x).\inf M_B(x), \\ \sup N_A(x) + \sup M_B(x) - \sup N_A(x).\sup M_B(x) \end{array} \right] \end{array} \right\rangle \mid x \in X \right\}$$

Now, we want to get that intervals are subsets of $[0,1]$.
$\inf M_A(x)$ and $\inf N_B(x) \in [0,1] \Rightarrow \inf M_A(x).\inf N_B(x) \in [0,1]$
$\sup M_A(x)$ and $\sup N_B(x) \in [0,1] \Rightarrow \sup M_A(x).\sup N_B(x) \in [0,1]$
$\inf M_A(x) \leq \sup M_A(x)$ and $\inf N_B(x) \leq \sup N_B(x)$
$\Rightarrow \inf M_A(x).\inf N_B(x) \leq \sup M_A(x).\sup N_B(x)$
$\Rightarrow [\inf M_A(x).\inf N_B(x), \sup M_A(x).\sup N_B(x)] \subset [0,1]$
$\inf N_A(x) + \inf M_B(x) \geq \inf N_A(x).\inf M_B(x) \Rightarrow$
$\inf N_A(x) + \inf M_B(x) - \inf N_A(x).\inf M_B(x) \geq 0$
Let $\inf N_A(x) + \inf M_B(x) - \inf N_A(x).\inf M_B(x) > 1$. Then,
$\inf M_B(x) - \inf N_A(x).\inf M_B(x) > 1 - \inf N_A(x) \Rightarrow$
$\inf M_B(x)(1 - \inf N_A(x)) > 1 - \inf N_A(x)$
$\Rightarrow \inf M_B(x) > 1$ This is a contradiction. Therefore,
$\inf N_A(x) + \inf M_B(x) - \inf N_A(x).\inf M_B(x) \leq 1$
$\Rightarrow \inf N_A(x) + \inf M_B(x) - \inf N_A(x).\inf M_B(x) \in [0,1]$
$\sup N_A(x) + \sup M_B(x) \geq \sup N_A(x).\sup M_B(x) \Rightarrow$
$\sup N_A(x) + \sup M_B(x) - \sup N_A(x).\sup M_B(x) \geq 0$
Let $\sup N_A(x) + \sup M_B(x) - \sup N_A(x).\sup M_B(x) > 1$. Then,
$\sup M_B(x) - \sup N_A(x).\sup M_B(x) > 1 - \sup N_A(x) \Rightarrow$
$\sup M_B(x)(1 - \sup N_A(x)) > 1 - \sup N_A(x)$
$\Rightarrow \sup M_B(x) > 1$ This is a contradiction. Therefore,
$\sup N_A(x) + \sup M_B(x) - \sup N_A(x).\sup M_B(x) \leq 1$
$\Rightarrow \sup N_A(x) + \sup M_B(x) - \sup N_A(x).\sup M_B(x) \in [0,1]$
$\inf N_A(x) \leq \sup N_A(x) \Rightarrow 1 - \sup N_A(x) \leq 1 - \inf N_A(x)$ and

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$\inf M_B(x) \leq \sup M_B(x) \Rightarrow 1 - \sup M_B(x) \leq 1 - \inf M_B(x)$

$\Rightarrow (1 - \sup N_A(x))(1 - \sup M_B(x)) \leq (1 - \inf N_A(x))(1 - \inf M_B(x))$

$\Rightarrow 1 - \sup N_A(x) - \sup M_B(x) + \sup N_A(x). \sup M_B(x) \leq 1 - \inf N_A(x) - \inf M_B(x) + \inf N_A(x). \inf M_B(x)$

$\Rightarrow -\sup N_A(x) - \sup M_B(x) + \sup N_A(x). \sup M_B(x) \leq -\inf N_A(x) - \inf M_B(x) + \inf N_A(x). \inf M_B(x)$

$\Rightarrow \sup N_A(x) + \sup M_B(x) - \sup N_A(x). \sup M_B(x) \geq \inf N_A(x) + \inf M_B(x) - \inf N_A(x). \inf M_B(x)$

$\Rightarrow \begin{bmatrix} \inf N_A(x) + \inf M_B(x) - \inf N_A(x). \inf M_B(x), \\ \sup N_A(x) + \sup M_B(x) - \sup N_A(x). \sup M_B(x) \end{bmatrix} \subset [0,1]$

Now, we show that the sum of the supremums of intervals is equal to 1 or less than 1.

