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Factors for Generalized Matrix Summability

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Abstract: In [1], Sulaiman has proved a theorem dealing with $\left|A\right|_k$ summability of the series $\sum a_n\lambda_n X_n$. In the present paper, generalized absolute matrix summability has been studied. The known theorem on $\left|A\right|_k$ summability has been generalized to the $\varphi-\left|A;\delta\right|_k$ summability method under some suitable conditions.

Genelleştirilmiş Matris Toplanabilme için Çarpanlar

Anahtar Kelimeler
Toplanabilme çarpanları,
Mutlak matris toplanabilme,
Sonsuz seriler,
Hölder eşitsizliği,
Minkowski eşitsizliği

Öz: Sulaiman [1], $\sum a_n\lambda_n X_n$ serisinin $\left|A\right|_k$ toplanabilmesi ile ilgili bir teorem ispatlamıştır. Bu makalede genelleştirilmiş mutlak matris toplanabilme çalışılmıştır. $\left|A\right|_k$ toplanabilme üzerine bilinen teorem uygun bazı şartlar altında $\varphi-\left|A;\delta\right|_k$ toplanabilme metoduna genelleştirilmiştir.

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1. Introduction

Let $\sum a_n$ be an infinite series with its partial sums (s_n). Let $A=(a_{nv})$ be a normal matrix, i.e., a lower triangular matrix of non-zero diagonal entries. Then A defines the sequence-to-sequence transformation, mapping the sequence $s=(s_n)$ to $As=(A_n(s))$, where

$$A_n(s)=\sum_{v=0}^{n} a_{nv}s_v, \quad n=0,1,...$$

Let (φ_n) be any sequence of positive real numbers. The series $\sum a_n$ is said to be summable $\varphi-\left|A;\delta\right|_k$, $k\geq 1$ and $\delta\geq 0$, if

$$\sum_{n=1}^{\infty}\varphi_n^{\delta k+k-1}\left|A_n(s)-A_{n-1}(s)\right|^k<\infty \quad \text{(see [2]).}$$

If we take $\delta=0$ and $\varphi_n=n$, then $\varphi-\left|A;\delta\right|_k$ summability reduces to $\left|A\right|_k$ summability [3].

Given any normal matrix $A=(a_{nv})$, two lower semimatrices $\bar{A}=(\bar{a}_{nv})$ and $\hat{A}=(\hat{a}_{nv})$ are defined as follows:

$$\bar{a}_{nv}=\sum_{i=v}^{n} a_{ni}, \quad n,v=0,1,... \tag{1}$$

$$\hat{a}_{00} = \bar{a}_{00} = a_{00}\ ,\ \hat{a}_{nv} = \bar{a}_{nv} - \bar{a}_{n-1,v}\ ,\ n = 1,2,... \tag{2}$$

$$A_n(s) = \sum_{v=0}^{n} a_{nv} s_v = \sum_{v=0}^{n} \bar{a}_{nv} a_v \quad \text{and} \quad \bar{\Delta} A_n(s) = \sum_{v=0}^{n} \hat{a}_{nv} a_v\ . \tag{3}$$

2. Material and Method

Absolute summability of an infinite series is an attractive topic in the field of summability. Many different studies on this topic have been done by researchers, see ([1, 4-29]). In 2013, Sulaiman [1] proved the following lemmas and theorem.

Lemma 1: If $\sum n^{-1}\lambda_n$ is convergent, then (λ_n) is non-negative and decreasing, $\lambda_n \log_n = O(1)$, and $n\Delta\lambda_n = O\left(1/(\log n)^2\right)$.

