ARAŞTIRMA MAKALESİ / RESEARCH ARTICLE

AGE AND WEALTH: AN ANALYSIS OF GLOBAL TOP WEALTH DISTRIBUTION

YAŞ VE SERVET: KÜRESEL EN ÜST SERVET DAĞILIMININ BİR ÇÖZÜMLEMESİ

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ABSTRACT

This paper uses a simple economic model of age and wealth to study the dynamics of global top wealth distribution. The main purpose is to investigate whether and to what extent age—as a demographic variable—affects wealth accumulation. The global top wealth distribution is represented by that of Forbes billionaires, the wealthiest people in the world as listed each year by the Forbes Magazine. After documenting the main distributional properties of age and wealth of Forbes billionaires in the year 2018, the paper introduces the model economy and derives two testable hypotheses. First, the wealth of an individual relative to the average is an exponential function of his/her age as dictated by the theoretical model. Second, the distribution of wealth across individuals has a Pareto shape. Estimation results show that (i) age differences alone explain nearly one fourth of variation in relative wealth, and (ii) global top wealth distribution has a Pareto tail. Results also show that (i) the exponential form of the age-wealth relationship originating from the long-run equilibrium of the model economy performs significantly well in comparison to the linear and quadratic forms, and (ii) the estimated values of Pareto tails indicate that there exists a large degree of wealth inequality even among Forbes billionaires themselves.

KEYWORDS: wealth accumulation, inequality, age distribution, dynamic optimization

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ÖZET

Bu makale, küresel en üst servet dağılımının dinamiklerini çalışmak için yaş ve servetin basit bir ekonomik modelini kullanmaktadır. Ana amaç, bir demografik belirleyen olarak yaşın, servet dağılımını etkileyip etkilemediğini ve ne ölçüde etkilediğini araştırmaktadır. Küresel en üst servet dağılımı, her yıl Forbes Dergisi tarafından dünyadaki en varlıklı insanlar olarak listelenen Forbes milyarderleri ile temsil edilmektedir. Forbes milyarderlerinin 2018 yılındaki yaş ve servetlerinin ana dağılım özellikleri ortaya konduktan sonra, makale model ekonomiyi tanıtmakta ve iki sınanabilir hipotez türetmektedir. İlk olarak, bireyin ortalamaya göreli olan servet düzeyi, onun yaşının üstel bir fonksiyonudur. İkincisi, servetin bireyler arasındaki dağılımı Pareto şekline sahiptir. Tahmin bulguları, (i) sadece yaş farklarının göreli servetteki değişimin yaklaşık dörtte birini açıkladığını ve (ii) küresel en üst servet dağılımının bir Pareto kuyruğu taşıdığını göstermektedir. Bulgular, aynı zamanda, (i) yaş-servet ilişkisinin model ekonominin uzun-dönem dengesinden türeyen üstel biçiminin, doğrusal ve karesel biçimler ile karşılaştırıldığında, çok daha iyi performans sergilediğini ve (ii) tahmin edilen Pareto kuyruğu değerlerinin, Forbes milyarderlerinin kendileri arasında bile çok yüksek bir servet eşitsizliği olduğuna işaret ettiğini göstermektedir.

ANAHTAR KELİMELER: servet birikimi, eşitsizlik, yaş dağılımı, dinamik optimizasyon

INTRODUCTION

A common subject matter in economics, sociology, and political science literatures is the distribution of wealth. Understanding the main patterns and regularities of wealth inequality through time, across the globe, and within a society has profound implications for social scientists. How people accumulate and use their wealth are related with a large set of issues ranging from optimal capital income taxation to the design and conduct of social policies (Benhabib and Bisin, 2018; Zucman, 2019), from political power to election campaigns (Boix, 2010; Scheve and Stasavage, 2017), and from social exclusion to social progress (Keister, 2014; Killewald et al., 2017).

The simplest economic models of income and wealth inequality provide stark predictions regarding the pace of wealth accumulation and the shape of within-country wealth distributions. In the Blanchard (1985)-Yaari (1965) growth model—also known as the *perpetual youth* model—individuals optimally accumulate their wealth under the risk of death at any age. In the continuous-time version outlined by Jones (2014), the average stock of wealth within a society grows exponentially at a fixed flow rate, and the emerging shape of the wealth distribution is of Pareto family. Hence, a skewed wealth distribution is a natural equilibrium outcome of a simple economic environment within which wealthy people takes optimal wealth accumulation decisions.[1]

The Blanchard-Yaari model is particularly appealing from a demographic point of view since individuals differ only in their initial stock of wealth and in their age. In the long-run equilibrium of the model with a stable population, the cross-section distribution of wealth is obtained through the cross-section distribution of age. More specifically, older wealth owners hold larger shares of wealth. In the theoretical model considered in this paper, wealth owners only have capital income, and they do not supply labor. This feature of the model makes it particularly well-suited for an analysis of global top wealth distribution because the tail properties of wealth distribution in models with finitely-lived agents are determined by the distributional properties of capital income, not of labor income (Benhabib et al., 2011).

