The density gradient in the atmosphere of the K-type component of 31 Cygni

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Özet: Bu araştırma 31 Cygni'nin 1951 de vukubulan son tutulmasında alınan spektrumlara dayanmaktadır. Elde edilen neticeler şöyle hulasa edilebilir.

Her bir rasad gününe tekabül eden, K-tip yıldızın kenarından itibaren olan yükseklik kilometre olarak hesap edilmiştir.

Rasad edilen her bir atmosferik seviye için büyüme eğrisi (curve of growth) turbulent hız $\xi = 0$ ve $\xi = 20$ km/sa. için ayrı ayrı inşa edilmiştir.

Büyüme eğrisinin hesabında demir için kullanılan f-değerleri King and King'in labaratuar neticelerinden alınmış ve H. Kopfermann ve G. Wessel'in eşeline irca etmek için 3.2 faktörü ile çarpılmıştır. Diğer elementler için f-değerleri muhtelif kaynaklardan elde edilmiştir,

Her bir atmosferik seviye için $\xi = 0$ ve $\xi = 20$ km/sa. alınarak inşa mütekabil büyüme eğrisinden her bir atomik ve iyonizasyon seviyelerindeki izafi atom sayıları tayin edilmiştir.

Atmosferin ortalama yoğunluk gradyeni büyüme eğrisi vasıtasıyla bulunur. Bu araştırmadaki atomik ve iyonik sevilerin çoğu için gradyan birbirine uygundur. Yalnız Ti I ve Mn I istisna teşkil ederler. Bundan başka $\xi = 0$ ve $\xi = 20$ km/sa için çıkarılan iki takım gradyen mütekabilen hemen hemen aynidir.

Eğer yoğunluklar $n = n_0 e^{-ah}$ formülü ile yüksekliğin fonksiyonu olarak ifade edilirse a için rasatlar $a = 0,81 \times 10^{-12}$ ortalama değerini verirler. Rasadlanan gradyen hidrostatik denge hipotezine göre hesaplanan gradyenle mukayese edilmiştir. Demir için hidrostatik denge hipotezi, rasatlanan gradyenden üç bin kerre büyük netice verir.

Mc Crea'nın turbulent teorisi tecrübe edilmiştir. Mc Crea'nın denkleminde turbulent hız için 20 km/sa konulursa, elde edilen gradyen yine dokuz misli büyüktür.

Bu sebepten, K-tipi dev yıldız atmosferinin muhtemelen hidrostatik dengede olmayıp, bunun yerine dinamik bir şeklin hakiki yapıya daha yakın olduğuna dair elimizde bir vesika daha mevcuttur.

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Abstract: This investigation is based upon spectrograms obtained during the last eclipse of 31 Cygni late in 1951. The results can be summarised as follows:

The heights in kilometers above the limb of the K-type star corresponding to each day of observation are calculated

The curves of growth were constructed for each observed atmospheric level separately both for the turbulent velocity $\xi = 0$ and $\xi = 20$ km/s.

For computing the curve of growth the f-values used for iron were taken from the laboratory results of King and King and multiplied by the factor 3.2 in order to reduce them to the scale of H. Kopfermann and G. Wessel. The f-values for the other elements were taken from various other sources.

For each atomic and ionization state the relative number of atoms were derermined for each atmospheric level from the curver of growth constructed for that level with $\xi = 0$ and $\xi = 20$ km/sec.

The mean density gradients in the atmosphers are readily evaluated by means of the curve of growth. For most of the atomic and ionic states included in this investigation the gradients are consistent with each other except for Ti I and Mn I. Moreover the two sets of gradients derived for $\xi = 0$ and $\xi = 20$ km/sec. respectively are nearly equal.

If the densities are expressed as a function of height by means of the formula $n = n_0 e^{-ah}$ for a the observations give a mean value $a = 0.81 \times 10^{-12}$ cm^{-1}. The observed gradient is compared with that calculated from the assumption of hydrostatic equilibrium. For Fe this hydrostatic equilibrium hypothesis leads to a gradient of an order three thousand times larger than the observed one.

Next Mc Crea's theory of turbulent support is tried. If the turbulent velocity 20 km/sec. is inserted in the Mc Crea's equation, the resulting gradient still is nine times too large.

