

ALLOCATION OF THE GENERAL PRODUCTION COSTS TO THE COST CENTERS BY LINEAR PROGRAMMING METHOD

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ABSTRACT:

In order to find out the cost of the products produced at a business enterprise, the cost which is accumulated at cost centers should be loaded on to products through appropriate allocation bases. Direct labor cost and direct material cost which are related with the products directly are loaded on to the cost of the products directly. However, so as to load the overhead on to the cost of the products which are related with the products indirectly, the cost that is accumulated at auxiliary cost centers should be loaded on to the real production centers by choosing one of the basic, graded, mathematical or continuous allocation methods. Each of the methods used at secondary allocation is developed to remove the drawbacks of the others. The cost which is accumulated at cost centers by restrictions and the purpose function that will be implemented is possible to be allocated to the real production centers by linear programming method by considering the fact that the cost centers profit from each as it is the case with the mathematical and continuous allocation methods.

Key words: Secondary Allocation, Cost Allocations To The Cost Centers, Linear Programming.

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GENEL ÜRETİM GİDERLERİNİN DOĞRUSAL PROGRAMLAMA YÖNTEMİYLE GİDER YERLERİNE DAĞITILMASI

ÖZET

Bir işletmede üretilen mamüllerin maliyetlerini bulabilmek için gider yerlerinde biriken giderlerin uygun dağıtım anahtarları vasıtasıyla üretilen mamüllere yüklenmesi gereklidir. Üretilen mamüllerle direkt olarak bağlantısı kurulan Direkt İlk Madde ve Malzeme, Direkt İşçilik giderleri üretilen ürün maliyetlerine doğrudan yüklenmektedir. Ancak üretilen ürünlerle dolaylı bir bağlantısı olan genel üretim giderlerinin mamül maliyetine yüklenebilmesi için birinci dağıtım yapıldıktan sonra yardımcı gider yerlerinde biriken giderlerin basit, kademeli, matematiksel veya turlama yöntemlerinden birisi seçilerek esas üretim yerlerine yüklenmesi sağlanır. II. Dağıtımda kullanılan bu yöntemlerden her birisi sırasıyla diğerinin sakıncasını gidermek için geliştirilmiştir. Doğrusal programlama yöntemiyle; matematiksel ve turlama yöntemlerinde olduğu gibi yardımcı gider yerlerinin birbirlerinden faydalanmaları göz önüne alınarak, oluşturulacak amaç fonksiyonu ve kısıtlarla yardımcı gider yerlerinde biriken giderlerin, esas üretim yerlerine dağıtımını yapılarak ayrıntılı analize bakılması mümkündür.

Anahtar Kelimeler: II.Dağıtım, Giderlerin Gider Yerlerine Dağıtımını, Doğrusal Programlama.

Jel Kodlar:

1. INTRODUCTION

The costs accumulated at auxillary cost centers after the initial allocation are allocated to the auxillary and real cost centers via allocation keys. Various methods are developed in order to allocate these costs accumulated at auxillary cost centers. These methods which are used at secondary allocation are basic, graded, mathematical and continuous allocation methods. Each of the methods used at secondary allocation is developed to remove the drawbacks of the others.

2. COST ALLOCATIONS TO THE COST CENTERS

2.1.Direct Allocation Method

If we analyze allocation methods; considering basic allocation method, it allocates the costs accumulated at the auxillary cost centers to the real

cost center. However, auxillary cost centers also profit from each other and basic method does not take this fact into consideration. Figure 1 shows how basic allocation method functions.

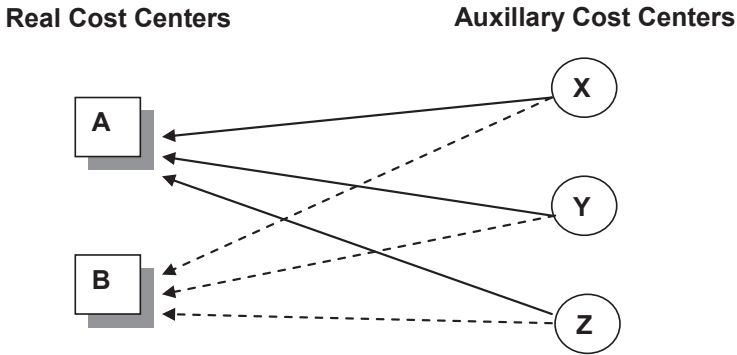


Figure 1: Direct Allocation Method (Büyükmirza 2011,209)

2.2.Step Method of Allocation

Step Method of Allocation, which is created to resolve the problem related to the basic allocation method, allocates the costs accumulated at the auxillary cost centers not only to real but also to the auxillary cost centers. An allocation course is set amongst auxillary cost centers in order to decide which auxillary cost center is to be allocated first. Hence, a profit ratio between these auxillary cost centers is evaluated. Costs are a loaded to the cost centers starting from which provides the most profit and receives the least. In order to be able to find out the profit ratio, a matrix is evaluated via allocation keys which are going to be used at each cost center.