This paper examines the version of the Blanchard-Yaari model outlined in Jones (2014) from both theoretical and empirical perspectives. Specifically, this paper analyzes the model to arrive at some key theoretical results and, then, tests two implications of the model using actual micro data on global top wealth owners. The first of these testable hypotheses is about the effects of age on the accumulated wealth of an individual; the theory dictates that the wealth of an individual relative to the average wealth in the society is an exponential function of his/her age. *Simply put, older individuals hold higher shares of wealth.* The second testable hypothesis is that the distribution of wealth across individuals is from a Pareto family. Hence, wealth follows a skewed distribution with a (minimum) threshold and with a fat (or heavy) right tail. *Roughly speaking, fewer individuals hold larger amounts wealth at top wealth levels.*

The main source of data is the *Forbes Magazine*'s 2018 list of world billionaires (Forbes, 2018). These 2,145 individuals from all around the world are selected by the *Forbes Magazine* as the wealthiest people in the planet since they each have a level of personal net worth that is equal to or larger than 1 billion US dollars in current (2018) terms. Hence, they represent the global top wealth distribution with a (minimum) threshold value of 1 billion US dollars. The wealthiest among all is Jeff Bezos—the founder and CEO of the global retail giant Amazon—with a personal wealth of 112 billion US dollars in 2018. At the bottom of the distribution, there are 85 individuals each having a personal wealth that is equal to the minimum value of 1 billion US dollars (Forbes, 2018).

The empirical framework of this paper focuses on (i) the exponential age-wealth profile emerging as a long-run equilibrium outcome of the Blanchard-Yaari model, and (ii) the cross-section distribution of wealth that is from the Pareto family of distributions. For both hypotheses, the model's predictions cannot be rejected at standard significance levels. More specifically, roughly one fourth of the variation in wealth relative to the average is explained by the variation in the ages of billionaires, and the cross-section distributions of wealth and relative wealth have Pareto tails. Besides, the failure to reject the equality of tail indexes of these two distributions provides an affirmative judgement on the internal validity of the model.

Related Literatures and Contributions

This paper is most directly related with two lines of literature. The first one is the literature that studies wealth inequality in relation with age, aging, and intergenerational linkages. The other literature is the one that focuses on Forbes billionaires or similar *rich lists* and on whether the distribution of wealth for such global top wealth samples fits a power law distribution such as that of Pareto.

The model developed by Wold and Whittle (1957) is the earliest work that relates demography to the distribution of wealth. These authors introduce a birth-death process to ensure that the distribution of wealth is stable, i.e., it converges to an ergodic distribution. The reason is simply that, at any time, the dying individuals' wealth is inherited by the newborn individuals within an overlapping generations framework. Yaari (1965) and Blanchard (1985) study life-cycle economies with constant probability of death but do not address wealth inequality. A directly related paper is those of Benhabib et al. (2016) who present a rigorous analysis of and empirical support for the wealth distribution properties of the Blanchard-Yaari model. That of Castañeda et al. (2003) is another related paper that shows that a carefully calibrated model with life-cycle elements such as retirement and bequests can account for observed earning and wealth inequalities in the US almost fully.

There exist studies that account for Pareto tails of wealth distribution by introducing heterogeneity through mechanisms in addition to age. Hubbard et al. (1995) develop a life-cycle model that successfully explain saving and wealth differences across different lifetime-income groups. Gokhale et al. (2001) develop a model of 88-period lives with several realistic elements including bequests, assortative mating, and heterogeneous skills. Their model implies a successful model-data match for the wealth at the retirement age in the US. Heer (2001) also develops a life-cycle model with accidental and voluntary bequests but finds that bequests do not fully account for wealth inequality, again, for the US economy. A recently-developed model by Sánchez-Romero et al. (2018) investigates the demographic determinants of wealth inequality, i.e., the effects of changes in fertility and life expectancy. The authors show that (i) fertility decline increases within-cohort wealth inequality but decreases across-cohort wealth inequality, and (ii) life expectancy increases have non-monotonic effects depending on the initial level of life expectancy. Several other papers relate wealth inequality with intergenerational links, including those of De Nardi (2004), Cagetti and De Nardi (2006), and—the last but not the least—Benhabib et al. (2011).

There also exists a demographic literature that associates fertility decline and the patterns of demographic transition with *intergenerational wealth flows* (Caldwell, 1976; Caldwell, 2005). Typically, in high fertility regimes, wealth flows move from children to parents in the form of child labor and/or (old age) insurance. However, in low fertility regimes, wealth is transferred from parents to children through investments in education and health and, more generally, as accidental or voluntary bequests. The model studied in this paper presumes an exogenous birth process (without education and health) where bequests are accidental.

The contribution of the present paper to the large literature on wealth inequality and its demographic aspects is to identify and estimate the structural parameters of a Blanchard-Yaari model with actual micro data on global top wealth owners.

The empirical literature focusing on billionaire samples or rich lists of various years studies mainly the shape of the wealth distributions. Early works on whether top wealth distributions have Pareto tails are centered on a few individual case study countries such as the US, the UK, Canada, China, and India. Ogwang's (2013) paper is the earliest work that investigates whether the global top wealth distribution is of Pareto family by using the Forbes data from 2000 to 2009. His results reject the existence of Pareto tails. Brzezinski (2014) documents that only around 35% of samples support the Pareto tails, and other distributions may be plausible from an empirical point of view. However, Capehart's (2014) results correct for the measurement errors originating from Forbes' practice of rounding and indicate that the existence of Pareto tails cannot be rejected. More recent studies including those of Vermeulen (2016, 2018), Schmidt (2017), and Cabla and Habarta (2019) also support the existence of Pareto tails. The present paper contributes to this line of literature by confirming the existence of Pareto tails for the wealth and relative wealth distributions of Forbes billionaires for the year 2018 *by using a theoretical model so that the value of the tail parameter is explained by* (*estimated*) *microeconomic foundations.*

Outline

The next section presents a descriptive analysis of age and wealth distributions of Forbes billionaires in the year 2018. The data used in that section is the main set of data for the empirical analysis presented in other sections of the paper. The section following the descriptive analyses introduces the model economy. The treatment in this theory section directly follows Jones (2014) but implements the analysis with an arguably better notation that uses for age and for wealth. After the presentation of the model economy, the testable implications of the theory are derived, one in the form of a nonlinear regression and the other in the form of a probability distribution. The section following the derivation of these testable implications presents the estimation results. The concluding section of the paper discusses the estimation results and shares some concluding remarks. A technical appendix presents a solution of the optimal control problem of wealth accumulation for the interested readers.

