A Statistical Study Of Some M-Type Variables Of Long Period

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Özet: Bu çalışmada, otokorrelasion metodu ve müteakip kuvvet spektrumu analizi kullanılarak, M-tipindeki uzun peryotlu yıldızların ışık eğrilerinin Analizi yapılmıştır. Bu metodun tatbikinde, üst tonların amplitütlerini esas tonun cinsinden elde etmekteyiz. Böylece ilk üst ton A (1) in izafi şiddeti ile log P arasındaki bağıntı incelendi. log P — A (1) dağılımında Pop I yıldızlarına ait eğik kol Pop II yıldızlarının tefrikine imkân vermektedir; fakat bu tarif çok kesin değildir.

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Abstract: In this paper the light curves of long period M-type stars were analysed. The method used was that of the autocorrelation and supsequent power spectrum analysis. Applying this method we obtain the amplitudes of the overtones expressed in that of the fundamental as unit. The relation between log P and the relative intensity of the first overtone A (1) was studied. Some indication who obtained that foı the Pop I stars the slope of the curve log P — A (I) may be different from the one valid with the Pop II stars, but as yet the evidence is not conclusive.

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1. Outline of the method

The autocorrelation method as developed py Kendall [1] and the subsequent power series analysis as introduced py Ashbrook, Duncombe and Woerkom [2] has been repeatedly applied in previous communication of this institute. In the special form used here it has been described by Kreiken and Ege [3]. For details the reader is referred to these papers. The autocorrelation coefficient is equal to

$$r_k = \frac{\Sigma u_i \, u_{i+k}}{\left\{\Sigma \, u_i^2 \, \Sigma \, u^2_{i+k}\right\}^{1/2}} \qquad \ldots . (1,1)$$

but is computed from the approximation

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exist but are masked by this large scatter. Therefore a further attempt seemed worth while in which only well determined light curves are used.

3. The material used in the present paper

For a number of stars almost continuous sets of observations have been obtained by the members of the «American Association of Variable Star Observers» and these observations have been edited by Campbell [6]. These observations cover the years 1935 to 1946. For 93 variables the magnitudes were plotted against the corresponding phases. In the resulting figures each interval of phase is covered by a large number of points and through these a free hand curve was drawn. In this way satisfactory curves could be obtained for 81 out of the 93 stars in which both the maxima and minima are well covered by the individual points. For the remaining 12 stars no satisfactory curves could de obtained. This point will be discussed in section 4. For the sake of brevity the author has not tried to give the light curve of the 81 stars mentioned before, but these stars are enumerated in table I. Also inserted in this table are the values A (1) and A (2) which resulted from the analysis of the individual curves. In the second column of the table the classification of the light curves according to Ludendorff has been entered. With the scheme of Ludendorff a type A light curve has a steep increase to maximum light followed by a slow decrease, with type B the light curve resembles a pure sinus curve, while finally type C is characterised by having a secondary maximum.

4. Irregularities in period

It is a well known fact that with the M type variables the periods are subject to certain variations. Actually with these stars the period is a certain mean period. With the majority of our stars the deviations of the individual periods from this mean are small, so that there were no serious objections against drawing mean curves for all cycles covered by the observations. Obviously internal variations between the successive cycles will lead to an increase of the scatter of the individual points along the mean curve. However in order to obtain a reliable light

$$r_k = \Sigma\,(u_i\,u_{i+k})/\Sigma\,u^2_i \qquad \cdots\cdot (2,1)$$

which is adequate if $N \gg K$. The corresponding power spectrum is obtained from

$$\pi\,(f) = \frac{2}{N}\,\Sigma\,r_k \cos 2\pi\,f\,k \qquad \cdots\cdot (3,1)$$

where f is the reciprocal of the trial period. Finally the relative intensity of the overtones with resbect to the fundamental. is computed from the relation.

$$A\,(f) = \sqrt{\frac{\pi\,(f)}{\pi\,(0)}} \qquad \cdots\cdot (4,1)$$

2. Subject of this investigation

Application of this method to pulsating variables of short period has given some interesting result. A summary of these results has been given by Kreiken and Ege [3]. Briefly stated, they find that if the values A (1) are plotted against the corresponding values log P the points representing the individual stars seem to be arrenged in sequences or at least within belts. With $\log P < 1{,}0$ the amplitude A (1) systematically decreases with log P while for $\log P > 1{,}0$ the amplitude A (1) increases. Also there is a fairly strong evidence that the distribution of the points representing Population I variables is different from that valid for the Population II stars. In a subsequent paper Kreiken and Eskioğlu [4] have tried to extend these results to the variables with longer periods ($\log P > 2{,}00$). They used all or nearly all stars contained in the compilation published by Mayall [5]. When they plot their values A (1) against log P they find that A (1) is nearly zero with $\log P = 2{,}00$ but systematically increases with increasing values of log P. As far as the relation between A (1) and log P is concerned they do not find any systematic differences between the variables of the two Population groups. The scatter of the individual points along the mean relation A (1) — log P is large, which they ascribe to the fact that in many cases the light curves of the variables are badly observed around minimum light. Therefore it could still be possible that systematic differences between Pop I and Pop II