$\sup M_A(x). \sup N_B(x) + \sup N_A(x) + \sup M_B(x) - \sup N_A(x). \sup M_B(x)$

$= \sup M_A(x). \sup N_B(x) + \sup N_A(x) + \sup M_B(x) - \sup N_A(x). \sup M_B(x) + 1 - 1$

$= \sup M_A(x). \sup N_B(x) + \sup N_A(x)(1 - \sup M_B(x)) - (1 - \sup M_B(x)) + 1$

$= \sup M_A(x). \sup N_B(x) + (\sup N_A(x) - 1).(1 - \sup M_B(x)) + 1$

$= \sup M_A(x). \sup N_B(x) - (1 - \sup N_A(x)).(1 - \sup M_B(x)) + 1$

$\left\{ \begin{array}{c} \text{We know that;} \\ \sup M_A(x) \leq 1 - \sup N_A(x) \text{ and } \sup N_B(x) \leq 1 - \sup M_B(x) \\ \Rightarrow \sup M_A(x). \sup N_B(x) \leq (1 - \sup N_A(x)).(1 - \sup M_B(x)) \\ \Rightarrow \sup M_A(x). \sup N_B(x) - (1 - \sup N_A(x)).(1 - \sup M_B(x)) \leq 0 \end{array} \right\}$

$\Rightarrow \sup M_A(x). \sup N_B(x) - (1 - \sup N_A(x).(1 - \sup M_B(x)) + 1 \leq 1$

$\Rightarrow \sup M_A(x). \sup N_B(x) + \sup N_A(x) + \sup M_B(x) - \sup N_A(x). \sup M_B(x) \leq 1$

We get that; $A -_. B \in IVIFS(X)$ □

Proposition 2. *For every IVIFSs A and B,*

$$A -_. B \subset A$$

Proof. Arbitrary IVIFSs A and B are given,

$\inf M_A(x). \inf N_B(x) \leq \inf M_A(x), \sup M_A(x). \sup N_B(x) \leq \sup M_A(x)$ and

Let $\inf N_A(x) + \inf M_B(x) - \inf N_A(x). \inf M_B(x) < \inf N_A(x)$ then,

$\inf M_B(x) - \inf N_A(x). \inf M_B(x) < 0$

$\Rightarrow \inf M_B(x)(1 - \inf N_A(x)) < 0$

$\Rightarrow 1 - \inf N_A(x) < 0 \Rightarrow \inf N_A(x) > 1$ This a contradiction. Therefore,

$\inf N_A(x) + \inf M_B(x) - \inf N_A(x). \inf M_B(x) \geq \inf N_A(x)$

Let $\sup N_A(x) + \sup M_B(x) - \sup N_A(x). \sup M_B(x) < \sup N_A(x)$ then,

$\sup M_B(x) - \sup N_A(x). \sup M_B(x) < 0$

$\Rightarrow \sup M_B(x)(1 - \sup N_A(x)) < 0$

$\Rightarrow 1 - \sup N_A(x) < 0 \Rightarrow \sup N_A(x) > 1$ This is a contradiction. Therefore,

$\sup N_A(x) + \sup M_B(x) - \sup N_A(x). \sup M_B(x) \geq \sup N_A(x)$ □

Example 1. *For two IVIFSs A and B;*

$$A -_\cap B \neq A -_. B$$

Solution 1. *Let universet set X be:*

$X = \{a, b, c, d\}$

IVIFSs A and B are:

$A = \{\langle a, [0, 0.5], [0.2, 0.4] \rangle, \langle b, [0.1, 0.3], [0.4, 0.5] \rangle, \langle c, [0.2, 0.7], [0, 0.1] \rangle, \langle d, [0.6, 0.8], [0.1, 0.2] \rangle\}$

$B = \{\langle a, [0.1, 0.3], [0.5, 0.7]\rangle, \langle b, [0.3, 0.6], [0.2, 0.3]\rangle, \langle c, [0.2, 0.6], [0.3, 0.4]\rangle, \langle d, [0.4, 0.5], [0.1, 0.3]\rangle\}$

below sets are:

$A -_\cap B = \left\{ \begin{array}{c} \langle a, [\min\{0, 0.5\}, \min\{0.5, 0.7\}], [\max\{0.2, 0.1\}, \max\{0.4, 0.3\}]\rangle, \\ \langle b, [\min\{0.1, 0.2\}, \min\{0.3, 0.3\}], [\max\{0.4, 0.3\}, \max\{0.5, 0.6\}]\rangle \\ \langle c, [\min\{0.2, 0.3\}, \min\{0.7, 0.4\}], [\max\{0, 0.2\}, \max\{0.1, 0.6\}]\rangle \\ \langle d, [\min\{0.6, 0.1\}, \min\{0.8, 0.3\}], [\max\{0.1, 0.4\}, \max\{0.2, 0.5\}]\rangle \end{array} \right\}$

