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Prešić Type Operators for a Pair Mappings

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Abstract. In this study, we extended the Prešić type contraction mapping using (δ, L)-weak contractive. We investigate Prešić type weak contraction mapping and obtain some fixed point results in Prešić type weak (almost) contraction mappings for a pair of mappings using Jungck type mappings. Additionally, we establish an example to show that the new results are applicable.

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1. Introduction

Fixed point theory has a significant role in diverse branches of mathematics. Banach [1] proved the basic fixed point theorem and results, that has formed a major part in fixed point theory. The Banach fixed point theorem has been generalized in different ways by many authors. Among these generalizations, weak contractive mapping is the most interesting one. Berinde [2–4] introduced (δ, L)-weak contractive (or almost contractive) mappings. Later, Berinde proved some generalizations of (δ, L)-weak contraction theorem. Prešić [9] introduced a contraction condition on finite product space and gave a fixed point theorem. Then, Ćirić and Prešić [5] and Rao et al. [10] have extended this fixed point theorem. Several authors have extended the contractive type given by Prešić, for example [7, 8, 11, 12].

Considering the convergence of certain sequences S.B. Prešić [9] generalized Banach contraction principle as follows:

Theorem 1.1. *Let (X, d) be a complete metric space, k be a positive integer and $T : X^k \to X$ be a mapping satisfying the following contractive type condition*

$$d(T(\vartheta_1, \vartheta_2, ..., \vartheta_k), T(\vartheta_2, \vartheta_3, ..., \vartheta_{k+1})) \leq q_1 d(\vartheta_1, \vartheta_2) + q_2 d(\vartheta_2, \vartheta_3) + ... + q_k d(\vartheta_k, \vartheta_{k+1}), \tag{1.1}$$

for every $\vartheta_1, \vartheta_2, ..., \vartheta_{k+1}$ in X, where $q_1, q_2, ..., q_k$ are non negative constants such that $q_1 + q_2 + ... + q_k < 1$. Then, there exist a unique point ϑ in X such that $T(\vartheta, \vartheta, ..., \vartheta) = \vartheta$. Moreover, if $\vartheta_1, \vartheta_2, ..., \vartheta_k$, are arbitrary points in X and for $n \in N$,

$$\vartheta_{n+k} = T(\vartheta_n, \vartheta_{n+1}, ..., \vartheta_{n+k-1}), \quad (n = 1, 2, ...)$$

then the sequence $\{\vartheta_n\}_{n=1}^{\infty}$ is convergent and

$$\lim \vartheta_n = T(\lim \vartheta_n, \lim \vartheta_n, ..., \lim \vartheta_n).$$

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Remark that in Theorem 1.1, if $k = 1$ the condition (1.1) coincides with the well-known Banach contraction mapping principles. So, theorem 1.1 is a generalization of the Banach fixed point theorem.

Ćirić and Prešić [5] generalized the above result as follows:

Theorem 1.2. *Let (X, d) be a complete metric space, k be a positive integer and $T : X^k \to X$ be a mapping satisfying the following contractive type condition*

$$d(T(\vartheta_1, \vartheta_2, ..., \vartheta_k), T(\vartheta_2, \vartheta_3, ..., \vartheta_{k+1})) \leq \lambda \max_{1\leq i\leq k}\{d(\vartheta_i, \vartheta_{i+1})\},$$

where $\lambda \in (0, 1)$ is constant and $\vartheta_1, \vartheta_2, ..., \vartheta_{k+1}$ are arbitrary elements in X. Then there exist a point ϑ in X such that $T(\vartheta, \vartheta, ..., \vartheta) = \vartheta$. Moreover, if $\vartheta_1, \vartheta_2, ..., \vartheta_k$, are arbitrary points in X and for $n \in N$,

$$\vartheta_{n+k} = T(\vartheta_n, \vartheta_{n+1}, ..., \vartheta_{n+k-1}), \quad (n = 1, 2, ...)$$

then the sequence $\{\vartheta_n\}_{n=1}^{\infty}$ is convergent and

$$\lim \vartheta_n = T(\lim \vartheta_n, \lim \vartheta_n, ..., \lim \vartheta_n).$$

If in addition we suppose that on a diagonal $\triangle \subset X^k$

$$d(T(\kappa, \kappa, ..., \kappa), T(\nu, \nu, ..., \nu)) < d(\kappa, \nu)$$

holds for all $\kappa, \nu \in X$, with $\kappa \neq \nu$, then ϑ is the unique point in X with $T(\vartheta, \vartheta, ..., \vartheta) = \vartheta$.