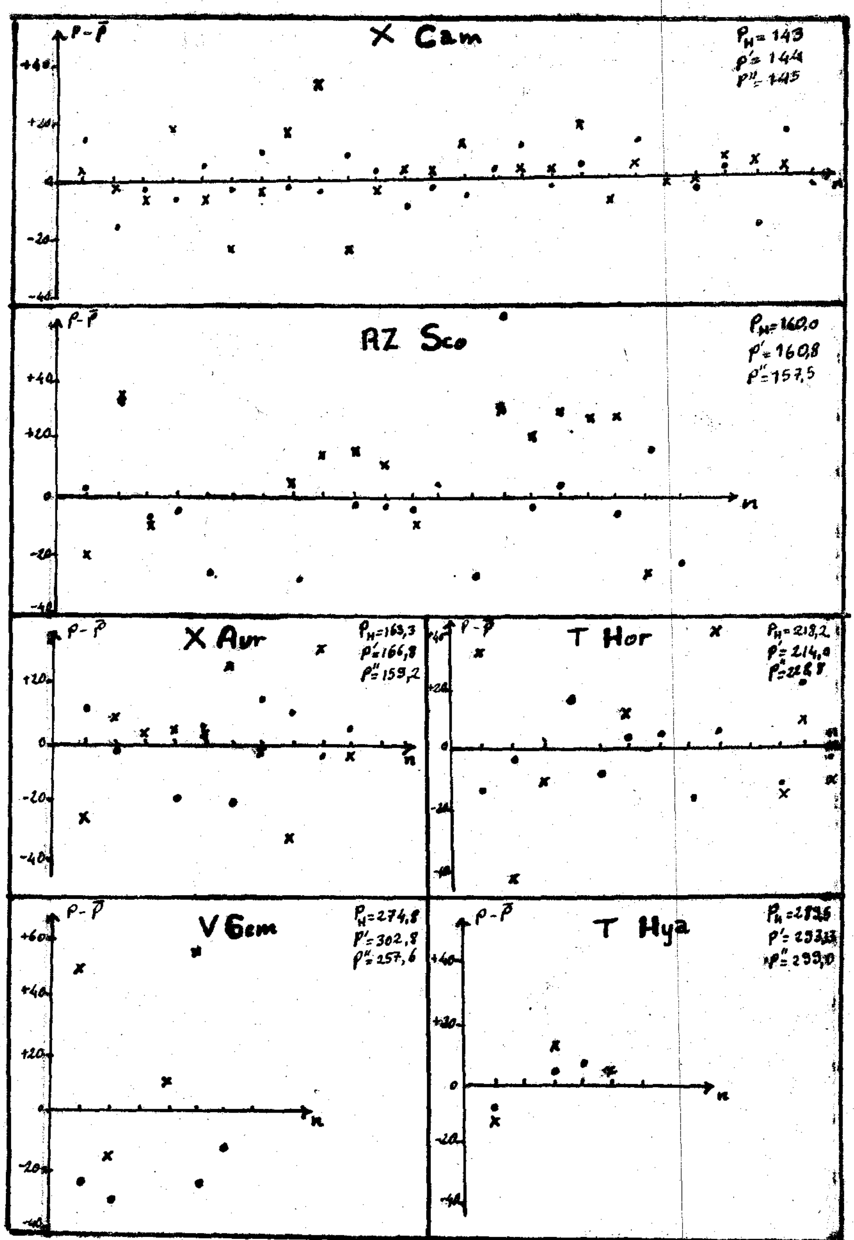

Fig. 1

P = Mean period

$P - \overline{P}$ = Deviation of individual period from the mean

P' = Period between consecutive maxima

P'' = Period between consecutive minima

P_H = Harvard period

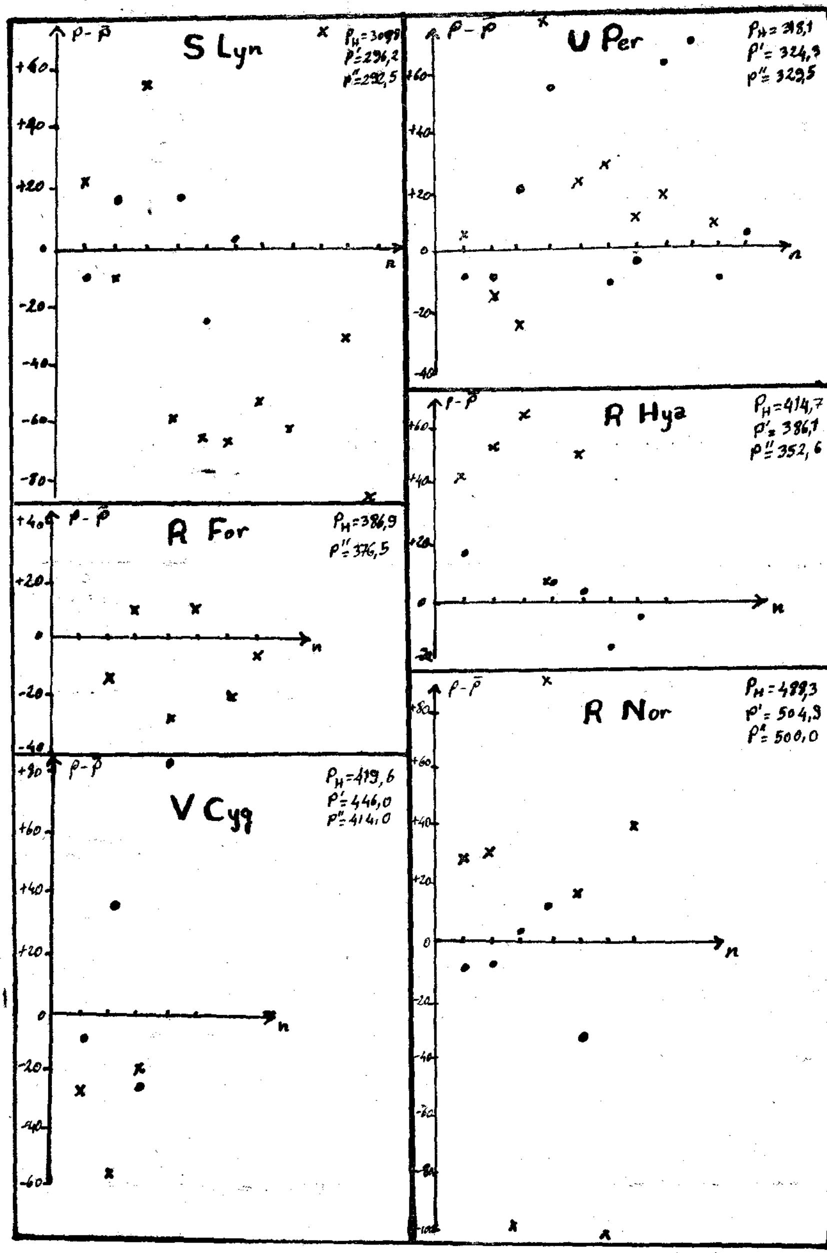

Fig. 1
(Continued)

Table I

Stars	Type acc. Ludendorf	log P	A(1)	A(2)
SS Her	B	2,03141	0,0812	0,0685
W Pup	B	2,08099	0,13[illegible]6	0,0824
R Vul	B	2,13672	0,0591	0,0591
R Mic	B	2,14176	0,1678	0,0938
R Vir	B	2,16256	0,1174	0,0860
S Car	A	2,17493	0,2262	0,1200
RY Oph	B	2,17782	0,0781	0,0588
RU Vul	B	2,19368	0,1280	0,0678
RS Cen	B	2,21590	0,0591	0,0100
T Her	B	2,21748	0,0916	0,0529
R Cet	B	2,22194	0,0842	0,0581
V Tau	B	2,22994	0,0781	0,0588
