A statistical study of pulsating stars.

First paper: *The variable stars in ω Centauri*
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Özet: ω Cent. de bulunan külliyetli miktarda değişen yıldız otokorelasyon ve müteakib kuvvet serisi metodu ile incelenmiştir. Bu suretle esas ve muhtelif üst tonların şiddeti elde edilmiştir. Bu çalışmanın mevzuu bilhassa ilk üst ton A(2) nin izafî şiddetidir. Muhtelif Cluster tipi değişen yıldızlar ve birkaç δ Cephei tipi yıldızlar için A(2) değerleri mütekabil peryotların logaritmasına göre izdüşürülmüştür. Elde edilen diyagramda bir tarafta Bailey *c* üst-tipi, diğer tarafta *a* ve *b* üst-tipleri olmak üzere, aralarındaki fark, A(2) seviyesindeki farkdan aşikârdır. Muhtelif üst-tipler arasında farklı seviyelerin mevcut olabileceğine dair bir emare mevcuttur.

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Abstract: A large number of the variables in ω Cent. has been analysed by the autocorrelation and subsequent power series method. In that way the intensities of the fundamental and of the various overtones are obtained. Subject of this study is especially the relative intensity A(2) of the first overtone. For the various types of cluster type variables and for the few δ Cephei stars the values of A(2) are plotted against the corresponding logarithm of the period. From the resulting diagram the difference between the Bailey sub type *c* on one side and the sub-types *a* and *c* on the other sides is evident from the difference in the level of A(2). There is an indication that within the different sub-types different sublevels may occur.

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§ 1. Introduction.

In a previous article[1] the autocorrelation method as developed by M. G. Kendall [2] was applied to several artificial light curves. The resulting autocorrelation curves were expanded into power series[3]. Evidence was given that if this method is applied to strictly periodical curves in the resulting power series the coefficients of terms with frequency ν automatically reduce to zero in all cases where the ratio ν : N is a non integer value.

Here N is the frequency of the fundamental. If the original curve is a sinus or cosinus curve, the coefficient of the terms with $\nu = 2N$; $\nu = 3N$ etc. also reduce to zero and we only retain the term with $\nu = N$. With other periodical curves the coefficients of the terms with $\nu = 2N$; $\nu = 3N$ are $\neq 0$ and therefore with such curves we obtain a series of overtones. The question was raised whether any physical significance can be attached to these overtones. There can be no doubt that in a purely mathematical way the series of events represented by the periodical curve can be represented by a power series. But that is not the point which is under discussing. We want to ascertain whether or not, when the periodical light curve of a variable can be represented by a power series containing the fundamental term and at least a few of the first overtones, these overtones correspond to separate pulsations in the atmosphere of the star.

As an example curves were considered resulting from a strictly periodic disturbance affecting the atmosphere of some hypothetical star. It was supposed that the disturbance repeated itself at regular intervals, while the pulsation which each separate disturbance sets up in the atmosphere is strongly damped. By a suitable choice of the various constants these artificial curves could be made closely to resemble the actual light curves of Cepheid and cluster type variables.

It is almost needless to state that for the present at least this resemblance is merely superficial because no relation has been established between the constants of the curve and the physical constants in the atmosphere of the star.

The interesting point is that when the autocorrelation plus power series method is applied to the artificial curves, apart from the fundamental we find a series of overtones of decreasing intensities.

From the way in which the artificial curves were constructed it is evident, that there is only one period which has any physical meaning. This is the fundamental period which is equal to the period of the disturbance. That the net result of the consecutive disturbances and the damping in the atmosphere can be represented by a power series is convenient, but it is evident that it would be erroneous to conclude that the atmosphere of the star is affected by more than one pulsation.

This point should of course not be overemphasised when the autocorrelation and power series method is applied to the actually observed light curves of variables.