Lemma 2: If $\sum n^{-1}\lambda_n X_n$ is convergent, and the conditions

$$n\Delta\lambda_n = O(\lambda_n) \ \ as \ \ n \to \infty,$$

(4)

$$\sum_{v=1}^{n} \lambda_v = O(n\lambda_n) \ \ as \ \ n \to \infty$$

(5)
are satisfied, then

$$n\lambda_n \Delta X_n = O(1), \tag{6}$$

$$\sum_{n=1}^{m} \lambda_n \Delta X_n = O(1) \ \ as \ \ m \to \infty, \tag{7}$$

$$\sum_{n=1}^{m} n\lambda_n \Delta^2 X_n = O(1) \ \ as \ \ m \to \infty. \tag{8}$$

Theorem 1: Let (λ_n), (X_n) be two sequences such that $\sum n^{-1}\lambda_n X_n$ is convergent, and the conditions (4) and (5) are satisfied. Let $A = (a_{nv})$ be a normal matrix with non-negative entries satisfying

$$\bar{a}_{n0} = 1, \ \ n = 0,1,..., \tag{9}$$

$$a_{n-1,v} \geq a_{nv} \ \ \text{for} \ \ n \geq v+1, \tag{10}$$

$$na_{nn} = O(1), \ \ 1 = O(na_{nn}), \tag{11}$$

$$\sum_{v=1}^{n-1} a_{vv}\hat{a}_{nv} = O(a_{nn}). \tag{12}$$

If $t_v^k = O(1)\ (C,1)$, where $t_v = \frac{1}{v+1}\sum_{r=1}^{v} r a_r$, then the series $\sum a_n \lambda_n X_n$ is summable $|A|_k$, $k \geq 1$.

3. Results

The aim of this paper is to generalize Theorem 1 for $\varphi - |A;\delta|_k$ summability method under some suitable conditions.

Theorem 2: Let (λ_n), (X_n) be two sequences such that $\sum n^{-1}\lambda_n X_n$ is convergent, and the conditions (4), (5), (9)-(12) are satisfied. Let (φ_n) be any sequence such that

$$\varphi_n a_{nn} = O(1), \ \ 1 = O(\varphi_n a_{nn}), \tag{13}$$

$$\sum_{n=v+1}^{m+1} \varphi_n^{\delta k} \left|\Delta_v(\hat{a}_{nv})\right| = O\left(\varphi_v^{\delta k-1}\right) \ \ as \ \ m \to \infty, \tag{14}$$

$$\sum_{n=v+1}^{m+1} \varphi_n^{\delta k} \hat{a}_{n,v+1} = O\left(\varphi_v^{\delta k}\right) \quad as \quad m \to \infty, \tag{15}$$

$$\sum_{v=1}^{n-1} a_{vv} \hat{a}_{n,v+1} = O\left(a_{nn}\right). \tag{16}$$

If $\varphi_v^{\delta k} t_v^k = O(1)\ (C,1)$, i.e. $\sum_{v=1}^{n} \varphi_v^{\delta k} t_v^k = O(n)$, where (t_v) as in Theorem 1, then the series $\sum a_n \lambda_n X_n$ is summable $\varphi - |A;\delta|_k$, $k \geq 1$ and $0 \leq \delta < 1/k$.

3.1. Proof of Theorem 2

Let $\theta_n = \lambda_n X_n$ and (M_n) be the A-transform of the series $\sum a_n \theta_n$. By (3), we get $\bar{\Delta} M_n = \sum_{v=1}^{n} \frac{\hat{a}_{nv} \theta_v}{v} v a_v$.

Here, by Abel's transformation, we get

$$\bar{\Delta} M_n = \sum_{v=1}^{n-1} \Delta_v \left(\frac{\hat{a}_{nv} \theta_v}{v} \right) \sum_{r=1}^{v} r a_r + \frac{\hat{a}_{nn} \theta_n}{n} \sum_{v=1}^{n} v a_v$$

$$= \sum_{v=1}^{n-1} \left(\frac{\Delta_v \left(\hat{a}_{nv}\right) \theta_v}{v+1} + \frac{\hat{a}_{n,v+1} \Delta \theta_v}{v+1} + \frac{\hat{a}_{nv} \theta_v}{v(v+1)} \right) (v+1) t_v + \frac{n+1}{n} a_{nn} \theta_n t_n$$

$$= \frac{n+1}{n} a_{nn} \theta_n t_n + \sum_{v=1}^{n-1} \Delta_v \left(\hat{a}_{nv}\right) \theta_v t_v + \sum_{v=1}^{n-1} \hat{a}_{n,v+1} \Delta \theta_v t_v + \sum_{v=1}^{n-1} \frac{\hat{a}_{nv} \theta_v t_v}{v} = M_{n,1} + M_{n,2} + M_{n,3} + M_{n,4}.$$

