

## FUSED AND ADAPTIVE MODIFIED WAVE ESTIMATORS FOR NAVIGATION SYSTEMS

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### ABSTRACT

Navigation systems have vital importance both in military and civil applications. However, the error associated with these systems reduces the efficiency of the navigation system. Kalman filter is a tool that is commonly used to address this problem. Recently, the modified wave estimator (MWE) has been proposed as an alternative to Kalman filter for navigation systems. Unlike the Kalman filter, the MWE defines the process noise as deterministic means. Both estimators have advantages and disadvantages with respect to each other. This paper proposes an adaptive MWE and an estimator that fuses estimations from the MWE and Kalman filter. The proposed estimators try to make use of advantages of each estimator in the best way possible. Performance of the MWE, Kalman filter, the fused estimator and adaptive MWE are compared through a navigation simulation and results are discussed.

**KEYWORDS:** Navigation, Data Fusion, Kalman, MWE.

### INTRODUCTION

Navigation is the calculation of relative position and velocity of a physical platform with respect to a reference coordinate frame or a coordinate grid [3] and navigation systems have vital importance in military and civil applications. They are employed to provide coordination and to increase efficiency at military and civil air, maritime or land vehicles. But, many applications require calculation of the angular velocity relative to the reference axis as well.

Error produced by navigation systems reduces the efficiency of the system. Thus, minimizing the error associated with navigation systems is a primary priority. The most commonly used and probably the most efficient method for this purpose is to

integrate the systems with complementary features, such as INS and GPS. Kalman filter is the most commonly used integration tool in navigation applications. However, recently, new approaches have been proposed, one of which is the Modified Wave Estimator [5]. In this approach, unlike Kalman filtering, the process noise is assumed deterministic.

In this paper, Kalman filter and the MWE are discussed as integration estimators in navigation systems. Also, a novel estimator that fuses estimations from Kalman filter and the MWE is proposed. The proposed estimator attempts to exploit the advantages of both estimators while trying to reduce the errors associated with navigation systems. Moreover, a novel adaptive structure for the MWE has also been proposed. The proposed adaptive approach takes the cycle time dependent structure of the MWE and requirements of navigation systems into account and tries to improve the performance while reducing the computational load.

This paper is organized as follows; the next section outlines the existing and proposed estimators for navigation systems where third section explains the simulation model. Simulation results are presented in section 4 and some concluding remarks are given in the last section.

## INTEGRATION ESTIMATORS

### Kalman Filter

Performance of integration methods for navigation systems are determined by the state estimator employed. Kalman filter is the traditional and the most commonly used state estimator [10, 11]. The filter produces recursive solution to the linear quadratic Gaussian problem. If the dynamics of the system is correctly modeled, it yields unbiased minimum mean square estimation for linear system and measurement models where the process and measurement noises are white and Gaussian.

Let's assume that the dynamic process of the navigation error could be defined as discrete Markov process. Then, the navigation error dynamics are described by

$$X(k+1) = FX(k) + \Gamma v(k) \quad (1)$$

where,  $X(k)$  is the state vector (the error in navigation),  $F$  and  $\Gamma$  are known state transition and disturbance transition matrices respectively and  $v(k)$  is unknown zero mean white Gaussian noise with known covariance  $Q$ . Measurements are linear combinations of states, corrupted with white noise given by

$$Z(k) = HX(k) + w(k) \quad (2)$$

where,  $H$  is the known measurement matrix and  $w(k)$  is zero mean white Gaussian noise with known covariance  $R$ .  $v(k)$  and  $w(k)$  are statistically independent. In general the state vector  $X(k)$  includes position, velocity, angle and bias variables. A good derivation of the Kalman filter equations can be found in [12], here, only the resulting standard Kalman filter equations are given.

$$\tilde{X}(k+1|k) = F\hat{X}(k) \quad (3)$$

$$v(k+1) = Z(k+1) - H\tilde{X}(k+1|k) \quad (4)$$

$$P(k+1|k) = FP(k)F^T + \Gamma Q \Gamma^T \quad (5)$$

$$S(k+1) = HP(k+1|k)H^T + R \quad (6)$$

$$W(k+1) = P(k+1|k)H^T S(k+1)^{-1} \quad (7)$$

$$\hat{X}(k+1|k+1) = \tilde{X}(k+1|k) + W(k+1)v(k+1) \quad (8)$$

$$P(k+1) = P(k+1|k) - W(k+1)S(k+1)W(k+1)^T \quad (9)$$

In Eqs 3-9  $\tilde{X}(k+1|k)$  and  $P(k+1|k)$  are the state prediction and its corresponding covariance respectively,  $\hat{X}(k+1|k+1)$  and  $P(k+1)$  are updated state estimation and its corresponding covariance respectively and  $v(k+1)$ ,  $S(k)$ ,  $W(k)$ , are the innovation, innovation covariance and the filter gain respectively.