Therefore, we now have additional evidance that the atmosphere of a K-type supergiant star possibly is not in hydrostatis equilibrium, but that instead of this a dynamical picture might be closest to the actual structure.

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Introduction

The discovery of turbulent motions in the atmosphere of supergiant stars, made it increasingly important to study the structure of such atmospheres. The best clues to the structure of supergiant atmospheres come from the studies of eclipsing systems. The eclipsing system 31 Cygni, where a K type supergiant eclipses its B-type companion, gives us an opportunity to study the structure of the atmosphere of a K-type supergiant. After emergence of the B-star from behind the K-star, the light of the B-type secondary, on its way to observer, passes through the turbulent atmosphere of the K-type supergiant primary. So the light of the B-type star will be absorbed by the atoms of different elements in the atmosphere of the K-type star. e.g. by that part of the atmosphere which happens to be in the line of sight. The intensity of this absorption gives us an opportunity to analyse the constitution of the atmosphere of the K-type star as a function of its height above the photosphere of the K-star.

Observational Material : This investigation is based upon the spectrograms obtained during the last eclips of 31 Cygni late in 1951. Spectroscopic observations have been made at Victoria with the 72-inch reflector and at Ann Arbor with the 37-inch the reflector. The Victoria plates were taken with greater resolution, a grating spectrograph giving 4.6 R/mm being used, while the Ann Arbor plates were largely obtained with a two prism instrument, dispersion 12.3 Å/mm at λ 3933. For this investigation 19 plates were studied which wers kindly placed at my disposal by Prof. Aller. Of these, four were taken during totality. The other have been taken in the interval between the beginning and the end of the eclipse and cover the whole atmosphere of the K-star. The plates taken during totality were traced with the Moll Microphotometer of the Michigan University Observatory and the others have been traced with the Lake Angelus's Microphotometer. The regions covered on the tracings were from the Hydrogen absorption line H up to ultraviolet at the end of the plates.

The eclipsing system 31 Cygni consists of a supergiant K-star and much smaller B-star, which in a period of about 3800 days, revolve around their comman center of gravity. During totality we observe the normal K-type spectrum. After totality when the B-star emerges from behind the K-type star, the light of the B-type star, on its way to observer, passes through the K-type star's atmospthere. So we observe the normal spectrum of the K-type giant with the spectrum of the B-star superimposed upon it. The latter is not quite the normal spectrum of the B-star. It shows, besides its normal absortion lines some additional lines produced by those atoms in the atmosphere of the K-star which instantanuously are in the line of sight. Since, however the normal spectrum of the K-star contains absorption lines at the same weve-lengths these additional lines coincide with the normal absorption lines of the K-type spectrum. So, the observed spectrum is a composite spectrum and we must consider how we can separate them.

To separate the two spectra we proceeded in the usual way. According to usual definition $r_k = I_k / I_{ck}$ is the value of r at any point in the normal K spectrum. In other words, r_k is the the ratio of the line intensity to the intensity of the continuous background at this point.

Similiarly, without the atmosphere of the K-star in front of the B-star we could write $r_b = I_b / I_{cb}$. Actually from the tracing, we measure the composite value r_{bk} which by definition is equal to

$$r_{bk} = \frac{I_k + I_b}{I_{ck} + I_{cb}}$$

For any line the equivalent width is given by the expression of

$$W = \int (1 - r)\, d\lambda$$

During totality, the total absorption as measured is

$$W_k = \int (1 - r_k)\, d\lambda$$

Let $W_b = \int (1 - r_b)\, d\lambda$ be the total absorption for the B spectrum. Let I_{cb} / I_{ck}, the ratio of the intensities of the continuous spectra of the two stars, be indicated by α. The measured composite spectrum will be

$$W_{bk} = \int (1 - r_{bk})\, d\lambda$$

Since,

$$r_{bk} = \frac{I_k + I_b}{I_{ck} + I_{cb}} = \frac{r_k + \alpha r_b}{1 + \alpha}$$

