# Tables for the rapid evolution of the effects of differential galactic rotation in radial direction

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**Özet:** Diferensiyel galaktik rotation için E. A. Kreiken [1] tarafından politropik gaz küreleri teorisine dayanılarak bulanan ve yazar [2] tarafından genelleştirilen yeni formüller, bugüne kadar kullanılan Oort formüllerine benzemektedir. Fakat fazla olarak $r^2$ ve $r^3$ terimlerini ihtiva etmektedir Radyal hızlara ait indirgeme hesaplarında bu yeni formüllerin uygulanmasını kolaylaştırmak için, katsayılara ait iki cetvel düzenlenmiştir. Birinci eetvel galaktik boylamlarına bağlı olarak $S_1 = A \sin 2l$ ve $S_2 = A_0 \sin l\,(1-2.6 \cos^2 l)$ ve ikinci cetvel $b$ galaktik enlemlerine bağlı olarak $\cos^2 b$ ve $\cos^3 b$ katsayılarını vermektedir.

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Recently Kreiken [1] from the theory of polytropic spheres has derived a set of relations representing the effects of differential galactic retation These rotations closely resemble derived from the Lindblad - Oort theory, but contain additional terms in $r^2$ and $r^3$. The term in $r^3$ is uninportant. The term in $r^2$ important only with large distances.

The relations given by Kreiken are valid for stars with galactic latitude $b = 0°$. For the other galactic latitudes the relations must be used as given by author [2] and which are:

$$\Delta V_r = A r \cos^2 b \sin 2l + A_0 r^2 \cos^3 b \sin l\,(1-2.6 \cos^2 l) \quad \cdots (1)$$

$$\Delta V_b = -\frac{1}{2} A r \sin 2b \sin 2l - \frac{1}{2} A_0 r^2 \sin 2b \cos b \sin l\,(1-2.6 \cos^2 l) \quad (2)$$

$$\Delta V_l = A r \cos 2l + B \cdot r + A_0 r^2 \cos l (3-2\cdot 6 \cos^2 l) + C r^3 (1-5.3 \cos^2 l) \quad (3)$$

Where $r$ is the distance of the star in parsecs and galactic longitude $l = l^{II} = (l^{I} - l_0^{I}) = (l^{I} - 328°)$. For the numerical values of the constants the theory of Kreiken gives

$$A = + 0.0189\ Km.\ sec^{-1} \cdot pc^{-1} \quad A_0 = - 2.3.10^{-6}\ Km.\ sec^{-1} \cdot pc^{-2}$$

$$B = - 0.0145\ Km.\ sec^{-1} \cdot pc^{-1} \quad C = + 0.28.10^{-9} \cdot Km.\ sec^{-1} \cdot pc^{-3}$$

In many investigations dealing with radial velocities and the propor motions of stars, the values given in the catalogues can not be used before corrections have been applied which account for the effects of differential galactic rotation. In order to reduce the amount of numerical calculations which this involves the present paper gives a table which, especially for the radial velocities, for each degree of longitude gives the numerical value of the coefficients of $r$ and $r^2$. For this the relation (1) is written in the form

$$\Delta V_r = S_1 \cdot r \cdot \cos^2 b + S_2 \cdot r^2 \cdot \cos^3 b \qquad (4)$$

where

$$S_1 = A \sin 2l \qquad S_2 = A_0 \sin l\,(1 - 2.6 \cos^2 l)$$

The table valid for $b = 0°$. For other galactic galactic latitudes the tabulated values must be multiplied with the appropriate values $\cos^2 b$ and $\cos^3 b$, which are given in the auxillarly table 2.

**Explanation of table 1 :** The first column contains the value $l^{I}$ = galactic longitude according to the older definitions. The values $l^{I}$ are arranged in groups so that in each group of values $l^{I}$, the sign of all numerical values of $S_1$ and $S_2$ remain a constant. Whether this sign is (+) or (—) is indicated at the top of the table. Each part of the table contains four groups of values $l^{I}$. For the values in each horizontal row the numerical values $S_1$ and $S_2$ are identical and as indicated in the right hand side of the table.

