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On a Markov Chain with Denumerable Number of States and Transition Probabilities Dependent on Probability States

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Abstract. The authors consider homogeneous Markov chain $\xi_t, t \geq 0$ with a denumerable number of states and transition probabilities dependent on the states of that chain. If the chain $\xi_t, t \geq 0$ is assumed to be ergodic for stationary distribution $\{p_k^{\pm}\}$, $k \geq 0$, it is established that a unique solution to the differential equations system relative to the generating functions $P^{\pm}(\theta)$, $|\theta| \leq 1$ of that distribution $\{p_k^{\pm}\}$, $k \geq 0$ exists. This condition is found in the form of the inequality $\|G\| \leq e^2$. It is based on Fubini's theorem from the theory of functions and on the existence of the bound $G \equiv G_\infty = G_n = \lim_{n\to\infty} Ee^{\theta-\eta}$, $E$is the identity matrix. Using the principle of the matrix theory by induction, we get that

$$G_n = E\sum_{k=0}^{n} \frac{(\theta-\eta)^k}{k!},\ \ n = 1, 2, 3, ..$$

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1. Problem Statement

If ξ_t , $t \geq 0$ is a homogeneous Markov chain with the phase space $\{0^+, 1^{\pm}, 2^{\pm}, ...\}$ and the following transition probabilities at$\Delta \downarrow 0$:

$$0^+ \xrightarrow{\Delta} \begin{cases} 0^+ : \ 1-\lambda_0^+\Delta + o\ (\Delta) \\ 1^+ : \ \lambda_0^+\Delta + o\ (\Delta) \end{cases}$$

$$k^+ \xrightarrow{\Delta} \begin{cases} k^+ : 1-\left(\lambda_k^+ + \mu + \nu\right)\Delta + o\ (\Delta) \\ (k-1)^+ : \ \ \mu\Delta + o\,(\Delta) \\ k^- : \ \ \ \ \nu\Delta + o\,(\Delta) \\ (k+1)^+ : \ \ \lambda_k^+\Delta + o\,(\Delta) \end{cases} \tag{1.1}$$

$$k^- \xrightarrow{\Delta} \begin{cases} k^- \ : 1-\left(\lambda_k^- +\pi\right)\Delta+o\left(\Delta\right) \\ \left(k+1\right)^- : \quad \lambda_k^-\Delta+o\ \left(\Delta\right) \\ k^+ : \qquad \pi\,\Delta+o\left(\Delta\right), \end{cases}$$

$$(\,k=1,\ 2\,,3,\ ...)$$

where λ_0^+ , $\lambda_k^\pm$ $(k\geq 0)$, $\mu,$ ν and π are the set nonnegative constants.

One encounters processes of this type when examining maintenance systems with one unreliable device, when the arrival intensity depends on the number in the system.

Before initiating the solution of the problem, let us expand the contents of the correlation (1.1).

The symbol

$$\begin{cases} \xi_1 : \ \ p_1 \\ \xi_2 \ : \ p_2 \\ \vdots \\ \xi_m : \ \ p_m \end{cases}$$

$(p_1\geq 0,\ \ p_1+...+p_m=1\,)$denotes the random value with distribution function

$$p_1P\left\{\xi_1<x\right\}+p_2P\left\{\xi_2<x\right\}+...+p_mP\left\{\xi_m<x\right\}.$$

In terms of the queuing theory, the condition 0^+means that at some instant the server is idle and fault free, and $k^\pm$, $k\geq 1$means that at some instant the device is fault free (faulty) and the number of requests in the system is k.

By rate of some characteristic of the system, we understand the mean number of steps per unit time related to that characteristic. In this case, $\lambda_k^\pm\,, k\geq 0$ means the request arrival rate in the system, $\mu$is the service rate of one request, and ν and π are the rates of the server shutdown and reactivation. It should be noted that the normalization condition takes place as well:

$$\sum_{k=0}^{\infty}\lambda_k^+ +\sum_{k=1}^{\infty}\lambda_k^- = 1.$$

2. Solution of the problem

Following the chart proposed in [1], we establish that if the chain $\xi_t,\ \ t\geq 0$ is assumed to be argotic, a unique solution to the differential equations system in relation to the generating functions $P^\pm\left(\theta\right)$, $\ |\theta|\ \leq 1$ of that distribution exists. Thus, assume that the chain ξ_t , $\ t\geq 0$ is ergotic,i.e. there is the limit