### Modified Wave Estimator

The MWE technique [5] assumes that input disturbances can be described as deterministic means for short time periods. It models input disturbances as known base functions with unknown intensities, which can be estimated.

It is possible to describe the navigation system error model in a deterministic sense during a short time interval  $NT$ , where  $T$  is the sampling period and  $NT$  is called the cycle time as shown in Fig 1. Here, the main issue is the selection of an appropriate cycle time. A small cycle time allows a more accurate representation of the system; however, it may not be sufficient for all the state vectors to converge. On the other hand, a large cycle time ensures convergence, but it may degrade the estimation accuracy. A more prudent approach is to segregate the state variables into two groups based on their degree of observability [5]. The observability condition is defined as the ability to determine the state variables from the taken measurements.

In this approach the first group consists of all the strongly observed states and the second group is composed of all the weakly observed states.

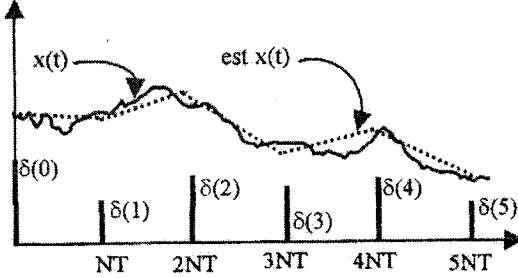


Figure 1: Modified wave estimator modeling of input disturbances

Through separation, the basic system model has two components in a wave cycle. The first component is the influence of the strongly observed states on themselves and the second component is the influence of the weakly observed states on the strongly observed states. Thus, the basic model of the system can be completely described as;

$$X(k+1) = X^0(k+1) + X^1(k+1) \quad (10)$$

With initial conditions  $X^0(0) = X(0)$  ve  $X^1(0) = 0$ .

$$\begin{aligned} X^0(k+1) &= F(k)X^0(k) \\ X^1(k+1) &= F(k)X^1(k) + Ge(k) \\ e(k+1) &= Le(k) \end{aligned} \quad (11)$$

where,  $X$  is the strongly observed state vector,  $X^0$  is the influence of strongly observed state vector on themselves ( $nxl$ ),  $X^1$  is the influence of weakly observed states on strongly observed states ( $nxl$ ),  $F$  is the state transition matrix of the strongly observed states ( $nxn$ ),  $G$  is the influence matrix expressing the effect that the weakly observed states has on the strongly observed states ( $nxk$ ),  $e$  is the weakly observed state vector ( $kxl$ ) and  $L$  is the state transition matrix of the weakly observed states ( $kxk$ ). Detailed information about the MWE and the segregation could be found in [5]. State and state covariance update along with the estimator gain equations are given in Eqs 12-15

$$(12)$$

$$\begin{aligned}
\hat{X}^0(k+1|k+1) &= F(k)\hat{X}^0(k|k) \\
&+ W(k+1)\left(Z(k+1) - HF(k)\hat{X}^0(k|k)\right) \\
W(k+1) &= \{F(k)P(k)F^T(k)H^T \\
&- F(k)\Psi(k)E[e_0 e_0^T]D^T(k+1)H^T\} \\
&\times \{HF(k)P(k)F^T(k)H^T \quad (13)
\end{aligned}$$

$$\begin{aligned}
&+ HD(k+1)E[e_0 e_0^T]D^T(k+1)H^T \\
&+ R - HF(k)\Psi(k)E[e_0 e_0^T]D^T(k+1)H^T \\
&- HD(k+1)E[e_0 e_0^T]\Psi^T(k)F^T(k)H^T\}^{-1} \\
P(k+1) &= (I - W(k+1)H)F(k)P(k)F^T(k) \quad (14) \\
&+ W(k+1)HD(k+1)E[e_0 e_0^T]\Psi^T(k)F^T(k)
\end{aligned}$$

$$\begin{aligned}
D(k+1) &= F(k)D(k) + GL(k|0) \\
\Psi(k+1) &= (I - W(k+1)H)F(k)\Psi(k) \quad (15) \\
&+ W(k+1)HD(k+1)
\end{aligned}$$

$$\prod_{i=1}^j (I - W(k+2-i)H)F(k-i) = I \quad \text{if } i > j$$

where the matrix  $D(k+1)$  propagates the influence of weakly observed states on strongly observed states throughout the wave cycle.