**Example :** We want to find the appropriate values for $l^{I} = 340°$. In the table 1 we look for $l^{I} = 340°$. At the top we find that $S_1$ is positive In the horizontal row we fine that the numerical values of $S_1$ is 0.0077. So the first term in the relation (4) is $+ 0.0077\, r \cos^2 b$. At the top we find that $S_2$ is also positive, while in the horizontal rew $l^{I} = 340°$, the numerical values of $S_2$ is found to be $0.0007.10^{-3}$. In the equation (4) the second term therefore reads $+ 0.0007.10^{-3} \cdot r^2 \cdot \cos^3 b$ and

$$\Delta V_r = + 0.0077\, r \cos^2 b + 0.000.10^{-3}\, r^2 \cos^3 b$$

In the same way for $l^I = 136°$ we find

$$\Delta V_r = - 0.0077\, r \cos^2 b + 0.0007.10\, r^2 \cos^3 b$$

For the table 2 no explanation is needed. All values $\cos^2 b$ and $\cos^3 b$ are to be taken positive.

## Bibliografya

[1] E. A. Kreiken, Representation of stellar systems by polytropic gas spheres, Annales d'Astrophysique, 24,3 (1961)

[2] A. Kızılırmak, Verallgemeinerung der Formeln für die differentielle galaktische Rotation, Astronomische Nachrichten, 286, H., S 128 (1961)

Table 1

| | | | | | | |
|---|---|---|---|---|---|---|
| $S_1$ | + | − | + | − | | |
| $S_2$ | + | + | − | − | | |
| | $l^I$ | $l^I$ | $l^I$ | $l^I$ | $S_1$ | $10^3.S_2$ |
| | 0° | 116° | 180° | 296° | 0,0170 | 0,0010 |
| | 1 | 115 | 181 | 295 | 0173 | 0011 |
| | 2 | 114 | 182 | 294 | 0175 | 0010 |
| | 3 | 113 | 183 | 293 | 0178 | 0010 |
| | 4 | 112 | 184 | 292 | 0180 | 0010 |
| | 5 | 111 | 185 | 291 | 0182 | 0009 |
| | 6 | 110 | 186 | 290 | 0183 | 0009 |
| | 7 | 109 | 187 | 289 | 0185 | 0008 |
| | 8 | 108 | 188 | 288 | 0186 | 0008 |
| | 9 | 107 | 189 | 287 | 0187 | 0007 |
| | 10 | 106 | 190 | 286 | 0188 | 0006 |
| | 11 | 105 | 191 | 285 | 0189 | 0006 |
| | 12 | 104 | 192 | 284 | 0189 | 0005 |
| | 13 | 103 | 193 | 283 | 0189 | 0005 |
| | 14 | 102 | 194 | 282 | 0189 | 0004 |
| | 15 | 101 | 195 | 281 | 0189 | 0003 |
| | 16 | 100 | 196 | 280 | 0188 | 0003 |
| | 17 | 99 | 197 | 279 | 0187 | 0002 |
| | 18 | 98 | 198 | 278 | 0186 | 0001 |
| | 19 | 97 | 199 | 277 | 0185 | 0000 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $S_1$ | + | − | + | − | | |
| $S_2$ | − | − | + | − | | |
| | $l^I$ | $l^I$ | $l^I$ | $l^I$ | $S_1$ | $10^3.S_2$ |
| | 20° | 96° | 200° | 276° | 0,0183 | 0,0000 |
| | 21 | 95 | 201 | 275 | 0182 | 0001 |
| | 22 | 94 | 202 | 274 | 0180 | 0002 |
| | 23 | 93 | 203 | 273 | 0178 | 0003 |
| | 24 | 92 | 204 | 272 | 0175 | 0004 |
| | 25 | 91 | 205 | 271 | 0173 | 0004 |
| | 26 | 90 | 206 | 270 | 0170 | 0005 |
| | 27 | 89 | 207 | 269 | 0167 | 0006 |
| | 28 | 88 | 208 | 268 | 0164 | 0007 |
| | 29 | 87 | 209 | 267 | 0160 | 0008 |
| | 30 | 86 | 210 | 266 | 0157 | 0008 |
| | 31 | 85 | 211 | 265 | 0153 | 0009 |
| | 32 | 84 | 212 | 264 | 0149 | 0010 |
| | 33 | 83 | 213 | 263 | 0145 | 0011 |
| | 34 | 82 | 214 | 262 | 0141 | 0012 |
| | 35 | 81 | 215 | 261 | 0136 | 0013 |
| | 36 | 80 | 216 | 260 | 0131 | 0013 |
| | 37 | 79 | 217 | 259 | 0126 | 0014 |
| | 38 | 78 | 218 | 258 | 0122 | 0015 |
| | 39 | 77 | 219 | 257 | 0116 | 0016 |

