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Investigation of Nonlinear Wave Solutions for Fusion and Fission Phenomenas

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Abstract: In this study, wave solutions of the (3+1) dimensional Jimbo-Miwa equation and two different phenomena of the solution, fusion and fission, are obtained using the modified exponential function method. In order to get more possible solutions, two different cases are investigated due to the nature of the modified exponential function method. When the resulting solutions are analyzed, trigonometric, hyperbolic and rational functions are obtained. It was checked whether the solution functions found by the Wolfram Mathematica software provided the (3+1) dimensional potential Jimbo-Miwa equation. Two and three dimensional graphs, contour and density graphs of the solution function were get by determining the appropriate parameters.

Füzyon ve Fizyon Fenomenleri İçin Doğrusal Olmayan Dalga Çözümlerinin İncelenmesi

Anahtar Kelimeler
Geliştirilmiş Üstel Fonksiyon Metodu, (3+1)-boyutlu Jimbo-Miwa denklemi, Dalga çözümleri

Öz: Bu çalışmada, (3+1) boyutlu Jimbo-Miwa denkleminin dalga çözümleri ve buna bağlı olarak da çözümün füzyon ve fisyon olmak üzere iki farklı olgusu modifiye üstel fonksiyon yöntemi kullanılarak elde edilmiştir. Daha olası çözümler elde etmek için modifiye edilmiş üstel fonksiyon yönteminin doğası gereği iki farklı durum incelenmiştir. Ortaya çıkan çözümler incelendiğinde trigonometrik, hiperbolik ve rasyonel fonksiyonlar elde edilmiştir. Wolfram Mathematica yazılımı tarafından bulunan çözüm fonksiyonlarının (3+1) boyutlu potansiyel Jimbo-Miwa denklemini sağlayıp sağlamadığı kontrol edildi. Uygun parametreler belirlenerek çözüm fonksiyonunun iki ve üç boyutlu grafikleri, kontur ve hassasiyet grafikleri elde edildi.

1. INTRODUCTION

Physics, engineering, health, etc. all events encountered in natural and applied sciences are represented by mathematical models. These types of models are stated in nonlinear partial differential equations. Therefore, it is of great importance to obtain the solutions of such equations. In scientific studies in the literature, there are various methods to investigate the solutions of such equations. Some of these methods in the literature; generalized tanh function method [1], the modified extended tanh-function method [2-3], the generalized Bernoulli sub-equation function method [4-6], The trial equation method [7-11], the first integral method [12], the modified exponential function method [13-17] and many more methods.

In this study, we consider the (3+1) dimensional Jimbo-Miwa equation given following [18-24],

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0. \qquad (1)$$

Equation (1) given as a mathematical model has been encountered in fusion and fission phenomena which is a subject of nuclear physics. In the literature, there are results obtained by using various methods regarding the solutions of equation (1) [18-24].

In the second part of this study, the modified exponential function method is introduced. To apply this method PDEs has been reduced to ordinary diferantial equations (ODEs). In the third chapter, the solutions obtained by applying the determined method (3+1)-dimensional Jimbo-Miwa equation and two- and three-dimensional graphs, contour graphs and density graphs of these results are presented for two cases. In the last part, the conclusion part is given.

2. ANALYSIS OF MODIFIED EXPONENTIAL FUNCTION METHOD

Let us consider the general form of the nonlinear partial differential equation for the modified exponential function method as follows;

$$P\left(U,U_x,U_y,U_z,U_t,U_{xx},U_{xt},U_{yy},U_{xxxz},\cdots\right)=0, \quad (2)$$

where $U=U(x,y,z,t)$ is unknown function.

Step 1. Taking the independent variables given in equation (1) into consideration, the wave transformation given below is considered,

$$U(x,y,z,t)=U(\xi),\xi=k\left(x+y+z-ct\right). \quad (3)$$

The terms k and c given here are constants in walking wave transformation. If the expression (3) is substituted by obtaining the derivative expressions in the equation (2) given as the general form of equation (1) by using the wave transformation,

$$N\left(U,U',(U')^2,U'',U''',\cdots\right)=0, \quad (4)$$

the general form of the nonlinear ordinary differential equation is gotten.