$\Rightarrow A -_\cap B = \left\{ \begin{array}{c} \langle a, [0, 0.5], [0.2, 0.4]\rangle, \langle b, [0.1, 0.3], [0.4, 0.6]\rangle, \\ \langle c, [0.2, 0.4], [0.2, 0.6]\rangle, \langle d, [0.1, 0.3], [0.4, 0.5]\rangle \end{array} \right\}$

and

$A -_. B = \left\{ \begin{array}{c} \langle a, [0.(0.5), (0.5).(0.7)], [0.2 + 0.1 - (0.2).(0.1), 0.4 + 0.3 - (0.4).(0.3)]\rangle, \\ \langle b, [(0.1).(0.2), (0.3).(0.3)], [0.4 + 0.3 - (0.4).(0.3), 0.5 + 0.6 - (0.5).(0.6)]\rangle \\ \langle c, [(0.2).(0.3), (0.7).(0.4)], [0 + 0.2 - 0.(0.2), 0.1 + 0.6 - (0.1).(0.6)]\rangle \\ \langle d, [(0.6).(0.1), (0.8).(0.3)], [0.1 + 0.4 - (0.1).(0.4), 0.2 + 0.5 - (0.2).(0.5)]\rangle \end{array} \right\}$

$\Rightarrow A -_. B = \left\{ \begin{array}{c} \langle a, [0, 0.35], [0.28, 0.58]\rangle, \langle b, [0.02, 0.09], [0.58, 0.8]\rangle, \\ \langle c, [0.06, 0.28], [0.2, 0.64]\rangle, \langle d, [0.06, 0.24], [0.46, 0.6]\rangle \end{array} \right\}$

It is clearly seen that; $A -_\cap B \neq A -_. B$

The result of above discussions is that $A -_. B$ is IVIFs. This set difference is not equal to $A -_\cap B$. Therefore, $X -_. A$ is the complement of IVIFSs A which is determined by $-.$ operation. It is shortly shown $A^{c.}$

Proposition 3. *For every IVIFSs A and B,*

$A -_. B \subset A -_\cap B$

Proof. Arbitrary IVIFSs A and B are given,

$\inf M_A(x). \inf N_B(x) \leq \inf M_A(x)$ and $\inf N_B(x) \Rightarrow$
$\inf M_A(x). \inf N_B(x) \leq \min\{\inf M_A(x), \inf N_B(x)\}$
$\sup M_A(x). \sup N_B(x) \leq \sup M_A(x)$ and $\sup N_B(x) \Rightarrow$
$\sup M_A(x). \sup N_B(x) \leq \min\{\sup M_A(x), \sup N_B(x)\}$
and
$\left\{ \begin{array}{l} \inf N_A(x) + \inf M_B(x) - \inf N_A(x). \inf M_B(x) \geq \inf N_A(x) \text{ and } \inf M_B(x) \\ \Rightarrow \inf N_A(x) + \inf M_B(x) - \inf N_A(x). \inf M_B(x) \geq \max\{\inf N_A(x), \inf M_B(x)\} \end{array} \right\}$
$\left\{ \begin{array}{l} \sup N_A(x) + \sup M_B(x) - \sup N_A(x). \sup M_B(x) \geq \sup N_A(x) \text{ and } \sup M_B(x) \\ \Rightarrow \sup N_A(x) + \sup M_B(x) - \sup N_A(x). \sup M_B(x) \geq \max\{\sup N_A(x), \sup M_B(x)\} \end{array} \right\}$
$\Rightarrow A -_. B \subset A -_\cap B$ $\square$

The normal complement of special sets are the same with $-.$ operation

Corollary 1. *For three special IVIFSs,*

  (1) $(0^*)^{c.} = X^*$ and $((0^*)^{c.})^{c.} = 0^*$
  (2) $(U^*)^{c.} = U^*$ and $((U^*)^{c.})^{c.} = U^*$
  (3) $(X^*)^{c.} = 0^*$ and $((X^*)^{c.})^{c.} = X^*$
  (4) For every IVIFSs A, $A^{c.} = A^c$ and $(A^{c.})^{c.} = A$

In below corollary, the normal set differences between special sets are the same between IVIFSs

Corollary 2. *For three special IVIFSs,*

  (1) $X^* -_. 0^* = X^*$
  (2) $X^* -_. U^* = U^*$
  (3) $0^* -_. X^* = 0^*$
  (4) $0^* -_. U^* = 0^*$

(5) $U^* - \cdot \, X^* = 0^*$

(6) $U^* - \cdot \, 0^* = U^*$

De Morgan Laws are given below.