GLOBAL TOP WEALTH DISTRIBUTION

The purpose of this section is to provide a descriptive analysis of global top wealth distribution. As mentioned above, the data source for the wealth levels of the wealthiest people across the globe is the *Forbes Magazine*'s 2018 database of billionaires, also known as *Forbes billionaires*. Specifically, the *Forbes Magazine* issues a list of the people whose personal net worth is equal to or greater than 1 billion (current) US dollars in a specific point in time.

The Forbes billionaire database is a subscription-based source of data that comes with detailed information on the person's age, sex, country of residence, and main sector of activity. However, the dataset with limited content can still be obtained from the Forbes website through manual web scraping. The wealth levels in (current) US dollars and the ages of wealthiest individuals across the globe are obtained in this way from the Forbes website (Forbes, 2018).

Table 1 presents a detailed summary of the age and wealth distributions of Forbes billionaires in the year 2018. These statistics weakly suggest that the distribution of age is close to a Gaussian distribution, but the null of normality is rejected at a p value of 0.0007 for age. The distribution of wealth is highly skewed and has a very large level of kurtosis. The distribution of wealth in 2018 US dollars is also characterized with a very large standard deviation, implying a coefficient of variation that is around 170%.[2] The (joint) normality of wealth is rejected at a p value near zero.

Table 1: Age and Wealth of Forbes Billionaires in 2018

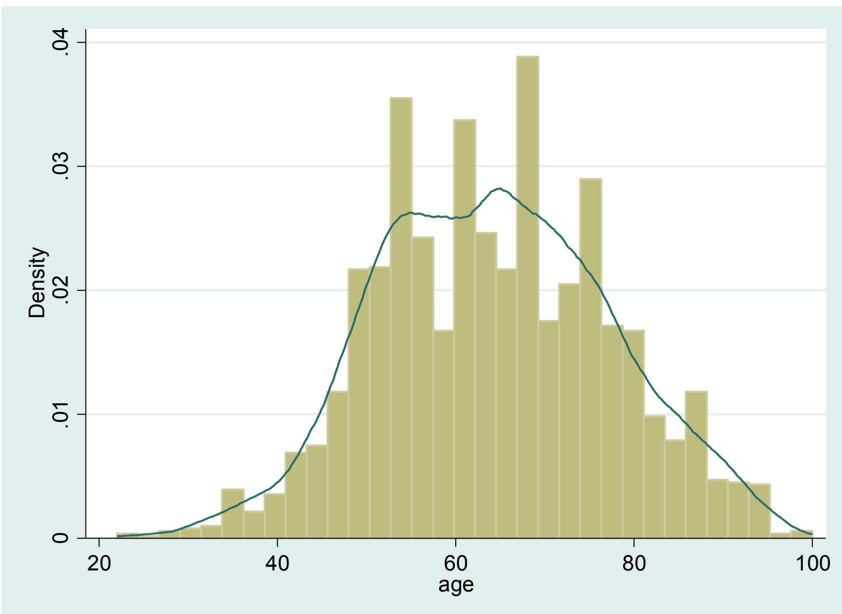
	Age	**Wealth**
	years	*2018 USD*
Mean	64.258	4.105
Standard Dev.	13.112	6.984
Skewness	0.045	7.156
Kurtosis	2.658	73.501
Null of Normality	$p = 0.0007$	$p \approx 0.0000$

Minimum		22	1.0
Maximum		100	112.0
Median		64	2.2
	Percentiles		
	5%	44	1.1
	25%	55	1.4
	75%	74	4.0
	95%	87	13.0
Sample size		2,145	2,145

Data Source: Forbes (2018).

The remainder of this section focuses on graphical analyses that give some information concerning the shapes of marginal and joint distributions of age and wealth of Forbes billionaires. Figure 1 shows the histogram and a kernel density estimate of age that uses an optimal bandwidth. The ages of a majority of Forbes billionaires are between 40 and 90 years. The histogram exhibits multiple modes that are visible between 50 and 80 years. The density estimate shows the existence of two modes around 52 and 64 years of age.