In all cases where the observed curves are not a pure sinus curve apart from the fundamental we will find a series of overtones. This quite apart from the question whether in the atmosphere of the star overtones do exist or not. Therefore if by our mathematical analysis overtones are thrown up, these overtones may either correspond to real additional pulsations or may have a purely formal value e. g. in mathematical terms describe the shape of the light curve.

Consequently when in the analysis of the observed curves, overtones are obtained, to these overtones only formal value can be attached. This does not exclude the possibility of a physical reality, but this physical reality must separately be proven from other evidence.

When the autocorrelation method and the subsequent development into a power series is applied to the observed light curves of cluster type and Cepheid variables, the great advantage is that we obtain a set of numerical values which in an objective way describe the shape of the observed curve.

The method also has certain limitations. In the first place the autocorrelation function corresponding to a certain curve does not uniquely describe that curve, nor does therefore the power series which is derived from the autocorrelation, function. Consequently if we describe the shape of the observed light curve by the numerical values of the coefficient obtained from the autocorrelation and power series analysis, this description is incomplete. It will be shown in section 3 that this incompleteness merely consists in the fact that the intensities of the fundamental and of the overtones of all observed curves are expressed on one and the same scale. This scale is such that the intensity of any pure sinus curve is equal to unity. This therefore means that in our analysis the true amplitudes are neglected. The fact that all observed light curves are reduced to one and the same scale, has several advantages which will appear in the following. Eventually the drawback that the actual amplitude is neglected, could be removed by multipying the coefficients obtained from the mathematical analysis with the mean value of the amplitude. Up till now we have not found it an advantage to do so. A second point is that there may be a faseshift between the various overtones and the fundamental, but these faseshifts are neglected in the analysis. This objection is not serious because for the present these eventual faseshifts are not important. More important is a further limitation which is not immediately evident. With the otocorrelation method large numbers of observations must be available which are spaced at equal time intervals and evenly cover all consecutive periods. The condition that the observations be spaced at equal time intervals offers no large difficulties. It is sufficient if large numbers of observations are available, so that a smooth curve can be drawn through the observed point. Then by simple interpolation we obtain the values at time intervals which are equally spaced. The real difficulty is in the second condition, which states that all consecutive periods must be evenly covered For evident reasons, especially with light curves with periods shorter than a few days, this condition can never be met. Variable star observers are accustomed to compile their results by given the normal curve. This normal light curve is based on observations covering different periods, but reduced to one and the same epoch.

This normal curve therefore represents a certain mean of the light curves of the separate periods. But cluster type and Cepheid variables are known often to be erratic and no light curve is completely regular e. g. small changes of shape and amplitude occur. Of several pulsating variables even the period is not quite a constant.

With a few of the latter variables the simultaneous existence has been suggested of two or more periods which are not wholly commensurate. As a result a beat period is set up causing a periodic variation in the length of the period and the shape of the light curve.

Such details might show up in the autocorrelation analysis if the available observations evenly cover all periods, but as this is not the case we have to content ourselves with the analysis of the normal curve. This implies that the normal light curve is assumed to repeat itself indefinitely over all periods, the shape, the amplitude and the length of the amplitude remaining constant. Consequently variations which are more or less secular, do not show up.

§ 2. Outline of program.

Up till now most attempts for a more detailed classification of the cluster type and Cepheid variables have mainly been based on the general expressive given by the outward shape of the light curve. In this way some notable successes have been obtained.

The method of autocorrelation and subsequent power analysis opens the possibility objectively to describe the shape of the different light curves by giving the numerical values of a few parameters. These few parameters are the intensities of the various overtones, leaving out of consideration the question whether these overtones have any physical significance or not. It is therefore tempting to try a classification of the pulsating variables on the basis of their periods and the above mentioned parameters. If such a classification is to have any value, fairly large numbers of variables must be analysed so as to give results which are statistically reliable.