Definition 1.3. For any $\vartheta_0 \in Z$, there exists a sequence $\{\vartheta_n\}_{n=0}^{\infty} \in Z$ such that $S\vartheta_{n+1} = T\vartheta_n$. The Jungck iteration is defined as the sequence $\{S\vartheta_n\}_{n=1}^{\infty}$ such that $S\vartheta_{n+1} = T\vartheta_n, \quad n \geq 0$. This procedure becomes Picard iteration when $Z = X$ and $S = I_d$ where I_d is the identity map on X. Similarly, Jungck [6] contraction maps are the maps S, T satisfying

$$d(T\vartheta, T\nu) \leqslant kd(S\vartheta, S\nu), \quad 0 \leqslant k < 1 \text{ for all } \vartheta, \nu \in Z. \tag{1.2}$$

If $Z = X$ and $S = I_d$, then maps satisfying (1.2) become the well known contraction maps.

Some authors [10] introduced fixed point theorems for Jungck type mapping as an extension of the Banach Contraction Principle.

In this paper, following by Ćirić and Prešić [5] Berinde [2–4] and Rao et al. [10], we obtain some new fixed point results for a pair of mappings using Jungck type mappings and we give an example which illustrate the effectiveness of our main theorem.

2. Main Results

In this section, we obtain a fixed point theorem for Prešić type almost contraction mapping using Jungck type mappings and introduce some new fixed point results.

Theorem 2.1. *Let (Z, d) be a complete metric space, r is a positive integer and let A, B be mappings of Z^{2r} to Z satisfying the following conditions*

$$\begin{aligned} d(A(\vartheta_1, \vartheta_2, ..., \vartheta_{2r-1}, \vartheta_{2r}),&B(\vartheta_2, \vartheta_3, ..., \vartheta_{2r}, \vartheta_{2r+1})) \\ &\leq \delta \max_{1\leq t\leq 2r}\{d(\vartheta_t, \vartheta_{t+1})\} \\ &\quad + \lambda \min_{1\leq t\leq 2r}\{d(\vartheta_{t+1}, A(\vartheta_1, \vartheta_2, ..., \vartheta_{2r-1}, \vartheta_{2r}))\}, \end{aligned} \tag{2.1}$$

for all $\vartheta_1, \vartheta_2, \cdots, \vartheta_{2r}, \vartheta_{2r+1} \in Z$ and similarly

$$\begin{aligned} d(B(s_1, s_2, ..., s_{2r-1}, s_{2r}),&A(s_2, s_3, ..., s_{2r}, s_{2r+1})) \\ &\leq \delta \max_{1\leq t\leq 2r}\{d(s_t, s_{t+1})\} \\ &\quad + \lambda \min_{1\leq t\leq 2r}\{d(s_{t+1}, B(s_1, s_2, ..., s_{2r-1}, s_{2r}))\}, \end{aligned}$$

for all $s_1, s_2, \cdots, s_{2r}, s_{2r+1} \in Z$, where $\delta \in [0, 1)$ and $\lambda \geq 0$. Suppose $\vartheta_1, \vartheta_2, \cdots, \vartheta_{2r}$ are arbitrary points in Z and for all $\upsilon \in \mathbb{N}$, let

$$\vartheta_{2r+2\upsilon-1} = A(\vartheta_{2\upsilon-1}, \vartheta_{2\upsilon}, ..., \vartheta_{2\upsilon+2r-2}),$$

and

$$\vartheta_{2r+2v} = B(\vartheta_{2v}, \vartheta_{2v+1}, ..., \vartheta_{2v+2r-1}).$$

Then, the sequence (ϑ_v) *is converges to some* $\kappa \in Z$ *such that*

$$A(\kappa, \kappa, ..., \kappa) = \kappa = B(\kappa, \kappa, ..., \kappa).$$

Proof. Let $\vartheta_1, \cdots, \vartheta_{2r}$ are arbitrary points in Z and for every $v \in \mathbb{N}$, let

$$\vartheta_{2r+2v-1} = A(\vartheta_{2v-1}, \vartheta_{2v}, ..., \vartheta_{2v+2r-2}),$$

and

$$\vartheta_{2r+2v} = B(\vartheta_{2v}, \vartheta_{2v+1}, ..., \vartheta_{2v+2r-1}).$$

