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Solutions of the Nonlinear Differential Equations by Use of Modified Kudryashov Method

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Abstract. Studies based on the non-linear physical problems have become very important in recent years. These problems are solved by using different mathematical approaches. In particular, the soliton solutions, compacton solutions, peakon solutions and other solutions have been found for such physical problems. Using a powerful method that is proposed to obtain exact solutions of nonlinear partial differential equations, we obtain some new solutions such as symmetric hyperbolic Fibonacci sin, cosine and tangent functions. Also, some basic properties of symmetric Fibonacci and Lucas functions are given in this research.

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1. INTRODUCTION

In the past few decades, there has been ongoing researches in various nonlinear evolution equations. A lot of methods have been used to handle nonlinear equations with constant coefficients and with time dependent coefficients. In this study, new exact solutions including 1-soliton and singular solutions in terms of symmetric hyperbolic Fibonacci functions are given using the modified Kudryashov method. The developed method is based on the well known Riccati equation. The obtained solutions are expressed in physical comments. We consider the Rosenau-Kawahara equation with power law nonlinearity [3,20]

$$u_t + au_x + bu^m u_x + cu_{xxx} + \lambda u_{xxxt} - \nu u_{xxxxx} = 0, \tag{1.1}$$

which is a nonlinear PDE appears in the study of fluid dynamics and falls in the category of NLEE. Here, in Eq. (1), a, b, c, λ, and ν are all constants and the parameter m dictates the power law nonlinearity. The first term is the evolution term. The coefficients of a, c and λ are dispersion terms while the coefficient of ν represents the viscous term [3,19]. Many authors have studied this equation to find new types of solutions. There are many methods that are used to obtain the integration of nonlinear partial differential equations. Some of them are the exp-function method [2,11-13], the trial equation method [4,6,8,10,15,16], the (G'/G)-expansion method [5], the Hirota's method [7,17], the auxiliary equation method [19], and many more. In this article, we modify Kudryashov's method [9] to raise the effectiveness of this method. Our aim is to derive new solutions to Eq (1) using the improved Kudryashov method [14]. Our key idea is that traditional base e of the exponential function is replaced by an arbitrary base $a \neq 1$. So, new exact solutions of nonlinear differential equations may be obtained by this simple modification.

2. The modified Kudryashov method

We consider the following nonlinear partial differential equation for a function q of two real variables, space x and time t:

$$P(t, x, q, q_t, q_x, q_{tt}, q_{tx}, q_{xx}, ...) = 0. \tag{2.1}$$

It is useful to summarize the main steps of modified Kudryashov method as follows:

Step 1. We seek the travelling wave solution of Eq. (2.1) of the form

$$q(x,t) = u(\xi), \quad \xi = B(x - wt), \tag{2.2}$$

where B is a free constant. We reduce Eq. (2.1) to a nonlinear ordinary differential equation of the form:

$$N(t, x, u, u', u'', ...) = 0, \tag{2.3}$$

where the prime denotes differentiation with respect to ξ. Suppose that the highest order nonlinear terms in Eq. (2.3) are $u^l(\xi)u^{(s)}(\xi)$ and $(u^{(p)})^k$.

Step 2. We suppose that the exact solutions of Eq. (2.3) can be obtained in the following form:

$$u(\xi) = y(\xi) = \sum_{j=0}^{N} a_j Q^j, \tag{2.4}$$

where $Q = \frac{1}{1 \pm a^{\xi}}$. We note that the function Q is solution of equation

$$Q_{\xi} = lna(Q^2 - Q). \tag{2.5}$$

Step 3. According to the proposed method, we assume that the solution of Eq. (2.3) can be expressed in the form

$$u(\xi) = a_N Q^N + \ldots \tag{2.6}$$

Calculation of value N in formula (2.6) that is the pole order for the general solution of Eq. (2.3). To determine the value of N we proceed analogously as in the classical Kudryashov method on balancing the highest order nonlinear terms in Eq. (2.3). More precisely, by straightforward calculations we have