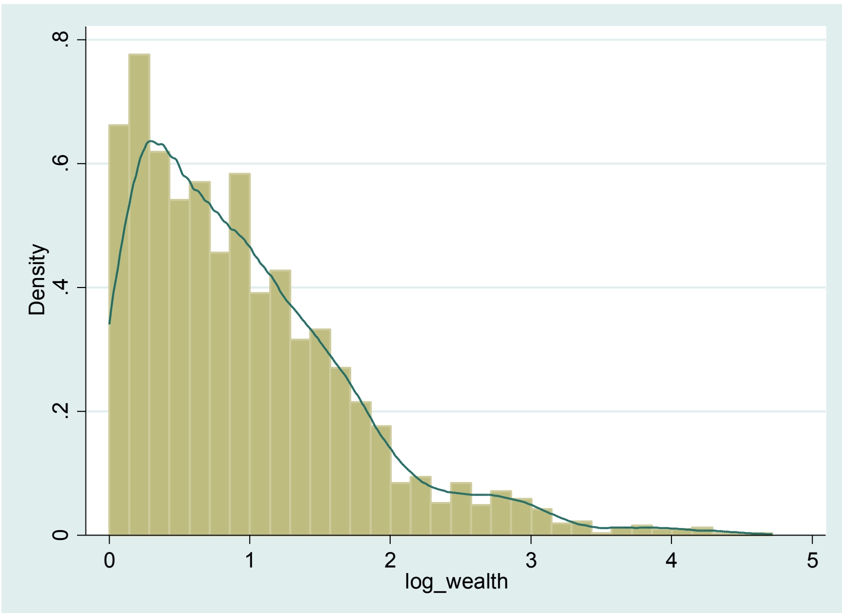
Figure 1: Age Distribution of Forbes Billionaires in 2018



Data Source: Forbes (2018).

Figure 2 shows the distribution of *the natural logarithm of wealth*—labeled log_wealth—for better readability of the graph given the extremely skewed distribution in absolute terms. The histogram has multiple modes as in the case of age distribution, and a vast majority of Forbes billionaires have wealth levels less than e^3 = 20 billion USD in 2018.

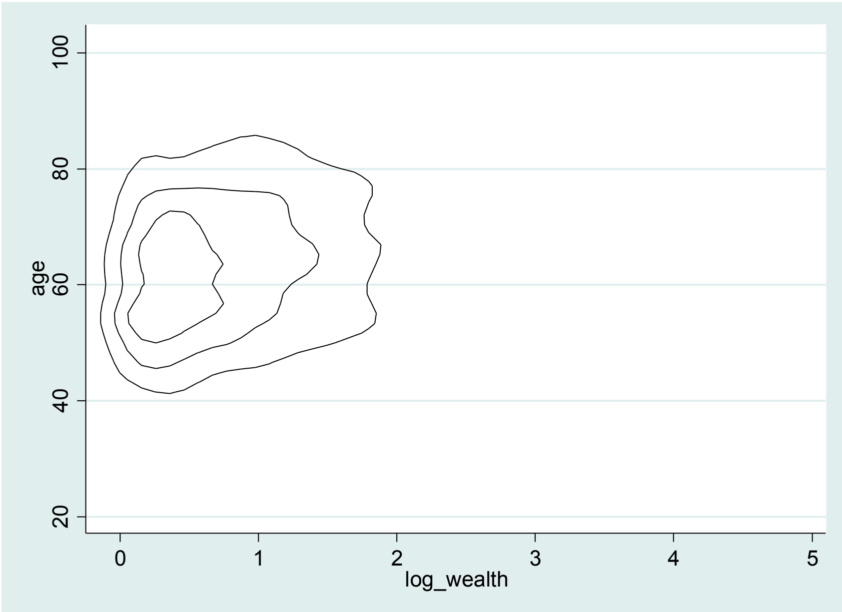
Figure 2: Wealth Distribution of Forbes Billionaires in 2018



Note: Wealth data is in natural logarithms. Data Source: Forbes (2018).

Finally, Figure 3 presents a bivariate kernel density estimate of age and log(wealth) for Forbes billionaires in the year 2018. This indicates that a large fraction of observations is concentrated at ages between 50 and 70 years and at wealth levels between 1.2 and 2.2 billion US dollars.

Figure 3: Joint Age-Wealth Distribution of Forbes Billionaires in 2018



Note: Wealth data is in natural logarithms. Data Source: Forbes (2018).

A SIMPLE THEORY OF AGE AND WEALTH

This section introduces the Blanchard-Yaari model of age and wealth by closely following Jones (2014). In this model, time is a continuous variable, and individuals that populate the economy exhibit heterogeneity with respect to their age and their wealth holdings. The fundamental assumptions of the Blanchard-Yaari model are the following:

1. The society has a stable age distribution with a constant, common death rate and a birth rate that is converging to a positive constant.
2. Individuals obtain utility only from consumption, and they are forward-looking. Hence, they have an incentive to take optimal saving decisions.
3. Individuals do not have altruistic preferences towards their children. That is, the welfare of children does not affect the parents' saving decisions.
4. The only source of income is the existing asset holdings; individuals deposit their wealth in a savings account that pays a fixed rate of return that is common and exogenously given.
5. The wealth of those dying at any time is given to the new members of the society.

6. The economy is on a balanced growth path such that the average stock of wealth exhibits exponential growth at a fixed rate.

