

On the Thermo Economical Optimization for Solar Hot Water Heating Systems

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Abstract

A thermo economic optimization analysis is presented based on steady state performance at annual average condition which yields simple algebraic formula for estimating the optimum heat storage tank capacity for solar energy applications. The P₁-P₂ method is used in the present study, together with the most favorable heat storage tank dimensioning criteria, for thermo economic analysis of solar hot water heating systems, SHW, including costs of all system elements. The optimum storage capacity and the most feasible collector area are calculated at which maximum net life cycle savings occurs for solar hot water heating systems. The validity of the optimization formulation was checked.

Keywords: Thermo economics, Solar hot water heating, Optimization, Storage volume

Güneş Sıcak Su Isıtma Sistemleri için Termo Ekonomik Optimizasyon Üzerine Bir Çalışma

Öz

Güneş enerjisi uygulamaları için optimum ısı depolama tankı kapasitesini tahmin etmek için basit cebirsel formül üreten yıllık ortalama koşulda kararlı durum performansına dayalı bir termo ekonomik optimizasyon analizi sunulmuştur. P₁-P₂ metodu, bu çalışmada, tüm sistem elemanlarının maliyetleri de dahil olmak üzere, sıcak kullanım suyu ısıtma sistemlerinin (SHW) termo ekonomik analizi için en uygun ısı depolama tankı boyutlandırma kriterleri ile birlikte kullanılmaktadır. Optimum depolama kapasitesi ve en uygun kollektör alanı, güneş enerjili sıcak su ısıtma sistemleri için maksimum net yaşam döngüsü tasarrufunun gerçekleştiği yerde hesaplanır. Optimizasyon formülasyonunun geçerliliği kontrol edildi.

Anahtar Kelimeler: Termo ekonomi, Güneş sıcak su ısıtma, Optimizasyon, Depolama hacmi

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1. INTRODUCTION

Optimization of SHW, as shown in Figure 1 is extremely important to maximize the savings from these systems. There are many parameters to optimize SHW in a thermo-economic way.

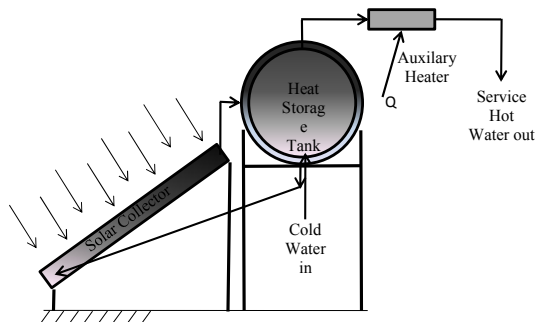


Figure 1. Schematic of SHW

Fixing and, therefore, eliminating all these arbitrary thermal and economic parameters and formulating the most important dependent parameter, the collector area, depending on the storage capacity for the most efficient operation of the SHW, can determine the optimal value for the production of thermal energy. The value of using alternative energy is constantly growing. Solar energy is generally used since it is a clean, renewable and environment conscious type of an alternative energy source. It is known that the efficiency of a solar collector directly depends on its performance, as well as on the temperature at the inlet of the circulating fluid, such as water and antifreeze solution. Before installing alternative energy generating systems, it is necessary to prepare a feasibility study. The main theme of this paper depends on this idea. For this purpose, a new method of thermo-economic optimization has been implemented and presented. An original formula has been developed for calculating the optimal amount of storage of solid waste, which achieves maximum net savings in the life cycle. A thorough search of the current literature are A deep investigation has been accomplished to determine thermal efficiency and cost analysis of solar water heater made in Rwanda [1]. Compared various optimization criteria for a solar domestic hot water system (SDHWS) and considered the energetic,

exergetic, environmental (CO₂ emissions) and financial (life cycle cost) analysis [2]. The proper design of renewable energy based systems is really important to provide their efficient and safe operation. Compared the results obtained during traditional static calculations, with the results of dynamic simulations [3]. Solar water heating (SWH) systems can provide a significant part of the heat energy that is required in the residential sector. The use of SWH systems is motivated by the desire to reduce energy consumption and especially to reduce a major source of greenhouse gas (GHG) emissions [4]. Presented an optimization method to design a solar water heating (SWH) system based on life cycle cost (LCC) [5]. Nowadays the main challenges and principles to increase energy efficiency in the buildings sector are: a very high energy performance of buildings, a significant extent of renewable energy sources and a minimum use of fossil fuels [6]. A solar water heating system for domestic use has been designed and constructed using locally available materials [7]. Solar Water Heaters intercept solar radiation and use it to heat water. Solar thermal collectors can be categorized by the temperature at which they efficiently deliver heat [8]. The design and development of experimental apparatus for demonstrating solar water heating is described. Solar water heating utilizing thermosiphon is attractive because it eliminates the need for a circulating pump [9].

