A statistical study of pulsating stars.

Eighth paper: *Variables in special lists.*

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Özet: Hertzsprung ve Joy tarafından iki hususî listede verilen Cepheid değişen yıldızları tetkik edildi. Muvakkatten *d* 1 ve *d* 2 seviyeleri olarak gösterilmiş olan iki ilâve seviyenin mevcut olduğu bulundu. Işık eğrileri modelinin değişim şekli P = 10 gün civarına çevrilmiştir. Bu noktadan evvel ışık eğrilerinde asimetri period ile azalır. Bu noktadan sonra asimetri peryot ile artar. Joy tarafından verilen ışık eğrilerinden görülürki ilâve bir *d* 3 seviyesi mevcut olabilir, fakat bu daha fazla delil olmadan kabul edilemez. Joy ayni şekilde tetkik edilen hız eğrilerini de verir. Muntazam değişimi teyid eden pozitif delil yoktur. Faz period ile artar. Bu artış muntazam olmayabilir. P = 10 gün civarında bir kesiklilik olabilir.

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Abstract: The Cepheid variables in two special lists by Hertzsprung and Joy are considered. Two additional levels are found to exist, which have provisionally been indicated as the levels *d* 1 and *d* 2. The way in which the pattern of the light curves changes, is inverted around P = 10 days. Before this point the assymetry of the light curve decreases with the period, beyond it the assymetry increases with the period. From the light curves given by Joy it would appear that an additional level *d* 3 might exist, but it cannot be accepted without further evidence. Joy also gives the velocity curves, which have been analysed in the same way. No positive evidence for any regularities is obtained. The phase increases with the period. This increase might not be regular. There might be a discontinuity around P = 10 days.

§ 1. Introduction

It was shown several years ago by H. Ludendorff [1] that with the light curves of Cepheids, there exists a marked dependence of their shapes upon the period. This dependence is clearly demonstrated in the series of light curves arranged by E. Hertzsprung [2] and in the more recent one arranged by L. Campbell and L. Jacchia [3]. According to Campbell and Jacchia for each period we find a definite pattern and this pattern gradually changes in shape if, step by step, we proceed from the shorter towards the longer periods This pattern system holds for all Cepheids, but is far from a precise law.

Up till now the method of autocorrelation and subsequent power series analysis has only been applied to variables in different globular clusters, that is to pulsating stars which generally speaking have short periods.

The result of the autocorrelation and subsequent power series analysis is that the shape of the light curves is described in terms of a few parameters, these parameters being the relative intensities of the various overtones. It is therefore worth while to extend this analysis towards the longer period in order to see whether the gradual change in pattern can also be traced in the numerical values given by the analysis.

I have therefore in the first place considered the series of 37 stars arranged by Hertzsprung [2] For obvious reasons the selection of the Cepheids in this list might not represent a fair sample of all Cepheids. The Cepheids have been selected in order to demonstrate a certain tendency viz. The gradual change of the pattern. Consequently the choice of the systems might have been slightly onesided In the following it will appear that this is not the case. Still I have thought it better to supplement our date, by analysing a large number of Cepheids from the list of A. H. Joy [4]. In his list Joy has collected the Cepheids for which he has been able to determine the (radial) velocity curve.

So in this list the choice of systems is quite independent from the variations in the shape of the light curve. If therefore any undue selection has been made, this will appear at once by comparing the results derived from the two lists.

Neither the list given by Hertzsprung, nor that given by Joy, contains any variables with period $<$ 1 day. This is not a serious disadvantage. For the shorter periods already sufficient data have become available by the analysis of the cluster stars. We know that in the numerical values yielded by the analysis, a definite trend is present which systematically depends on the period. Especially with the overtone A (2) we have been able to represent this trend by a series of levels and provisionel sublevels, which will also be used in the following.

§ 2. The variables in the list of Hertzsprung.