Therefore, we get

$$\begin{aligned} d(\vartheta_{2r+1}, \vartheta_{2r+2}) =& d(A(\vartheta_1, \vartheta_2, ..., \vartheta_{2r}), B(\vartheta_2, \vartheta_3, ..., \vartheta_{2r+1})) \\ \leq& \delta \max_{1\leq t\leq 2r}\{d(\vartheta_t, \vartheta_{t+1})\} \\ &+ \lambda \min_{1\leq t\leq 2r}\{d(\vartheta_{t+1}, A(\vartheta_1, \vartheta_2, ..., \vartheta_{2r}))\}. \end{aligned}$$

By using the fact that $\vartheta_{2r+1} = A(\vartheta_1, \vartheta_2, ..., \vartheta_{2r})$, we obtain

$$\min_{1\leq t\leq 2r}\{d(\vartheta_{t+1}, A(\vartheta_1, \vartheta_2, ..., \vartheta_{2r}))\} = 0.$$

Then, we get

$$\begin{aligned} d(\vartheta_{2r+1}, \vartheta_{2r+2}) \leq& \delta \max_{1\leq t\leq 2r}\{d(\vartheta_t, \vartheta_{t+1})\} \\ &+ \lambda \min_{1\leq t\leq 2r}\{d(\vartheta_{t+1}, A(\vartheta_1, \vartheta_2, ..., \vartheta_{2r}))\} \\ =& \delta \max_{1\leq t\leq 2r}\{d(\vartheta_t, \vartheta_{t+1})\}. \end{aligned}$$

Similarly, we can write

$$\begin{aligned} d(\vartheta_{2r+2}, \vartheta_{2r+3}) =& d(B(\vartheta_2, \vartheta_3, ..., \vartheta_{2r+1}), A(\vartheta_3, \vartheta_4, ..., \vartheta_{2r+2})) \\ \leq& \delta \max_{2\leq t\leq 2r+1}\{d(\vartheta_t, \vartheta_{t+1})\} \\ &+ \lambda \min_{2\leq t\leq 2r+1}\{d(\vartheta_{t+1}, B(\vartheta_2, \vartheta_3, ..., \vartheta_{2r+1}))\}. \end{aligned}$$

By using the fact that $\vartheta_{2r+2} = B(\vartheta_2, \vartheta_3, ..., \vartheta_{2r+1})$, we obtain

$$\min_{2\leq t\leq 2r+1}\{d(\vartheta_{t+1}, B(\vartheta_2, \vartheta_3, ..., \vartheta_{2r+1}))\} = 0.$$

Then, we have

$$\begin{aligned} d(\vartheta_{2r+2}, \vartheta_{2r+3}) \leq& \delta \max_{2\leq t\leq 2r+1}\{d(\vartheta_t, \vartheta_{t+1})\} \\ &+ \lambda \min_{2\leq t\leq 2r+1}\{d(\vartheta_{t+1}, B(\vartheta_2, \vartheta_3, ..., \vartheta_{2r+1}))\} \\ =& \delta \max_{2\leq t\leq 2r+1}\{d(\vartheta_t, \vartheta_{t+1})\}. \end{aligned}$$

Continuous this condition, we obtain

$$\begin{aligned} d(\vartheta_{2v+2r}, \vartheta_{2v+2r+1}) =& d(B(\vartheta_{2v}, \vartheta_{2v+1}, ..., \vartheta_{2v+2r-1}), A(\vartheta_{2v+1}, \vartheta_{2v+2}, ..., \vartheta_{2v+2r})) \\ \leq& \delta \max_{n\leq t\leq 2v+2r-1}\{d(\vartheta_t, \vartheta_{t+1})\} \\ &+ \lambda \min_{2v\leq t\leq 2v+2r-1}\{d(\vartheta_{t+1}, B(\vartheta_{2v}, \vartheta_{2v+1}, ..., \vartheta_{2v+2r-1}))\}. \end{aligned}$$

By using the fact that $\vartheta_{2v+2r} = B(\vartheta_{2v}, \vartheta_{2v+1}, ..., \vartheta_{2v+2r-1})$, we obtain

$$\min_{2v\leq t\leq 2v+2r-1}\{d(\vartheta_{t+1}, B(\vartheta_{2v}, \vartheta_{2v+1}, ..., \vartheta_{2v+2r-1}))\} = 0.$$