$$W_{bk} = \int \frac{(1 - r_k) + \alpha(1 - r_b)}{1 + \alpha}\, d\lambda$$

Hence we have

$$W_b = W_{bk} + \frac{1}{\alpha}(W_{bk} - W_k) \qquad (1)$$

On the right hand side of the equation W_{bk} is the measured total absorption of the composite line and W_k the total absoɪption in the normal K-spectrum (found from spectrograms taken during totality. If we follow the considerations of Christie and O. Wilson, α is computed from the equation

$$\alpha = \frac{W_{bk}}{W_k} - 1$$

The values α as computed from the foregoing equation were plotted against λ and through the points, obtained in this way, a mean curve was drawn. The drawing of the continuous background and the measurements of the absorption lines in the spectral regions adjoining the wave-length λ 3800 Å were rather difficult and in this region the values show a considerable scattering. The mean curve which was drawn is that which fits the scattered points as smoothly as possible. The values which were needed for computing W_b (the total absorption in the B spectrum) were read from this graph.

The lines selected for this purpose and the equivalent widths W_b which we obtained from the relation (1) for different times of observation are tabulated in Table I. Obviously these are the basic values for our further discussion.

Calculation of height above the limb of the K-type star: At a given time of observation the height of the line of sight to the center of the B-star above the limb of the K-star depends on the time which has elapsed between this time of observation and the moment of egress at fourth contact. For calculation of this height we need to know :

1. The transverse velocities of the stars during eclipse.
2. The inclination i of the orbit plane to the line of sight.
3. The ratio K of the radii of two stars.

From the known elements of this star the following approximate values are amply sufficient for our present purpose :

$$R_K = 150. \qquad K^{-1} = 30 \qquad i = 90^\circ$$

while the transverse velocity is taken to be uniform. Then, heights may be calculated with sufficient accuracy by simple taking them to be proportional to the time intervals.

In addition to the above data, the following figures are used:
Duration of eclipse = 61 days
Epoch of Minimum = J. D. 2433902

Table I

The values of W_b

Wave-length	Element	Oct. 18	Oct. 20	Oct. 21	Oct. 22	Oct. 25	Oct. 26	Oct. 28
4045.82	Fe I	1.67						
4034.49	Mn I	.03	—.13	.00	—.13	—.02	—.07	.15
4033.06	Mn I	.27	.27	.35	.18	.00	.10	.15
4030.76	Mn I	.23	.44	.23	.54	.15	.39	.42
4005.25	Fe I	.58	.51	.72	.68	.47	.52	.62
3998.84	Ti I	.17	.13	.26	.16	.20	.16	.28
3989.76	Ti I	.31	.29	.33	.25	.09	.09	.17
3982.54	Ti I	.00	.04	.01	.10	.04	.06	—.01
3961.52	Al I	.14	.16	.18	.11	.05	.04	.14
3944.01	Al I	.17	.16	.06	.13	.02	—.01	.01
3930.30	Fe I	.20	.20	.11	.02	—.02	—.08	.03
3927.93	Fe I	.05	.11	—.05	.08	.02	.01	—.05
3922.91	Fe I	.12	.11	—.04	.08	.08	.00	.06
3920.26	Fe I	.06	.14	—.07	.12	.10	—.01	.05
3913.46	Ti II	.27	.20	.11	.11	.11	.04	—.02
3900.55	Ti II	.36	.24	.24	.12	.02	.02	.02
3895.66	Fe I	.27	.26	.19	.19	.07	.09	.13
3865.53	Fe I	.03	04	.01	.04	.03	.09	.09
3859.91	Fe I	.25	.17	.14	.16	.05	.06	.08
3856.37	Fe I	.35	.27	.21	.15	.15	.28	.22
3849.97	Fe I	.05	.03	—.03	.00	—.06	.03	.12
3838.29	Mg I	.80	.48	.34	.27	.08	.18	.01
3832.30	Mg I	.78	.32	.23	.23	.00	—.14	—.11
3829.95	Mg I	.25	.23	.13	.12	.06	—.01	.10
3827.82	Fe I	.17	.26	.06	.02	.15	.12	.12
3825.88	Fe I	.17	.25	.17	.13	.07	.14	.13
3824.44	Fe I	.25	.12	.11	.04	.04	—.02	—.02
3820.43	Fe I	.59	.42	.11	.20	.10	.03	—.14
3815.84	Fe I	.17	.17	.07	.01	.07	.10	.10
3812.97	Fe I	.23	.09	.03	.15	.06	.10	.15
3763.79	Fe I	.58	.33	.13	.28	.25	.24	.14
3761.32	Ti II	.69	.51	.45	.51	.57	.57	.39
3759.29	Ti II	.81	.75	.29	.60	.22	.44	.41
3758.24	Fe I	.50	.41	—.03	.16	.01	.14	.05