The analysis has been performed in exactly the same way as before. We find the intensities of the fundamental π (1) and the relative intensities of the overtones A (2), A (3) and A (4). If the light curve is a pure harmonic $\pi (1) = 1$. In nearly all cases $\pi < 1$ and the larger the difference, the more the actual shape of the light curve deviates from a sinus or cosinus curve. The values A (2), A (3) and A (4) formally describe the shape of the curve but for the present no real physical significance can be ascribed to this set of overtones. The results of the analysis are collected in table 1, where the stars are arranged in order of increasing period. In order to make a rapid identification possible, I have indicated the stars by their original designation in Hertzsprung's list. As the number of stars with very long periods is rather short, in table 1 I have inserted one additional long period Cepheid viz. *l* Car.

In fig. 1. the tabulated values A (2) are plotted against the corresponding values of log P. The stars in Hertzsprung's list are indicated by open circles. In the left hand side of the figure are indicated by levels and provisional sublevels *c* 1, *c* 2 *c* 3, *a* 1 and *a* 2. Partly these sublevels are indicated by full drawn lines, partly by dotted lines. Along the full drawn interval, stars were actually to be found centered in the various globular clusters.

The dotted parts of the curves merely indicate a possible extension to the longer periods, but along those parts no stars have actually been observed. It is to be noticed that a few stars from Hertzsprung's list do seem to be on or very near to one of the dotted intervals. From our analysis it therefore would appear that these stars might be considered as forming an extension of the RR Lyrae stars towards the longer periods and therefore actually might be RR Lyrae stars. With the exception of these few possible RR Lyrae variables for the periods < 10 days, the stars are arranged along a line, with a steep slope from the left to the right, that is from the shorter

TABLE 1

Stars in Hertzsprung's list in B. A. N. 96

Design	log P	π (1)	A (2)	A (3)	A (4)
SU Cas	0.290	0.988	.071	.045	.000
95 j	.459	.965	.148	.105	.014
95 g	.464	.727	.509	.249	.145
SZ Tau	.498	.987	.089	.055	.063
95 u	.507	.980	.224	.000	.100
RT Aur	.572	.843	.379	.167	.100
95 E	.627	.869	.385	.179	.105
T Vul	.647	.808	.322	.195	.078
95 p	.662	.884	.311	.161	.078
V Lac	.697	.851	.373	.197	0.45
95 t	.724	.905	.286	.138	.055
δ Cep	.730	.865	.362	.152	071
X Lac	.736	.956	.195	084	.010
95 l	.753	.903	.300	.134	.010
95 h	.795	.851	.326	.205	.106
RR Lac	.807	.891	.333	.105	.045
CS Car	.823	.850	.316	.217	.032
r Aql	.856	.869	.367	.148	.000
95 0	.883	.891	.217	.257	.071
ER Car	.888	.929	.243	.089	.095
S Sge	.923	.937	.202	.122	.084
AQ Car	.990	.952	.095	.089	.128
ζ Gem	1.006	1.000	.000	.000	.000
95 r	1.012	.973	.032	.100	.110
95 q	1.018	.997	.045	.045	.032
95 s	1.030	0.976	0.145	0.071	0.045
Z Lac	1.037	1.000	.032	.064	.117
RY Cas	1.084	.946	.232	.055	.000
95 m	1.129	.993	.077	.045	.000
SV Vel	1.149	.887	.261	.164	.105
TX Cyg	1.168	.781	.373	.226	.153
RW Cas	1.170	.851	.274	.141	.190
H. V1225	1.213	.944	.219	.078	.114
WZ Car	1.362	.815	.389	.197	.095
95 n	1.366	.729	.442	.326	.207
T Mon	1.431	.859	.486	.173	.078
U Car	1.588	.765	.489	.241	.114
—	—	—	—	—	—
l Car	1.550	.909	.293	.148	.032

towards the longer periods. In the figure this additional level has been indicated as d 1, but there is a fairly large scatter of the individual points around this level.