Therefore a program was drawn up in which is foreseen the analysis of considerable numbers of variable stars in:

a) different globular clusters;
b) different galactic clouds
c) which for some reason were collected in a few specific lists
d) in extragalactic systems

while also

e) a few variables were considered of which the period is not constant and of which the light curves are subject to changes.

It is hoped that in this way a sufficient material will be obtained for a detailed discussion and for intracomparison of the variables in the different systems.

The numerical results of this analysis will be given in this and in a number of subsequent papers. In this first paper especially the variables in the w Cent. cluster are considered.

§ 3. Brief summary of the method.

The method by which the light curves are analysed were introduced in the study of variable starts by Ashbrook, Duncombe and van Woerkom [3] while the implications of their method have been discussed elsewhere [4].

In this case the period is divided into 24 equal intervals numbered from 0 to 23 and from the normal light curve we read the magnitude $m_0, m_1, m_2 \cdots m_{23}$ corresponding to the beginning of these time intervals. To the values m_0, m_1 etc. a correction is applied to the amount $\Delta = -\sum_{i=0}^{i=23} m_i$ so that the mean value of the residuals $j_i = m_i + \Delta$ is equal to zero. Next the autocorrelation coefficients $r_0, r_1, r_2 \cdots r_{23}$ are computed from the relation

$$r_k = \frac{\sum_{i=0}^{i=23} (j_i \cdot j_{i+k})}{\left\{ \sum_{i=0}^{i=23} j_i^2 \sum_{i=0}^{i=23} j_{i+k} \right\}^{\frac{1}{2}}} = \frac{\sum_{i=0}^{i=23} (j_i \cdot j_{i+k})}{\sum_{i=0}^{i=23} j_i^2} = \frac{1}{C} \sum_{i=0}^{i=23} (j_i \cdot j_{i+k}) \quad (1.3)$$

where C for all values of K is a constant of which the numerical value is equal to $C = \Sigma j_i^2$

Subsequently the coefficients in the power spectrum are computed from the relation

$$\pi(f) = \frac{2}{N} \sum_{K=0}^{K=23} r_k \cos 2\pi f k \quad \cdots\cdots (2.3)$$

where f is the reciprocal of the trial period, but for reason mentioned before for f only the values 1, 2, 3, 4, 6 etc. have been used e.g. values which are commensurate with 24. The value $f = 1$ corresponds to the fundamental period and therefore is equal to the observed period of the light curve.

The power spectrum gives the mean squared amplitude of each frequency. The values considered in the following are the values

$$A(f) = \sqrt{\frac{\pi(f)}{\pi(1)}} \quad \cdots\cdots (3.3)$$

or, in other words, the ratios.

$$\frac{\text{mean amplitude of overtone}}{\text{mean amplitude of fundamental}}$$

From the shape of the relation (1.3) it is evident that the true amplitude of the light curve is eliminated so that all curves are reduced to one and the same scale. Consequently also the values $\pi(f)$ as defined by the equation (2.3) are on the same scale.

By means of the relation (3.3) the intensities of the overtones are expressed in terms of the intensity of the fundamental as unit.

It will be evident that the maximum and minimum values which $\pi(1)$ can have are 1 and 0 respectively. If $\pi(1) \rightarrow 1$ this indicates that the observed curve deviates only slightly from a sinus curve. In all cases where $\pi(1) < 1$ overtones are present.

The case $\pi(1) \rightarrow 0$ is an obvious absurdity. It would indicate that the fundamental period is of very small importance and no variable star observer would identificate the period of his variable with such a fundamental.

§ 4. The variable stars in ω Cent.

The light curves of the variable stars in the ω Centauri cluster have been studied by W. Chr. Martin [4]. As this cluster contains a very large number of variable stars, it would be too laborious a work to analyse all of them because the analyse of each curve involves a good deal of numerical computations. Therefore a number of variables were selected in such a way, that all periods are well represented. That is to say, that both with the very short and with the very long periods all stars were used.

