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Research Article

Calculating the Position and Radius of the Center of a Curve Whose Horizontal Distance and Deflection Angle Are Measured from a Station in the Line of the Braces to the Circle Arc

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Abstract

In the production of the existing map, which is the basis for the construction of the zoning plans, and in the first facility cadastre, structures such as pools, buildings and minarets with a circle base are encountered. From a geodetic station point, it is possible to obtain the position and radius of the center of this structure whose base is circular with two consecutive beam lengths and polar measurements marked on such a structure. As a matter of fact, this issue has been seen in a study on the subject. In this study, the geodetic position of the center of the structure and the radius of the structure were tried to be obtained from a station point by using the horizontal angle between the tangent points of a structure with a circular base and the horizontal distance measured in the direction of the bisector of this angle. Since some publications of foreign origin do not enlighten the subject of research, it was deemed necessary to investigate the subject. In this study, first a theoretical explanation of the research subject was made, a numerical application was made on the subject and the findings and opinions obtained were stated.

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Radius Calculation,
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BİR İSTASYONDAN SAPMA AÇISI VE AÇIORTAYI DOĞRULTUSUNDA DAİRE YAYINA KADAR YATAY UZAKLIĞI ÖLÇÜLEN BİR KURBUN YARIÇAPININ VE MERKEZİNİN KONUMUNUN HESABI

Özet

İmar planlarının yapımına altlık olan halihazır harita üretiminde ve ilk tesis kadastrosunda arazide, tabanı daire şeklinde olan havuz, bina ve minare gibi yapılara rastlanılmaktadır. Jeodezik bir istasyon noktasından, böyle bir yapı üzerinde işaretlenen ardışık iki kiriş uzunlukları ve kutupsal ölçmeler ile tabanı daire şeklinde olan bu yapının merkezinin konumunu ve yarıçapını elde etmek mümkündür. Nitekim bu husus, konuyla ilgili bir çalışmada görülmüştür. Bu

Anahtar Kelimeler
Yarıçap Hesaplama,
Daire Merkezi Koordinatlarını Bulma,
Kutupsal Ölçümler

çalışmada ise, bir istasyon noktasından, tabanı daire şeklinde olan bir yapının teğet noktaları arasındaki yatay açı ile bu açının açıortayı doğrultusunda yapıya kadar ölçülen yatay uzaklık yardımıyla, yapı merkezinin jeodezik konumu ve yapının yarıçapı elde edilmeye çalışılmıştır. Bu konuda yabancı kaynaklı bazı yayınlar da araştırma konusunu aydınlatmadığı için konunun araştırılması gerekli görülmüştür. Bu çalışmada, önce araştırma konusunun teorik açıklaması yapılmış, konuyla ilgili sayısal uygulama yapılmış ve elde edilen bulgular ve kanaatler belirtilmiştir.

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INTRODUCTION

In the production of the existing map and the first facility cadastre, which is the basis for the construction of the zoning plans; In the land, structures such as pools, buildings and minarets with a circular base are encountered. It is possible to measure two consecutive beam lengths and horizontal angles marked on the structure in order to obtain the geodetic position and radius of the center of this structure after connecting to a connection point with an electronic tachometer installed at a station point [1].

Alternative solution methods can be produced to obtain the coordinate and radius of the center of the building without measuring two consecutive beams in this structure for a structure with a circular base. One of these methods is to measure the tangent length from the station point to the structure and the horizontal angle between the tangent directions.

The other method is to measure the horizontal angle between the direction's tangent to the structure from the station point and the horizontal distance to the structure in the direction of the bisector of this angle. The solution of the problem is easy with the first method. However, there are two studies [2, 3] in the literature regarding the solution of the problem with the second method.

It was seen that these studies were not sufficient to enlighten the subject and therefore it was decided to investigate the subject. In this study; first, a theoretical solution proposal on the subject was presented, numerical applications on the subject were made and the findings and opinions obtained were stated.

CALCULATION OF THE POSITION OF THE RADIUS AND CENTER OF THE CIRCLE ARC MEASURING TANGENT LENGTH AND DEFLECTION ANGLE FROM A STATION

With the electronic tachometer installed at P_2 point in the field, first connection to P_1 point was made. After that, the tangent points B and C of the structure with a circle base were observed and the horizontal distances P_2B and P_2C and the angles β_B and β_C were measured (Figure 1).