As already previously remarked by Hertzsprung (l. c.) around P = 10 days all light curves are fairly symmetrical and this is also very evident from the distribution of the points in our figure 1. In order to avoid creating the impression that the run of the levels is determined with a high degree of accuracy, up till now all levels have simply been represented by straight lines. Beyond log P = 1 it is no longer possible simply to use a straight line. Starting from P = 10 days. there is a strong and continuous increase of the values A (2), but the variation

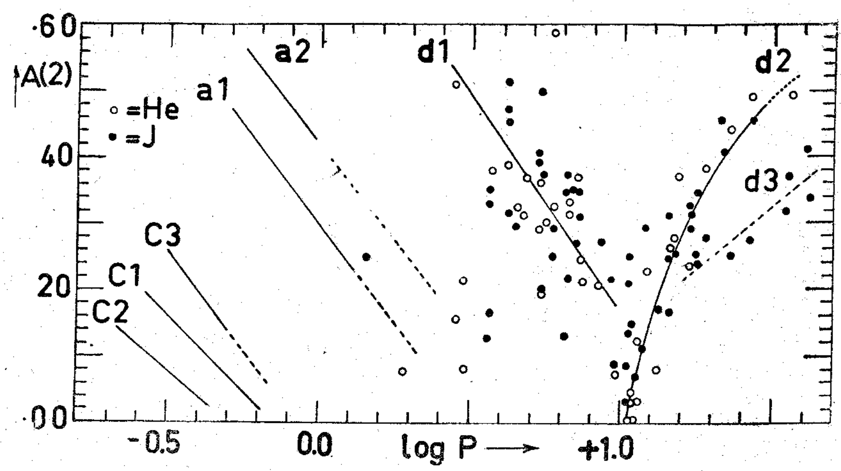

Fig. 1

of A (2) with log P can no longer be represented by a straight line. The individual points are arranged along a curve of which the slope decreases as the period increases. It is to be remarked that near P = 10 days the way in which the sublevels must be drawn, is open to some doubt.

In figure 1 the curves are drawn in such a way as in my opinion to represent at best the distribution of the observed points, but others might prefer to draw lines in the shape of the letter v 1 or add a sloping line to this letter, thus v/.

Anyhow, it is quite evident that with the Cepheids the period P = 10 days is of fundamental importance. At this point the whole pattern of the changes is inversed. Before $P = 10$ days with each of the separate levels there is an increase of the degree of symmetry with increasing period.

Beyond $P = 10$ days the symmetry decreases as the period increases. With $P < 10$ days, each level overlaps with the next one in the sense that the longest periods belonging to a given level are longer than the shortest periods belonging to the next one. For the periods $P > 10$ days as to this no statements can be made bacause as yet the stars of the longest periods have not been analysed. The hypothetical sublevel *d* 3 will be discussed later on in connection with the stars from the list of Joy.

§ 3. The variables in the list of Joy.

The light curves which he has used were collected by Joy from various sources, which will not be enumerated here. For his sources we refer to his original paper (Joy 1. c.). Together with the light curves, Joy gives the velocity curves as derived from his observations so that it is possible at any phase to compare the magnitude and the velocity.

It is interesting to see to what extend the conclusions derived from the light curves are corroborated by those obtained from the velocity curve. Therefore in table 2 in which the stars ara arranged in order of increasing period, I have not only given the values π_m (1), A_m (2), A_m (3) and A_m (4) as derived from the light curve but also the corresponding values π_v (1), A_v (2), A_v (3) and A (4) derived from the velocity curve.