With the intermediate periods a selection had to be made, which as far as possible was carried out in an entirely arbitrary way. But here also some extra systems were added having periods about equal to the one where the transition from the one subtype to another occurs. The result is that the distribution of the periods of the variables which we have used is not representative for the true distribution of the periods in the cluster. On the other hand as far as the distribution of the values $\pi(f)$ and $A(f)$ is concerned, our data should be fully representative. These data are collected in table 1. The arrangement of this table is discussed in the next session.

TABLE 1.

List of the variables in the ω Centauri cluster which have been considered in the present paper. For explanation of the meaning of the various columns see section 5.

No.	log P	type	π (1)	A (2)	A (3)	A (4)	$\frac{M_1+m}{r}$	Remarks
65	0.797—2	—	0.928	0.219	0.095	0.045	15.00	
94	0.254—1	*c*	.943	.195	.031	.055	14.80	
98	0.449—1	*c′*	.908	.164	.089	.081	14.83	
19	0.477—1	*c*	.955	.105	.159	.055	14.95	
121	0.483—1	*c*	.959	.155	.063	.071	14.65	
127	0.484—1	*c*	.977	.084	.063	—	14.73	
163	0.496—1	*c*	.903	.095	.089	—	14.65	
103	0.517—1	*c*	.852	.237	.195	.145	14.63	
124	0.521—1	*c′*	.938	.161	.114	.077	14.67	
137	0.524—1	*c′*	.955	.135	.077	.031	14.64	
101	0.533—1	*c′*	.957	.110	.071	.126	14.72	
83	0.553—1	*c′*	.959	.114	.118	.045	14.72	
158	0.565—1	*c*	.972	.071	.063	.071	14.53	
145	0.572—1	*c*	.914	.267	.095	.095	14.64	
89	0.574—1	*c*	.971	.110	.084	.063	14.72	
21	0.581—1	*c′*	.965	.055	.114	.071	14.51	
70	0.592—1	*c′*	.982	.077	.100	.000	14.70	
160	0.599—1	*c′*	.986	.045	.077	.000	14.72	
87	0.599—1	*c*	.974	.110	.095	.055	14.65	
95	0.607—1	*c*	.986	.071	.032	.045	14.74	
155	0.617—1	*c*	.972	.100	.071	.045	14.66	
117	0.625—1	*c*	.942	.195	.055	.071	14.66	
24	0.655—1	*c′*	.991	.095	.089	—	14.65	
112	0.676—1	*a*	.677	.468	.370	.247	14.42	irregular
47	0.686—1	*c*	.989	.071	.063	.032	14.50	
130	0.693—1	*a*	.752	.448	.261	.179	14.85	irregular
74	0.702—1	*a*	.733	.483	.316	.184	14.50	
59	0.714—1	*a*	.777	.430	.274	.130	14.69	irregular
68	0.728—1	*c*	.988	.032	.045	.000	14.41	
120	0.740—1	*a*	.835	.435	.226	.114	14.74	irregular
67	0.751—1	*a*	.737	.447	.290	.212	14.73	irregular
44	0.753—1	*a*	.719	.480	.303	.176	14.80	
51	0.759—1	*a*	.685	.451	.333	.268	14.51	
84	0.763—1	*a*	.776	.459	.261	.071	14.49	
45	0.770—1	*a*	.717	.468	.336	.187	14.57	irregular
108	0.774—1	*a*	.679	.519	.341	.214	14.82	
90	0.474—1	*a*	.674	.554	.327	.148	14.27	

TABLE 1. (continued)