So, we get

$$d(\vartheta_{2v+2r}, \vartheta_{2v+2r+1}) \leq \delta \max_{n\leq t\leq 2v+2r-1}\{d(\vartheta_t, \vartheta_{t+1})\}.$$

For simplicity set $w_v = d(\vartheta_v, \vartheta_{v+1})$ for all $v \in \mathbb{N}$, $M = \max\{\frac{w_1}{s}, \frac{w_2}{s^2}, ..., \frac{w_{2r}}{s^{2r}}\}$. We shall prove by induction that for each $v = 1, 2, ...2r$:

$$w_v \leq Mh^v, \tag{2.2}$$

where $h = \delta^{\frac{1}{2r}}$. Thus, we obtain

$$\begin{aligned}
w_{2r+1} &= d(\vartheta_{2r+1}, \vartheta_{2r+2})\\
&= d(A(\vartheta_1, ..., \vartheta_{2r-1}, \vartheta_{2r}), B(\vartheta_2, ..., \vartheta_{2r}, \vartheta_{2r+1}))\\
&\leq \delta \max_{1\leq t\leq 2r}\{d(\vartheta_t, \vartheta_{t+1})\}\\
&= \delta \max\{w_1, w_2, ..., w_{2r}\}\\
&\leq \delta \max\{Mh, Mh^2, ..., Mh^{2r}\}\\
&= \delta Mh\\
&= Mh^{2r+1}
\end{aligned}$$

and hence $w_{2r+1} \leq Mh^{2r+1}$. Likewise

$$\begin{aligned}
w_{2r+2} &= d(\vartheta_{2r+2}, \vartheta_{2r+3})\\
&= d(B(\vartheta_2, ..., \vartheta_{2r}, \vartheta_{2r+1}), A(\vartheta_3, ..., \vartheta_{2r+1}, \vartheta_{2r+2}))\\
&\leq \delta \max_{2\leq t\leq 2r+1}\{d(\vartheta_t, \vartheta_{t+1})\}\\
&= \delta \max\{w_2, w_3, ..., w_{2r+1}\}\\
&\leq \delta \max\{Mh^2, Mh^3, ..., Mh^{2r+1}\}\\
&= \delta Mh^2\\
&= Mh^{2r+2}
\end{aligned}$$

and thus, $w_{2r+2} \leq Mh^{2r+2}$. So, our hypothesis are true. Using (2.2) for any $v, m \in \mathbb{N}$ we have the following argument:

$$\begin{aligned}
d(\vartheta_v, \vartheta_m) &\leq d(\vartheta_v, \vartheta_{v+1}) + d(\vartheta_{v+1}, \vartheta_{v+2}) + \cdots + d(\vartheta_{m-1}, \vartheta_m)\\
&\leq w_v + w_{v+1} + \cdots + w_{m-1}\\
&\leq Mh^v + Mh^{v+1} + ... + Mh^{m-1}\\
&\leq Mh^v(1 + h + h^2 + ...)\\
&= \frac{Mh^v}{1-h}.
\end{aligned}$$

As $h = \delta^{\frac{1}{2r}} < 1$, thus, $\frac{Mh^v}{1-h} \to 0$ as $v \to \infty$. Thus, (ϑ_v) is a Cauchy sequence. There exists a point $\kappa \in Z$ such that $\lim_{v\to\infty} \vartheta_v = \kappa$ because Z is a complete space.

Now we prove that κ is a fixed point of A and B.

$$\begin{aligned}
&d(A(\kappa, \kappa, ..., \kappa), \vartheta_{2v+2r-1}) = d(A(\kappa, \kappa, ..., \kappa), A(\vartheta_{2v-1}, \vartheta_{2v}, ..., \vartheta_{2v+2r-2}))\\
&\leq d(A(\kappa, \kappa, ..., \kappa), B(\kappa, ..., \kappa, \vartheta_{2v-1}))+\\
&\quad d(B(\kappa, ..., \kappa, \vartheta_{2v-1}), A(\kappa, ..., \kappa, \vartheta_{2v-1}, \vartheta_{2v}))+\\
&\quad d(A(\kappa, ..., \kappa, \vartheta_{2v-1}, \vartheta_{2v}), B(\kappa, ..., \kappa, \vartheta_{2v-1}, \vartheta_{2v}, \vartheta_{2v+1}))\\
&\quad + ... + d(B(\kappa, \vartheta_{2v-1}, ..., \vartheta_{2v+2r-3}), A(\vartheta_{2v-1}, \vartheta_{2v} ..., \vartheta_{2v+2r-2}))
\end{aligned}$$