**Table 1 (Continued)**

| | | | | | | |
|---|---|---|---|---|---|---|
| $S_1$ | + | − | + | − | | |
| $S_2$ | − | − | + | + | | |
| | $l^{II}$ | $l^{II}$ | $l^{II}$ | $l^{II}$ | $S_1$ | $10^3.S_2$ |
| | 40° | 76° | 220° | 256° | 0,0111 | 0016 |
| | 41 | 75 | 221 | 255 | 0106 | 0017 |
| | 42 | 74 | 222 | 254 | 0100 | 0018 |
| | 43 | 73 | 223 | 253 | 0095 | 0018 |
| | 44 | 72 | 224 | 252 | 0089 | 0019 |
| | 45 | 71 | 225 | 251 | 0083 | 0019 |
| | 46 | 70 | 226 | 250 | 0077 | 0020 |
| | 47 | 69 | 227 | 249 | 0071 | 0021 |
| | 48 | 68 | 228 | 248 | 0065 | 0021 |
| | 49 | 67 | 229 | 247 | 0058 | 0022 |
| | 50 | 66 | 230 | 246 | 0052 | 0022 |
| | 51 | 65 | 231 | 245 | 0046 | 0022 |
| | 52 | 64 | 232 | 244 | 0040 | 0022 |
| | 53 | 63 | 233 | 243 | 0033 | 0023 |
| | 54 | 62 | 234 | 242 | 0026 | 0023 |
| | 55 | 61 | 235 | 241 | 0020 | 0023 |
| | 56 | 60 | 236 | 240 | 0013 | 0023 |
| | 57 | 59 | 237 | 239 | 0007 | 0023 |
| | 58 | 58 | 238 | 238 | 0000 | 0023 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $S_1$ | + | − | + | − | | |
| $S_2$ | + | + | − | − | | |
| | $l^{II}$ | $l^{II}$ | $l^{II}$ | $l^{II}$ | $S_1$ | $10.^3 S_2$ |
| | — | — | — | — | — | — |
| | — | — | — | — | — | — |
| | — | — | — | — | — | — |
| | 328° | 148° | 148° | 328° | 0,0000 | 0,0000 |
| | 329 | 147 | 149 | 327 | 0007 | 0000 |
| | 330 | 146 | 150 | 326 | 0013 | 0002 |
| | 331 | 145 | 151 | 325 | 0020 | 0002 |
| | 332 | 144 | 152 | 324 | 0026 | 0003 |
| | 333 | 143 | 153 | 323 | 0033 | 0003 |
| | 334 | 142 | 154 | 322 | 0040 | 0003 |
| | 335 | 141 | 155 | 321 | 0046 | 0005 |
| | 336 | 140 | 156 | 320 | 0052 | 0005 |
| | 337 | 139 | 157 | 319 | 0058 | 0006 |
| | 338 | 138 | 158 | 318 | 0065 | 0006 |
| | 339 | 137 | 159 | 317 | 0071 | 0006 |
| | 340 | 136 | 160 | 316 | 0077 | 0007 |
| | 341 | 135 | 161 | 315 | 0083 | 0007 |
| | 342 | 134 | 162 | 314 | 0089 | 0009 |
| | 343 | 133 | 163 | 313 | 0095 | 0008 |
| | 344 | 132 | 164 | 312 | 0100 | 0008 |