No.	log P	type	π (1)	A (2)	A (3)	A (4)	$\frac{M_1+m}{2}$	Remarks
79	0.784—1	*a*	.677	.522	.319	.207	14.62	
20	0.790—1	*a*	.807	.406	.217	—	14.60	
27	0.790—1	*b*	.687	.539	.316	.217	14.84	
32	0.792—1	*a*	.748	.442	.235	.182	14.54	
146	0.801—1	*a*	.692	.515	.307	.182	14.32	
86	0.810—1	*a*	.718	.502	.285	.217	14.57	
134	0.815—1	*a*	.692	.526	.290	.214	14.57	
52	0.819—1	*a*	.680	.504	.272	.224	13.91	
114	0.829—1	*a*	.656	.607	.224	.214	14.38	
149	0.834—1	*a*	.678	.496	.394	.217	14.52	
97	0.840—1	*a*	.738	.465	.292	.187	14.64	
116	0.857—1	*b*	.807	.423	.224	.122	14.44	
34	0.866—1	*a*	.750	.430	.266	.158	14.65	
109	0.872—1	*a*	.700	.549	.326	.063	14.51	
111	0.883—1	*b*	.741	.469	.272	.122	14.49	
54	0.888—1	*b*	.794	.464	.232	.114	14.64	
15	0.909—1	*b*	.782	.447	.232	.118	14.56	
143	0.914—1	*b*	.735	.498	.239	.084	14.50	
128	0.922—1	*b*	.862	.371	.152	.071	14.56	
144	0.922—1	*b*	.846	.336	.228	.032	14.57	
3	0.925—1	*b*	.774	.457	.207	.114	14.65	
104	0.938—1	*b*	.905	.285	.114	.105	14.74	
91	0.952—1	*b*	.840	.373	.182	.114	14 58	
150	0.954—1	*b*	.878	.298	.167	.032	14.50	
43	0.063	δ	.916	.319	.118	—	13.98	
92	0.129	δ	.915	.217	.045	.089	14.34	
60	0.130	δ	.924	.222	—	—	13.90	
61	0.356	δ	.917	.228	.105	.071	14.10	
48	0.650	δ	.914	.285	.063	.071	13.52	
29	1.679	δ	.939	.217	.239	.055	12.97	

§ 5. Arrangement of the table.

After the aforegoing explanations it is sufficient briefly to indicate the meaning of the different columns of the table.

In the table the variables are arranged in order of increasing period.

Column 1. Gives the number by which the variable is designated in Martin's list [5].

Column 2. Gives the logarithm of the period.

Column 3. Gives the Bailey subtype viz. the type *a*, *b* or *c*. To the three Bailey subtypes Martin has added a fourth one, by subdividing the *c* stars into the types *c* and *c'*.

Column 4. Gives the values $\pi(1)$ as computed from the equation (2. 3) and therefore indicates the intensity of the fundamental expressed on a scale on which the intensity of a pure sinus curve is equal to 1.

Columns 5, 6 and 7. Give the values A(2), A(3) and A(4) respectively as computed from (3. 3). They therefore indicate the relative importance of the various overtones. It is to be observed that the numerical values of $\pi(f)$ rapidly decrease with increasing values of f. Consequently in most cases only little confidence can be placed on the numerical values of A(4).

In some instances they have been omitted altogether.

The series of values A(2), A(3), etc. strongly vary from one system to the other. With the systems for which $\pi(1)$ is large, even the values $\pi(3)$ may become too small to give reliable values of A(3). Consequently only the values A(2) are suitable for a thorough statistical discussion.

Column 8. Gives the median magnitude $(M+m):2$, where M is the maximum and m the minimum value of the magnitude. The values of the median magnitude were borrowed from the table given by Martin (1.c.).

Column 9. In this column several remarks are collected concerning the systems which are considered.

§ 6. Discussion of the results.

In figure 1, the values A(2) as given in the table, are plotted against the corresponding value log P. Different symbols were used to indicate the various Bailey subtypes.

For indicating the *c* stars, black dots surrounded by a circle were used the latter, especially to indicate the subtype *c′* introduced by Martin. The one star with abnormally short period (No. 67 with log P = 0.797-2) is also indicated by a black dot, but it should be inferred from this that it belongs to Bailey's type *c*. For the subtypes *a* and *b* different crosses were used, while the δ Cephei stars have been indicated by triangles.