Step 2: According to this method, the solution function of equation (1) is as follows.

$$U(\xi)=\frac{\sum_{i=0}^{n}A_i\left[\exp\left(-\Omega(\xi)\right)\right]^i}{\sum_{j=0}^{m}B_j\left[\exp\left(-\Omega(\xi)\right)\right]^j}= \quad (5)$$
$$=\frac{A_0+A_1\exp\left(-\Omega\right)+\cdots+A_n\exp\left(n\left(-\Omega\right)\right)}{B_0+B_1\exp\left(-\Omega\right)+\cdots+B_m\exp\left(m\left(-\Omega\right)\right)},$$

where $A_i,B_j,\left(0\le i\le n,0\le j\le m\right)$ are constants. The balancing procedure is applied to the nonlinear ordinary differential equation (4) obtained by applying the wave transformation. In other words, by equating the term with the highest order derivative in the equation (4) and the nonlinear term, the relation between m and n is obtained. Equality is found by giving value to the constants in this relation. Thus, the upper limits of the sum symbols in equation (5) are determined.

$$\Omega'(\xi)=\exp\left(-\Omega(\xi)\right)+\mu\exp\left(\Omega(\xi)\right)+\lambda. \quad (6)$$

The omega function given below is a function used as the power of the exponential function in the solution function. When we solve the Eq. (6), the following families are obtained by He et al [13]:

Family 1: When $\mu\ne 0,\ \lambda^2-4\mu>0,$

$$\Omega(\xi)=\ln\left(\frac{-\sqrt{\lambda^2-4\mu}}{2\mu}\tanh\left(\frac{\sqrt{\lambda^2-4\mu}}{2}(\xi+E)\right)-\frac{\lambda}{2\mu}\right). \quad (7)$$

Family 2: When $\mu\ne 0,\ \lambda^2-4\mu<0,$

$$\Omega(\xi)=\ln\left(\frac{\sqrt{-\lambda^2+4\mu}}{2\mu}\tan\left(\frac{\sqrt{-\lambda^2+4\mu}}{2}(\xi+E)\right)-\frac{\lambda}{2\mu}\right). \quad (8)$$

Family 3: When $\mu=0,\lambda\ne 0$ and $\lambda^2-4\mu>0,$

$$\Omega(\xi)=-\ln\left(\frac{\lambda}{\exp\left(\lambda(\xi+E)\right)-1}\right). \quad (9)$$

Family 4: When $\mu\ne 0,\lambda\ne 0$ and $\lambda^2-4\mu=0,$

$$\Omega(\xi)=\ln\left(-\frac{2\lambda(\xi+E)+4}{\lambda^2(\xi+E)}\right). \quad (10)$$

Family 5: When $\mu=0,\ \lambda=0$ and $\lambda^2-4\mu=0,$

$$\Omega(\xi)=\ln\left(\xi+E\right). \quad (11)$$

where $A_0,A_1,\ldots,A_n,B_0,B_1,\ldots,B_m,E,\lambda,\mu$ are constants.

Step 3: After equation (6) is solved, when the equation (5) is written in its place, an algebraic equation system consisting of coefficients is get. When this system of equations is solved with the help of the Mathematica program, the travelling wave solutions that provide the equation (1) are obtained.

3. APPLICATION

Using the traveling wave transformation (3) for equation (1), the following nonlinear ordinary differential equation is obtained,

$$k^2U''' + 3k\left(U'\right)^2 - (2c+3)U' = 0. \quad (12)$$

If $U' = V$ transform is applied in equation (12), we obtain,

$$k^2V'' + 3kV^2 - (2c+3)V = 0. \quad (13)$$

In equation (13), if the equalization term is applied between V'' and V^2 according to the balance procedure,

$$M + 2 = N. \quad (14)$$

If $M = 1$ so as to satisfy the equation (14), $N = 3$ is obtained. In this case, the upper limits of the total symbols in the sought solution function in equation (5) are determined. Accordingly, the terms required in the equation (15) are given below.