### Fused Estimator

The basic principle of the newly proposed fused estimator is to make use of good parts of the MWE and Kalman filter. Kalman filter produces better estimates for the strongly observed state, i.e., position, whereas the estimation accuracy is higher for weakly observed states, i.e., velocity and angle, for the MWE approach [5]. However, despite the improved performance for the weakly observed states, the MWE suffers from delay (which is equal to the cycle time) in estimation process and high computational burden. With the recent technological developments in computer technology, the computational burden might not be an issue. In fact when the accuracy of all the states is of importance advantages of both estimators could be exploited by employing them both and fusing their estimates in an appropriate manner. Data fusion techniques are frequently used in navigation and target tracking systems where the reliability is of utmost importance. Please see [13-15] for more information on data fusion.

The fusion equation under the error independence assumption for the estimated states is given by [16];

$$\hat{x}(t) = P \left[ P_1^{-1} \hat{x}_1 + P_2^{-1} \hat{x}_2 + \dots + P_n^{-1} \hat{x}_n \right] \quad (16)$$

where the covariance fusion equation is,

$$P = \left( P_1^{-1} + P_2^{-1} + \dots + P_n^{-1} \right)^{-1} \quad (17)$$

If the above data fusion method is applied to two minimum mean square estimators (Kalman and the MWE in this case), the state and its covariance are described by,

$$\hat{X} = P \left( P_{kalman}^{-1} \hat{X}_{kalman} + P_{GDK}^{-1} \hat{X}_{GDK} \right) \quad (18)$$

$$P = \left( P_{kalman}^{-1} + P_{GDK}^{-1} \right)^{-1} \quad (19)$$

### Adaptive Modified Wave Estimator

Selection of the cycle time determines the performance of the MWE in terms of error reduction. For instance, longer cycle times will unnecessarily increase the computational burden if the error is steadily changing. On the other hand shorter cycle times will have problem in dealing with sudden changes in error. Unfortunately, in the MWE cycle time has to be pre-selected and there is no mechanism to change it according to error characteristics. Thus, adaptively changing cycle time with the error characteristics would both improve the estimations and reduce the computational burden. Moreover, a navigation system with such an estimator should better react to different measurement units employed for different applications.

The question is how to vary the cycle time. Although the MWE has been proposed as an alternative to Kalman filter it possesses some of Kalman filter's important features. One of them is the ability to tell how well it is doing through its covariance. Thus if a metric could be defined based on the covariance it could be used to adaptively vary the cycle time. In [1] a statistical test namely, normalized innovation squared (NIS), was defined for Kalman filter to monitor the filter's performance where a small NIS indicates that the filter model matches system thus produces better. On the other hand growing NIS means that the system is no longer matched by the filter. NIS could also be defined for the MWE and by monitoring it and comparing with a predetermined threshold, the cycle time could be adaptively varied. Under the consistent filter assumptions the NIS has a chi-square distribution

with  $n_z$  degree of freedom where  $n_z$ , is the dimension of the measurement. Thus, an acceptance region could be determined for a given hypothesis from chi-square tables. Varying the cycle time adaptively helps the MWE produce better estimates with reduced computational load.

Innovation covariance is defined by Eq 20 where NIS for the MWE is given by Eq 21,

$$\begin{aligned}
 S(k+1) &= HFP(k+1|k+1)F^T H^T \\
 &+ HD(k+1)E[e_0 e_0^T]D^T(k+1)H^T \\
 &+ R - HF\Psi(k)E[e_0 e_0^T]D^T(k+1)H^T \\
 &- HD(k+1)E[e_0 e_0^T]\Psi^T(k)F^T H^T \\
 e_v(k) &= v^T(k)S(k)^{-1}v(k)
 \end{aligned} \tag{20}$$

If NIS, calculated through Eq 21 is below the predetermined threshold then the estimator is doing well and a smaller data set is enough for estimation, thus the cycle time can be reduced to save computational power. On the other hand when NIS goes above the threshold the cycle time must be increased for better estimation.