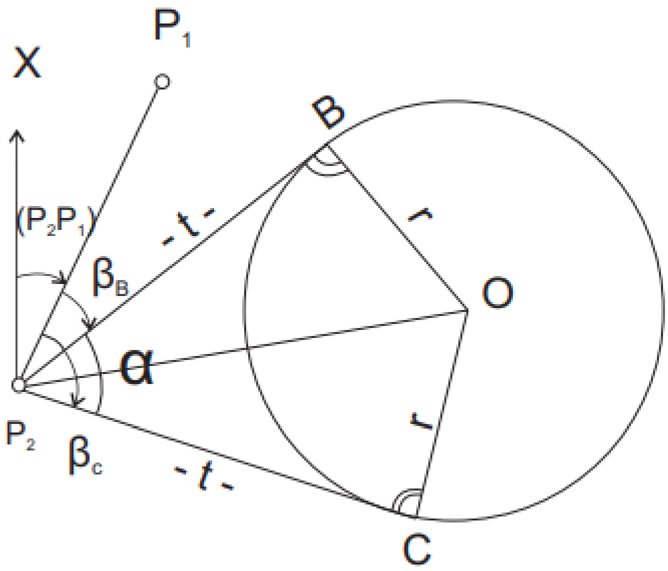

Figure 1. Measuring Tangent Length and Deflection Angle from a Station

Considering the coordinates of P_2 and P_1 points (P_2P_1) the bearing angle is obtained from the following relations [4-6];

$\Delta Y_{P2P1}=Y_{P1}-Y_{P2}$, $\Delta X_{P2P1}=X_{P1}-X_{P2}$

$(P_2P_1)=\arctan(\frac{\Delta Y_{P2P1}}{\Delta X_{P2P1}})$ (1)

In Figure 1, the radius r in the right triangle P_2BO is obtained from the following relations [7].

$\alpha=\beta_C - \beta_B$ (2)

$r=P_2B*\tan(\alpha/2)$ (3)

P_2O horizontal length and (P_2O) bearing angle are obtained from the following equations [8];

$P_2O=\sqrt{(P2B^2+r^2)}$ (4)

$(P_2O)= (P_2P_1) + \beta_B+(\alpha/2)$

$(P2O) = (P2P1) + \beta B + (\alpha / 2)$ (5)

The coordinate of the O circle center (Y_0, X_0) is obtained from the following relations [9, 10].

$Y_O =Y_{P2} + P_2O*\sin(P_2O)$, $X_O =X_{P2} + P_2O*\cos(P_2O)$ (6)

The following pathway is followed for control:

In Figure 1 (P_2B) the bearing angle and the coordinate of the B point are calculated with the following equations.

$(P_2B)= (P_2P_1)+ \beta_B$ (7)

$Y_B = Y_{P2} + P_2B*\sin(P_2B)$, $X_B = X_{P2} + P_2B*\cos(P_2B)$ (8)

(BO) bearing angle and O coordinate of circle center (Y_0, X_0) are obtained from the following equations in a controlled manner.

(BO)= (BP$_2$P$_1$)-100 (9)

$Y_O = Y_B + r*\sin(BO)$, $X_O = X_B + r*\cos(BO)$ (10)

CALCULATING THE POSITION AND RADIUS OF THE CENTER OF A CURVE WHOSE HORIZONTAL DISTANCE AND DEFLECTION ANGLE ARE MEASURED FROM A STATION IN THE LINE OF THE BRACES TO THE CIRCLE ARC

After connecting to P_1 point with the electronic tachometer installed at P_2 point in the field, the horizontal distance $S = P_2D$ in the direction of the bisector with angles β_B and β_C was measured by looking at the tangent points B and C of the structure with a circular base (Figure 2).

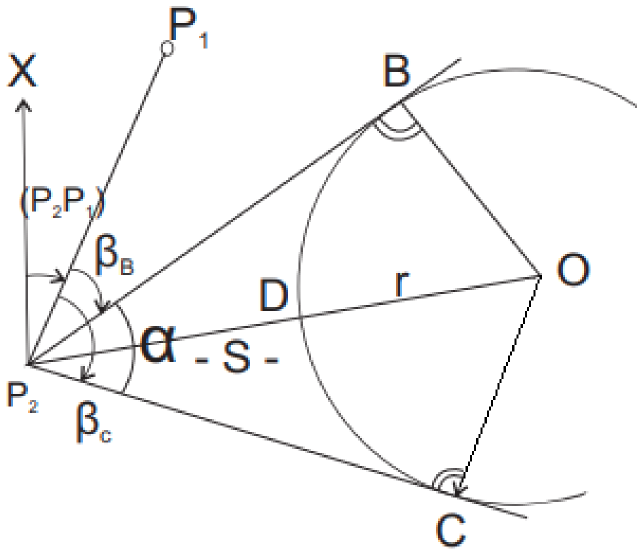

Figure 2. Measuring from a Station in the Line of the Braces to the Circle Arc

In Figure 2, the following equation is written for the horizontal distance P_2B.