and by (2.1), we obtain

$$\begin{aligned}
&d(A(\kappa,\kappa,...,\kappa),\vartheta_{2v+2r-1})\\
&\leq \delta d(\kappa,\vartheta_{2v-1}) + \lambda \min\{d(\vartheta_{2v-1},A(\kappa,\kappa,...,\kappa)),d(\kappa,A(\kappa,\kappa,...,\kappa))\},\\
&\quad + \delta \max\{d(\kappa,\vartheta_{2v-1}),d(\vartheta_{2v-1},\vartheta_{2v})\}\\
&\quad + \lambda \min\{d(\vartheta_{2v},B(\kappa,...,\kappa,\vartheta_{2v-1})),d(\vartheta_{2v-1},B(\kappa,...,\kappa,\vartheta_{2v-1})),\\
&\quad d(\kappa,B(\kappa,...,\kappa,\vartheta_{2v-1}))\},\\
&\quad + ... + \delta \max\{d(\kappa,\vartheta_{2v-1}),d(\vartheta_{2v-1},\vartheta_{2v}),...,d(\vartheta_{2v+2r-3},\vartheta_{2v+2r-2})\}\\
&\quad + \lambda \min\{d(\vartheta_{2v-1},B(\kappa,\vartheta_{2v-1},...,\vartheta_{2v+2r-3})),\\
&\quad d(\vartheta_{2v},B(\kappa,\vartheta_{2v-1},...,\vartheta_{2v+2r-3})),...,d(\vartheta_{2v+2r-2},B(\kappa,\vartheta_{2v-1},...,\vartheta_{2v+2r-3}))\}.
\end{aligned}$$

Letting $v\to\infty$ in the above inequality, we obtain

$$d(A(\kappa,\kappa,...,\kappa),\kappa) \leq \frac{(2r-1)}{2}\lambda d(A(\kappa,\kappa,...,\kappa),\kappa) + \frac{(2r-1)}{2}\lambda d(B(\kappa,\kappa,...,\kappa),\kappa). \tag{2.3}$$

Similarly,

$$\begin{aligned}
&d(B(\kappa,\kappa,...,\kappa),\vartheta_{2v+2r}) = d(B(\kappa,\kappa,...,\kappa),B(\vartheta_{2v},\vartheta_{2v+1},...,\vartheta_{2v+2r-1}))\\
&\leq d(B(\kappa,\kappa,...,\kappa),A(\kappa,...,\kappa,\vartheta_{2v}))+\\
&\quad d(A(\kappa,...,\kappa,\vartheta_{2v}),B(\kappa,...,\kappa,\vartheta_{2v},\vartheta_{2v+1}))+\\
&\quad d(B(\kappa,...,\kappa,\vartheta_{2v},\vartheta_{2v+1}),A(\kappa,...,\kappa,\vartheta_{2v},\vartheta_{2v+1},\vartheta_{2v+2}))\\
&\quad + ... + d(A(\kappa,\vartheta_{2v},...,\vartheta_{2v+2r-2}),B(\vartheta_{2v},\vartheta_{2v+1}...,\vartheta_{2v+2r-1}))
\end{aligned}$$

and by (2.1), we obtain

$$\begin{aligned}
&d(B(\kappa,\kappa,...,\kappa),\vartheta_{2v+2r})\\
&\leq \delta d(\kappa,\vartheta_{2v}) + \lambda \min\{d(\vartheta_{2v},B(\kappa,\kappa,...,\kappa)),d(\kappa,B(\kappa,\kappa,...,\kappa))\},\\
&\quad + \delta \max\{d(\kappa,\vartheta_{2v}),d(\vartheta_{2v},\vartheta_{2v+1})\}\\
&\quad + \lambda \min\{d(\vartheta_{2v},A(\kappa,...,\kappa,\vartheta_{2v})),d(\vartheta_{2v+1},A(\kappa,...,\kappa,\vartheta_{2v})),\\
&\quad d(\kappa,A(\kappa,...,\kappa,\vartheta_{2v}))\},\\
&\quad + ... + \delta \max\{d(\kappa,\vartheta_{2v}),d(\vartheta_{2v},\vartheta_{2v+1}),...,d(\vartheta_{2v+2r-2},\vartheta_{2v+2r-1})\}\\
&\quad + \lambda \min\{d(\vartheta_{2v},A(\kappa,\vartheta_{2v},...,\vartheta_{2v+2r-2})),\\
&\quad d(\vartheta_{2v+1},A(\kappa,\vartheta_{2v},...,\vartheta_{2v+2r-2})),...,d(\vartheta_{2v+2r-1},A(\kappa,\vartheta_{2v},...,\vartheta_{2v+2r-2}))\}.
\end{aligned}$$