First, by using the facts that $\varphi_n a_{nn} = O(1)$ and $n a_{nn} = O(1)$, we have

$$\sum_{n=1}^{m} \varphi_n^{\delta k + k - 1} \left| M_{n,1} \right|^k = \sum_{n=1}^{m} \varphi_n^{\delta k + k - 1} \left| \frac{n+1}{n} a_{nn} \theta_n t_n \right|^k$$

$$= O(1) \sum_{n=1}^{m} \varphi_n^{\delta k + k - 1} a_{nn}^k \theta_n^k t_n^k$$

$$= O(1) \sum_{n=1}^{m} \varphi_n^{\delta k} \left(\varphi_n a_{nn}\right)^{k-1} a_{nn} \theta_n^k t_n^k$$

$$= O(1) \sum_{n=1}^{m} \varphi_n^{\delta k} a_{nn} \theta_n^k t_n^k$$

$$= O(1) \sum_{n=1}^{m} \frac{\varphi_n^{\delta k} \theta_n \theta_n^{k-1} t_n^k}{n}.$$

Here $\theta_n^{k-1} = O(1)$, then we get

$$\sum_{n=1}^{m} \varphi_n^{\delta k + k - 1} \left| M_{n,1} \right|^k = O(1) \sum_{n=1}^{m} \frac{\varphi_n^{\delta k} \theta_n t_n^k}{n}.$$

Applying Abel's transformation, we have

$$\sum_{n=1}^{m} \varphi_n^{\delta k + k - 1} \left| M_{n,1} \right|^k = O(1) \sum_{n=1}^{m-1} \left(\sum_{r=1}^{n} \varphi_r^{\delta k} t_r^k \right) \Delta \left(\frac{\theta_n}{n} \right) + O(1) \left(\sum_{n=1}^{m} \varphi_n^{\delta k} t_n^k \right) \frac{\theta_m}{m}.$$

Here

$$\Delta \left(\frac{\theta_n}{n} \right) = \frac{\theta_n}{n} - \frac{\theta_{n+1}}{n+1} = \frac{n \Delta \theta_n + \theta_n}{n(n+1)} < \frac{\Delta \theta_n}{n+1} + \frac{\theta_n}{n^2}.$$

Then, we have

$$\sum_{n=1}^{m}\varphi_n^{\delta k+k-1}\left|M_{n,1}\right|^k = O(1)\sum_{n=1}^{m-1} n\left(\frac{\Delta\theta_n}{n+1}+\frac{\theta_n}{n^2}\right)+O(1)\theta_m$$

$$= O(1)\sum_{n=1}^{m-1}\Delta\theta_n + O(1)\sum_{n=1}^{m-1}\frac{\lambda_n X_n}{n} + O(1)\lambda_m X_m = O(1) \quad as \quad m\to\infty,$$

by using the hypotheses of Theorem 2 and Lemma 2.

Now, using Hölder's inequality with indices k and k', where $k>1$ and $\frac{1}{k}+\frac{1}{k'}=1,$ we have

$$\sum_{n=2}^{m+1}\varphi_n^{\delta k+k-1}\left|M_{n,2}\right|^k = \sum_{n=2}^{m+1}\varphi_n^{\delta k+k-1}\left|\sum_{v=1}^{n-1}\Delta_v\left(\hat{a}_{nv}\right)\theta_v t_v\right|^k$$

$$\leq \sum_{n=2}^{m+1}\varphi_n^{\delta k+k-1}\sum_{v=1}^{n-1}\left|\Delta_v\left(\hat{a}_{nv}\right)\right|\theta_v^k t_v^k\left\{\sum_{v=1}^{n-1}\left|\Delta_v\left(\hat{a}_{nv}\right)\right|\right\}^{k-1}.$$

By using (2), (1), (10) and (9), it follows that

$$\sum_{v=1}^{n-1}\left|\Delta_v(\hat{a}_{nv})\right| = \sum_{v=1}^{n-1}(a_{n-1,v}-a_{nv}) = \sum_{v=0}^{n-1}a_{n-1,v}-a_{n-1,0}-\sum_{v=0}^{n}a_{nv}+a_{n0}+a_{nn} = \bar{a}_{n-1,0}-a_{n-1,0}-\bar{a}_{n0}+a_{n0}+a_{nn}\leq a_{nn}.$$