X Cet	B	2,24724	0,2236	0,1330
SS Oph	B	2,25455	0,1766	0,1000
Z Cet	B	2,26576	0,0905	0,1204
R Ari	B	2,27068	0,1118	0,0830
RT Cyg	B	2,27967	0,0616	0,0932
W Cen	B	2,30211	0,1889	0,1122
T Pic	B	2,30211	0,0579	0,0579
R Per	B	2,32181	0,0800	0,0748
SU Vir	B	2,32263	0,1382	0,0600
Y And	B	2,34301	0,1234	0,0886
T Col	B	2,35276	0,0579	0,0579
U Cet	B	2,37107	0,0400	0,0400
S Tuc	B	2,38093	0,1044	0,0624
Y Per	B	2,40037	0,1584	0,1486
S Hya	B	2,40877	0,1206	0,0624
V And	A	2,41128	0,1558	0,0655
U Tuc	B	2,41880	0,0400	0,0400
X Gem	B	2,42210	0,0848	0,0685
V Ori	B	2,42878	0,0183	0,0183
V Cnc	B	2,43521	0,1200	0,0800
V Leo	A	2,43586	0,3288	0,0591
W Aur	B	2,43791	0,2265	0,0529
R Ret	B	2,44295	0,2059	0,1118
T And	B	2,44840	0,0787	0,0030