Demographic Structure

Time starts at the initial point $t = 0$ and diverges to $+\infty$. Let $i \in [0, N_t]$ index the individuals in the economy at instant (or time) t. The demographic structure is described in an abstract way by a simple birth-death process without migration. This is basically the stable population theory, whose origins date back to Leonhard Euler and Alfred Lotka, and a recent formulation can be found in Tuljapurkar (2008). In the stable population theory, the birth and death rates—defined as total numbers of births and deaths relative to total population N_t, respectively—are fixed. Specifically,

$$B_t = B_0 \exp(\pi t) \tag{1}$$

denotes the total number of births at time t where $B_0 > 0$ and are $\pi \geq 0$ fixed numbers. Deaths, on the other hand, occur as a result of a Poisson process with a fixed arrival rate $\delta \in (0,1)$. Thus, at time t, a fraction δ of N_t individuals die. The total population N_t evolves as in

$$\mathrm{d}N_t = B_t \mathrm{d}t - \delta N_t \mathrm{d}t. \tag{2}$$

where for any variable x_t, dx_t denotes the total differential with respect to time. For such a birth-death process to return a stable age distribution with constant population growth, the birth rate B_t/N_t should converge to a constant $\beta > 0$ that satisfies

$$\beta = \pi + \delta. \tag{3}$$

Thus, in the long run, a fraction δ of births compensates for the deaths, and the remaining fraction causes the population to grow at the instantaneous rate $\pi \geq 0$. Jones (2014) also proves that the cross-section distribution of age in this population satisfies

$$\Pr[\text{Age} > a] = \exp(-\beta a). \tag{4}$$

Thus, the fraction of agents whose ages are strictly larger than a declines with a, and the pace of the decline naturally increases with the birth rate β.

Endowments and Preferences

Individual i owns a stock of wealth denoted by $w_{it}(a)$ at age a at time t. Individuals in this population do not work, and the only source of income for each individual is the (net) return on $w_{it}(a)$. Put differently, there is only capital income in this economy.

Individuals derive instantaneous utility from the consumption of a final good. Let c_{it}(a) denote the flow of consumption of individual *i* at time *t* when she is at age *a*. Notice that, for the individual, time *t* and age *a* are identical variables.

Suppose that, for simplicity, the instantaneous utility function is the natural logarithm. The individuals seek to maximize a discounted sum of utility flows over an infinite horizon. Formally, we have

$$U_{it}(0) = \int_0^{+\infty} \ln[c_{it}(a)] \exp[-(\rho + \delta)a] \mathrm{d}a \qquad (5)$$

where $\rho>0$ is the pure rate of discounting. Notice that the welfare is defined as if the individual lives indefinitely. In such models, under the assumption that the individual knows that the death process is Poisson with the arrival rate δ, the expected utility over a finite but stochastic lifetime is well-approximated by the infinite sum in (5). Naturally, the flow death rate δ causes the effective rate of discounting to be greater than ρ as individuals care less about the future utility flows under the risk of death. The lower limit "0" of the integral above signifies the age at which individual *i* enters the economy.

Optimal Wealth Accumulation

The remainder of the analysis focuses on a partial equilibrium reasoning where each individual accumulates her wealth by maximizing the lifetime utility $U_{it}(0)$ subject to the flow budget constraint

$$\mathrm{d}w_{it}(a) + c_{it}(a)\mathrm{d}a \leq (r - \tau)w_{it}(a)\mathrm{d}a \qquad (6)$$

where $w_{i0}(0)>0$ is exogenously given. Here, the left-hand side denotes the sum of saving dw_{it}(a) and consumption flow c_{it}(a)da. The right-hand side denotes the capital income flow $(r-\tau)w_{it}(a)da$. On the right-hand side, r ε (0,1) denotes the real interest rate, and τ ε (0,1) is the marginal capital income tax rate. Both are fixed numbers.

As shown in the Appendix A, this simple optimal control problem has a unique solution characterized by the (optimal) paths

$$c_{it}(a) = (\rho + \delta)w_{it}(a) \qquad (7)$$

$$\mathrm{d}w_{it}(a) = (r - \tau - \rho - \delta)w_{it}(a)\mathrm{d}a. \qquad (8)$$

Thus, each individual consumes a fixed fraction of her wealth as determined by the effective discount rate (ρ+δ) provided the wealth stock grows exponentially at the instantaneous rate (r−τ−ρ–δ) provided that this rate is strictly positive. Using (8), we can write the stock of wealth that will be held by an individual of age *a* at time *t* as in

$$w_{it}(a) = w_{it-a}(0)\exp[(r - \tau - \rho - \delta)a] \qquad (9)$$

where $w_{it-a}(0)=w_{i0}(0)>0$ denotes the initially inherited stock of wealth of individual *i* at age 0. In the remainder of the analysis, we can drop the subscript i without loss of generality since the individual heterogeneity is represented by age a. That is, individuals' identities are immaterial as their differences are fully represented by age and wealth differences.

"Intergenerational" Transfers, Growth, and the Distribution of Wealth

We are now ready to derive the cross-section distribution of wealth in this economy. The first task is to specify how the wealth of the dying individuals at time t is redistributed among the newborns, i.e., the new members of the society who would start accumulating wealthat time t.

Let W_t denote the total stock of wealth in the economy at time t, and suppose that each newborn individual has an equal share of the wealth stock that is being left by the dying individuals. Hence, we have

$$w_t(0) = \frac{\delta W_t}{(\pi+\delta)N_t} = \theta \bar{w}_t \qquad (10)$$

where $w_t(0)$ denotes the initial (a=0) stock of wealth for those entering the economy at time t, $\bar{w}_t = W_t/N_t$ is the average stock of wealth at time t, and $\theta=\delta/(\pi+\delta)\in(0,1)$ is a fixed ratio. Note that, if population growth rate is equal to zero ($\pi=0$), the newborns must be inheriting the entire stock of wealth of the dead ($\theta=1$). Hence, even if individuals' preferences are not altruistic, the newborns enter the club of billionaires with a positive level of wealth.