$$u'(\xi) = a_N N Q^{N+1} + \ldots, \tag{2.7}$$

$$u''(\xi) = a_N N(N+1) Q^{N+2} + \ldots, \tag{2.8}$$

$$u^{(s)}(\xi) = a_N N(N+1)\ldots(N+s-1) Q^{N+s} + \ldots, \tag{2.9}$$

$$u^l u^{(s)}(\xi) = \overline{a_N} N(N+1)\ldots(N+s-1) Q^{(l+1)N+s} + \ldots, \tag{2.10}$$

$$(u^{(p)})^k(\xi) = (a_N N(N+1)\ldots(N+s-1))^k \, Q^{k(N+p)} + \ldots, \tag{2.11}$$

where a_N and $\overline{a_N}$ are constant coefficients. Balancing the highest order nonlinear terms of Eq. (2.10) and Eq. (2.11), we have

$$(l+1)N + s = k(N+p), \tag{2.12}$$

so

$$N = \frac{s-kp}{k-l-1}. \tag{2.13}$$

Step 4. Substituting Eq. (2.4) into Eq. (2.3) yields a polynomial $R(Q)$ of Q. Setting the coefficients of $R(Q)$ to zero, we get a system of algebraic equations. Solving this system, we shall determine $w(t)$ and the variable coefficients of $a_0(t), a_1(t), ..., a_N(t)$. Thus, we obtain the exact solutions to Eq. (2.1).

3. Application of the method to the Rosenau-Kawahara equation

We first assume that Eq (1) has solutions of the form

$$u(x,t) = v(\xi), \quad \xi = B(x-wt) \tag{3.1}$$

and substituting it into Eq.(1), we can reduce to the ordinary differential equation which can be written as

$$-Bwv'(\xi) + aBv'(\xi) + bBv^m v'(\xi) + cB^3 v'''(\xi) - \lambda B^5 v''''(\xi) + \nu B^5 v'''''(\xi) = 0. \tag{3.2}$$

Upon integration, Eq (3.2) is converted to

$$-Bwv(\xi) + aBv(\xi) + bB\frac{v^{m+1}}{m+1} + cB^3 v''(\xi) - \lambda B^5 v'''(\xi) + \nu B^5 v''''(\xi) = 0, \tag{3.3}$$

where C is the integration constant. For simplicity we take $C = 0$. We use the transformation

$$v(\xi) = V^{\frac{2}{m}}(\xi) \tag{3.4}$$

which will convert to Eq (3.3) into

$$(w-a)Bm^4(m+1)V^4 - bBm^4V^6 - cB^3m^2(m+1)V^2(V')^2 - 2cB^3m^3(m+1)V^3V''$$
$$+ B^5(\lambda w+\nu)(m+1)(4-2m)(2-2m)(2-3m)(m+1)(V')^4 + 2B^5(\lambda w+\nu)m^3(m+1)V^3V''''$$
$$+ B^5(\lambda w+\nu)m(m+1)(24m^2-72m+48)V(V')^2V'' + B^5(\lambda w+\nu)m^2(m+1)(16-8m)V^2V'V'''$$
$$-B^5(\lambda w+\nu)(6m^4-6m^3-12m^2)V^2(V'')^2 = 0 \tag{3.5}$$

We take

$$V(\xi) = y(\xi) = \sum_{n=0}^{N} a_n Q^N \tag{3.6}$$

where $Q = \frac{1}{1 \pm a^{\xi}}$. We note that the function Q is solution of equation

$$Q_\xi = lna(Q^2 - Q) \tag{3.7}$$

Considering the homogeneous balance with $V^3(V')^2$ and V^6 in Eq (3.5) gives

$$6N = 5N + 2, \tag{3.8}$$

$$N = 2. \tag{3.9}$$

Therefore we have

$$V(\xi) = y(\xi) = \sum_{N=0}^{2} a_N Q^N = a_0 + a_1 Q + a_2 Q^2, \tag{3.10}$$

and we substitute derivatives of the function $y(\xi)$ with respect to ξ. The required derivatives in Eq (3.5) are obtained