Some of the current literature showed that there were several studies on thermodynamic optimization of the various components of the SHW [10, 11, 12, 13, 14 and 15] and thermo economic optimization of the collector area [16] for fixed storage size per unit collector area excluding the volume dependent first cost of the storage tank. All of these studies do not consider the thermal performances and economics of the system elements all together. A practical method, P₁-P₂ method [11], is used for optimizing the storage tank size of SHW in supplying the thermal energy. Variable parameters used in formulating the thermo economically optimum SHW volume are listed as technical life of the SHW system, first cost of the collector and storage tank per unit size,

annual interest rate, present net price of energy, annual energy price escalation rate, annual average operating time, daily mean operating time, average value of ambient temperature, overall heat loss coefficient of the collector, overall equivalent heat loss coefficient of the heat storage tank, additional independent first expenses of the system for control systems etc., daily average solar energy incoming onto tilted collector surface, collector heat removal factor, transmittance-absorptivity product of the inclined solar collector surface, density and specific heat of water, resale value of the system and the ratio of annual maintenance and operation cost to the original first cost. In addition, the optimal net savings from SHW and payback period is achieved algebraically in the present way of compiling. The optimal amount of accumulated heat of SHW and the corresponding optimal collector area for this storage volume, the optimal net savings from SHW and the payback period can be easily calculated in a few minutes using practical formulas. Designed and presented original formulas by considering three types of tanks as cylindrical, cubic and spherical.

2. MATHEMATICAL FORMULATION

The minimum heat transfer area can be determined by using the method illustrated in [17] for the cylindrical solar heat storage tank as shown in Figure 1 as (Equation 1):

$$A_{\min} = 5.83.V^{2/3} = c.V^{2/3} \tag{1}$$

For a cubic tank similar procedure can be applied which yields (Equation 2):

$$A_{\min} = 6.V^{2/3} = c.V^{2/3} \tag{2}$$

Likewise, if the SHW has a spherical tank its minimum heat transfer area is equal to (Equation 3):

$$A_{\min} = 4.84.V^{2/3} = c.V^{2/3} \tag{3}$$

It is seen that $c = 6$ for cubical tank, 5.83 for cylindrical tank and 4.84 for spherical tank meaning that the minimum heat loss occurs in spherical tank per unit storage volume in comparison with the others.

The net amount of heat that can be stored in the SHW tank can be determined by Equation 4.

$$Q_u = \rho.C_p.V.\Delta T_s \Rightarrow \Delta T_s = \frac{Q_u}{\rho.C_p.V} \tag{4}$$

Energy balance equation for the insulated storage tank (assumed to be at lumped temperature) can be written as in Equation 5.

$$Q_c = Q_u + Q_L \tag{5}$$

Equation 5 can be expanded to get Equation 6 assuming cold water supply temperature is equal to surrounding temperature and average solar energy storage tank temperature is approximately equal to the cold fluid inlet temperature to the collector (Equation 6):

$$F_R.(\tau.\alpha).H_T.A_c.\Delta t - A_c.F_R.U_L.\Delta t.\Delta T = \rho.C_p.V.\Delta T_s + U_t.c.\Delta T.\Delta t.V^{2/3} \tag{6}$$

Area of collector can be formulated after combination of Equations 4 and 6:

The net saving function for disposing of waste heat from SHW can be recorded using the well-known P₁-P₂ method [18] equipment cost estimation parameters for the solar collector area and storage tank as follows (Equation 7):

$$A_c = \frac{\rho.C_p.V.Q_u + c.U_t.Q_u.V^{2/3}}{F_R.(\tau.\alpha).H_T.F_R.(c.\Delta T) - F_R.U_L.Q_u} \tag{7}$$

$$S = \frac{P_1.C_E.Q_u.H}{\Delta t} - P_2.[(C_A.A_c) + C_{EX}] - P_2.C_V.V^{2/3} \tag{8}$$