For the model to be consistent with the actual experience of the world economy, average stock of wealth should be growing as in

$$\bar{w}_t = \bar{w}_0 \exp(\gamma t) \qquad (11)$$

where $\gamma>0$ is the instantaneous rate of growth.

Note from (10) that, when an individual is born at time $(t-a)$, she is endowed with the initial wealth $\theta\bar{w}_{t-a}$. Hence , we have

$$w_{t-a}(0) = \theta \bar{w}_{t-a}. \qquad (12)$$

We also know from (11) that, at time $(t-a)$, the average stock of wealth can be rewritten as in

$$\bar{w}_{t-a} = \bar{w}_0 \exp(\gamma t - \gamma a) = \bar{w}_0 \exp(\gamma t)\exp(-\gamma a) = \bar{w}_t \exp(-\gamma a). \qquad (13)$$

Substituting $\bar{w}_{t-a}$ from (13) in (12) and, then, substituting $w_{t-a}(0)$ in (9) allow us to define the cross-section distribution of wealth as in

$$w_t(a) = \theta \bar{w}_t \exp[(r-\tau-\rho-\delta-\gamma)a] \qquad (14)$$

so that the wealth stock of an individual of age a at time t is expressed as a function of the average stock w_t of wealth at time t and of an exponential function of age a.

TESTABLE HYPOTHESES

The simple model introduced above has two implications that can be directly tested with the cross-section data on age and wealth. The first one originates directly from (14) where individual stock of wealth relative to the average is an exponential function of the individual's age. Rewriting (14) as a nonlinear cross-section regression of a sample *I={1,2,…,n}* of n individuals at some time t returns

$$s_i = \frac{w(a_i)}{\bar{w}} = \omega_0 \exp(\omega_1 a_i) + \varepsilon_i \qquad (15)$$

where si that denotes relative wealth is the dependent variable, (ω_0,ω_1) are regression coefficients uniquely identifying θ and (r−τ–ρ–δ–γ) respectively, and εi is a typical error term.

The other testable hypothesis is concerned with the shape of the wealth distribution in such an economy. Let $F_t(w)=Pr[w_t(a)>w]$ denote the counter-cumulative distribution function of wealth at time *t*. From (14), we simply have

$$F_t(w) = \Pr[\theta \bar{w}_t \exp[(r-\tau-\rho-\delta-\gamma)a] > w] \qquad (16)$$

which can be rewritten as in

$$F_t(w) = \Pr\left[a > \left(\frac{1}{r-\tau-\rho-\delta-\gamma}\right)\ln\left(\frac{w}{\theta \bar{w}_t}\right)\right] \qquad (17)$$

connecting the exponential age distribution in (4) to the wealth level *w*. Using (4) accordingly implies that the cross-section distribution of wealth is of Pareto as in

$$F_t(w) = \left(\frac{w}{\theta \bar{w}_t}\right)^{-\frac{\pi+\delta}{r-\tau-\rho-\delta-\gamma}}. \qquad (18)$$

A generalized Pareto distribution is thus estimated via Maximum Likelihood to investigate whether the cross-section distribution of wealth of Forbes billionaires in 2018 has a Pareto tail. The estimated counter-cumulative function is of the form

$$F(w) = \left[1 + \frac{\xi(w-w^{\min})}{\sigma}\right]^{-\frac{1}{\xi}} \qquad (19)$$

where $w^{min}=1$ is the threshold of 1 billion US dollars, η>0 is the scale parameter, and ξ>0 is the parameter that determines the shape of the distribution. Here, the parameter of interest is the shape parameter ξ that satisfies

$$\xi = \frac{r-\tau-\rho-\delta-\gamma}{\pi+\delta} \qquad (20)$$

from a structural point of view. The inverse of the shape parameter $\xi>0$ is known as the Pareto tail index and generally denoted by $\alpha=\xi^{-1}>0$. Lower levels of α and higher levels of ξ correspond to thicker tails of the Pareto distribution, meaning that larger shares of wealth are owned by the individuals located at the higher percentiles of the distribution. More specifically, if the distribution takes the form of the simple Pareto law with $F(Wealth>w)=\kappa\omega^{-\alpha}$ for some $\kappa>0$, then the top p-th percentile's wealth share can be approximated by $(p/100)^{(\alpha-1)/\alpha}$.

ESTIMATION RESULTS

Table 2 documents the structural estimation results for the nonlinear regression defined in (15). For comparison, it also shows the linear regression results when age and the square of age are taken as explanatory variables of relative wealth s_i.