Table I (*Continued*)

Stars		Type acc. Ludendorf	log P	A( )	A( )
S	Gem	B	2,46464	0,2924	0,1417
V	Oct	B	2,48029	0,1452	0,0858
X	Hya	B	2,48044	0,1574	0,0806
U	Cnc	A	2,48416	0,5620	0,2668
X	Cen	A	2,49707	0,2253	0,1029
S	Cet	A	2,50596	0,2222	0,0714
R	Tau	A	2,50934	0,1384	0,0714
S	Col	A	2,51244	0,2190	0,1157
Y	Dra	A	2,51835	0,3820	0,1881
S	Cam	B	2,51388	0,2381	0,1228
V	Mon	A	2,52388	0,2195	0,1029
R	CMi	B	2,52930	0,1951	0,0982
X	And	A	2,53958	0,2844	0,1408
RZ	Per	C	2,54888	0,1959	0,0937
S	Scl	B	2,56289	0,1952	0,0932
T	Cam	C	2,57124	0,1356	0,0793
T	Lep	B	2,57530	0,0911	0,0800
S	Vir	B	2,57646	0,0781	0,0574
R	Lyn	A	2,57830	0,1685	0,0798
W	Cnc	A	2,59278	0,3067	0,1546
W	And	A	2,59879	0,1962	0,0998
R	Hor	B	2,60314	0,2297	0,1179
S	Gru	A	2,60347	0,2701	0,1204
R	Oct	A	2,60831	0,3470	0,1760
W	Cas	B	2,60863	0,1396	0,0824
U	Her	C	2,60895	0,1931	0,1414
U	Aur	A	2,61077	0,1703	0,0932
R	And	A	2,61162	0,2300	0,1110
U	CMi	C	2,61426	0,2610	0,1510
S	Ori	C	2,61838	0,2040	0,1220
T	Lyn	B	2,62221	0,0400	0,0490
R	Cyg	A	2,62961	0,2330	0,0960
S	Pic	A	2,63114	0,1729	0,1005
R	Cas	A	2,63577	0,1680	0,1080
RW	And	A	2,63458	0,2493	0,1256
RZ	Peg	A	2,64375	0,1590	0,0888

Table I (*Continued*)

Stars		Type acc. Ludendorf	log P	A(1)	A(2)
T	Cas	C	2,64826	0,3740	0,1890
RV	Cen	B	2,64982	0,3761	0,1765
R	Aur	B	2,66918	0,1374	0,0877
RU	Her	A	2,68431	0,1774	0,1449
W	Aql	B	2,69958	0,3009	0,1717
V	Cam	A	2,71542	0,4240	0,2471
R	Cen	C	2,73263	1,1330	0,4900
S	Cas	B	2,78661	1.1805	0,0100

curve the scatter of the individual points must remain within reasonable limits *e. g.* so that for all cycles a common mean light curve remains clearly discernable. The twelve stars mentioned in section 3 do not meet this condition. With all of these twelve stars the duration of the individual cycles is subject to strong variations and the scatter around the mean period is large. This is very evident from figure 1 where for each of these twelve stars the lengths of the individual cycles has been compared with the mean beriod. It is to be observed that the duration of the individual cycles has been estimated in two different ways.

(a) once between two consecutive maxima,

(b) once between two consecutive minima.

From the figure it is evident that the period is not even approximately a constant so that one and the same light curve for all successive cycles can have no physical meaning. From the figure it is also evident that in some cases the variation of the period seems to be systematical. In any case it will be understood why these variables have been excluded from our considerations.