P₁-P₂ method is selected since it is an easy method and widely used in solar energy economics. P₁ is the ratio of the energy cost savings during the life cycle to the energy savings in the first year, and P₂ is the ratio of the life cycle costs incurred as a result of the additional investments to the initial investments. Combining Equations 7 and 8 yields:

$$S = \frac{P_1 \cdot C_E \cdot Q_u \cdot H}{\Delta t} - P_2 \cdot C_A \left[\frac{\frac{\rho C_p V Q_u}{\Delta t} + c U_t Q_u V^{2/3}}{\rho C_p V H_T F_R (\tau \alpha) - F_R U_L Q_u} \right] - P_2 \cdot C_{EX} - P_2 \cdot C_V \cdot V^{2/3} \quad (9)$$

Equation 9 can be rearranged to:

$$S = S_F - P_2 \cdot C_A \left[\frac{\frac{\rho C_p V Q_u}{\Delta t} + c U_t Q_u V^{2/3}}{\rho C_p V H_T F_R (\tau \alpha) - F_R U_L Q_u} \right] - P_2 \cdot C_V \cdot V^{2/3} \quad (10)$$

S_F is given as in the Equation 11:

$$S_F = \frac{P_1 \cdot C_E \cdot Q_u \cdot H}{\Delta t} - P_2 \cdot C_{EX} \quad (11)$$

Equation 10 can be reformulated as Equation 12:

$$S = S_F - P_2 \cdot C_A \left[\frac{a_1 V + a_2 V^{2/3}}{a_3 V - a_4} \right] - P_2 \cdot C_V \cdot V^{2/3} \quad (12)$$

Where a₁, a₂, a₃, a₄ and S_F are constants. By taking the derivative of Equation 12 with respect to volume, V, gives the following equation:

$$\frac{\partial S}{\partial V} = -P_2 \cdot C_A \left[\frac{(a_1 + \frac{2}{3} a_2 V^{-1/3})(a_3 V - a_4) - a_3 (a_1 V + a_2 V^{2/3})}{(a_3 V - a_4)^2} \right] - \frac{2}{3} P_2 \cdot C_V \cdot V^{-1/3} = 0 \quad (13)$$

Resultant equation can be simplified to Equation 14.

$$b_1 \cdot V_{opt}^{-2/3} - b_2 \cdot V_{opt}^{-1/3} - b_3 \cdot V_{opt}^{1/3} = b_4 \quad (14)$$

Where:

$$b_1 = \frac{2 \cdot C_V}{3 \cdot C_A} \quad (15)$$

Trial error method is applied to solve Equation 14 to get V_{opt}. Convergence is achieved with few iteration. One can get the second derivative of the net savings function with respect to V, (∂²S/∂V²), and the result is found to be always negative, which indicates a local maximum point.

$$b_2 = \frac{4 F_R U_L Q_u C_V}{3 C_A \rho C_p F_R (\tau \alpha) H_T} + \frac{U_t c Q_u}{3 \rho C_p F_R (\tau \alpha) H_T} \quad (16)$$

$$b_3 = \frac{2 F_R U_L U_t c}{3} \left(\frac{Q_u}{\rho C_p F_R H_T (\tau \alpha)} \right)^2 \quad (17)$$

$$- \frac{2 C_V}{3 C_A} \left(\frac{F_R U_L Q_u}{\rho C_p F_R (\tau \alpha) H_T} \right)^2$$

$$b_4 = \frac{F_R U_L}{\rho C_p \Delta t} \left(\frac{Q_u}{F_R \tau \alpha H_T} \right)^2 \quad (18)$$

The payback period of the SHW can be evaluated by equating the net savings function, S, to zero as (Equation 19):

$$S = S_F - P_2 \cdot C_A \left[\frac{a_1 V_{opt} + a_2 V_{opt}^{2/3}}{a_3 V_{opt} - a_4} \right] - P_2 \cdot C_V \cdot V_{opt}^{2/3} = 0 \quad (19)$$