For this it is sufficient in the relation

$$r_k(m) = \frac{\sum_{i=0}^{i=23} \Delta m_i . \Delta m_{i+k}}{\left\{(\Sigma \Delta m_i^2)(\Sigma \Delta_{i+k}^2)\right\}^{1/2}} = \frac{\sum_{i=0}^{i=23} \Delta m_i . \Delta m_{i+k}}{\sum_{i=0}^{i=23} \Delta m_i^2} \quad \ldots (1)$$

for Δm_i to substitute $\Delta \nu_i$, thus:

$$r_k(\nu) = \frac{\sum_{i=}^{i=23} \Delta \nu_i . \Delta \nu_{i+k}}{\sum_{i=0}^{i=23} \Delta \nu_i^2} \quad \ldots (2)$$

TABLE 2

The variables from the list of Joy which were analysed in the present paper

Design	log P	$\pi_m(1)$	$A_m(2)$	$A_m(3)$	$A_m(4)$	$\pi_m(1)$	$A_v(2)$	$A_v(3)$	$A_v(4)$	Max. corr.	$\Delta\varphi$
SW Tau	0.199	0.910	0.255	0.134	0.063	0.789	0.434	0.251	0.105	0.936	− .083
SS Scu	.565	.962	.131	.110	.089	.835	.403	.141	.077	.922	+ .167
AD Gem	.579	.898	.322	.032	.000	.932	.224	.032	.062	.948	+ .083
Y Aur	587	.826	.407	.167	.100	.931	.251	.000	.089	.969	− .041
CS Ori	.590	.812	.349	.243	.118	.936	.249	.055	000	.929	+ .083
ST Tau	.605	.875	.313	.155	.084	.939	.228	.084	.045	.904	.000
SY Cas	.610	.661	.515	.326	.000	.987	.000	.000	.078	.857	+ .041
BF Oph	.610	.744	.455	.257	.105	.969	.161	.063	.045	.874	− .041
UZ Cas	.629	.809	.474	.032	.084	.879	.310	.148	.000	.900	+ .041
RY CMa	.668	.901	.295	.130	.020	.920	279	.030	.000	.984	+ .146
V350 Sgr	.712	.841	.397	.130	.078	.869	.352	.127	0.78	.968	+ .041
X Lac	.736	.924	.200	.155	.122	.915	.261	.032	.141	.974	+ .083
SW Cas	.736	.750	.500	.239	.045	898	.317	.100	.000	.968	.000
RW Cma	.758	.828	.405	.167	.000	.880	.341	.118	.010	.963	+ .123
VW Cas	.777	.867	.378	.126	.089	.988	.045	.000	.000	.929	+ .083
FM Aql	.786	.928	.241	.000	.000	927	.263	071	.055	.949	+ .292
VV Cas	.793	.861	.298	.228	.095	.957	.168	.095	.089	.949	+ .208
RS Cas	.799	.683	.592	.285	.040	.868	.381	.210	.030	.924	+ .041
TY Scu	.807	.978	.131	.055	.000	.976	.131	.055	.000	.997	+ .041
XX Sgr	.807	.837	.352	.145	.126	.843	.382	.190	.071	.965	+ .083
BB Sgr	.822	.858	.379	.055	089	.949	.226	000	.000	.972	.000
U Sgr	.829	.878	.349	.000	.055	.777	.474	.226	.032	.959	+ .083
AO Aur	.830	.953	.212	.032	.055	.689	.587	.305	.145	.895	+ .167
V336 Aql	.864	.894	.311	.131	.063	.966	.190	.032	.000	.973	− .041
RS Ori	.879	.910	.270	.122	.063	.811	.414	.179	.084	.960	+ .041
RX Cam	0.898	.887	.352	.122	.010	.866	.360	.055	.010	.949	+ .083
TX Mon	.940	.911	.265	.000	.118	.969	.032	.095	.161	.901	− .063
FN Aql	.977	.959	.084	.045	.000	.971	.089	.109	.084	.974	+ .125
YZ Sgr	.980	.862	.212	.170	.110	.836	.406	.170	.100	.867	+ .083
SY Aur	1.006	.947	.134	.141	.089	819	.420	.152	.000	.946	− .041

TABLE 2 (concluded)