5. The relation between log P and A (1)

In figure 2 the indivudual values A (1) are plotted against the corresponding values log P. From the figure it would appear that our variables can roughly be devided into two groups. With the first group the period are within the interval $\log P = 2{,}0$ to $2{,}462$ while the values A (1) seem to be scattered along a horizontal line. The second group contains the variables for which $\log P > 2{,}462$ and with this second group in the mean there is a systematic increase of A (1) with increasing value of the period. It seems possible that the first group consists out of Pop II variables while the second mainly contains Pop I stars, The velocities of M type variables have been studied by Tulder and Oort [7]. Among the variables with $\log P < 2{,}462$ high velocities prevail and we have to deal with Pop II stars. With longer periods Tulder and Oort find systematic decrease of velocity and stars with very long period have very small velocities. It can hardly be doubted that these latter are Pop I stars. With the variables considered by Tulder and Oort there seem to be a gradual transition from extreme Pop

II over intermediate types to extreme Pop I type variables.

In our figure 2 no such gradual transition is apparent. The change from a scattering along a horizontal line to one along a sloping line seems to be rather abrupt. We suspect it to be connected with the transition from Pop II to Pop I variables, but even now the evidence is not conclusive. It must be remarked that notwithstanding a carefull choice of favourable light curves for any particular value log P the scatter of the indivi-

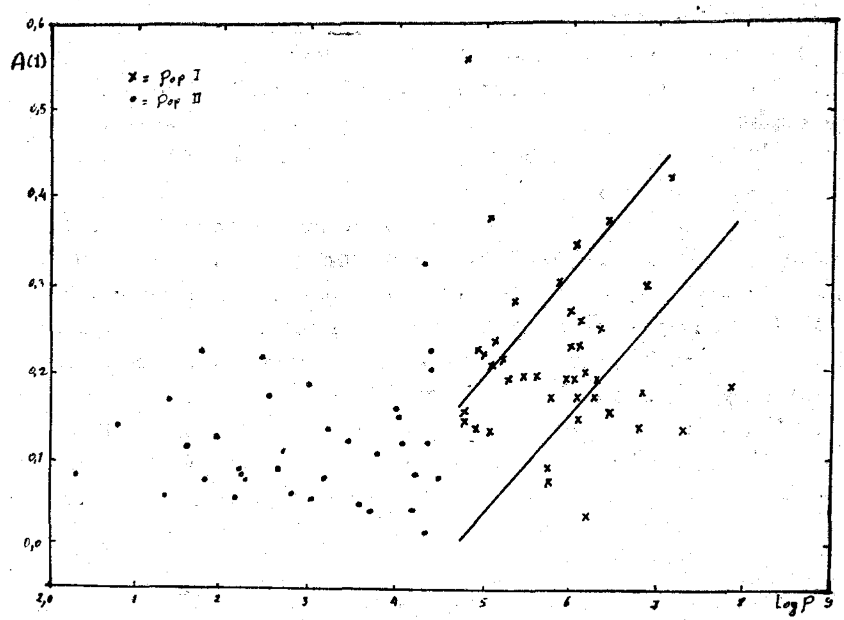

Fig. 2

Individual values of A (1) table 1 plotted against log P. The points for R Cen and S Cas are outside the figure

dual values A (1) remains large. It has to be remembered that the large amplitude of the M variables are mainly due to the blending effects of the absorption lines of Titanium oxide. To a large extend this effects of blending may mask the secondary variations in the light curves of these stars. It seems reasonable that this is the case and it would therefore seem that the large scatter of the individual poinst in our figure mainly represent variations in the effect of blending.

This would imply that our figure 2 represent the ultimate result which can be obtained for the M variables with the help of the autocorrelation method. Obviously this conclusion is correct only when visual light curves or anyhow light curves are used which are greatly affected by the blending of absorption lines. The situation would be different if bolometric curves were used, but it seems hardly possible that such curves could be obtained.

6. Conclusions

Our results can be summarised as follows.

a) The autocorrelation method and subsequent power analysis was applied to 81 M type variables with favourable light curve.

b) The resulting values A (1) were plotted against the corresponding values log P (figure 2)

c) There seems to be a slight difference between the distribution of the points corresponding to Pop I variables and those corresponding to Pop II stars but the evidence is not conclusive.

d) It is suggested that the large scatter of the individual points in figure 2 is due to the unequal effects of the blending by absorption line mainly of Titanium oxide.

e) If this latter suggestion is correct figure 2 represents the best results which the autocorrelation method can give for these stars and there is no reason to attempt further improvements.

Literature

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