### Simulation Model

This section outlines the error model for the navigation system used in the simulations. The INS system for a single axis may be described as [17]

$$\begin{aligned}
 \delta \dot{P}_E &= \delta V_E \\
 \delta \dot{V}_E &= -g\phi_N + B_E \\
 \dot{\phi}_N &= \frac{\delta V_E}{R} + \delta\omega_N \\
 \dot{B}_E &= -\beta_1 B_E + \sqrt{2\sigma_1^2 \beta_1} w_1 \\
 \delta \dot{\omega}_N &= -\beta_2 \delta\omega_N + \sqrt{2\sigma_2^2 \beta_2} w_2
 \end{aligned} \tag{22}$$

where,  $\delta P_E$  is the position error on the given axis ( $m$ ),  $\delta V_E$  is the velocity error on the same axis ( $m/s$ ),  $\phi_N$  is the attitude error ( $rad$ ),  $B_E$  is the accelerometer bias

( $m/s^2$ ),  $\delta\omega_N$  is the gyro drift rate ( $rad/s$ ),  $\sigma_1, \beta_1$  are the parameters of the shaping filter to represent the accelerometer bias ( $m/s^2$  and  $s^{-1}$  respectively),  $\sigma_2, \beta_2$  are the parameters of the shaping filter to represent the gyro drift ( $m/s^2$  and  $s^{-1}$  respectively) and  $w_1, w_2$  are white noise.

The gyro drift rate and accelerometer bias are modeled through shaping filters employing first order Gauss-Markov process with different parameters for each. Measurements are the position difference between the INS and GPS where GPS's position error is assumed to measurement error.

$$\begin{aligned}
 P_{INS} &= P_{True} + \delta P \\
 P_{GPS} &= P_{True} + v \\
 z_k &= P_{INS} - P_{GPS} = \delta P + v_k
 \end{aligned} \tag{23}$$

## SIMULATION RESULTS

In the simulations, the INS accelerometer and gyro noises are assumed to be zero mean white, Gaussian with  $2500 \times 10^{-12} (\mu g)^2/Hz$  and  $1 \times 10^{-9} (deg/s)^2/Hz$  variances respectively where the GPS noise has also been taken as zero mean white, Gaussian with  $100m^2$  variance. For the adaptive MWE, the acceptance region, that the chosen cycle time is correct, hypothesis has been determined to be  $0.5 \leq e_v(k) \leq 1.5$ . Smaller cycle time has been used as long as the calculated NIS is in this region and the NIS has been increased as soon as NIS fell outside of this region.

Table 1 presents the performance improvement of the MWE with respect to Kalman filter for 25 Monte Carlo runs. As the cycle time increases so does the estimation performance of the MWE, however, after a certain point longer cycle time allows more noise into the system then the performance starts to degrade. As it can be seen in Table 1 the maximum improvement is achieved when the cycle time is 250 s after which the performance deteriorates.

RMS position, velocity and angle errors for the Kalman filter, MWE and fused estimator are given in Tables 2 through 4 respectively for different cycle times. When compared to Kalman filter and the MWE, the fused estimator produces better estimates in terms of RMS position and velocity errors for all cycle times whereas it is outperformed by the MWE for the weakest observed state that is the angle. The improvement in terms of position and velocity is minimum at the optimum cycle time, 250 s. As for the angle estimation, the fused estimator provides approximately 40% improvement over Kalman filter but is outperformed by the MWE by 25% on



average. Improvements achieved by the fused estimator over Kalman filter and the MWE are given in Table 9.

Table 1: Performance improvement by the MWE over Kalman filter

Cycle Time	Position Improv.	Velocity Improv.	Angle Improv.
50 sn	8.52%	18.58%	10.82%
100 sn	17.44%	27.99%	37.51%
150 sn	0.39%	23.53%	47.65%
200 sn	11.18%	33.59%	47.20%
250 sn	18.55%	39.44%	48.61%
300 sn	15.18%	31.82%	41.61%
350 sn	3.87%	24.73%	39.27%
400 sn	5.06%	25.38%	39.16%

Table 2: RMS position errors (m) for different cycle times

Cycle Time (s)	Meas. Error (m)	Kalman Filter	MWE	Fused Estimator
50	1.9794	0,50875	0,46539	0,35888
100	1.9794	0,508	0,41942	0,35684
150	1.9794	0,50714	0,50516	0,35934
200	1.9794	0,51075	0,45363	0,33669
250	1.9794	0,49977	0,40706	0,32269
300	1.9794	0,51103	0,43344	0,34504
350	1.9794	0,50246	0,48299	0,34129
400	1.9794	0,50879	0,48304	0,34678