Proposition 4. *For every IVIFSs A and B,*

(1) $(A \cap B)^{c \cdot} = A^{c \cdot} \cup B^{c \cdot}$

(2) $(A \cup B)^{c \cdot} = A^{c \cdot} \cap B^{c \cdot}$

Proof. Arbitrary IVIFSs A and B are given,

1. $(A \cap B)^{c \cdot} = \left\{ \left\langle \begin{matrix} x, [1. \max\{\inf N_A(x), \inf N_B(x)\}, 1. \max\{\sup N_A(x), \sup N_B(x)\}], \\ \left[\begin{matrix} 0 + \min\{\inf M_A(x), \inf M_B(x)\} - 0. \min\{\inf M_A(x), \inf M_B(x)\}, \\ 0 + \min\{\sup M_A(x), \sup M_B(x)\} - 0 + \min\{\sup M_A(x), \sup M_B(x)\} \end{matrix} \right] \end{matrix} \right\rangle \mid x \in X \right\}$

$\Rightarrow (A \cap B)^{c \cdot} = \left\{ \left\langle \begin{matrix} x, [\max\{\inf N_A(x), \inf N_B(x)\}, \max\{\sup N_A(x), \sup N_B(x)\}], \\ [\min\{\inf M_A(x), \inf M_B(x)\}, \min\{\sup M_A(x), \sup M_B(x)\}] \end{matrix} \right\rangle \mid x \in X \right\}$

$= A^{c \cdot} \cup B^{c \cdot}$

2. $(A \cup B)^{c \cdot} = \left\{ \left\langle \begin{matrix} x, [1. \min\{\inf N_A(x), \inf N_B(x)\}, 1. \min\{\sup N_A(x), \sup N_B(x)\}], \\ \left[\begin{matrix} 0 + \max\{\inf M_A(x), \inf M_B(x)\} - 0. \max\{\inf M_A(x), \inf M_B(x)\}, \\ 0 + \max\{\sup M_A(x), \sup M_B(x)\} - 0. \max\{\sup M_A(x), \sup M_B(x)\} \end{matrix} \right] \end{matrix} \right\rangle \mid x \in X \right\}$

$\Rightarrow (A \cup B)^{c \cdot} = \left\{ \left\langle \begin{matrix} x, [\min\{\inf N_A(x), \inf N_B(x)\}, \min\{\sup N_A(x), \sup N_B(x)\}], \\ [\max\{\inf M_A(x), \inf M_B(x)\}, \max\{\sup M_A(x), \sup M_B(x)\}] \end{matrix} \right\rangle \mid x \in X \right\}$

$= A^{c \cdot} \cap B^{c \cdot}$ $\square$

In this below proposition, it is the same as crisp set.

Proposition 5. *For every IVIFSs A . B and C,*

(1) $A - \cdot (B \cup C) = (A - \cdot B) \cap (A - \cdot C)$

(2) $A - \cdot (B \cap C) = (A - \cdot B) \cup (A - \cdot C)$

Proof. Arbitrary IVIFSs A . B and C are given,

1. $A - \cdot (B \cup C) = \left\{ \left\langle \begin{matrix} x, \left[\begin{matrix} \inf M_A(x). \min\{\inf N_B(x)), \inf N_C(x)\}, \\ \sup M_A(x). \min\{\sup N_B(x), \sup N_C(x)\} \end{matrix} \right], \\ \left[\begin{matrix} \inf N_A(x) + \max\{\inf M_B(x), \inf M_C(x)\} - \\ \inf N_A(x). \max\{\inf M_B(x), \inf M_C(x)\}, \\ \sup N_A(x) + \max\{\sup M_B(x), \sup M_C(x)\} - \\ \sup N_A(x). \max\{\sup M_B(x), \sup M_C(x)\} \end{matrix} \right] \end{matrix} \right\rangle \mid x \in X \right\}$