$$\lim_{t\to\infty}\ P\ \left\{\xi_t=k^\pm\right\}\ =p_k^\pm\quad(k\geq 0)$$

According to (1.1), it is easy to obtain the following system of difference-differential equations for the probabilities $p_k^\pm\left(t\right)=P\ \left\{\xi_t=k^\pm\right\}$, $\ k\geq 0$

$$(2.1)\quad \left\{ \begin{array}{l} \frac{dp_0^+(t)}{dt} = -\lambda_0^+ p_0^+ (t) + \mu \, p_1^+ (t) \, , \\ \frac{dp_k^+(t)}{dt} = -\left(\lambda_k^+ + \mu + \nu\right) \, p_k^+ \, (t) + \mu \, p_{k+1}^+ (t) + \pi \, p_k^- \, (t) + \lambda_{k-1}^+ p_{k-1}^+ (t) \, , \\ \frac{dp_k^-(t)}{dt} = -\left(\lambda_k^- - \pi\right) \, p_k^- \, (t) + \nu \, p_k^+ (t) + \delta_{k1} \, \lambda_{k-1}^- p_{k-1}^- (t) \, , \end{array} \right.$$

$$(k = 1, 2, 3, ...)$$

where

$$\delta_{k1} = \left\{ \begin{array}{ll} 0, & k = 1 \\ 1, & k \neq 1 \end{array} \right.$$

In (2.1), proceeding to the limit $t \to \infty$, we have the following system of algebraic equations relative to $p_k^\pm$, $k \geq 0$:

$$(2.2)\quad \left\{ \begin{array}{l} -\lambda_0^+ p_0^+ + \mu \, p_1^+ = 0 \\ -\left(\lambda_k^+ + \mu + \nu\right) \, p_k^+ + \mu \, p_{k+1}^+ + \pi \, p_k^- + \lambda_{k-1}^+ p_{k-1}^+ = 0 \\ -\left(\lambda_k^- + \pi\right) \, p_k^- + \nu \, p_k^+ + \delta_{k1} \, p_{k-1}^- \lambda_{k-1}^- = 0, \end{array} \right.$$

$$(k = 1, 2, 3, ...)$$

Let us then proceed to solving the system (2.2) by introducing the following denotation

$$z_k^+ = \lambda_k^+ p_k^+ \quad (k \geq 0) \, , \qquad z_k^- = \lambda_k^- p_k^- \quad (k \geq 1)$$

Adding together the second and the third relations of the system (2.2), we have

$$-\left(z_k^+ + z_k^-\right) + \left(z_{k-1}^+ + \delta_{k1} z_{k-1}^-\right) = \mu \, p_k^+ - \mu \, p_{k+1}^+$$

$$(k = 1, 2, ...)$$

Hence, at $k = 1$

$$z_1^+ + z_1^- = \mu \, p_2^+ \, ,$$

and at $k \geq 2$

$$(2.3)\quad -\left(z_k^+ + z_k^-\right) + \left(z_{k-1}^+ + z_{k-1}^-\right) = \mu \, p_k^+ - \mu \, p_{k+1}^+$$

If we denote

$$z_k^+ + z_k^- \ = s_k^+ \quad (k \geq 1) \ \ ,$$

then (2.3) takes the following form

$$(2.4)\quad s_{k-1}^+ - s_k^+ = \mu \, p_k^+ - \mu_{k+1}^+ \, (k \geq 2)$$

$$s_1^+ = \mu \, p_2^+$$

Using successive substitution, we can see that

$$s_k^+ = \mu\, p_{k+1}^+ \,, \qquad z_k^+ + z_k^- = \mu\, p_{k+1}^+ \tag{2.5}$$

$$(k = 1, 2, ...)$$

Further, subtracting the third relation from the second relation of the system (2.2), we have

$$-\left(z_k^+ - z_k^-\right) + \left(z_{k-1}^+ - \delta_{k1} z_{k-1}^-\right) = 2\,\nu\, p_k^+ - 2\pi\, p_k^- + \mu\, p_k^+ - \mu\, p_{k+1}^+ \qquad (k \geq 1)$$