Letting $v\to\infty$, we get

$$d(B(\kappa,\kappa,...,\kappa),\kappa) \leq \frac{(2r-1)}{2}\lambda d(B(\kappa,\kappa,...,\kappa),\kappa) + \frac{(2r-1)}{2}\lambda d(A(\kappa,\kappa,...,\kappa),\kappa). \tag{2.4}$$

Using (2.3) and (2.4), we obtain $B(\kappa,\kappa,...,\kappa) = \kappa = A(\kappa,\kappa,...,\kappa)$. Therefore, this completes the proof of the theorem. □

Let us give an example to show the application of fixed point theorem for Prešić type almost contraction mapping using Jungck type mappings.

Example 2.2. Let $Z = \{\vartheta_s = s^2 + s,\ \ s\in\mathbb{N}\}\cup\{0\}$ and $d(w,v) =\mid w-v\mid$. Obviously, (Z,d) be a complete metric spaces, let $r = 1$ and define $A,B : Z^2 \to Z$ by $A(\vartheta_s,\omega_s) = \frac{\vartheta_s+\omega_s}{4}$ and $B(\vartheta_s,\omega_s) = \frac{\vartheta_s+\omega_s}{2}$ $\ for\ all\ \vartheta_s,\omega_s \in Z$. Subsequently $\lambda = 1 \geq 0$ and $\delta = \frac{1}{2} \in [0,1)$.

Then for all $\vartheta_s,\omega_s \in Z$ with $\vartheta_s = \vartheta_{2e-1}$ and $\omega_s = \vartheta_{2e}$, we obtain

$$\begin{aligned}
d(A(\vartheta_{2e-1},\vartheta_{2e}),B(\vartheta_{2e},\vartheta_{2e+1})) \leq&\ \delta \max\{d(\vartheta_{2e-1},\vartheta_{2e}),d(\vartheta_{2e},\vartheta_{2e+1})\}\\
&+ \lambda \min\{d(\vartheta_{2e},A(\vartheta_{2e-1},\vartheta_{2e})),d(\vartheta_{2e+1},A(\vartheta_{2e-1},\vartheta_{2e}))\}
\end{aligned} \tag{2.5}$$

for $\delta \in [0,1)$. Then,

$$\lim_{e\to\infty} \frac{d(A(\vartheta_{2e-1},\vartheta_{2e}), B(\vartheta_{2e},\vartheta_{2e+1})) - \lambda \min\{d(\vartheta_{2e}, A(\vartheta_{2e-1},\vartheta_{2e})), d(\vartheta_{2e+1}, A(\vartheta_{2e-1},\vartheta_{2e}))\}}{\max\{d(\vartheta_r,\vartheta_{r+1}), d(\vartheta_{r+1},\vartheta_{r+2})\}}$$
$$= \lim_{e\to\infty} \frac{2e+1}{4e+2} = \frac{1}{2}.$$

Thus, the inequality (2.5) is satisfied with $\lambda = 1$ and $\delta = \frac{1}{2} \in [0,1)$. So, Theorem 2.1 shows that A and B have a fixed point, that is, $A(0,0) = B(0,0) = 0$.

Corollary 2.3. *Let (Z,d) be a complete metric space, r is a positive integer and let A, B be mappings of Z^{2r} to Z satisfying the following conditions*