Several of the systems in ω Cent. are indicated by Martin as being irregular. Of these the periods and the light curves appear to vary from one period to the other. Of these irregular systems several are contained in our table (see column 9 of the table). To indicate these irregular variables, separate symbols, e.g. open circles, were used.

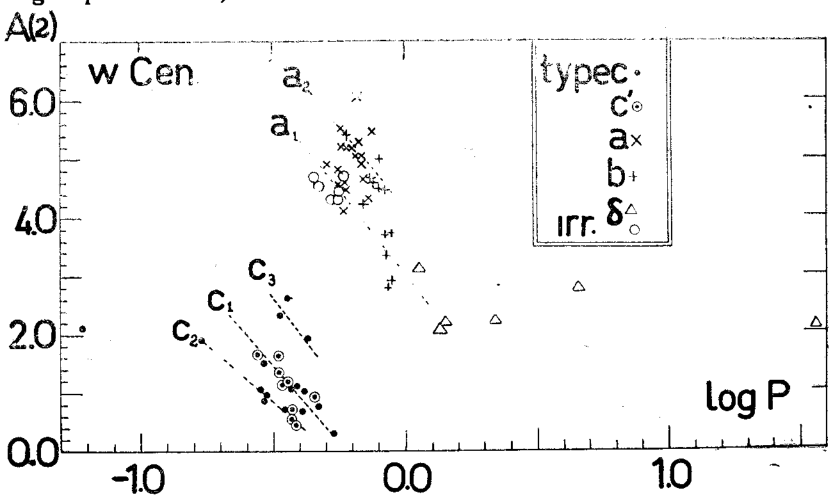

Fig. 1.

The resulting diagram shows some very interesting features. The level of the *c* type systems is well separated from the level of the *a* and *b* type systems. The most interesting point is that both with the *c* and the *a* and *b* type, the points seem to be arranged along several straight lines which are more or less parallel, thus possibly indicating a number of sublevels. As appears from the figure, these eventual sublevels are not very well separated and before they can be accepted as definite, it is better to await the further material which will result from the analysis of the variables in the other clusters and in the various galactic clouds.

For the present we will accept them as a working hypothesis. We will provisionally indicate the three sublevels which appear with the c stars, as the levels c_1, c_2 and c_3 respectively and the two levels which appaer with the a and b stars as the levels a_1 and a_2.

It is to be noticed that Martin's subtype c' is almost entirely confided to the sublevel c_1, while a few appear on the lower end of the level c_2. The c type systems are divided over the levels c_1, c_2 and c_3. The short period stars N 67 cannot be identified with any of these three levels.

The sublevel a_2 almost exclusively contains a type systems and only at the lower end some b type systems are present.

At its lower end the a_1 level contains b stars and at its upper end a type stars, but it is interesting to notice that the majority of these stars is indicated as being irregular.

The distribution of the irregular variables in our diagram seems to indicate that they form a small very compact group[6].

Altogether Martin denotes 16 of his variables as being irregular. The distribution of those systems which are contained in our table is so interesting, that the remaining ones are now being analysed.

Most of the short period δ Cephei variables seem to coincide with the a_1 level, and this might indicate that partly at least they are an extension towards longer periods of one type of cluster variables.

The number of Cepheids with longer period is so small, that no comment can be made about their distribution.

§ 7. Final remarks.

A possible splitting up of the Bailey types in subtypes of different levels suggests that there may also be a systematic difference between the magnitudes on the various sublevels. As pointed out by Martin, the magnitudes of the variables in ω Cent. are strongly affected by the Eberhard effect, the stars near the center of the cluster being systematically too bright. Consequently it is not possible for the different levels separately to determine a reliable period luminosity relation. So even if some magnitude effects were found, no confidence whatever could be placed on these results.

Moreover, the splitting up into separate levels should first be more fully confirmed and for the present I have therefore not further considered the question, whether the sublevels have any further special physical characteristics.

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