When implementing the graded allocation method; after an auxillary cost center is allocated, other cost centers are not allocated again to this particular cost center. There is no repetition in this process of cost allocation. This shows why the graded method is defective. Figure 2 shows how step method of allocation method functions.

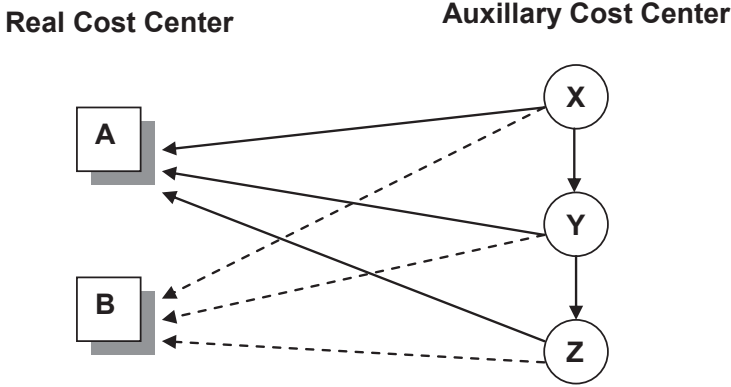


Figure 2: Step Method of Allocation (Büyükmirza 2011,212)

2.3.Continuous and Mathematical Allocation Methods

On the other hand, continuous and mathematical allocation methods, which are created to resolve the problem related to the graded allocation method, operates a retrospective allocation in the interest of auxillary cost centers while the costs accumulated at the auxillary cost centers are being allocated. In order to implement a retrospective cost allocation; it is intended that the total cost obtained out of other auxillary cost centers is evaluated, by considering the fact that auxillary cost centers profit from each other. Results from both continuous and mathemtaical allocation methods are the same while only the procedures differ. Figure 3 shows how continuous and mathematical allocation methods function.

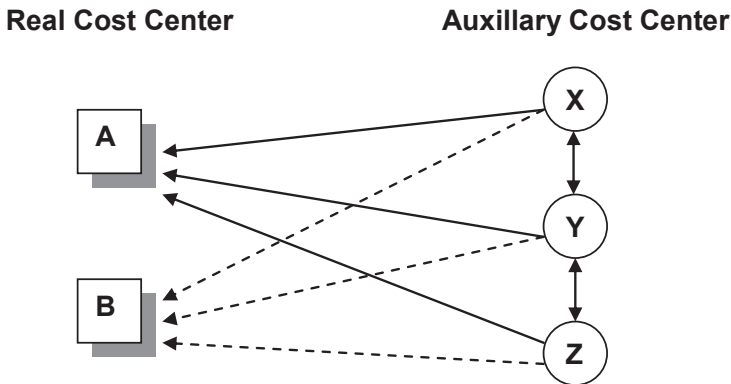


Figure 3: Continuous and Mathematical Allocation Methods (Büyükmirza 2011,215)

There is another method that could be used to allocate cost centers. By linear programming method, implementing a purpose function and restrictions, all of the costs accumulated at auxiliary cost centers could be accumulated to the real and auxiliary cost centers.

3. LITERATURE REVIEW

Literature of accounting, which studies cost distribution methods using methods like Mathematical programs and game theory, dates back to around half a century ago. Shubik (1962), was the first person who applied the Shapley value to cost distributions.¹ In 1965, Manes and Smith showed that common cost production process was in line with Kuhn-Tucker Analysis. Kaplan and Thompson, with their 1971 study, developed the rules aiming to distribute the general production costs of a firm with linear programming, and in this framework, they established a distribution algorithm by focusing on a decision of optimal mixture of outcomes. Baker and Taylor (1979), developed a model of linear programming intended to buy services from outside, and thereby, aimed at both showing how much of the services bought as part of general production costs needs to be obtained and whether they are affordable, or not. Williams and Butler (2002), studied the subjects of productivity and justice within the scope of the distribution of general production costs among the users who share the facilities and they developed a linear programming model in order to minimize the costs. On the other hand, Moghaddam and Michelot (2009), used linear programming methods with the purpose of distribution general production costs and the suggested a two-phased methodology based on the information on marginal costs and the production flexibility of input factors using Simplex table. As a result, depending on the literature in question that developed distribution methods for general production costs using the methods of linear programming and mathematical modeling, this study aims to develop a linear programming method intended to the distribution of general production costs.