Hence, at $k = 1$
$z_1^+ - z_1^- = 2\pi\, p_1^- + \mu\, p_2^+ - 2\nu\, p_1^+$, and at $k \geq 2$

$$-\left(z_k^+ - z_k^-\right) + \left(z_{k-1}^+ - z_{k-1}^-\right) = 2\,\nu\, p_k^+ - 2\pi\, p_k^- + \mu\, p_k^+ - \mu\, p_{k+1}^+ \tag{2.6}$$

If we denote

$$s_k^- = z_k^+ - z_k^- \qquad (k \geq 1)\,,$$

then (2.6) takes the following form

$$s_{k-1}^- - s_k^- = 2\,\nu\, p_k^+ - 2\pi\, p_k^- + \mu\, p_k^+ - \mu\, p_{k+1}^+ \qquad (k \geq 2)$$

$$s_1^- = 2\pi\, p_1^- + \mu\, p_2^+ - 2\nu p_1^+$$

Now we see that s_k^- $(k \geq 1)$ takes the following form

$$s_k^- = 2\pi \sum_{i=1}^{k} p_i^- - 2\nu \sum_{i=1}^{k} p_i^+ + \mu\, p_{k+1}^+\,,$$

i. e.

$$z_k^+ - z_k^- = 2\pi \sum_{i=1}^{k} p_i^- - 2\nu \sum_{i=1}^{k} p_i^+ + \mu\, p_{k+1}^+$$

$$(k = 1, 2, ...)$$

As a result, the system(2.2) is reduced to the following equivalent system of algebraic equations:

$$\begin{cases} \lambda_k^+ p_k^+ + \lambda_k^-\, p_k^- = \mu\, p_{k+1}^+\,, \\ \lambda_k^+ p_k^+ - \lambda_k^- p_k^- = \mu\, p_{k+1}^+ + 2\pi \sum\limits_{i=1}^{k} p_i^- - 2\nu \sum\limits_{i=1}^{k} p_i^+ \end{cases} \tag{2.7}$$

$$k = 1, 2, ...$$

or

(2.8)
$$\begin{cases} \lambda_k^+ p_k^+ = \mu\, p_{k+1}^+ + \pi \sum\limits_{i=1}^{k} p_i^- - \nu \sum\limits_{i=1}^{k} p_i^+ \\ \lambda_k^- p_k^- = \nu \sum\limits_{i=1}^{k} p_i^+ - \pi \sum\limits_{i=1}^{k} p_i^- \ , \qquad k \geq 1 \end{cases}$$

For the sake of clarity, we will assume that

$$\lambda_k^{\pm} = \lambda^{\pm}\ [1 + (k-1)\ z] \qquad 0 \leq z \leq 1, \qquad k \geq 1$$

If we denote

$$P^{\pm}(\theta) = \sum_{k=0}^{\infty} p_k^{\pm} \theta^{\,k} \qquad |\theta| \leq 1,$$

it is easy to show that

$$\sum_{k=0}^{\infty} \lambda^+\ [1 + (k-1)\ z]\ p_k^+ \theta^k = \mu \sum_{k=0}^{\infty} p_{k+1}^+ \theta^k +$$

$$+\pi \sum_{k=0}^{\infty} \sum_{i=1}^{k} p_i^- \theta^k - \nu \sum_{k=0}^{\infty} \sum_{i=1}^{k} p_i^+ \theta^k$$

In these denotations, the system (2.8) takes the following form

$$\begin{cases} \lambda^+ P^+(\theta) + \lambda^+ z \sum\limits_{k=1}^{\infty} (k-1)\ p_k^+ \theta^k = \frac{\mu}{\theta}\, P^+(\theta) - \lambda_0^+ p_0^+ + \pi \frac{P^-(\theta)}{1-\theta} - \nu \frac{P^+(\theta)}{1-\theta}\ , \\ \lambda^- P^-(\theta) + \lambda^- z \sum\limits_{k=1}^{\infty} (k-1)\ p_k^- \theta^k = \nu \frac{P^+(\theta)}{1-\theta} - \pi \frac{P^-(\theta)}{1-\theta} \end{cases}$$