Table 3: RMS velocity errors (m/s) for different cycle times

Cycle Time (s)	Meas. Error	Kalman Filter	MWE	Fused Estimator
50	-	0,021042	0,017133	0,011286
100	-	0,020902	0,015052	0,01224
150	-	0,020814	0,015916	0,014138
200	-	0,021179	0,014064	0,012802
250	-	0,020761	0,012572	0,011891
300	-	0,021096	0,014383	0,012636
350	-	0,020712	0,01559	0,012792
400	-	0,02096	0,01564	0,012687

Table 4: RMS angle errors (deg) for different cycle times

Cycle Time (s)	Meas. Error	Kalman Filter	MWE	Fused Estimator
50	-	0,0027322	0,0024367	0,002317
100	-	0,0027033	0,0016893	0,0020426
150	-	0,0027201	0,0014241	0,0017836
200	-	0,0027658	0,0014604	0,0018015
250	-	0,0027244	0,0014	0,001685
300	-	0,0027678	0,0016161	0,0020066
350	-	0,002686	0,0016312	0,0024622
400	-	0,0027147	0,0016517	0,0020219

Tables 5 and 6 present simulation times for the MWE and Kalman filter respectively for different cycle times.

Table 5: Simulation times for the MWE

Cycle Time (s)	Times Function Called	Simulation Time (s)
50	1100	6.905
100	1100	13.020
150	1100	19.361
200	1100	26.106
250	1100	33.411
300	1100	39.335
350	1100	45.434
400	1100	52.396

Table 6: Simulation time for Kalman filter

Cycle Time (s)	Times Function Called	Simulation Time (s)
-	1100	0.126

The longer the simulation time the bigger the computational load which increases with the increased

cycle time. Computational load for the fused estimator is only slightly higher than the MWE, thus, Kalman filter has the best performance in terms of computational load. In summary, fusing the estimates of two estimators produces considerably better estimates in position, velocity and angle when compared Kalman filter and in position and velocity when compared to the MWE. However, the MWE's angle estimations are better for almost all cycle times. Having revealed that, one could only fuse the strongly observed states, namely position and velocity, and use the angle estimations by the MWE for even more improved performance.

When Table 1 is examined, it will be seen that the MWE provides 15%, 31% and 42% improvement in position, velocity and angle respectively for 300 s over Kalman filter. When the cycle time is halved the improvement over Kalman filter in position, velocity and angle becomes 0.4%, 24% and 47% respectively. If the simulation times given in Table 5 are also examined it will be observed that the simulation times are 19.361 s and 39.335 s for 150 s and 300 s cycle times respectively. If a MWE that switches between 150 s and 300 s cycle times adaptively better performance with less computational load could be achieved. Such an adaptive MWE has been designed and its performance in terms of RMS errors and simulation time is investigated. Simulation results have revealed that the adaptive MWE achieves the performance of the MWE at 300 s cycle time in 9.196 s less time (i.e., approximately 24% less computational load). RMS errors for different adaptive MWEs that switch between different cycle times are presented in Table 7 whereas simulation times for these estimators are given in Table 8.

Table 7: RMS errors for the adaptive MWE

	400s – 200s	300s – 150s	200s – 100s
Position (m)	0,41136	0,43169	0,44558
Velocity (m/s)	0,012738	0,014007	0,014371
Angle (deg)	0.0014	0,0015	0.0016

Table 8: Simulation time for the Adaptive MWE

Cycle Time (s)	Times Function Called	Simulation Time (s)
200-100	1100	23.265
300-150	1100	30.139
400-200	1100	45.326

Table 9: Performance improvement of the fused estimator for position, velocity and angle

Cycle Time	Position Improvement		Velocity Improvement		Angle Improvement	
	Over Kalman Filter	Over MWE	Over Kalman Filter	Over MWE	Over Kalman Filter	Over MWE
50 s	29.46%	22.89%	46.36%	34.13%	15.20%	4.91%
100 s	29.76%	14.92%	41.44%	18.68%	24.44%	-20.91%
150 s	29.14%	28.87%	32.07%	11.17%	34.43%	-25.24%
200 s	34.08%	25.78%	39.55%	8.97%	34.87%	-23.36%
250 s	35.43%	20.73%	42.72%	5.42%	38.15%	-20.36%
300 s	32.48%	20.39%	40.10%	12.15%	27.50%	-24.16%
350 s	32.08%	29.34%	38.24%	17.95%	8.33%	-50.94%
400 s	31.84%	28.21%	39.47%	18.88%	25.52%	-22.41%

The adaptive MWE produces better estimates than Kalman filter, for instance when the adaptive estimator switches between 200 s and 400 s the improvement is 19%, 39% and 49% in position, velocity and angle respectively over Kalman filter. When the MWE is allowed to switch between cycle times it achieves, the adaptive structure helps achieve performance of the longer cycle time (better performance) in less time. Besides the advantage of achieving the performance of a longer cycle time with reduced computational load, the adaptive MWE also overcomes the problem of increased noise for longer cycle times in the system.