4. COST ALLOCATIONS TO THE COST CENTERS USING THE METHOD OF LINEAR PROGRAMMING

If costs are accumulated to the cost centers by implementing linear prog-

¹ Shapley value is a method in the literature of game theory, which aims to predict beforehand the profit each player expects to get (Roth and Verrechia, 1979).

ramming method, the total cost that is to be allocated for each auxillary cost centers would be evaluated to the same result as continuous and mathematical allocation methods. The total cost which is calculated considering the profit ratio is allocated to the auxillary cost centers via allocation keys. Hence, all costs are allocated to the real cost centers. Accordingly, figure 3 also shows how the linear programming method functions and allocates the costs to the cost centers. The idea will be explained by the following example.

4.1. Application

All the parts which is necessary for a 2.75" rocket launcher manufactured in ELROKSAN² factory are produced at three different cost centers. These cost centers are Fuel Station (100 Cost center), Engine Pipe (104 Cost center), Mechanical Parts (101 Cost center) cost centers. Auxillary cost centers are as follows: Maintenance, Refectory, Hating and Head Office. Below, accumulated costs and allocation keys after the initial allocation for each cost center are given.

Table 1: Direct costs and allocation keys for second production period of year 2000

Cost Center	Direct Cost	Maintenance (Hours)	Staff	Area (m ²)
Fuel Station (100 Cost center)	102.000 \$	150	60	1500
Engine Pipe (104 Cost center)	225.000 \$	100	80	1000
Mechanical Parts (101 Cost center)	127.000 \$	125	100	1200
Meintinance	33.000 \$	10	10	100
Refectory	8.000 \$	40	15	300
Heating	45.000 \$	30	5	80
Head Office	4.000 \$	15	20	700

The costs accumulated at maintenance, refectory, heating and head office cost centers will be allocated to the real cost centers via selected proper allocation keys. Purpose function and restrictions should be defined at first in order to implement the allocation.

In order to determine the problem, each field in both real and auxillary cost centers is defined with a variable. As shown in Table 2, applying a matrix will help both understanding the problem and writing the restrictions.

² ELROKSAN is a defense industry company that produces weapons systems to meet the needs of defense industry.

Table 2: Defining Each Element at Real and Auxillary Production Centers as Variables

Real Production Center			Auxillary Production Center			
Fuel Station	Engine Pipe	Mechanical Parts	Maintenance	Refectory	Heating	Head Office
$X_{11} = 102.000$	$X_{21} = 225.000$	$X_{31} = 127.000$	$X_{41} = 33.000$	$X_{51} = 8.000$	$X_{61} = 45.000$	$X_{71} = 4.000$
$X_{12} = ?$	$X_{22} = ?$	$X_{32} = ?$	$X_{42} = ?$	$X_{52} = ?$	$X_{62} = ?$	$X_{72} = ?$
$X_{13} = ?$	$X_{23} = ?$	$X_{33} = ?$	$X_{43} = ?$	$X_{53} = ?$	$X_{63} = ?$	$X_{73} = ?$
$X_{14} = ?$	$X_{24} = ?$	$X_{34} = ?$	$X_{44} = ?$	$X_{54} = ?$	$X_{64} = ?$	$X_{74} = ?$
$X_{15} = ?$	$X_{25} = ?$	$X_{35} = ?$	$X_{45} = ?$	$X_{55} = ?$	$X_{65} = ?$	$X_{75} = ?$
$X_{16} = ?$	$X_{26} = ?$	$X_{36} = ?$	$X_{46} = ?$	$X_{56} = ?$	$X_{66} = ?$	$X_{76} = ?$

The variables indicated with a question mark on Table 2 show the distribution of costs in auxiliary costs places and the amounts that need to be distributed in every costs distribution place. The sum of second distribution will be shown with line Ex_{16} indicated on Table 2. According to this, in order for the Second distribution to be made, the function of the purpose and limitations to be developed must be determined as follows:

Purpose Function

Purpose is the distribution of accumulating costs in auxiliary costs places (Maintenance, Mess hall, Heating, and General Management costs place) in accordance with the extent to which they benefit from each other. Because, at the end of this procedure, there won't be any accumulated costs remaining in the auxiliary costs place, purpose function will be a model that equalize the total value of these variables.

$$\text{Min } Z = \sum_{i=4}^7 x_{ij} = 0, \quad j = i - 2 \quad (1)$$

Restrictions

a. Costs accumulated at real and auxillary cost centers after initial allocation should be defined as restrictions.

$$x_{i1} = a_u, \quad i = 1,2,3,\dots,7 \quad (2)$$

b. Due to the fact that the costs accumulated at auxillary cost centers after the allocation will be allocated via allocation keys, these keys should be added to the model as a restriction.