Desing	log P	$\pi_m(1)$	$A_m(2)$	$A_m(3)$	$A_m(4)$	$\pi_v(1)$	$A_v(2)$	$A_v(3)$	$A_v(4)$	Max. corr.	$\Delta\varphi$
BZ Cyg	1.006	.955	.205	.063	.032	.855	.379	.173	.045	.968	+.041
An Aur	1.012	.982	.105	.077	.032	.951	.207	.177	.095	.985	−.167
Y Scu	1.014	.994	.032	.045	.000	.968	.000	.000	.000	.974	+.104
VX Per	1.037	.981	.000	.107	.046	.919	.268	.089	.130	.944	+.146
Z Lac	1.037	.882	.247	.130	.161	.975	.155	.071	.000	.965	+.041
SV Per	1.046	.969:	.164:	.055:	.045:	.774:	.495:	.272:	.000:	.884	+.167
RX Aur	1.065	.964	.071	.100	.089	.724	.570	.000	.000	.847	+.041
RY Cas	1.084	.871	.295	.192	.095	.976	.100	071	.055	.935	+.041
SS CMa	1.092	.958	.118	.130	.110	.841	.366	.161	.105	.963	+.250
TT Aql	1.138	.984	.162	.000	.077	.892	.295	.130	.063	.956	+.083
TX Cyg	1.167	.928	.249	.063	.063	.963	.205	.000	.055	.986	+.041
RW Cas	1.170	.828	.405	.167	.000	.880	.341	.118	.010	.963	+.125
SZ Cyg	1.179	.951	.164	.176	.110	.708	.513	.279	.185	.790	+.250
SV Mon	1.182	.939	.249	.078	.055	.868	.356	.089	.000	.985	+.083
RW Cam	1.215	.842	.316	.100	.141	.944	.227	.084	.000	.912	+.083
CD Cyg	1.232	.850	.322	.211	.095	.834	.409	.063	.089	.880	+.041
W Vir	1.238	.906	.288	.095	.077	.913	.297	.045	.000	.968	+.083
SZ Aql	1.231	.811	.349	.214	.207	.956	.214	.071	.000	.944	+.125
YZ Aur	1.260	.929	.249	.095	.063	.972	.122	.055	.045	.993	+.125
VX Cyg	1.303	.774	.459	.224	.110	.982	.000	.071	.000	.882	+.250
RU Sco	1.308	0.906	0.270	0.110	0.089	0.929	0.288	0.000	0.032	.980	+.041
WZ Sgr	1.339	.767	.407	.228	.207	.882	.358	.100	.055	.939	+.083
RX Lib	1.396	.921	.251	.151	.045	.934	.257	.000	.045	.967	+.392
X Pup	1.415	.750	.456	.281	.145	.780	.456	.268	.161	.981	+.125
RS Pup	1.616	.826	.406	.182	.126	.750	.477	.266	.114	.964	+.041
SV Vul	1.655	.840	.383	.190	.063	.820	.443	.239	.161	.962	+.062

Next in the relation

$$\pi(f) = \frac{2}{N}\sum_{0}^{N} r_k \cos 2\pi f k \qquad \cdots(3)$$

instead of the value $r_k(m)$ the corresponding value $r_k(\nu)$ is inserted and this will give us the values $\pi_\nu(1)$, $\pi_\nu(2)$, $\pi_\nu(3)$ etc. Finally in the same way as before the values $A(f)$ are obtained from the relation

$$A_\nu\,(f) = \sqrt{\frac{\pi_\nu(f)}{\pi_\nu(1)}} \qquad \cdots(4)$$

I have also tried to determine the functional correlation between magnitude and velocity. This functional correlation is given by

$$r_k(m,\nu) = \frac{\sum_{i=0}^{i=23} \Delta m_i \, \Delta\nu_{i+k}}{\left\{\left(\sum_{i=0}^{i=23} \Delta m_i^2\right)\left(\sum_{i=0}^{i=23} \Delta\nu_{i+k}\right)\right\}^{1/2}}$$

$$= \frac{\sum_{i=0}^{i=23} \Delta m_i \, \Delta\nu_{i+k}}{\left\{\left(\sum_{i=0}^{i=23} \Delta m_i^2\right)\left(\sum_{i=0}^{i=23} \Delta\nu_i^2\right)\right\}^{1/2}} \qquad \cdots(5)$$