Since

$$\sum_{k=1}^{\infty} (k-1)\ p_k^+ \theta^k = \theta \frac{dP^+(\theta)}{d\theta} - P^+(\theta)\ ,$$

then

$$\begin{cases} \lambda^+ P^+(\theta) + \lambda^+ z \left[\theta \frac{d\,P^+(\theta)}{d\,\theta} - P^+(\theta)\right] = \frac{\mu}{\theta}\, P^+(\theta) - \lambda_0^+ p_0^+ + \pi \frac{P^-(\theta)}{1-\theta} - \nu \frac{P^+(\theta)}{1-\theta}\ , \qquad |\theta| < 1 \\ \lambda^- P^-(\theta) + \lambda^- z \left[\theta \frac{d\,P^-(\theta)}{d\,\theta} - P^-(\theta)\right] = \nu \frac{P^+(\theta)}{1-\theta} - \pi \frac{P^-(\theta)}{1-\theta}\ , \qquad |\theta| < 1 \end{cases}$$

By multiplying the both parts of the obtained relations by $(1-\theta)$, we get the system:

(2.9)
$$\begin{cases} \lambda^+ z\, \theta\ (1-\theta)\ \frac{dP^+(\theta)}{d\,\theta} + \left[\lambda^+\ (1-\theta)\ (1-z) - \frac{\mu}{\theta} + \mu + \nu\right] P^+(\theta) = \pi\, P^-(\theta) - \lambda_0^+\ (1-\theta)\ p_0^+\ , \\ \lambda^- z\, \theta\ (1-\theta)\ \frac{dP^-(\theta)}{d\theta} + [\lambda^-\ (1-\theta) + \pi]\ P^-(\theta) = [\lambda^- z\ (1-\theta) + \nu]\ P^+(\theta) \end{cases}$$

At $\theta \to 1+0$, each of those relations will give us the following:

$$\pi P^{-}(1) = \nu P^{+}(1)$$

It should be noted that

$$P^{+}(1) + P^{-}(1) + p_0^{+} = 1.$$

If we introduce the following denotation

$$a^{\pm}(z,\theta) = \lambda^{\pm} z\,\theta\,(1-\theta)$$

$$b(z,\theta) = \lambda^{+}(1-\theta)(1-z) - \frac{\mu}{\theta} + \mu + \nu$$

$$c(z,\theta) = \lambda^{-} z\,(1-\theta) + \nu,$$

$$l(\theta) = \lambda^{-}(1-\theta) + \pi$$

In these denotations, the system (2.9) takes the following form

$$\text{(2.10)} \qquad \begin{cases} a^{+}(z,\theta)\frac{dP^{+}(\theta)}{d\theta} + b(z,\theta)\,P^{+}(\theta) = \pi P^{-}(\theta) - \lambda_0^{+}(1-\theta)\,p_0^{+} \\ a^{-}(z,\theta)\frac{dP^{-}(\theta)}{d\theta} + l(\theta)\,P^{-}(\theta) = c(z,\theta)\,P^{+}(\theta) \end{cases}$$

Let us now establish the existence and uniqueness of solution to the system (2.10) by first rewriting it like this

$$\text{(2.11)} \qquad \begin{cases} \frac{dP^{+}(\theta)}{d\theta} = a(\theta)\,P^{+}(\theta) + b(\theta)\,P^{-}(\theta) + c(\theta)\,p_0^{-} \\ \frac{dP^{-}(\theta)}{d\theta} = d(\theta)\,P^{+}(\theta) + e(\theta)\,P^{-}(\theta), \end{cases}$$

where

$$a(\theta) = -\frac{b(z,\theta)}{a^{+}(z,\theta)}, \qquad b(\theta) = \frac{\pi}{a^{+}(z,\theta)}, \qquad c(\theta) = -\frac{\lambda_0^{+}(1-\theta)}{a^{+}(z,\theta)},$$

$$d(\theta) = \frac{c(z,\theta)}{a^{-}(z,\theta)}, \qquad e(\theta) = -\frac{l(\theta)}{a^{-}(z,\theta)}$$

If we introduce the following denotations

$$P(\theta) = \begin{pmatrix} P^{+}(\theta) \\ P^{-}(\theta) \end{pmatrix}, \quad A(\theta) = \begin{pmatrix} a(\theta) & b(\theta) \\ d(\theta) & e(\theta) \end{pmatrix}, \quad f(\theta) = \begin{pmatrix} c(\theta)\,p_0^{+} \\ 0 \end{pmatrix}$$

the system (2.11) will look as follows in the matrix form [2]

$$\frac{dP(\theta)}{d\theta} = A(\theta)\,P(\theta) + f(\theta)$$

or

$$dP(\theta) = A(\theta)\,P(\theta)\,d\theta + f(\theta)\,d\theta,$$

whence

$$P(\theta) - P(0) = \int_0^\theta A(\xi) P(\xi)\, d\xi + \int_0^\theta f(\xi)\, d\xi,$$

$$P(\theta) = P(0) + \int_0^\theta f(\xi)\, d\xi + \int_0^\theta A(\xi) P(\xi)\, d\xi$$