Table 3: Defining Allocation Keys as Variables

Maintenance (Hours)	Staff	Area m ²
Y ₁₁ = 150	Y ₂₁ = 60	Y ₃₁ = 1500
Y ₁₂ = 100	Y ₂₂ = 80	Y ₃₂ = 1000
Y ₁₃ = 125	Y ₂₃ = 100	Y ₃₃ = 1200
Y ₁₄ = 10	Y ₂₄ = 10	Y ₃₄ = 100
Y ₁₅ = 40	Y ₂₅ = 15	Y ₃₅ = 300
Y ₁₆ = 30	Y ₂₆ = 5	Y ₃₆ = 80
Y ₁₇ = 15	Y ₂₇ = 20	Y ₃₇ = 700

c. Defining the multiplication of cost per unit and each cost that is to be allocated according to the allocation key which is related to each cost center. (cost per unit X allocation key)³,

$$x_{i1} = \left(x_{42} / \sum_{\substack{j=1 \\ j \neq 4}}^7 y_{1j} \right) \cdot y_{1i} \quad i = 1, 2, 3, \dots, 7 \quad i \neq 4 \tag{3}$$

$$x_{ki} = a_u \cdot x_{i+2,i} \quad (k = 1, 2, \dots, 7; i = 3, 4, 5, \quad k \neq i + 2) \tag{4}$$

d. Equality of total costs that is to be allocated to the cost centers and optimum sum of costs which are evaluated via linear programming for each cost centers.

$$x_{jj-2} = \frac{\sum_{i=1}^7 x_{ij-2}}{2} \quad (i = 1, 2, \dots, 7; j = 4, 5, 6, 7) \tag{5}$$

e. Addition of the fact that the sum of costs evaluated from optimum allocation could be bigger than or equal to the initial allocation, as a restriction.

$$x_{jj-2} \geq x_{j1} \quad (j = 4, 5, 6, 7) \tag{6}$$

f. Subtracion of sum of the allocation costs which are obtained from both initial allocation and other auxillary cost centers in order to evaluate the sum of secondary allocation.

$$x_{jj-2} = \frac{\sum_{i=1}^6 x_{ji}}{2} \quad (i = 1, 2, \dots, 6; j = 4, 5, 6, 7) \tag{7}$$

g. After the secondary allocation, there should not be any costs to be allocated left at auxiliary cost centers; in other words, it should be equal to zero. Therefore, after the allocation, sum of the auxiliary cost centers being zero should be added to the model as a restriction. While evaluating the purpose function it is defined that the sum of these variables should be zero. By implementing this restriction, we both secured that there would be no cost to be allocated at auxiliary cost centers and the purpose function is zero.

Each variable can be evaluated by entering the purpose function and its restrictions to an operational research software. Values of the variables will not change after the solution, because some of the values are already known.

After the secondary allocation; in order to be able to solve the costs accumulated at the real production centers, if we enter our purpose function and restrictions to the software "Solver", the program would carry out the purpose function and the restrictions in the model and would search for a feasible area (Anderson ve diğerleri.2010:38-39) . The software would report if it obtains a solution that satisfies our all restrictions and the purpose function.⁴ After obtaining the solution; the answer, sensitivity analysis and restrictions should be checked under the reports section. The solution and the answer from the reports section would be equal because the cells which are going to be changed were determined during the implementation of the model.

4.2. Result of Analysis

Table 4 shows total amount of costs which are supposed to be allocated to each cost center within the framework of the purpose function and restrictions which are determined considering the fact that auxiliary cost centers profit each other.

Tablo 4: Solver Response Report

Name	Initial Value	Final Value
Maintenance Maintenance	- \$	35.021 \$
Refectory Refectory	- \$	14.765 \$
Heating Heating	- \$	47.797 \$
Head Office Head Office	- \$	13.186 \$

Sensitivity analysis of the report shows how much is the value of the purpose function sensitive against the left side values of the restrictions and

the change in the values of the variables. The opportunity cost calculated for non-basic variables (which are zero at optimum solution) is the amount of change in the value of the purpose function, which occurs when there is a one unit of increase in the related value (Ulucan 2004)

Due to the fact that the Maintenance, Refectory, Heating, Head Office auxiliary cost centers are basic variables, their opportunity cost would be equal to zero. Because there isn't any non-basic variable, a one unit of increase in a non-basic variable would not decrease the value of the purpose function.