$$V(\xi) = \frac{\psi}{\varphi} = \frac{A_0 + A_1e^{-\Omega(\xi)} + A_2e^{-2\Omega(\xi)} + A_3e^{-3\Omega(\xi)}}{B_0 + B_1e^{-\Omega(\xi)}},$$

$$V'(\xi) = \frac{\psi'\varphi - \psi\varphi'}{\varphi^2}, \quad (15)$$

$$V''(\xi) = \frac{\psi''\varphi^3 - \varphi^2\psi'\varphi' - (\psi\varphi'' + \psi'\varphi')\varphi^2 + 2(\psi')^2\psi\varphi}{\varphi^4}.$$

Case 1:

$$A_0 = -\frac{1}{3}k(\lambda^2 + 2\,\mu)B_0,$$

$$A_1 = -\frac{1}{3}k(6\,\lambda B_0 + (\lambda^2 + 2\mu)B_1),$$

$$A_2 = -2\,k\,(B_0 + \lambda B_1),\; A_3 = -2\,k\,B_1,$$

$$c = \frac{1}{2}\left(k^2\left(-\left(\lambda^2 - 4\mu\right)\right) - 3\right).$$

By using the obtained coefficients, the traveling wave solutions of equation (1) have been analyzed considering the following family cases.

Family-1:

$$V_{1,1}(\xi) = -\frac{k\left(\lambda^2 - 4\mu\right)sech^2\left(\frac{1}{2}\phi\right)\left(\lambda\beta + \alpha - 4\mu\right)}{3\left(\sqrt{\lambda^2 - 4\mu}\,tanh\left(\frac{1}{2}\phi\right) + \lambda\right)^2}. \quad (16)$$

Where $\alpha = \left(\lambda^2 - 2\mu\right)Cosh[\phi]$, $\beta = \sqrt{\lambda^2 - 4\mu}Sinh[\phi]$, $\phi = \sqrt{\lambda^2 - 4\mu}\left(EE + \xi\right)$.

Integrate ξ on both sides of the equation $U' = V$,

$$U_{1,1}(\xi) = \frac{1}{3}k\left(\lambda^2 - 4\mu\right)\left(\frac{3\lambda}{2\mu cosh(\phi) + \lambda^2 - 2\mu} - EE - \xi\right) + \frac{2k\mu\beta}{2\mu cosh(\phi) + \lambda^2 - 2\mu}. \quad (17)$$

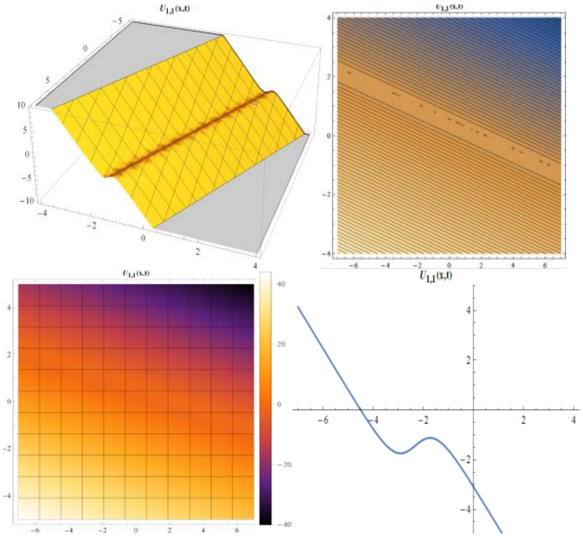

Figure 1. The three dimensional graph, contour graph, density graphs of Eq.(17) for the values $c = -4, A_3 = 2, B_1 = 1, \lambda = 3, \mu = 1, y = -1, z = 1, k = -1, \mathrm{EE} = 0.82$ and two-dimensional graph for $t = 1$.