$$P_2B=\sqrt{((S+r)^2-r^2)}=\sqrt{(S^2+2Sr+r^2-r^2)}=\sqrt{(S^2+2Sr)} \quad (10)$$

Write the following equation for P_2B from equation (3) (Cuomo, 1997).

$$P_2B=r/\tan(\alpha/2) \quad (11)$$

If the equations (10) and (11) are equalized and squared, the following equation is obtained;

$$(S^2+2Sr)=r^2/(\tan\alpha/2)^2 \quad (12)$$

$$\frac{r^2}{\left(\tan\frac{\alpha}{2}\right)^2}-2Sr-S^2=0 \quad (13)$$

The roots of this equation are expressed as;

$$r_{1,2}=\frac{2S\mp\sqrt{(4S^2+4S^2/(\tan\alpha/2)^2)}}{2/(\tan\alpha/2)^2}\ \frac{2S\mp\sqrt{4S^2(1+1/(\tan\alpha/2)^2)}}{2/(\tan\alpha/2)^2}=\frac{2S\mp2S\sqrt{(1+1/(\tan\alpha/2)^2)}}{2/(\tan\alpha/2)^2} \quad (14)$$

Since the radius r of the circle is usually smaller than S, only the following equation is written for r_1 in equation (14);

$$r_1 = \frac{2S+2S\sqrt{(1+1/(tan\alpha/2)^2)}}{2/(tan\alpha/2)^2} \qquad (15)$$

P_2O edge required to calculate the coordinate of the circle center is found from the following equation.

$P_2O = P_2D + r$

The bearing angle (P_2O) is calculated by equation (5) and the coordinate of O by equation (6). For control, after calculating r from equation (15), the horizontal length P_2B is calculated from equation (11). The coordinate of the point B with the relations (7) and (8) and the coordinate of the O by the relations (9), (10) are calculated in a controlled manner. In the measurement of the horizontal distance from the station point to the circle-based structure in the direction of the bisector, an error occurs due to the thickness of the reflector head. In order to eliminate this error, a special paper reflector is placed at the target point.

This reflector is usually made of thin cardboard and measures 6 cm*6 cm (Figure 3). If the special paper reflector cannot be found, the offset value written on the used reflector must be entered into the memory of the electronic tachometer (Figure 4). Offset value of the reflector is the horizontal distance of a reflector head from the center of the reflector.

Figure 3- Special paper reflector [11] Figure 4- Reflector head where Offset is written [12]

NUMERICAL APPLICATIONS

Calculating the Coordinate and Radius of The Circle Center with The Help of The Tangent Length and the Angle Between the Tangent Points Measured from A Station

In accordance with Figure 1, after connecting to P_1 connection point with the electronic tachometer installed at station P_2, polar measurements were made in Table 1 by looking at the tangent points B and C of the structure with a circular base. Using the coordinates of the points and the measurements made, calculate the coordinate of the center of the circle and the radius of the circle.

Table 1- Coordinates of polar coordinates and polygons made from point P_2

Station number	Number of points of observation	Horizontal Angle	Horizontal Distance	Y	X
P_2				2501.42	3510.45
	P_1	0.0000		2586.13	3850.66
	B	83.2256	50.990		
	C	85.7223	50.990		

Solution;

$\Delta Y_{P2P1} = Y_{P1} - Y_{P2}$ = + 84.71 m, $\Delta X_{P2P1} = X_{P1} - X_{P2}$=340.21 m

$(P_2P_1)=\arctan(\frac{84.71}{340.21}) = 15^g.5384$

$\alpha=\beta_C - \beta_B$=2.4967, r=P_2B*tan(α/2)=50.99*tan(2.4967/2)=0.999999=1.00 m

$(P_2O)= (P_2P_1)+ \beta_B+(\alpha/2)$=15.5384+83.2256+(2.4967/2)=$100^g.0124$

$P_2O=\sqrt{(P_2B^2 + r^2)} = \sqrt{(50.99^2 + 1^2)}$ =50.99988 m=51.00 m

$Y_O = Y_{P2} + P_2O*\sin(P_2O)$= 2501.42+51.00*sin100.0124=2552.420 m

$X_O = X_{P2} + P_2O*\cos(P_2O)$=3510.45+51.00*cos100.0124=3510.440 m

Control;