Tablo 5: Solver Sensitivity Analysis Report

Name	Final Value:	Decreased Gradient
Maintenance Maintenance	35.021 \$	- \$
Refectory Refectory	14.765 \$	- \$
Heating Heating	47.797 \$	- \$
Head Office Head Office	13.186 \$	- \$

Lagrange Multiplier value is equal to the amount of increase in the value of the purpose function, which occurs when there is a one unit of increase in the value of the right hand side of the restriction. The example below shows how much does the value of the purpose function increase when there is a one unit of increase in the costs accumulated at the auxiliary cost centers.

Tablo 6: Solver Sensitivity Analysis Report (Restrictions)

Name	Final Value:	Lagrange Multiplier
Head Office Maintenance	488 \$	- \$
Secondary Allocation Refectory	- \$	- \$
Secondary Allocation Heating	- \$	- \$
Maintenance Refectory		0
Refectory Quality Control		0
Quality Control Refectory	2.987 \$	- \$
Refectory Maintenance	537 \$	- \$
Head Office Head Office	13.186 \$	- \$

Limits Report: Shows how much are the values able to be increased or decreased without violating the restrictions.

Limits Report lists the optimal, the lowest and highest values that could be

used without breaking the limitations on every value cell. As can be seen, in order to reach the aimed result (the value that makes purpose function zero) Maintenance cost has taken the highest value of 35.021 without breaking the limitations on the maximum value.

Table 7: Limits Report Analisis Report

Adjustable Name	Value:	Lower	Target	Upper	Target
		Boundary	Result	Boundary	Result
Maintenance	35.021 \$	35.021 \$	- \$	35.021 \$	- \$
Maintenance	14.765 TL				
Quality Control Heating	47.797 TL				
Head Office Head Office	13.186 TL				

Table 8: Allocation of the General Production Centers to the Cost Centers by Linear Programming Method

Cost Center	Real Production Center			Auxillary Production Center			
	Fuel Station	Engine Pipe	Mechanical Parts	Maintenance	Regretory	Heating	Head Office
Initial Allocation	102.000 \$	225.000 \$	127.000 \$	33.000 \$	8.000 \$	45.000 \$	4.000 \$
Maintenance	11.420 \$	7.613 \$	9.517 \$	(35.021 \$)	3.045 \$	2.284 \$	1.142 \$
Regretory	3.221 \$	4.295 \$	5.369 \$	537 \$	(14.765 \$)	268 \$	1.074 \$
Quality Control	14.936 \$	9.958 \$	11.949 \$	996 \$	2.987 \$	(47.797 \$)	6.970 \$
Head Office	2.930 \$	3.907 \$	4.884 \$	488 \$	733 \$	244 \$	(13.186 \$)
Secondary Allocation	134.508 \$	250.773 \$	158.719 \$	0 \$	0 \$	0 \$	0 \$

Costs that are accumulated at auxillary cost centers and to be allocated, and results of the problem solved with three different methods; namely, mathematical, continuous and linear programming methods; are given the table below. There is no difference between the results of the methods.

Table 9: Comparison of Results of the Methods

Method/Cost	Continuous Method	Linear Programming Method	Mathematical Method
Maintenance	35.021 TL	35.021 TL	35.021 TL
Regretory	14.765 TL	14.765 TL	14.765 TL
Quality Control	47.796 TL	47.797 TL	47.796 TL
Head Office	13.186 TL	13.186 TL	13.186 TL
Total	110.768 TL	110.769 TL	110.768 TL

5. CONCLUSION

In order for general production costs to be added to the cost of the product, the costs that accumulate in costs places are added to the main costs places by choosing one of the simple, gradual, mathematical and ranging methods. With linear programming method; as it is in the mathematical and ranging methods, the purpose function that will be established by considering the co-benefiting of auxiliary costs places can be distributed to the main production sites.

Another advantage of linear programming method over other methods is that it shows, with the help of sensitivity analysis, how sensitive the purpose function is to right side values and to the changes in the variable values. The progress that a one-digit increase on the right side value about the shadow price value will make on the purpose function value is demonstrated. It has also been shown how much increase will a one digit increase in the costs of the auxiliary costs places cause on purpose function value. Besides, it also presents how much could these optimal values be decreased or increased without changing the limitations and by listing the optimal, lowest and highest values in a one digit increase on the right side of the related limitation of shadow price value. Therefore, the decision makers could see how the results will change in case a change might occur in the parameters.

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