Family 2:

$$V_{1,2}(\xi) = \frac{k\left(\lambda^2 - 4\mu\right)sec^2\left(\frac{1}{2}\tau\right)\left(\varsigma - \omega + 4\mu\right)}{3\left(\lambda - \sqrt{4\mu - \lambda^2}\,tan\left(\frac{1}{2}\tau\right)\right)^2}. \quad (18)$$

Where $\varsigma = \lambda\sqrt{-\lambda^2 + 4\mu}Sin[\tau], \omega = \left(\lambda^2 - 2\mu\right)Cos[\tau]$, $\tau = \sqrt{-\lambda^2 + 4\mu}\left(EE + \xi\right)$.

$$U_{1,2}(\xi)=\frac{1}{3}k\left(\lambda^2-4\mu\right)\left(\frac{3\lambda}{2\mu cos\left(\tau\right)+\lambda^2-2\mu}-EE-\xi\right)-\frac{2k\mu\sqrt{4\mu-\lambda^2}sin\left(\tau\right)}{2\mu cos\left(\tau\right)+\lambda^2-2\mu}. \tag{19}$$

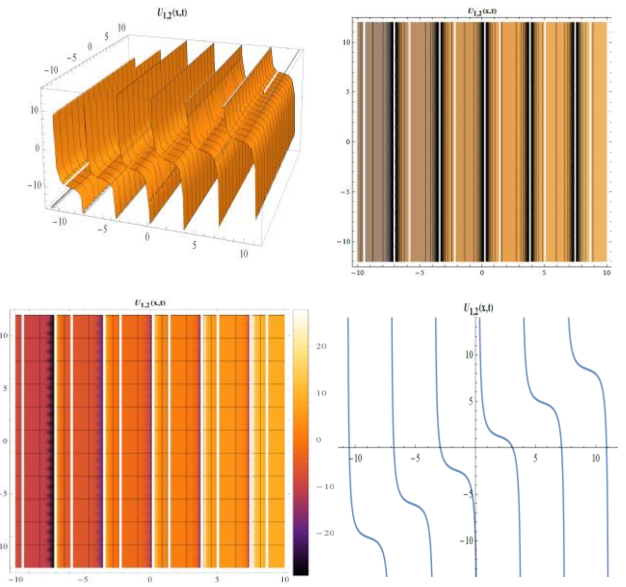

Figure 2. The three dimensional graph, contour graph, density graphs of Eq.(19) for the values $c=0, A_3=2, B_1=1, \lambda=1, \mu=1, y=-0.01, z=0.01, k=-1, \mathrm{EE}=0.82$ and two-dimensional graph for $t=1$.

Family 3:

$$V_{1,3}(\xi)=-\frac{1}{6}k\lambda^2(cosh(\lambda(EE+\xi))+2)\,csch^2\left(\frac{1}{2}\lambda(EE+\xi)\right). \tag{20}$$

Integrating equation (20) according to ξ,

$$U_{1,3}(\xi)=k\lambda coth\left(\frac{1}{2}\lambda(EE+\xi)\right)-\frac{1}{3}k\lambda^2(EE+\xi). \tag{21}$$

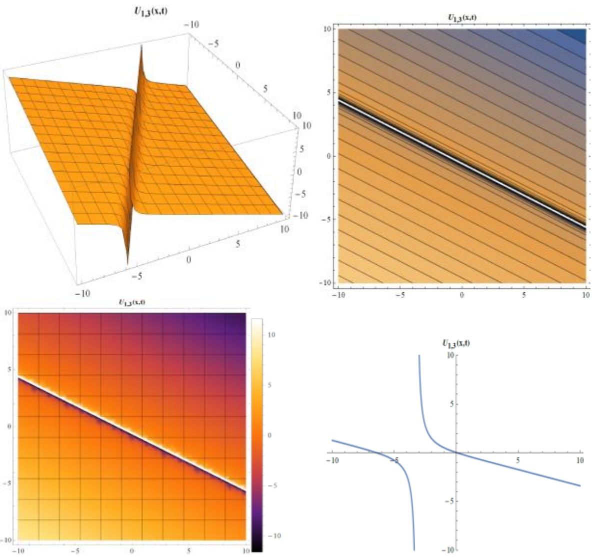

Figure 3. The three dimensional graph, contour graph, density graphs of Eq.(21) for the values $c=-2, \lambda=1, \mu=0, y=1, z=1, k=-1, \mathrm{EE}=0.82$ and two-dimensional graph for $t=1$.