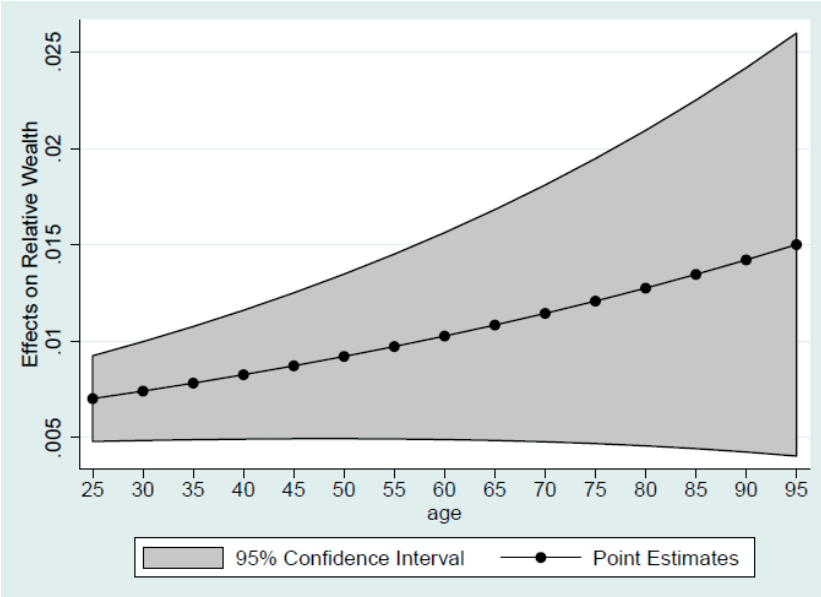
These results imply that the structural, nonlinear model in (15) clearly outperform the reduced-form model that assumes linearity. The uniquely identified parameter θ shows that, in the long run, new billionaires inherit nearly half of the average stock of wealth in the economy. On the other hand, the uniquely identified parameter $r-\tau-\rho-\delta-\gamma$ shows that the fixed growth rate of individual wealth in absolute terms, i.e., $r-\tau-\rho-\delta$, is larger than 1%. In fact, if the growth rate of the average stock of wealth is approximated by 1.5% or 2% annual economic growth rates of advanced capitalist economies, then individual wealth stocks should be growing at rates larger than 2.5% or 3% per annum. In terms of explanatory power, the structural estimates of (15) show that the demographic mechanism explains only around one fourth of variation in relative wealth with age.[3]

Table 2: Age as a Determinant of Relative Wealth

Structural Estimates		**Reduced-Form Estimates**		
$s_i = \omega_0 \exp(\omega_1 a_i) + \varepsilon_i$		$s_i = \psi_0 + \psi_1 a_i + \psi_2 a_i^2 + \varepsilon_i$		
	[1]		[2]	[3]
$\omega_0 = \theta$	0.49229*** (0.10005)	constant	0.33890* (0.20124)	1.44624* (0.78604)
$\omega_1 = r - \tau - \rho - \delta - \gamma$	0.01086*** (0.00305)	age	0.01028*** (0.00316)	–0.02549 (0.02448)
		age-squared		0.00027 (0.00018)
Observations	2,145	Observations	2,145	2,145
R-squared	0.26	R-squared	0.0058	0.0067
SSR	6,165.2	SSR	6,167.9	6,206.9
		F stat. *p value*	0.0012	0.0016

Notes: Robust standard errors are in parentheses. The superscripts *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively. SSR stands for the sum of squared residuals. The reported R-squared values are adjusted.

Figure 4: Marginal Effects of Age on Relative Wealth



Note : Results are based on the robust estimates of the exponential form presented in Table 2.

Figure 4 shows the marginal effects of age on relative wealth for five-year age intervals. Because of the exponential form, the marginal effects grow with age, implying that aging causes the older wealthy individuals to climb to the top of the wealth distribution faster. In terms of magnitudes, one year of aging leads to around 0.0075 increase in relative wealth at age 25. This corresponds to an absolute increase in individual wealth that is equal to 0.75% of the average stock of wealth. For an individual at age 95, the increase is equal to 0.015 units with a much wider confidence margin.

Table 3: Fitted Pareto Distributions of Wealth and Relative Wealth

Wealth (w_i)		**Relative Wealth** (s_i)	
$F(w) = \left[1 + \frac{\xi_w(w - w^{\min})}{\sigma_w}\right]^{-1/\xi_w}$		$F(s) = \left[1 + \frac{\xi_s(s - s^{\min})}{\sigma_s}\right]^{-1/\xi_s}$	
ξ_w	0.50169*** (0.03097)	ξ_s	0.55869*** (0.03305)
$\ln(\sigma_w)$	0.47769*** (0.03398)	$\ln(\sigma_s)$	−1.05339*** (0.03770)
Observations	2,060	Observations	2,145
log Likelihood	–4,077.5	log Likelihood	−1,083.8
$w^{\min}$	1 billion USD	$s^{\min}$	0.2435

Notes: Models are estimated with the pseudo-likelihood method. Robust standard errors are in parentheses. The superscripts *, **, and *** denote statistical significance at 10%, 5%, and 1%, respectively.

Table 3 documents the Maximum Likelihood estimates of the cross-section wealth distribution specified in (19). It also documents the results of the analysis for relative wealth since (16) implies that the cross-section distribution of relative wealth s_i should also be of Pareto with the same tail parameter $\xi_w=\xi_s=\xi$ but with different location and scale parameters.

Results indicate that both the wealth and the relative wealth distributions in the year 2018 have Pareto tails with statistically significant shape and scale parameters. For wealth, the tail index is equal to $\alpha^w=1/\xi_w=1.993$. This roughly indicates that the top 1-percentile wealth share is around 10%, the top 10-percentile wealth share is around 30%, and the top 50-percentile wealth share is around 70%. Hence, it is fair to conclude that the 2018 distribution of global top wealth levels is itself highly unequal. For relative wealth, the tail index is estimated to be equal to $\alpha^s=1/\xi_s=1.789$.

A simple t test value of 1.26 cannot reject the null hypothesis of H_0: $\xi_w=\xi_s$, i.e., the equality of the shape parameters of wealth and relative wealth distributions. The failure to reject this cross-equation restriction supports the internal validity of the theoretical model. Put differently, the shape parameters must not be statistically significantly different from each other since the relative wealth distribution originates from the wealth distribution defined in (16) where the average stock of wealth is taken as given.