Thus,

$$\sum_{n=2}^{m+1}\varphi_n^{\delta k+k-1}\left|M_{n,2}\right|^k \leq \sum_{n=2}^{m+1}\varphi_n^{\delta k}\left(\varphi_n a_{nn}\right)^{k-1}\sum_{v=1}^{n-1}\left|\Delta_v\left(\hat{a}_{nv}\right)\right|\theta_v^k t_v^k$$

$$= O(1)\sum_{v=1}^{m}\theta_v^k t_v^k\sum_{n=v+1}^{m+1}\varphi_n^{\delta k}\left|\Delta_v(\hat{a}_{nv})\right|$$

$$= O(1)\sum_{v=1}^{m}\varphi_v^{\delta k}a_{vv}\theta_v^k t_v^k = O(1) \quad as \quad m\to\infty,$$

as in $M_{n,1}$.

Again using Hölder's inequality, and using the conditions (16), (13), (15) and (11), it follows that

$$\sum_{n=2}^{m+1}\varphi_n^{\delta k+k-1}\left|M_{n,3}\right|^k = \sum_{n=2}^{m+1}\varphi_n^{\delta k+k-1}\left|\sum_{v=1}^{n-1}\hat{a}_{n,v+1}\Delta\theta_v t_v\right|^k$$

$$\leq \sum_{n=2}^{m+1}\varphi_n^{\delta k+k-1}\sum_{v=1}^{n-1}\hat{a}_{n,v+1}\left(\Delta\theta_v\right)^k t_v^k a_{vv}^{1-k}\left\{\sum_{v=1}^{n-1}a_{vv}\hat{a}_{n,v+1}\right\}^{k-1}$$

$$= O(1)\sum_{n=2}^{m+1}\varphi_n^{\delta k}\left(\varphi_n a_{nn}\right)^{k-1}\sum_{v=1}^{n-1}\hat{a}_{n,v+1}\left(\Delta\theta_v\right)^k t_v^k a_{vv}^{1-k}$$

$$= O(1)\sum_{v=1}^{m}\left(\Delta\theta_v\right)^k t_v^k a_{vv}^{1-k}\sum_{n=v+1}^{m+1}\varphi_n^{\delta k}\hat{a}_{n,v+1}$$

$$= O(1)\sum_{v=1}^{m}\varphi_v^{\delta k}\left(\Delta\theta_v\right)^k t_v^k v^{k-1}.$$

Since $\left(v\Delta\theta_v\right)^{k-1} = O(1)$, we obtain

$$\sum_{n=2}^{m+1}\varphi_n^{\delta k+k-1}\left|M_{n,3}\right|^k = O(1)\sum_{v=1}^{m}\varphi_v^{\delta k}\left(\Delta\theta_v\right)t_v^k$$

$$= O(1)\sum_{v=1}^{m}\varphi_v^{\delta k}t_v^k\left(\Delta\lambda_v X_v + \lambda_{v+1}\Delta X_v\right)$$

$$= O(1)\sum_{v=1}^{m}\varphi_v^{\delta k}t_v^k\Delta\lambda_v X_v + O(1)\sum_{v=1}^{m}\varphi_v^{\delta k}t_v^k\lambda_{v+1}\Delta X_v.$$

For the first part, using the condition (4), and using Abel's transformation, it follows that

$$\sum_{v=1}^{m}\varphi_v^{\delta k}t_v^k\Delta\lambda_v X_v = O(1)\sum_{v=1}^{m-1}\left(\sum_{r=1}^{v}\varphi_r^{\delta k}t_r^k\right)\Delta\left(\frac{\lambda_v X_v}{v}\right)+O(1)\left(\sum_{v=1}^{m}\varphi_v^{\delta k}t_v^k\right)\frac{\lambda_m X_m}{m}.$$