Family 4:

$$V_{1,4}(\xi)=\frac{1}{6}k\left(\lambda^2\left(1-\frac{12}{(\lambda(EE+\xi)+2)^2}\right)-4\mu\right). \tag{22}$$

Integrating equation (22) according to ξ, we get,

$$U_{1,4}(\xi)=\frac{1}{6}k\left(\lambda(\lambda(EE+\xi)+2)+\frac{12\lambda}{\lambda(EE+\xi)+2}-4\mu\xi\right). \tag{23}$$

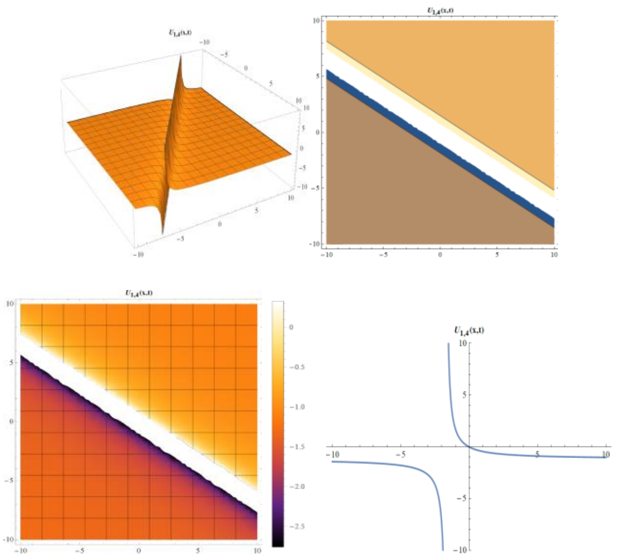

Figure 4. The three dimensional graph, contour graph, density graphs of Eq.(23) for the values $c=-1.5, \lambda=2, \mu=1, y=1, z=1, k=-1, \mathrm{EE}=0.82$ and two-dimensional graph for $t=1$.

Family 5:

$$V_{1,5}(\xi)=-\frac{2k}{(EE+\xi)^2}. \tag{24}$$

Integrating equation (24) according to ξ, we obtain,

$$U_{1,5}(\xi)=\frac{2k}{EE+\xi}. \tag{25}$$

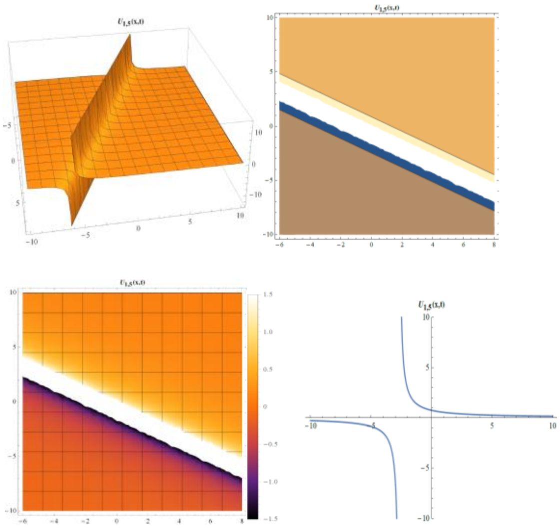

Figure 5. The three dimensional graph, contour graph, density graphs of Eq.(25) for the values $c=-1.5, \lambda=0, \mu=0, y=1, z=1, k=-1$, $EE=0.82$ and two-dimensional graph for $t=1$.

Case 2:

$$A_0=-\frac{B_0\left(2c+3k^2\lambda^2+3\right)}{6k},$$

$$A_1=-\frac{B_1\left(2c+3k^2\lambda^2+3\right)}{6k}-2B_0k\lambda,$$

$$A_2=-2k\left(B_1\lambda+B_0\right),\quad A_3=-2B_1k,$$

$$\mu=\frac{2c+k^2\lambda^2+3}{4k^2}.$$

By using the obtained coefficients, the traveling wave solutions of the equation (1) are analyzed by considering the following family cases.