And if i = d, economic parameter P₁ [9] and payback period N_p can be determined as Equation 20:

$$P_1 = \frac{N}{1+i} \quad (20)$$

$$N_p = \frac{\left[C_{EX} + P_2 \cdot C_V \cdot V_{opt}^{2/3} + P_2 \cdot C_A \cdot \left[\frac{a_1 \cdot V_{opt} + a_2 \cdot V_{opt}^{2/3}}{a_3 \cdot V_{opt} - a_4} \right] \right]}{C_E \cdot H \cdot Q_u} \cdot (1+i) \quad (21)$$

Or when $i \neq d$, then the value of the P_1 [9 and 10] and N_p , in years, is calculated by the following equations:

$$P_1 = \frac{1}{d-i} \cdot \left[1 - \left(\frac{1+i}{1+d} \right)^N \right] \quad (22)$$

and

$$N_p = \frac{\ln \left\{ 1 - \frac{(d-i) \left[C_{EX} + P_2 \cdot C_V \cdot V_{opt}^{2/3} + P_2 \cdot C_A \cdot \left(\frac{a_1 \cdot V_{opt} + a_2 \cdot V_{opt}^{2/3}}{a_3 \cdot V_{opt} - a_4} \right) \right]}{C_E \cdot H \cdot Q_u} \right\}}{\ln \left(\frac{1+i}{1+d} \right)} \quad (23)$$

4. RESULTS AND DISCUSSION

For a typical SHW problem illustrated in [17], it is assumed that $i = d = 0.1$, $C_E = 7.5 \times 10^{-5}$ \$/(W.hr), $H = 2000$ hr/yr, $N = 15$ yr, $c = 6$, $U_t = 5$ W/(m².K), $F_R \cdot (\tau \cdot \alpha) \cdot H_T = 290$ W/m², $F_R \cdot U_L = 4$ W/(m².K), $\rho = 1000$ kg/m³, $C_p = 4200$ J/(kg.K), $\Delta t = 36000$ s, $Q_u = 2.1 \times 10^8$ J, $C_A = 200$ \$/m², $C_V = 300$ \$/m³, $C_{EX} = 1000$ \$, $M_s = 0$, $R_v = 0$. The optimum storage size of the SHW is calculated to be approximately 7 m³ using Equation 14. The savings value for this particular example is shown in Figure 2. It can be concluded that in SHW applications there is a local maximum value. Excessive storage size SHW will not be cost effective compared with the optimal, despite the large storage capacity of heat. On the other hand, the value of A_{opt} is determined to be approximately 25.3 m² for this specific sample problem using Equation 7. Optimum collector area of 38 m² is determined by using the optimization method available in [16] based on TRNSYS simulation

code in f-Chart method [18 and 19] with storage volume per unit collector area of 75 L/m².

Optimal collector area strongly depends upon this parameter in f-Chart method. A lower optimal amount of storage size is obtained by the present optimization method for the sample set of data. It is obtained that the most important parameter affecting the optimal sizes of storage volume and collector area is the ratio of unit cost of storage tank into unit cost of the collector area, C_V/C_A . A set of C_V/C_A values are used to see the effect of these cost parameters on optimal storage volume and collector area values as shown in Figure 3.

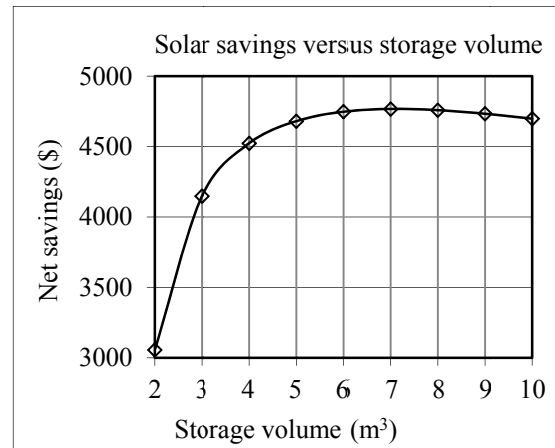


Figure 2. Net savings versus volume of SHW

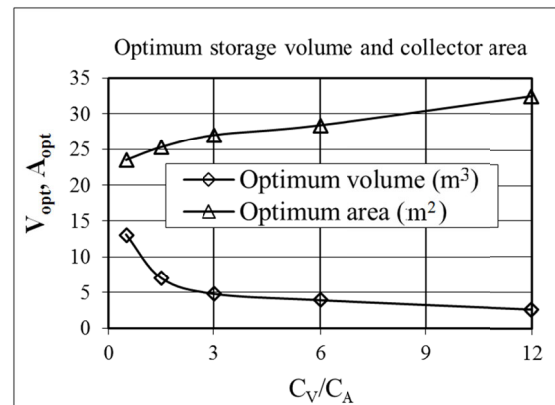


Figure 3. Optimum storage capacity and corresponding optimum collector area versus C_V/C_A