$$y_\xi = \ln a\left(-a_1 Q + a_1 Q^2 - 2a_2 Q^2 + 2a_2 Q^3\right), \tag{3.11}$$

$$y_{\xi\xi} = \ln^2 a\left(a_1 Q - 3a_1 Q^2 + 2a_1 Q^3 + 4a_2 Q^2 - 10a_2 Q^3 + 6a_2 Q^4\right), \tag{3.12}$$

$$y_{\xi\xi\xi} = \ln^3 a\left(-a_1 Q + 7a_1 Q^2 - 12a_1 Q^3 + 6a_1 Q^4 - 8a_2 Q^2 + 38a_2 Q^3 - 54a_2 Q^4 + 24a_2 Q^5\right), \tag{3.13}$$

$$y_{\xi\xi\xi} = \ln^4 a\Big\{a_1 Q - 15a_1 Q^2 + 50a_1 Q^3 - 60a_1 Q^4 + 24a_1 Q^5 + 16a_2 Q^2 - 130a_2 Q^3 + 330a_2 Q^4 - 336a_2 Q^5 + 120a_2 Q^6\Big\}. \tag{3.14}$$

As result of this we have the system of algebraic equations can be solved with Mathematica. Solving the system of algebraic equations, we obtain the coefficients a_0, a_1 and a_2 as follows:

$$a_0 = 0, \quad a_1 = -\sqrt{-\frac{(m+1)(m+4)(3m+4)\left((a\lambda+v)(m^2+4m+8)+\varphi\right)}{b\lambda(m+2)(m^2+4m+8)}},$$

$$a_2 = \sqrt{-\frac{(m+1)(m+4)(3m+4)\left((a\lambda+v)(m^2+4m+8)+\varphi\right)}{b\lambda(m+2)(m^2+4m+8)}},$$

$$B = -\frac{m}{2\ln a(m+2)}\sqrt{\frac{(a\lambda+v)(m^2+4m+8)+\varphi}{2c\lambda}},$$

$$w = -\frac{(v-a\lambda)(m^2+4m+8)+\varphi}{2\lambda(m^2+4m+8)}, \tag{3.15}$$

where a, b, c, λ, v are arbitrary constants and $\varphi = \sqrt{16\lambda c^2(m+2)^2 + (a\lambda+v)^2(m^2+4m+8)^2}$. Substituting Eq (3.15) into (3.10) and we have

$$V(\xi) = -\sqrt{-\frac{(m+1)(m+4)(3m+4)\left((a\lambda+v)(m^2+4m+8)+\varphi\right)}{b\lambda(m+2)(m^2+4m+8)}}\left(\frac{1}{1\pm a^\xi} - \frac{1}{(1\pm a^\xi)^2}\right) \tag{3.16}$$

where $\xi = -\frac{m}{2\ln a(m+2)}\sqrt{\frac{(a\lambda+v)(m^2+4m+8)+\varphi}{2c\lambda}}\left(x + \frac{(v-a\lambda)(m^2+4m+8)+\varphi}{2\lambda(m^2+4m+8)}t\right)$. Substituting Eq. (3.16) into (3.4) and applying several simple transformations to these solutions, we obtain new exact solutions to Eq (1)

$$u_1(x,t) = \frac{A_1}{\mathrm{cFs}^{\frac{4}{m}}[B(x-wt)]}, \tag{3.17}$$

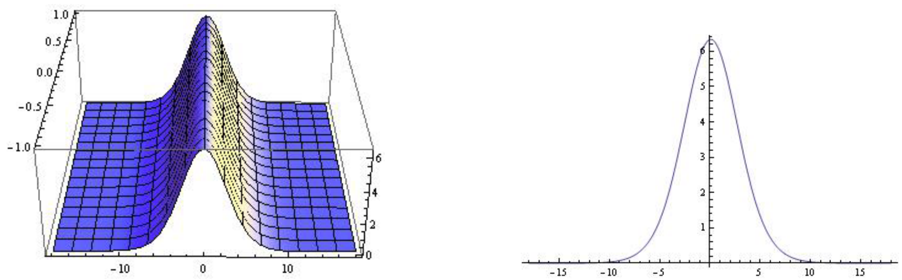

Figure 1. Solution of $u_1(x,t)$ is shown at $a = \frac{1}{8}$, $b = -\frac{25}{296}$, $c = \frac{5}{14}$, $\lambda = 16$, $\nu = -2$, $m = \frac{4}{5}$ and the second graph represents the exact approximate solution of Eq. (31) for $t = 1$.