If we denote

$$F(\theta) = P(0) + \int_0^\theta f(\xi)\, d\xi$$

then

$$P(\theta) = \int_0^\theta A(\xi) P(\xi)\, d\xi + F(\theta) \tag{2.12}$$

Let us carry out the iteration process [4] in the relation(2.12):

$$P_{n+1}(\theta) = \int_0^\theta A(\xi) P_n(\xi)\, d\xi + F(\theta)$$

$$n = 0, 1, 2, ...$$

Taking the following as the initial value $P_0(\theta)$:

$$P_0(\theta) = F(\theta),$$

we have

$$P_1(\theta) = \int_0^\theta A(\xi) P_0(\xi)\, d\xi + F(\theta) = F(\theta) + \int_0^\theta A(\xi) F(\xi)\, d\xi,$$

$$P_2(\theta) = \int_0^\theta A(\xi) P_1(\xi)\, d\xi + F(\theta) = F(\theta) + \int_0^\theta A(\xi) \left[F(\xi) + \int_0^\xi A(\eta) F(\eta)\, d\eta \right] d\xi =$$

$$= F(\theta) + \int_0^\theta A(\xi) F(\xi)\, d\xi + \int_0^\theta A(\xi)\, d\xi \int_0^\xi A(\eta) F(\eta)\, d\eta$$

In the latter, according to Fubini's theorem [3], we have the following by rearranging the integrals:

$$P_2(\theta) = F(\theta) + \int_0^\theta A(\xi) F(\xi)\, d\xi + \int_0^\theta d\eta \int_\eta^\theta A(\xi) A(\eta) F(\eta)\, d\xi =$$

$$= F(\theta) + \int_0^\theta A(\eta) F(\eta)\, d\eta + \int_0^\theta \left[\int_\eta^\theta A(\xi)\, d\xi\right] A(\eta) F(\eta)\, d\eta =$$

$$= F(\theta) + \int_0^\theta \left[A(\eta) + \int_\eta^\theta A(\xi)\, d\xi\, A(\eta)\right] F(\eta)\, d\eta.$$

At $n = 3$

$$P_3(\theta) = \int_0^\theta A(\xi) P_2(\xi)\, d\xi + F(\theta) = F(\theta) +$$

$$+ \int_0^\theta A(\xi)\, d\xi \left\{ F(\xi) + \int_0^\xi \left[A(\eta) + \int_\eta^\xi A(t)\, dt\, A(\eta)\right] F(\eta)\, d\eta \right\} =$$

$$= F(\theta) + \int_0^\theta A(\xi) F(\xi)\, d\xi + \int_0^\theta A(\xi)\, d\xi \left\{ \int_0^\xi \left[A(\eta) + \int_\eta^\xi A(t)\, dt\, A(\eta)\right] F(\eta)\, d\eta \right\} =$$

$$= F(\theta) + \int_0^\theta A(\eta) F(\eta)\, d\eta + \int_0^\theta d\eta \int_\eta^\theta A(\xi) \left[A(\eta) + \int_\eta^\xi A(t)\, dt\, A(\eta)\right] F(\eta)\, d\xi =$$

$$= F(\theta) + \int_0^\theta \left\{ A(\eta) + \int_\eta^\theta A(\xi) \left[A(\eta) + \int_\eta^\xi A(t)\, dt A(\eta)\right] d\xi \right\} F(\eta)\, d\eta$$

Assuming that $A(\eta) = E$ we have

$$P_2(\theta) = F(\theta) + \int_0^\theta \left(E + \int_\eta^\theta E\, d\xi\, E\right) F(\eta)\, d\eta = F(\theta) + \int_0^\theta \left[E + E(\theta - \eta)\right] F(\eta)\, d\eta$$

$$P_3(\theta) = F(\theta) + \int_0^\theta \left\{ E + \int_\eta^\theta E\, d\xi\, [E + \int_\eta^\xi E\, dt\, E \right\} F(\eta)\, d\eta$$

$$P_4(\theta) = F(\theta) + \int_0^\theta \left\{ E + \int_\eta^\theta E\, d\xi \left[E + \int_\eta^\xi E\, dt\, E \left(E + \int_\eta^t E\, d\tau\right)\right] \right\} F(\eta)\, d\eta$$

and so on.