Family-1:

$$V_{2,1}(\xi)=-\frac{2c+3}{6k}-\frac{k\left(\lambda\beta+\lambda^2-4\mu\right)^2}{2\left(\beta+\lambda\right)^2}. \tag{26}$$

Where $\beta=\sqrt{\lambda^2-4\mu}\alpha$, $\alpha=tanh\left(\frac{1}{2}\phi\right)$,

$\phi=\sqrt{\lambda^2-4\mu}\left(EE+\xi\right)$.

Integrate ξ on both sides of the equation $U'=V$,

$$U_{2,1}(\xi)=-\frac{\begin{array}{c}\beta\left(2(2c+3)\mu\xi+3\lambda k^2\left(\lambda^2-8\mu\right)\right)+\\ 2(2c+3)\lambda\mu\xi++12k^2\mu tanh^{-1}\left(\alpha\right)\\ \left(\sqrt{\lambda^2-4\mu}(\beta+\lambda)\right)+3k^2\left(\lambda^2-4\mu\right)^2\end{array}}{12k\mu\left(\beta+\lambda\right)}. \tag{27}$$

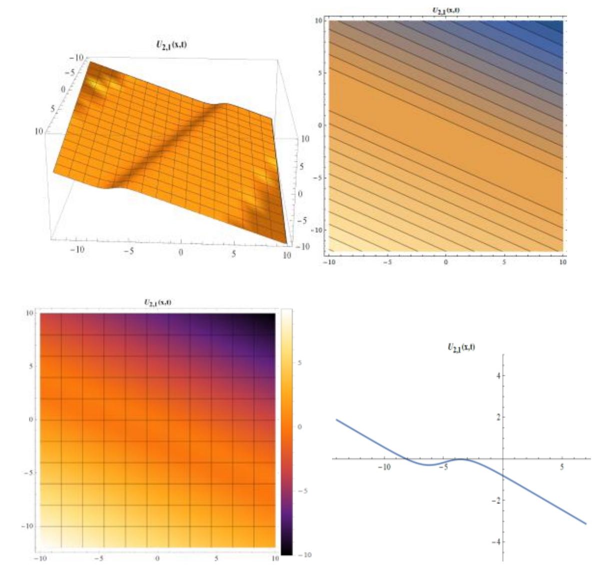

Figure 6. The three dimensional graph, contour graph, density graphs of Eq.(27) for the values $c=-2, \lambda=2, \mu=\frac{15}{16}, y=1, z=1, k=2$, $EE=0.75$ and two-dimensional graph for $t=1$.

Family 2:

$$V_{2,2}(\xi)=-\frac{2c+3}{6k}-\frac{k\left(-\lambda\varsigma+\lambda^2-4\mu\right)^2}{2\left(\lambda-\varsigma\right)^2}. \tag{28}$$

Where $\varsigma=\sqrt{\lambda^2-4\mu}tanh\left(\frac{1}{2}\tau\right)$, $\omega=\left(2\mu\right)Cos\left[\tau\right]$,

$\tau=\sqrt{-\lambda^2+4\mu}\left(EE+\xi\right)$.

$$U_{2,2}(\xi)=-\frac{(2c+3)\xi}{6k}-\frac{1}{2}k(EE+\xi)\left(\lambda^2-4\mu\right)+\frac{k\lambda\left(\lambda^2-4\mu\right)}{\omega+\lambda^2-2\mu}-\frac{2k\mu\sqrt{4\mu-\lambda^2}sin\left(\tau\right)}{\omega+\lambda^2-2\mu}. \tag{29}$$