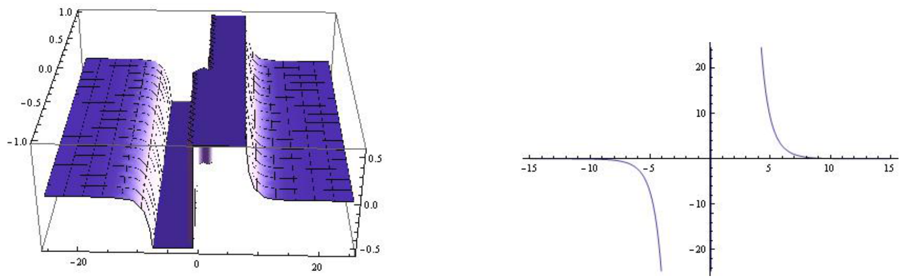

Figure 2. Solution of $u_2(x,t)$ is shown at $a = \frac{1}{8}$, $b = -\frac{25}{296}$, $c = \frac{5}{14}$, $\lambda = 16$, $\nu = -2$, $m = \frac{4}{5}$ and the second graph represents the exact approximate solution of Eq. (32) for $t = 1$.

$$u_2(x,t) = \frac{A_2}{\mathrm{sFs}^{\frac{4}{m}}[B(x-wt)]}, \tag{3.18}$$

where

$$A_\epsilon = \left((-1)^\epsilon \sqrt{-\frac{(m+1)(m+4)(3m+4)\left((a\lambda+\nu)(m^2+4m+8)+\varphi\right)}{b\lambda(m+2)(m^2+4m+8)}}\right)^{\frac{2}{m}}, (\epsilon = 1, 2.),$$

$$B = -\frac{m}{2\ln a(m+2)}\sqrt{\frac{(a\lambda+\nu)(m^2+4m+8)+\varphi}{2c\lambda}},$$

and

$$w = -\frac{(\nu - a\lambda)(m^2+4m+8)+\varphi}{2\lambda(m^2+4m+8)}.$$

Here, A_1, A_2 represent the amplitude of the solitons, while B is the inverse width of the solitons and w represents the velocity of the solitons. Also, Eq. (3.18) is a singular soliton solution for Eq. (1).

4. Remarks and Conclusion

Our aim in this section is to show that general Exp_a-function with Kudryashov method could be used to one solutions in the form of symmetrical hyperbolic Fibonacci and Lucas functions. We highlight briefly the definitions of symmetrical hyperbolic Fibonacci and Lucas functions. Also Stakhov and Rozin [1,18] defined all details of symmetrical hyperbolic Fibonacci and Lucas functions. We only give formulas here. Symmetrical Fibonacci sin, cosine and tangent are respectively defined as

$$sFs(x) = \frac{a^x - a^{-x}}{\sqrt{5}}, \quad cFs(x) = \frac{a^x + a^{-x}}{\sqrt{5}}, \quad tFs(x) = \frac{a^x - a^{-x}}{a^x + a^{-x}}. \tag{4.1}$$

Analogously, symmetrical Lucas sin and cosine are respectively defined as

$$sLs(x) = a^x - a^{-x}, \quad cLs(x) = a^x + a^{-x}, \tag{4.2}$$

where $a = \frac{1+\sqrt{5}}{2}$, which is known in the literature as Golden Mean [18]. So we can find more general (or more larger classes of) solutions in applying the general Exp_a-function method with Symmetrical Fibonacci functions. If we take $a = e$ then we can find other solutions also.

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