Let us denote

$$G_0\ (\eta,\ \theta) \equiv E$$

$$G_1\ (\eta\,,\theta) = E + \int\limits_{\eta_\theta}^{\theta} G_0\ (\eta,\ \xi)\ d\xi\,,$$

$$G_2\ (\eta\,,\theta) = E + \int\limits_{\eta}^{\theta} G_1\,(\eta,\ \xi)\ d\xi$$

and so on.

In the general form

$$G_n\,(\eta,\ \theta)\ = E + \int\limits_{\eta}^{\theta} G_{n-1}\ (\eta,\ \xi)\ d\xi$$

$$n = 1, 2, ...$$

we have

$$G_1\ (\eta\,,\theta) = E + \int\limits_{\eta}^{\theta} E\,d\xi = E + E\ \ (\theta - \eta)$$

$$G_2\ (\eta\,,\theta) = E + \int\limits_{\eta}^{\theta} [E + E\ \ (\xi - \eta)]\ d\xi =$$

$$= E + E\,(\theta - \eta) + E \cdot \left.\frac{(\xi - \eta)^2}{2\,!}\right|_{\xi=\eta}^{\theta} = E + E\ (\theta - \eta) + E\,\frac{(\theta - \eta)^2}{2\,!}$$

For $G_3\ (\eta\,,\ \theta)$, we will get

$$G_3\,(\eta\,,\,\theta) = E + E\ (\theta - \eta) + E\frac{(\theta - \eta)^2}{2!} + E\frac{(\theta - \eta)^3}{3!}$$

As a result, by induction we have

$$G_n = E\sum_{k=0}^{n}\frac{(\theta - \eta)^k}{k!}\,, \quad n = 1, 2, ...$$

In the latter, at $n \to \infty$ for $G \equiv G_\infty$, we have

$$G \equiv G_\infty = \lim_{n\to\infty} G_n = E\,e^{\theta - \eta}.$$

Considering that $A\ (\eta) \equiv E$, e.g. for $P_4\ (\theta)$ and $P_5\,(\theta)$, we have

$$P_4(\theta) = F(\theta) + E\int_0^\theta G_3(\eta,\theta)\, F(\eta)\, d\eta$$

$$P_5(\theta) = F(\theta) + E\int_0^\theta G_4(\eta,\theta)\, F(\eta)\, d\eta$$

Again, by induction we have

$$P_n(\theta) = F(\theta) + E\int_0^\theta G_{n-1}(\eta,\theta)\, d\eta, \qquad n \geq 2 \tag{2.13}$$

In the relation (2.13), at $n \to \infty$, we will get

$$P(\theta) = \lim P_n(\theta) = F(\theta) + E\int_0^\theta e^{\theta-\eta} F(\eta)\, d\eta$$

Further, if $\|\cdot\|$ is the norm sign, then

$$\| G_0(\eta,\theta) \| = 1$$

$$\| G_1(\eta,\theta) \| = 1 + (\theta-\eta)$$

$$\| G_2(\eta,\theta) \| = 1 + \frac{(\theta-\eta)}{1!} + \frac{(\theta-\eta)^2}{2!}$$

. . . .

In the general form

$$\| G_n(\eta,\theta) \| = \sum_{k=0}^{n} \frac{(\theta-\eta)^k}{k!}$$

$$n = 0, 1, 2, ...$$

In the latter, at $n \to \infty$ for $G \equiv G_\infty$, we have

$$\| G \| = e^{\theta-\eta} \leq e^2.$$

The inequality $\|G\| \leq e^2$guarantees the existence and uniqueness of the solution to the system of differential equations (2.10) relative to the generating functions $P^{\pm}(\theta)$, $|\theta| \leq 1$ of the distribution $\{p_k^{\pm}\}$, $k \geq 1$.

3. Summary

In the paper the existence and uniqueness of the solution to the system of differential equations with respect to the generating functions $P^{\pm}(\theta)$, $|\theta| \leq 1$is established under the ergodic conditions of ξ_t, $t \geq 0$ chain with the stationary distribution $\{p_k^{\pm}\}$, $k \geq 1$.

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