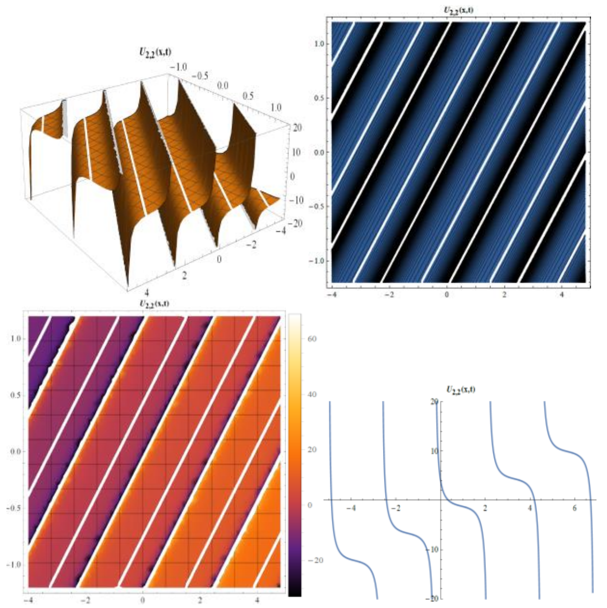

Figure 7. The three dimensional graph, contour graph, density graphs of Eq.(29) for the values $c = 2, \lambda = 1, \mu = \frac{11}{16}, y = -1, z = 1, k = 2, \mathrm{EE} = 0.75$ and two-dimensional graph for $t = 1$.

Family 3:

$$V_{2,3}(\xi) = \frac{2c + 3k^2\lambda^2 \coth^2\left(\frac{1}{2}\lambda(EE+\xi)\right) + 3}{6k}. \quad (30)$$

Integrating equation (30) according to ξ,

$$U_{2,3}(\xi) = k\lambda\left(\coth\left(\frac{1}{2}\lambda(EE+\xi)\right) - \tanh^{-1}\left(\tanh\left(\frac{1}{2}\lambda(EE+\xi)\right)\right)\right) - \frac{(2c+3)\xi}{6k}. \quad (31)$$

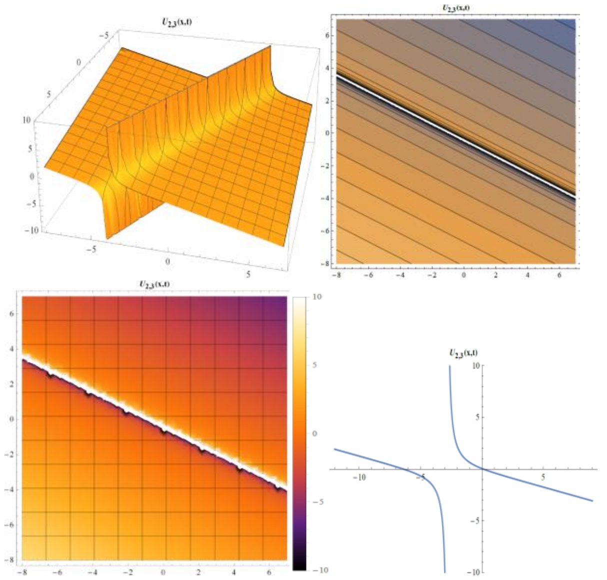

Figure 8. The three dimensional graph, contour graph, density graphs of Eq.(31) for the values $c = -2, \lambda = 1, \mu = 0, y = -1, z = 1, k = 1, \mathrm{EE} = 0.85$ and two-dimensional graph for $t = 1$.

Family 4:

$$V_{2,4}(\xi) = -\frac{2c+3}{6k} - \frac{2k\lambda^2}{(\lambda(EE+\xi)+2)^2}. \quad (32)$$

Integrating equation (32) according to ξ, we get,

$$U_{2,4}(\xi) = \frac{2k\lambda}{\lambda(EE+\xi)+2} - \frac{(2c+3)\xi}{6k}. \quad (33)$$