Now K indicates over how many intervals the light curve has been shifted forward relatively to the velocity curve. The numerical value of $r_K\ (m_2 v)$ depends on K. It will attain maximum value for that value of K for which there is the best correlation between the light curve and the velocity curve. For the different variables in table one only this maximum value of the serial correlation $r_K\ (m_2 v)$ has been given.

The values $\Delta\varphi$ in the final column of the table indicate at what value K the maximum correlation occurs. The values $\Delta\varphi$ have been arranged in such a way that $\Delta\varphi = 0$ for the ideal case where the velocity curve is the negative replica of the light curve, the largest negative and positive velocities corresponding to maximum and minimum light respectively while intermediate velocities correspond to intermediate magnitudes etc. / Relatively to this ideal case the magnitude curve has to be shifted in forward direction $\Delta\varphi$ is taken to be positive. If the magnitude curve has to be shifted backwards $\Delta\varphi$ is taken to be negative. There would be a difficulty if $K = 12$, but actually the values of K which are obtained are almost exclusively in the interval $K = O - + 4$ and $K = + 22 - - + 24$. So in the first case K and $\Delta\varphi$ are positive and in the latter case negative. In our calculations the light curves and velocity curves were divided into 24 equal intervals. The values $\Delta\varphi$ are expressed in fractions of the period, so numerically $\Delta\varphi = K : 24$.

The values A_m (2) of table 2 have also been plotted in figure 1, and are represented by the black disk in that figure It is seen that the distribution of these additional points corresponds well with those from the list of Hertzsprung. In this case also we find a few systems which might be RR Lyrae stars with a comparatively long period. The majority of the variables with $P < 10$ days are scattered along the level d 1 but at the upper left hand of this level a few systems have a strong positive deviation. Near $P = 10$ days the observed light curves are mostly fairly symmetrical. Beyond $P = 10$ days the majority of the systems fit well along the level d 2 of which at which at the upper end the shape remains slightly undetermined.

For a small group of Cepheids with $P > 10$ days the individual points do not fit into the level d 2, but rather seem to be arranged along the straight line d 3, which at an early stage branches off from the main level. It certainly is too early to state that here we have to deal with an additional level or even sublevel. Only a few systems fall along d 3, and if only a few additional systems were found to fall in the interval between d 2 and d 3, the figure would certainly create quite a different impression.

In this context it should be mentioned, that our table 2 does not contain all the variable systems given by Joy. A complete analysis of the curves involves a great deal of calculations. Therefore only a part of the systems enumerated by Joy were analysed. The selection was made in such a way that all periods were represented. That is to say, of the shortest and the longest periods nearly all stars were analysed, but with the intermediate periods only 50 — 60 % of the available systems were used.

On the other hand care was taken to include such controversional systems as W Virginis. This star is known to have a rather peculiar light curve while for a Cepheid of this period it has a very high galactic latitude.

According to the results of our present analysis there is nothing very abnormal about this system. It fits well into the $d2$ level.

§ 4. The relation between the light curves and the velocity curves.

A.H. Joy (1 c.) concluded that there is a very close correlation between the light curves and the velocity curves of the Cepheids. This conclusion has been confirmed by all later observers. In this case we are able numerically to express the degree of observations.

In the system used here $r_K = 1\,000$ indicates complete correlation. From the values in table 2 it appears that nearly with all systems the degree of correlation is very large, the distribution of the values r_K (m_2v) max. being as indicated in table 3.

TABLE 3

Distribution of the values r_K (m_2v) max.