$$\begin{aligned} d(A(\vartheta_1,\vartheta_2,...,\vartheta_{2r-1},\vartheta_{2r}),&B(\vartheta_2,\vartheta_3,...,\vartheta_{2r},\vartheta_{2r+1})) \\ &\leq \delta_1 d(\vartheta_1,\vartheta_2) + \delta_2 d(\vartheta_2,\vartheta_3) + ... + \delta_{2r} d(\vartheta_{2r},\vartheta_{2r+1}) \\ &\quad + \lambda_1 d(\vartheta_2, A(\vartheta_1,\vartheta_2,...,\vartheta_{2r-1},\vartheta_{2r})) + \lambda_2 d(\vartheta_3, A(\vartheta_1,\vartheta_2,...,\vartheta_{2r-1},\vartheta_{2r})) + ... \\ &\quad + \lambda_{2r} d(\vartheta_{2r+1}, A(\vartheta_1,\vartheta_2,...,\vartheta_{2r-1},\vartheta_{2r})), \end{aligned}$$

for all $\vartheta_1,\vartheta_2,\cdots,\vartheta_{2r},\vartheta_{2r+1} \in Z$ and similarly

$$\begin{aligned} d(B(s_1,s_2,...,s_{2r-1},s_{2r}),&A(s_2,s_3,...,s_{2r},s_{2r+1})) \\ &\leq \delta_1 d(s_1,s_2) + \delta_2 d(s_2,s_3) + ... + \delta_{2r} d(s_{2r},s_{2r+1}) \\ &\quad + \lambda_1 d(s_2, B(s_1,s_2,...,s_{2r-1},s_{2r})) + \lambda_2 d(s_3, B(s_1,s_2,...,s_{2r-1},s_{2r})) + ... \\ &\quad + \lambda_{2r} d(s_{2r+1}, B(s_1,s_2,...,s_{2r-1},s_{2r})), \end{aligned}$$

for all $s_1,s_2,\cdots,s_{2r},s_{2r+1} \in Z$, where $\delta_1+\delta_2+...+\delta_{2r} = \delta \in [0,1)$ and $\lambda_1+\lambda_2+...+\lambda_{2r} = \lambda \geq 0$. Then, the sequence (ϑ_v) is convergent to a $\kappa \in Z$ such that

$$A(\kappa,\kappa,...,\kappa) = \kappa = B(\kappa,\kappa,...,\kappa).$$

Corollary 2.4. *Let (Z,d) be a complete metric space, r is a positive integer and let A, B be mappings of Z^r to Z satisfying the following conditions*

$$\begin{aligned} d(A(\vartheta_1,\vartheta_2,...,\vartheta_{r-1},\vartheta_r),&B(\vartheta_2,\vartheta_3,...,\vartheta_r,\vartheta_{r+1})) \\ &\leq \delta \max_{1\leq t\leq r}\{d(\vartheta_t,\vartheta_{t+1})\} \\ &\quad + \lambda \min_{1\leq t\leq r}\{d(\vartheta_{t+1}, A(\vartheta_1,\vartheta_2,...,\vartheta_{r-1},\vartheta_r))\}, \end{aligned}$$

for all $\vartheta_1,\vartheta_2,\cdots,\vartheta_r,\vartheta_{r+1} \in Z$ and similarly

$$\begin{aligned} d(B(s_1,s_2,...,s_{r-1},s_r),&A(s_2,s_3,...,s_r,s_{r+1})) \\ &\leq \delta \max_{1\leq t\leq r}\{d(s_t,s_{t+1})\} \\ &\quad + \lambda \min_{1\leq t\leq r}\{d(s_{t+1}, B(s_1,s_2,...,s_{r-1},s_r))\}, \end{aligned}$$

for all $s_1,s_2,\cdots,s_r,s_{r+1} \in Z$, where $\delta \in (0,1)$ and $\lambda \geq 0$. Suppose $\vartheta_1,\vartheta_2,\cdots,\vartheta_r$ are arbitrary points in Z and for all $v \in \mathbb{N}$, let

$$\vartheta_{r+v-1} = A(\vartheta_{v-1},\vartheta_v,...,\vartheta_{v+r-2}),$$

and

$$\vartheta_{r+v} = B(\vartheta_v,\vartheta_{v+1},...,\vartheta_{v+r-1}).$$

Then the sequence (ϑ_v) is convergent to some $\kappa \in Z$ such that

$$A(\kappa,\kappa,...,\kappa) = \kappa = B(\kappa,\kappa,...,\kappa).$$

3. Conclusions

In this paper we introduce a new fixed point theorem by combining the ideas of'Ćirić and Prešić [5] Berinde [2–4] and Rao et al. [10]. We prove a fixed point theorem and some fixed point results in Prešić type weak (almost) contraction mappings using Jungck type mappings. Subsequently, we give an example to show that the new theorem and results are applicable.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this article.

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