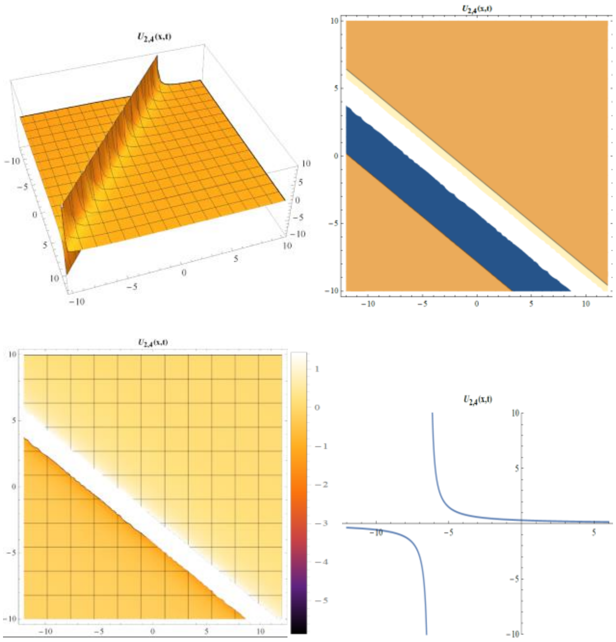

Figure 9. The three dimensional graph, contour graph, density graphs of Eq.(33) for the values $c = -1.5, \lambda = 1, \mu = 0.25, y = 1, z = 1, k = 1, \mathrm{EE} = 0.82$ and two-dimensional graph for $t = 1$.

Family 5:

$$V_{2,5}(\xi) = -\frac{2c+3}{6k} - \frac{2k}{(EE+\xi)^2}. \quad (34)$$

Integrating equation (34) according to ξ, we obtain,

$$U_{2,5}(\xi) = \frac{2k}{EE+\xi} - \frac{(2c+3)\xi}{6k}. \quad (35)$$

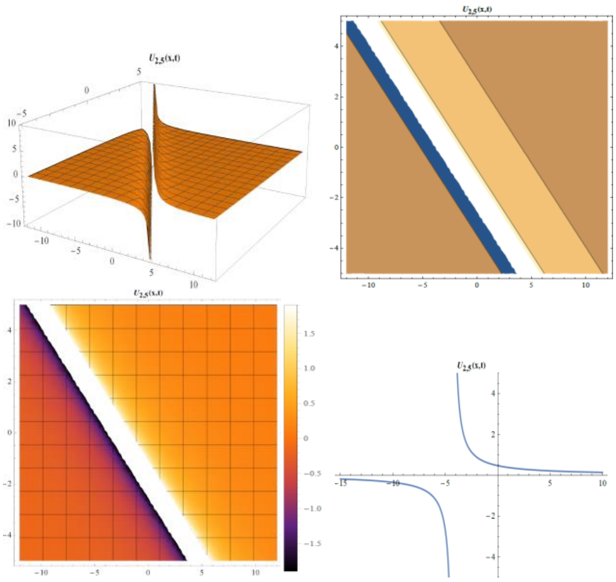

Figure 10. The three dimensional graph, contour graph, density graphs of Eq.(35) for the values $c = -1.5, \lambda = 0, \mu = 0, y = 1, z = 1, k = 1, \text{EE} = 0.82$ and two-dimensional graph for $t=1$.

4. CONCLUSION

In this article, MEFM was applied to the (3+1)-dimensional Jimbo-Miva equation considered as a nonlinear mathematical model. In this research, it was seen that the analytical solutions found under the conditions obtained according to the method provided the equation (1). When the analytical solution functions found are analyzed, it is seen that there are hyperbolic and trigonometric functions with periodic functions and rational functions. In particular, obtaining periodic functions is an advantage because such solution functions allow interpretation for an infinite range thanks to the understanding within a specific range of the behaviors represented by the mathematical model. All calculations and graphics were made using Mathematica software program. It has been observed that two and three dimensional graphs obtained by determining the appropriate parameters are suitable for the physical behavior of wave solutions. In addition, contour graphics and density graphics were found with the help of software program for wave solutions. Using this method, if more cases are investigated and different coefficient values are taken, different traveling wave solutions can be get. When the obtained solutions are analyzed, it can be stated that MEFM is effective and reliable in revealing analytical solutions of nonlinear partial differential equations. The obtaining results can help us learn more about the fusion and fission events.

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