As C_V/C_A increases, optimum V/A ratio decreases and approaches to the result of the f-Chart method. The primary difference between the present results and that obtained from the f-Chart based collector area optimization method [16] is basically additional cost of storage size that was used in the present formulation. A secondary source for this difference is classical f-Chart daily load pattern that was used in [16] and uniform load profile that was assumed in the present work. On the other hand, steady state assumption was used in the present work whereas f-Chart method is an averaged method and is based on unsteady TRNSYS simulation program. Number of payback years is determined to be 9.5 years by using the optimum value of heat storage tank volume in Equation 21.

5. CONCLUSION

It is clear that at the optimum point for SHW there are good thermal and economic indicators. Solar water heating systems should be designed close to this optimum point. Results indicated that the economics of solar hot water heating systems is extremely important and the cost of all system elements must be considered in calculating the solar energy economics. The presented formulas may seem useful for developers and manufacturers, especially for those associated with SHW.

6. NOMENCLATURE

A Length of the side of cubical tank, [m]
 A Heat transfer area of storage tank, [m²]
 a₁ Constant, (a₁ = ρ.C_p.Q_u/Δt)
 a₂ Constant, (a₂ = c.U_t.Q_u)
 a₃ Constant, a₃ = F_R.(τ.α).ρ.C_p.H_T
 a₄ Constant, a₄ = F_R.U_L.Q_u
 A_c Area of collector, [m²]
 A_{min} Minimum heat transfer area of storage tank, [m²]
 A_{opt} Optimum area of collector, [m²]
 b₁₋₄ Constants defined in Equations 14, 17 and 18,
 c Constant connecting to heat transfer area of heat storage tank into its volume,

C_A Area dependent first cost of the collector, [\$/m²]
 C_E Cost of energy supplied by auxiliary heater, [\$(W.hr)]
 C_{EX} Area independent first cost of solar system, [\$]
 C_p Specific heat of water, [J/(kg.K)]
 C_V First cost of solar energy storage tank per unit volume, [\$/m³]
 d Market discount rate in fraction,
 F_R Collector heat removal factor,
 H Annual time of operation of solar energy system, [h/yr]
 H_T Annual seasonal average of instantaneous solar energy incoming onto the tilted collector surface, [W/m²]
 i Energy price escalation rate in fraction,
 M_s Ratio of annual maintenance and operation cost to first original cost,
 N Technical life of the solar system, [yr]
 N_p Payback time, [yr]
 P₁ Ratio of the life cycle energy cost savings to the first year energy cost savings, [yr]
 P₂ Ratio of the life cycle expenditures incurred because of the additional capital investment to the initial investment,
 Q_L Heat loss from heat storage tank, [W]
 Q_c Net solar energy input to collector, [W]
 Q_s Net useful energy that is stored by heat storage tank, [W]
 Q_u Daily net useful energy that is gained by solar system, [J]
 R_v Ratio of resale value into the first original cost,
 S Net savings gained from solar energy, [\$]
 S_F Constant defined in Equation 21, [\$]
 SHW Solar Hot Water,
 T_a Mean temperature of ambient air, [K]
 T_{c,i} Mean inlet temperature of collector fluid, [K]
 T_s Mean lumped temperature of storage tank, [K]
 U_L Overall heat loss coefficient of collector, [W/(m².K)]
 U_t Equivalent overall heat loss coefficient of storage tank, [W/(m².K)]
 V Volume of storage tank, [m³]
 V_{opt} Optimum volume of storage tank, [m³]

- ΔT Temperature difference between collector fluid at the inlet and ambient air or temperature difference between the storage tank and ambient air, [K]
- ΔT_s Temperature difference between mean storage tank and cold water supply, [K]
- Δt Daily mean operating time for solar system, [s]
- ρ Density of water in storage tank, [kg/m³]
- ($\tau \cdot \alpha$) Average transmittance absorptivity product of tilted solar collector surface.

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