$= \left\{ \left\langle \begin{matrix} x, \left[\begin{matrix} \min\{\inf M_A(x). \inf N_B(x), \inf M_A(x). \inf N_C(x)\}, \\ \min\{\sup M_A(x). \sup N_B(x), \sup M_A(x). \sup N_C(x)\} \end{matrix} \right], \\ \left[\max \left\{ \begin{matrix} \inf N_A(x) + \inf M_B(x) - \inf N_A(x). \inf M_B(x), \\ \inf N_A(x) + \inf M_C(x) - \inf N_A(x). \inf M_C(x) \end{matrix} \right\}, \\ \max \left\{ \begin{matrix} \sup N_A(x) + \sup M_B(x) - \sup N_A(x). \sup M_B(x), \\ \sup N_A(x) + \sup M_C(x) - \sup N_A(x). \sup M_C(x) \end{matrix} \right\} \right] \end{matrix} \right\rangle \mid x \in X \right\}$

$= (A - \cdot B) \cap (A - \cdot C)$

2. $A - \cdot (B \cap C) = \left\{ \left\langle \begin{matrix} x, \left[\begin{matrix} \inf M_A(x). \max\{\inf N_B(x)), \inf N_C(x)\}, \\ \sup M_A(x). \max\{\sup N_B(x), \sup N_C(x)\} \end{matrix} \right], \\ \left[\begin{matrix} \inf N_A(x) + \min\{\inf M_B(x), \inf M_C(x)\} - \\ \inf N_A(x). \min\{\inf M_B(x), \inf M_C(x)\}, \\ \sup N_A(x) + \min\{\sup M_B(x)), \sup M_C(x)\} - \\ \sup N_A(x). \min\{\sup M_B(x)), \sup M_C(x)\}) \end{matrix} \right] \end{matrix} \right\rangle \mid x \in X \right\}$

$$= \left\{ \left\langle x, \begin{bmatrix} \max\left\{\inf M_A(x).\inf N_B(x), \inf M_A(x).\inf N_C(x)\right\}, \\ \max\left\{\sup M_A(x).\sup N_B(x), \sup M_A(x).\sup N_C(x)\right\} \end{bmatrix}, \right. \right.$$
$$\left. \left. \begin{bmatrix} \min\left\{ \begin{array}{c} \inf N_A(x) + \inf M_B(x) - \inf N_A(x).\inf M_B(x), \\ \inf N_A(x) + \inf M_C(x) - \inf N_A(x).\inf M_C(x) \end{array} \right\}, \\ \min\left\{ \begin{array}{c} \sup N_A(x) + \sup M_B(x) - \sup N_A(x).\sup M_B(x), \\ \sup N_A(x) + \sup M_C(x) - \sup N_A(x).\sup M_C(x) \end{array} \right\} \end{bmatrix} \right\rangle \mid x \in X \right\}$$

$$= (A -_. B) \cup (A -_. C) \qquad \square$$

Proposition 6. *For every IVIFSs A . B and C,*

   (1) $(A \cap B) -_. C = (A -_. C) \cap (B -_. C)$
   (2) $(A \cup B) -_. C = (A -_. C) \cup (B -_. C)$

Proof. Arbitrary IVIFSs A . B and C are given,

$$1. \ (A \cap B) -_. C = \left\{ \left\langle x, \begin{bmatrix} \min\left\{\inf M_A(x), \inf M_B(x)\right\}.\inf N_C(x), \\ \min\left\{\sup M_A(x), \sup M_B(x)\right\}.\sup N_C(x) \end{bmatrix} \right. \right.$$
$$\left. \left. \begin{bmatrix} \max\left\{\inf N_A(x), \inf N_B(x)\right\} + \inf M_C(x) - \\ \max\left\{\inf N_A(x), \inf N_B(x)\right\}.\inf M_C(x), \\ \max\left\{\sup N_A(x), \sup N_B(x)\right\} + \sup M_C(x) - \\ \max\left\{\sup N_A(x), \sup N_B(x)\right\}.\sup M_C(x) \end{bmatrix} \right\rangle \mid x \in X \right\}$$

$$= \left\{ \left\langle x, \begin{bmatrix} \min\left\{\inf M_A(x).\inf N_C(x)), \inf M_B(x).\inf N_C(x)\right\}, \\ \min\left\{\sup M_A(x).\sup N_C(x)), \sup M_B(x).\sup N_C(x)\right\} \end{bmatrix}, \right. \right.$$
$$\left. \left. \begin{bmatrix} \max\left\{ \begin{array}{c} \inf N_A(x) + \inf M_C(x) - \inf N_A(x).\inf M_C(x), \\ \inf N_B(x) + \inf M_C(x) - \inf N_B(x).\inf M_C(x) \end{array} \right\}, \\ \max\left\{ \begin{array}{c} \sup N_A(x) + \sup M_C(x) - \sup N_A(x).\sup M_C(x), \\ \sup N_B(x) + \sup M_C(x) - \sup N_B(x).\sup M_C(x) \end{array} \right\} \end{bmatrix} \right\rangle \mid x \in X \right\}$$