Limits	1.000	.975	.950	.925	.900	.875	.850	.825	.800	.775
Numbers	10	20	11	6	4	3	1	0	1	

For only a fraction of the total number of stars is the maximum correlation smaller than 0.900.

When considering the shape of the light curve we could describe $\pi(1)$ is the first order term, which describes the general shape of the light curve, e.g. the skewness of this curve. The terms A(2), A(3) and A(4) which indicate the intensities of the overtones, can be described as the terms of second and higher order. It is therefore apparent that the correlation between the first order terms is therefore apparent that the correlation between the first order terms is high. This is no longer true with the terms of higher order. Figure 2 gives a plot of the values $A_v(2)$ in table 2 against the corresponding values $A_m(2)$. The distribution of the points in the plane seems to be quite haphazard and no correlation whatever is apparent.

So whereas there can be no doubt as to the close relation of the radial velocity and the variations in magnitude, a complete explanation of the shape of the light curve on the basis of the observed velocities will be difficult.

With a view to figure 2 it will not be surprising that a plot of the values $A_v(2)$ against the logarithms of the periods does not yield any positive results.

With any value log P the distribution of the values $A_v(2)$ seems to be quite an haphazard one and no systematic trends are apparent. No separate figure is given here. Finally a few words should be said about the values $\Delta\varphi$.

Obviously the values $\Delta\varphi$ are closely related to the values determined by Joy for the lag of the velocity curve relative to the light curve. They are however not identical to these values.

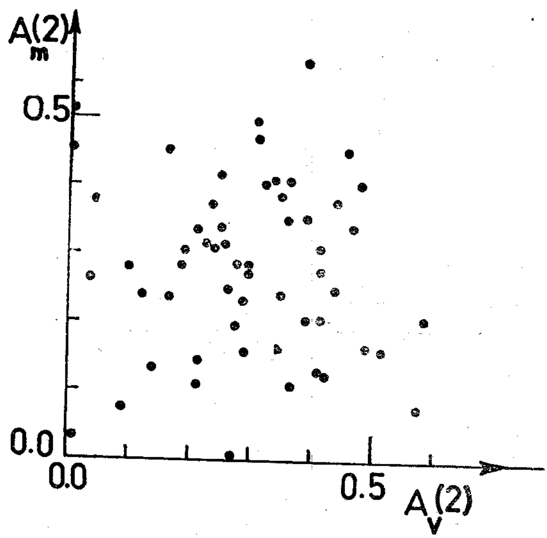

Fig. 2

The values of Joy are determined by the comparison of the epochs of maximum and minimum velocity with the corresponding epoch of the light curve. The values $\Delta\varphi$ are based on a comparison of the complete curves. Whether this is an advantage or a disadvantage is debatable. When both the linght curve and the velocity curve are well determined over the whole range of the period, I think that values based on the complete curve are to be preferred. If either of the curves is not well determined, direct comparison of the instants of maximum velocity and minimum light and minimum velocity and maximum light might give more reliable results. On the whole the conclusion which can be drawn from the distribution of the individual values $\Delta\varphi$ do not deviate very much from those previously obtained by Joy. There seems to be a tendency of the mean values $\overline{\Delta\varphi}$ to increase with increasing period.

We can only consider mean values $\overline{\Delta\varphi}$ because the scatter of the individual values $\Delta\varphi$ is considerable. A survey of the run of the the numbers $\overline{\Delta\varphi}$ is given in table 4. In order of increasing period the variables have been collected in groups of 5 stars each, except for the final group which contains 6 stars.

TABLE 4.