**Table 1 (Continued)**

| | | | | | | |
|---|---|---|---|---|---|---|
| $S_1$ | + | − | + | − | | |
| $S_2$ | + | + | − | − | | |
| | $l^{I}$ | $l^{I}$ | $l^{I}$ | $l^{I}$ | $S_1$ | $10^3 . S_2$ |
| | 345° | 131° | 165° | 311° | 0,0106 | 0,0010 |
| | 346 | 130 | 166 | 310 | 0111 | 0009 |
| | 347 | 129 | 167 | 309 | 0116 | 0009 |
| | 348 | 128 | 168 | 308 | 0122 | 0010 |
| | 349 | 127 | 169 | 307 | 0126 | 0010 |
| | 350 | 126 | 170 | 306 | 0131 | 0011 |
| | 351 | 125 | 171 | 305 | 0136 | 0011 |
| | 352 | 124 | 172 | 304 | 0141 | 0010 |
| | 353 | 123 | 173 | 303 | 0145 | 0011 |
| | 354 | 122 | 174 | 302 | 0149 | 0011 |
| | 355 | 121 | 175 | 301 | 0153 | 0011 |
| | 356 | 120 | 176 | 300 | 0157 | 0011 |
| | 357 | 119 | 177 | 299 | 0160 | 0011 |
| | 358 | 118 | 178 | 298 | 0164 | 0011 |
| | 359 | 117 | 179 | 297 | 0167 | 0011 |

**Table 2**

| b | cos² b | cos³ b | b | cos² b | cos³ b | b | cos² b | cos³ b |
|---|---|---|---|---|---|---|---|---|
| 1° | 1.000 | 1.000 | 31° | 0.734 | 0.629 | 61° | 0.235 | 0.114 |
| 2 | .998 | .997 | 32 | .719 | .610 | 62 | .221 | .104 |
| 3 | .998 | .997 | 33 | .704 | .591 | 63 | .206 | .093 |
| 4 | .996 | .994 | 34 | .687 | .570 | 64 | .192 | .084 |
| 5 | .992 | .988 | 35 | .671 | .549 | 65 | .179 | .076 |
| 6 | .988 | .982 | 36 | .654 | .530 | 66 | .166 | .067 |
| 7 | .982 | .973 | 37 | .638 | .510 | 67 | .153 | .060 |
| 8 | .980 | .970 | 38 | .621 | .489 | 68 | .141 | .053 |
| 9 | .976 | .964 | 39 | .594 | .458 | 69 | .128 | .046 |
| 10 | .970 | .956 | 40 | .587 | .450 | 70 | .117 | .040 |
| 11 | .964 | .947 | 41 | .570 | .430 | 71 | .106 | .035 |
| 12 | .956 | 935 | 42 | .552 | .410 | 72 | .095 | .030 |
| 13 | .949 | .924 | 43 | .534 | .391 | 73 | .085 | .025 |
| 14 | .941 | .913 | 44 | .517 | .372 | 74 | .076 | .021 |
| 15 | .933 | .901 | 45 | .500 | .353 | 75 | .067 | .017 |
| 16 | .924 | .888 | 46 | .483 | .336 | 76 | .059 | .014 |
| 17 | .914 | .874 | 47 | .465 | .317 | 77 | .051 | .011 |
| 18 | .904 | .860 | 48 | .448 | .299 | 78 | .043 | .009 |
| 19 | .895 | .847 | 49 | .430 | .282 | 79 | .036 | .007 |
| 20 | .884 | .831 | 50 | .413 | .266 | 80 | .030 | .005 |
| 21 | .872 | .815 | 51 | .396 | .249 | 81 | .024 | .004 |
| 22 | .859 | .797 | 52 | .379 | .234 | 82 | .019 | .003 |
| 23 | .846 | .779 | 53 | .362 | .218 | 83 | .015 | .002 |
| 24 | .834 | .761 | 54 | .346 | .203 | 84 | 0.011 | .001 |
| 25 | .821 | .744 | 55 | .329 | .189 | 85 | 0.008 | .001 |
| 26 | .808 | .727 | 56 | .312 | .175 | 86 | 0.005 | 0.000 |
| 27 | .794 | .707 | 57 | .297 | .162 | 87 | 0.003 | 0.000 |
| 28 | .780 | .689 | 58 | .281 | .149 | 88 | 0.001 | 0.000 |
| 29 | .766 | .670 | 59 | .265 | .137 | 89 | 0.000 | 0.000 |
| 30 | .750 | .649 | 60 | .250 | .125 | 90 | 0.000 | 0.000 |

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