$(P_2B)= (P_2P_1)+ \beta_B$ =98.7640 ,

$Y_B = Y_{P2} + P_2B*\sin(P_2B)$= 2501.42+50.99*sin98.7640 =2552.400 m

$X_B = X_{P2} + P_2B*\cos(P_2B)$= 3510.45+50.99*cos98.7640 =3511.440 m

$(BP_2)= (P_2B)\pm200$ =298.7640

$(BO)=(BP_2)-100^g$ =198.7640

$Y_O = Y_B + r*\sin(BO)$= 2552.400+1.00*sin198.7640=2552.420 m

$X_O = X_B + r*\cos(BO)$=3511.440+51.00*cos198.7640=3510.440 m

Solution of the Problem by the help of the horizontal distance to the structure in the direction of the angle and the bisector at the tangent points of the structure measured from the station point

In accordance with Figure 2, after the connection was made to P1 point with the electronic tachometer installed at station P2, polar measurements were made at the tangent points B and C of the circular structure and at point D in the bisector direction, in Table 2. Using the coordinates of the points and the measurements made, calculate the coordinate of the center of the circle and the radius of the circle.

Table 2- Coordinates of polar coordinates and polygons made from point P_2

Station number	Number of points of observation	Horizontal Angle	Horizontal Distance	Y	X
P_2				2501.42	3510.45
	P_1	0.0000		2586.13	3850.66
	B	83.2256			
	C	85.7223			
	D	84.4740	50.000		

Solution;

$\Delta Y_{P2P1} = Y_{P1} - Y_{P2}$ = + 84.71 m, $\Delta X_{P2P1} = X_{P1} - X_{P2}$=340.21 m

(P_2P_1)=arctan($\frac{84.71}{340.21}$) $= 15^g.5384$

α=85.7223-83.2256=2^g.4967, α/2=1^g.24835

$r_1 = \frac{2S+2S\sqrt{(1+1/(tan\alpha/2)^2)}}{2/(tan\alpha/2)^2} = \frac{100+100\sqrt{(1+1/(tan2.4967/2)^2)}}{2/(tan2.4967/2)^2}$ =1.0003 m

$P_2O= P_2D+r$=50.000+1.000=51.000m

$(P_2O)= (P_2P_1)+ \beta_B+(\alpha/2)$=15.5384+83.2256+(2.4967/2)=100^g.0124

$Y_O = Y_{P2} + P_2O*\sin(P_2O)$= 2501.42+51.00*sin100.0124=2552.420 m

$X_O = X_{P2} + P_2O*\cos(P_2O)$=3510.45+51.00*cos100.0124=3510.440 m

Control;

$(P_2B)= (P_2P_1)+ \beta_B$ =98.7640 ,

$Y_B = Y_{P2} + P_2B*\sin(P_2B)$= 2501.42+50.99*sin98.7640 =2552.400 m

$X_B = X_{P2} + P_2B*\cos(P_2B)$= 3510.45+50.99*cos98.7640 =3511..440 m

$(BP_2)= (P_2B)\pm 200$ =298.7640

$(BO)=(BP_2)-100^g$ =198.7640

$Y_O = Y_B + r*\sin(BO)$= 2552.400+1.00*sin198.7640=2552.420 m

$X_O = X_B + r*\cos(BO)$=3511.440+51.00*cos198.7640=3510.440 m

CONCLUSIONS and RECOMMENDATIONS

With the electronic tachometer installed at the station point, a measurement error occurs due to the thickness of the reflector in the measurement of the horizontal distance to the circle-based structure in the bisector direction. In order to eliminate this error, a paper reflector should be used at the target point, if there is no paper reflector, the offset value of the reflector used should be entered into the memory of the device. The angle of deviation between the tangent points of the circle-based structure measured at the station point should be measured at least for one complete series in order to zero the effect of the collimation error on the measurement. In measuring the tangent lengths of the circle-based structure from a station point, the measurements should be millimetric precision and the measurement differences due to the inhomogeneity of the plaster thickness in the structure should be at minimum value. In the calculation of the position of the circle center and the radius of a circle-based structure; it has been observed that the method of measuring tangent length and divergence angle is much easier than other methods. It can be said that the calculation of the circle center and radius with the method specified here is easier than the beam measurement method.

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