The relation between $\overline{\Delta\varphi}$ and log P

$\overline{\log P}$	$\overline{\Delta\varphi}$	n	$\overline{\log P}$	$\overline{\Delta\varphi}$	n	$\overline{\log P}$	$\overline{\Delta\varphi}$	n
0.504	+ 0.942	5	0.845	+ 0.050	5	1.183	+ .116	5
0.624	+ 0.037	5	0.960	+ 0.038	5	1.255	+ .125	5
0.744	+ 0.066	5	1.021	+ 0.033	5	1.455	+ .124	6
0.798	+ 0.123	5	1.085	+ 0.116	5	—	—	—

With the values $\overline{\Delta\varphi}$ it is not perfectly clear whether there is a continuous increase with the period or that near log P = 1.000 a discontinuity occurs. With the exception of the value $\overline{\Delta\varphi} = + 0.123$ for $\overline{\log P} = 0.798$ for log P < 1.000 the mean values $\overline{\Delta\varphi}$ are nearly constant, the overall mean being + 0.045. On the other hand for log P>1.085 the values $\overline{\Delta\varphi}$ also are nearly constant, but for these periods the overall mean is $\overline{\Delta\varphi} = + 0.120$ (see also figure 3).

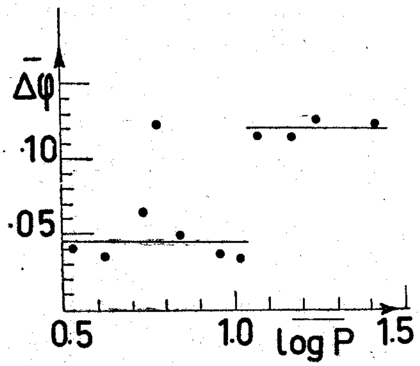

Fig. 3

There a break in the continuity may occur near log P = 1.000 and with a view to the conclusions drawn from figure 1, the occurence of such a break might be significant. I do not think however, that the evidence as to the reality of this break is in any way conclusive.

§ 5. Intracomparison of results.

The way in which the different observers draw a mean light curve through the observed magnitudes is by no means a uniform one. Some authors prefer to draw a smooth curve through the normal points while others attach more importance to the irregularities which they observe and give more irregular curves. Most authors leave open that part of a light curve which is not completely covered by observations or indicate it by a dotted line. Some authors however think the general shape of the Cepheid light curves to be sufficiently established and do not hesitate to give a complete curve even if not all phases of the curve are equally well observed. It is evident that the values $\pi(1)$, A(2), A(3) and A(4) used in this series of paper do not represent an absolute set of values. Actually they must be strongly influenced by the way in which the mean light curve is constructed, but it is difficult exactly to evaluate the resulting amount of error. In an analysis as carried out in this and in the previous paper, it is certainly preferable always to use sets of observations of variables which all have been treated in a homogeneous way. The two sets of values used in table 1 and 2 do not completely meet this condition. That this leads to deviation is apparent if we compare the values found for the variables common to both lists. These are the five variables collected in table 5. On the left hand side of the table I have repeated the values $\pi(1)$ A(2), A(3) and A(4) as

TABLE 5.

Comparison of results

Design	π (1)	A (2)	A (3)	A (4)	π (1)	A (2)	A (3)	A (4)
X Lac.	.956	.195	.084	.010	.924	.200	.155	.122
Z Lac	1.000	.032	.064	.117	.882	.247	.130	.161
R Y Cas	.946	.232	.055	.000	.871	.295	.192	.095
TX Cyg	.781	.373	.226	.153	.928	.249	.063	.063
RW Cass	.851	.274	.141	.190	.828	.405	.167	.000

obtained from the light curves used by Hertzsprung while the right hand side contains the corresponding values obtained from the light curves used by Joy.

There is little difference between the values $\pi(1)$. The values of A(2) are of the same degree magnitude. Only very little correlation remains between the two sets of values of third and fourth order A(3) and A(4).

All our considerations have been based on the values A(2). It is therefore apparent that these values are affected by a certain probable error.

It might therefore be that in figure 1, the scatter of the individual points is due to the influence of probable error and that actually the shift in the pattern of the light curve is more strictly uniform than is suggested by the figure. To decide this point additional evidence is needed.

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