Then, by using the hypotheses of Theorem 2 and Lemma 2, we obtain

$$\sum_{v=1}^{m}\varphi_v^{\delta k}t_v^k\Delta\lambda_v X_v = O(1)\sum_{v=1}^{m-1}v\left(\frac{\lambda_v X_v}{v^2}+\frac{\Delta\lambda_v X_v}{v}+\frac{\lambda_{v+1}\Delta X_v}{v}\right)+O(1)\lambda_m X_m$$

$$= O(1)\sum_{v=1}^{m-1}\frac{\lambda_v X_v}{v}+O(1)\sum_{v=1}^{m-1}\Delta\lambda_v X_v+O(1)\sum_{v=1}^{m-1}\lambda_{v+1}\Delta X_v+O(1)\lambda_m X_m$$

$$= O(1) \quad as \quad m\to\infty.$$

Also, for the second part, again using Abel's transformation, and the conditions (4), (7), (8), (6), we get

$$\sum_{v=1}^{m}\varphi_v^{\delta k}t_v^k\lambda_v\Delta X_v = \sum_{v=1}^{m-1}\left(\sum_{r=1}^{v}\varphi_r^{\delta k}t_r^k\right)\Delta\left(\lambda_v\Delta X_v\right)+\left(\sum_{v=1}^{m}\varphi_v^{\delta k}t_v^k\right)\lambda_m\Delta X_m$$

$$= O(1)\sum_{v=1}^{m-1}v\left(\Delta\lambda_v\Delta X_v+\lambda_{v+1}\Delta^2 X_v\right)+O(1)m\lambda_m\Delta X_m$$

$$= O(1)\sum_{v=1}^{m-1}\lambda_v\Delta X_v+\sum_{v=1}^{m-1}v\lambda_{v+1}\Delta^2 X_v+O(1)m\lambda_m\Delta X_m$$

$$= O(1) \quad as \quad m\to\infty.$$

Therefore, we get $\sum_{n=2}^{m+1}\varphi_n^{\delta k+k-1}\left|M_{n,3}\right|^k = O(1)$ as $m\to\infty$.

Finally, using (12), (11), (13) and (15), we have

$$\sum_{n=2}^{m+1}\varphi_n^{\delta k+k-1}\left|M_{n,4}\right|^k = \sum_{n=2}^{m+1}\varphi_n^{\delta k+k-1}\left|\sum_{v=1}^{n-1}\frac{\hat{a}_{nv}\theta_v t_v}{v}\right|^k$$

$$\leq \sum_{n=2}^{m+1}\varphi_n^{\delta k+k-1}\sum_{v=1}^{n-1}\left(\frac{1}{v}\right)^k\hat{a}_{nv}\theta_v^k t_v^k a_{vv}^{1-k}\left\{\sum_{v=1}^{n-1}a_{vv}\hat{a}_{nv}\right\}^{k-1}$$

$$= O(1)\sum_{n=2}^{m+1}\varphi_n^{\delta k}\left(\varphi_n a_{nn}\right)^{k-1}\sum_{v=1}^{n-1}\hat{a}_{nv}\theta_v^k t_v^k a_{vv}$$

$$= O(1)\sum_{v=1}^{m}\theta_v^k t_v^k a_{vv}\sum_{n=v+1}^{m+1}\varphi_n^{\delta k}\hat{a}_{nv}$$

$$= O(1)\sum_{v=1}^{m}\varphi_v^{\delta k}a_{vv}\theta_v^k t_v^k.$$

Then, as in $M_{n,1}$, we obtain

$$\sum_{n=2}^{m+1}\varphi_n^{\delta k+k-1}\left|M_{n,4}\right|^k = O(1) \quad as \quad m\to\infty.$$

Thus, we obtain $\sum_{n=1}^{\infty}\varphi_n^{\delta k+k-1}\left|M_{n,r}\right|^k<\infty$ for $r=1,2,3$ and $r=4$, which complete the proof of Theorem 2.

4. Discussion and Conclusion

In this paper, a general theorem on $\varphi-\left|A;\delta\right|_k$ summability factors of an infinite series has been obtained and it reduces to Theorem 1 in case of $\delta=0$ and $\varphi_n=n$. Therefore, the conditions (13)-(16) are automatically satisfied.

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