DISCUSSION AND CONCLUSION

Results presented above show that the simple age-wealth model is an accurate representation of observed age-wealth profiles of Forbes billionaires in the year 2018. Relative wealth, defined simply as the ratio of individual to average wealth within the "universe" of the Forbes billionaires, increases exponentially with age along the life-cycle. The estimated values of structural parameters also indicate that (i) the new members enter the billionaire club by inheriting nearly half of the average wealth in any particular time, and (ii) the growth rate of wealth in absolute terms is about one percentage point larger than the growth rate of the "global" economy of billionaires.

Results concerning the shape of the wealth and relative wealth distributions clearly confirm the existence of Pareto tails. The estimated tail indexes indicate high degrees of wealth inequality among the global billionaires themselves.

While the estimated structural parameters are statistically significant, a roughly three fourth of the variation in relative wealth is left unexplained. Other factors should account for some part of the observed differences in relative wealth. These include the sex, the main sector of business activity, and the country of residence of the billionaires. Thus, future work may exploit the variation in such factors with a larger set of data.

Future work may also benefit from establishing a longitudinal dataset of Forbes billionaires for several different years. Such an extended dataset would be quite informative for a structural econometric analysis of the transition to the long-run equilibrium of the model. It would not be necessary to assume that demographic and economic parameters are constant, and researchers would identify the time-variant cohort effects that partially describe how the global top wealth distribution evolves in time. Surely, the longitudinal analysis would also allow the identification of entering and exiting members of the distribution for each year, and let researchers investigate billionaire-specific qualifications that determine the probability of entrance and exit. The design of optimal taxation and redistribution policies would benefit from structural estimates of gross and net returns that originate from such a set of micro data on the wealthiest people in the planet.

NOTES

1. Jones (2015) uses this continuous-time version of the model to study the top income and top wealth inequality dynamics in growing economies.
2. The coefficient of variation is defined as the ratio of the (sample) standard deviation of a random variable to the absolute value of its (sample) mean. It is a percentage measure of (sample) dispersion.
3. For comparison purposes, the same exponential form has been estimated with the square of age as an explanatory variable (instead of age). Results omitted here for space considerations do not differ much in terms of statistical significance and explanatory power.

APPENDIX A: THE OPTIMAL CONTROL PROBLEM

This appendix solves the optimal wealth accumulation problem described in the main text. First note that any path that satisfies the first-order necessary conditions (FONCs) will be the optimal path since the instantaneous utility function $ln[c(a)]$ is strictly concave and the budget constraint is concave.

The current-value Hamiltonian function associated with the problem reads

$$\mathcal{H} = \ln[c(a)] + q(a)[(r-\tau)w(a) - c(a)] \qquad \text{(A.1)}$$

where the effective discount rate is $(\rho+\delta)>0$. The variable $q(a)$ is the shadow price of wealth and satisfies $q(a)\geq 0$. Since $c(a)=0$ will not be optimal for any a given $\lim_{c\to 0}\ln(c)=-\infty$, the FONCs to be satisfied along the optimal path are simply

$$\frac{1}{c(a)} = q(a) \qquad \text{(A.2)}$$

$$\mathrm{d}q(a) - (\rho+\delta)q(a)\mathrm{d}a = -(r-\tau)q(a)\mathrm{d}a \qquad \text{(A.3)}$$

$$\mathrm{d}w(a) = (r-\tau)w(a)\mathrm{d}a - c(a)\mathrm{d}a \qquad \text{(A.4)}$$

$$\lim_{a\to+\infty} q(a)\exp[-(\rho+\delta)a]w(a) = 0. \qquad \text{(A.5)}$$

Note that we must now be searching for a solution candidate—a path—that satisfies (A.2)-(A.5). Denoting the instantaneous growth rate of any variable with $(dx/da)/x$, the FONC in (A.3) implies that $q(a)$ decreases at the instantaneous rate $(r-\tau-\rho-\delta)$:

$$\mathrm{d}q(a) = -[(r-\tau) - (\rho+\delta)]q(a)\mathrm{d}a. \qquad \text{(A.6)}$$

The FONC in (A.4) on the other hand implies that, for $w(a)$ to grow at a constant rate, $c(a)$ should be growing at the same rate; the right-hand side of (A.7) below would be constant if the left-hand side is constant:

$$\frac{\mathrm{d}w(a)/\mathrm{d}a}{w(a)} = (r-\tau) - \frac{c(a)}{w(a)}. \qquad \text{(A.7)}$$

But we know from the FONC in (A.2) that $c(a)$ will grow exactly at the same rate that $q(a)$ decreases. Hence, we have

$$\frac{\mathrm{d}w(a)/\mathrm{d}a}{w(a)} = \frac{\mathrm{d}c(a)/\mathrm{d}a}{c(a)} = r-\tau-\rho-\delta = (r-\tau) - \frac{c(a)}{w(a)}. \qquad \text{(A.8)}$$

The last equality in (A.8) implies that, for all a, the consumption-to-wealth ratio must satisfy

$$\frac{c(a)}{w(a)} = \rho+\delta. \qquad \text{(A.9)}$$

Along a path that satisfies (A.9), the FONC in (A.5) is fulfilled since $q(a)w(a)$ is a constant at all a and $\exp[-(\rho+\delta)a]$ converges to nil.

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