$$= (A \cap B) -_. C = (A -_. C) \cap (B -_. C)$$

$$2. \ (A \cup B) -_. C = \left\{ \left\langle x, \begin{bmatrix} \max\left\{\inf M_A(x), \inf M_B(x)\right\}.\inf N_C(x), \\ \max\left\{\sup M_A(x), \sup M_B(x)\right\}.\sup N_C(x) \end{bmatrix} \right. \right.$$
$$\left. \left. \begin{bmatrix} \min\left\{\inf N_A(x), \inf N_B(x)\right\} + \inf M_C(x) - \\ \min\left\{\inf N_A(x), \inf N_B(x)\right\}.\inf M_C(x), \\ \min\left\{\sup N_A(x), \sup N_B(x)\right\} + \sup M_C(x) - \\ \min\left\{\sup N_A(x), \sup N_B(x)\right\}.\sup M_C(x) \end{bmatrix} \right\rangle \mid x \in X \right\} \text{we}$$

$$= \left\{ \left\langle x, \begin{bmatrix} \max\left\{\inf M_A(x).\inf N_C(x)), \inf M_B(x).\inf N_C(x)\right\}, \\ \max\left\{\sup M_A(x).\sup N_C(x)), \sup M_B(x).\sup N_C(x)\right\} \end{bmatrix}, \right. \right.$$
$$\left. \left. \begin{bmatrix} \min\left\{ \begin{array}{c} \inf N_A(x) + \inf M_C(x) - \inf N_A(x).\inf M_C(x), \\ \inf N_B(x) + \inf M_C(x) - \inf N_B(x).\inf M_C(x) \end{array} \right\}, \\ \min\left\{ \begin{array}{c} \sup N_A(x) + \sup M_C(x) - \sup N_A(x).\sup M_C(x), \\ \sup N_B(x) + \sup M_C(x) - \sup N_B(x).\sup M_C(x) \end{array} \right\} \end{bmatrix} \right\rangle \mid x \in X \right\}$$

$$= (A -_. C) \cup (B -_. C) \qquad \square$$

Corollary 3. *For every IVIFSs A and B,*

$$(A -_. B) \cup A = A$$

Proposition 7. *For every IVIFSs A ,B and C,*

$$C \subset B \Rightarrow A -_. B \subset A -_. C$$

Proof. Arbitrary IVIFSs A and B are given,

   $\inf M_A(x).\inf N_B(x) \leq \inf M_A(x).\inf N_C(x)$ and $\sup M_A(x).\sup N_B(x) \leq \sup M_A(x).\sup N_C(x)$

   Let $\inf N_A(x) + \inf M_B(x) - \inf N_A(x).\inf M_B(x) < \inf N_A(x) + \inf M_C(x) - \inf N_A(x).\inf M_C(x)$ then,

$\inf M_B(x)(1 - \inf N_A(x)) < \inf M_C(x)(1 - \inf N_A(x)) \Rightarrow \inf M_B(x) < \inf M_C(x)$
This is a contradiction. Therefore,

$\Rightarrow \inf N_A(x) + \inf M_B(x) - \inf N_A(x). \inf M_B(x) \geq \inf N_A(x) + \inf M_C(x) - \inf N_A(x). \inf M_C(x)$

Let $\sup N_A(x) + \sup M_B(x) - \sup N_A(x). \sup M_B(x) < \sup N_A(x) + \sup M_C(x) - \sup N_A(x). \sup M_C(x)$ then,

$\sup M_B(x)(1 - \sup N_A(x)) < \sup M_C(x)(1 - \sup N_A(x)) \Rightarrow \sup M_B(x) < \sup M_C(x)$
This is a contradiction. Therefore,

$\Rightarrow \sup N_A(x) + \sup M_B(x) - \sup N_A(x). \sup M_B(x) \geq \sup N_A(x) + \sup M_C(x) - \sup N_A(x). \sup M_C(x)$

We get that; $\Rightarrow A -_{.} B \subset A -_{.} C$ $\square$

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