## CONCLUSIONS

Two novel estimators, namely the fused estimator and adaptive MWE, for navigation systems are proposed. The fused estimator produces better performance than both Kalman filter and the MWE in terms of strongly observed state estimations whereas it is outperformed by the MWE while estimating the weakly observed state, i.e., angle. However, the fused estimator does not provide any improvement in terms of computational load. On the other hand the adaptive MWE is better than both Kalman filter and the MWE in terms of estimation performance overcomes the problems that MWE faces.

## ÖZET

Seyrüsefer sistemleri hem askeri hem de sivil uygulamalarda önemli yere sahiptir ancak sistemde oluşan hatalar seyrüsefer sistemlerinin etkinliğini azaltabilmektedir. Kalman filtresi oluşan bu hataları azaltmak için sıklıkla kullanılan bir araçtır. Son zamanlarda Geliştirilmiş Dalga Kestiricisi (GDK) seyrüsefer sistemlerinde hata giderici olarak kullanılmak üzere Kalman filtresine alternatif olarak sunulmuştur. Kalman filtresinden farklı olarak GDK süreç gürültüsünü deterministik ortalamalar şeklinde tanımlar. Her iki kestiricinin birbirlerine göre avantaj ve dezavantajları vardır. Bu makalede her iki kestiricinin avantajlarından mümkün olduğunca yararlanabilmek için bu kestiricilerin kestirimlerini birleştiren yeni bir kestirici ile daha iyi performans ve daha düşük işlem yüküne sahip uyarlamalı bir GDK önerilmiştir. Önerilen kestiriciler, Kalman filtresi ve standart GDK'nın performansları benzetim yoluyla karşılaştırılmış ve elde edilen sonuçların analizi verilmiştir.

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(i) Name of the paper, (ii) Complete name(s) of the author(s), (iii) Name and address of the university, laboratory or institute at which the research has been carried out.

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Each paper should always be preceded by an abstract. The abstract should be short and self-contained and all essential points of the paper should be mentioned.

### 3. Sections and Subsections

Principle sections such as introduction or formulation of the problem should be numbered consecutively (1. Introduction, 2. Formulation of the problem, ..., etc.) Subsections should be numbered 1.1, 1.2, ..., etc.

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References including comments must be numbered consecutively in order of first appearance in the text. The reference number should be put in brackets [1] where referred to in the text. References should be listed at the end of the manuscript in the numbered order in which they appear in the text. The method of citation should be as follows:

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Example: M. Chen and P.M. Zerwas, Phys. Rev. D 12 (1975) 187.



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Theses: (i) Surname and initials of the author, (ii) Type of degree (Ph.D., M.Sc.), (iii) Name and address of institute where the work has been carried out, (iv) Year.

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All illustrations not including tables (photographs and other films, drawings, graphs, etc) must be labeled as "Figure".

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All tables and figures must have a number (Table 1, Figure 1) and a caption or legend. If there is only one table or figure simply label it "Table" or "Figure".

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All captions and legends must also be double-space typed on a separate sheet and labeled according to which table or figure they belong.

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Tables must be clearly typed, each on a separate sheet, and double-spaced. Dimensions including caption, title, column heads, and footnotes must not exceed 13x16 cm. Tables may be continued onto another sheet if necessary but dimensions still apply.

Figures must be originals and drawn clearly in Indian ink on white paper or printed on smooth tracing/drawing paper. Reduced photocopies are not acceptable. Photographs must be clear, black and white, and printed on glossy paper.

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All appendices must be typed on separate sheets and should be numbered consecutively with capital Roman numerals.

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If you are able to initially prepare your manuscript in a MS Word Programme (Macintosh or PC) file including the figures translated into the picture environment of Encapsulated PostScript format (EPS), we advise that you do so. Then, only if and after your manuscript is accepted for publication, we will ask you to submit a revised disk copy of your manuscript which will enable us to more efficiently and accurately prepare proofs. (This is not a requirement but is highly encouraged.)

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