Table I (continued)

The values of W_b

Wave-length	Element	Oct. 30	Oct. 31	Nov. 6	Nov. 10	Nov. 11	Nov. 21
4045.82	Fe I						
4034.49	Mn I	.24	.03	.00	.03	.08	.05
4033.07	Mn I	.15	.35	.30	.15	.15	.17
4030.76	Mn I	.34	.42	—.10	.30	.07	—.10
4005.25	Fe I	.59	.53	.32	.39	.47	.46
3998.64	Ti I	.32	.19	.21	.19	.17	.13
3989.76	Ti I	—.03	.15	.11	.13	.09	.01
3982.54	Ti I	.04	—.15	—.07	—.03	—.07	—.05
3961.52	Al I	.00	.00	.04	.05	—.02	—.03
3944.01	Al I	—.01	.01	—.01	—.01	—.04	—.06
3930.30	Fe I	—.07	—.02	—.08	—.07	—.08	—.07
3927.93	Fe I	.02	.16	.08	—.06	.00	—.05
3922.91	Fe I	.00	.11	.04	—.04	—.01	—.01
3920.26	Fe I	.02	—.09	.04	.04	.00	—.02
3913.46	Ti II	—.02	.00	—.02	.00	—.02	—.05
3900.55	Ti II	.01	.02	.01	—.03	.01	—.02
3895.66	Fe I	.13	.00	.03	.02	.05	.02
3865.53	Fe I	.07	.09	.07	.10	.10	.04
3859.91	Fe I	.07	—0.8	—.05	—.02	—.14	—.12
3856.37	Fe I	.24	.17	.15	.14	.05	.16
3849.97	Fe I	.06	.03	—.07	—.01	—.07	—.01
3838.29	Mg I	.27	.17	.00	.01	—.02	.04
3832.30	Mg I	—.01	—.01	—.08	—.08	—.01	—.05
3829.35	Mg I	.05	—.01	—.01	.00	—.01	—.01
3827.82	Fe I	.02	.10	.02	—.01	.03	—.05
3825.88	Fe I	.00	.05	.00	.04	.00	.00
3824.44	Fe I	.01	—.04	—.08	—.08	—.04	—.05
3820.43	Fe I	.07	—.04	.04	—.06	—.02	
3815.84	Fe I	.05	.10	.01	—.06	—.03	.01
3812.97	Fe I	.00	.09	.00	.03	.09	.03
3763.79	Fe I	.32	.21	.22	.21	.14	.07
3761.32	Ti II	.17	.29	.04	—.05	—.05	.04
3759.29	Ti II	.18	.23	—.01	.06	.06	—.04
3758.24	Fe I	.15	.14	.01	.08	.08	—.11

Transverse speed is:

$$2\left(150 + \frac{150}{30}\right) \times 7 \times 10^5 : 61 \text{ km/day} = 3{\cdot}56 \times 10^6 \text{ km/d}$$

If t indicates the time interval between the time of observation and the fourth contact for egress, the heights in kilometers is given by the formula :

$$h = 3{\cdot}5 \times 10^6 + 3{\cdot}56 \times 10^6 \; \Delta t \quad km$$

In the table II, I have tabulated the plates used in this investigation the dates of observation and the height in kilometers as computed from the aforegoing equation.

Table II

Spectrograms, Times of observations and Calculated heights

Plates	Date 1951	U. T	J. D. 24334	t	$h \times 10^{-7}$ km
19206-A	Oct. 18	00.8	937.53	5.03	2.14
19213-A	Oct. 20	01.0	939.54	7.04	2.86
19217-A	Oct. 21	04.5	940.68	8.18	3.26
19219-A	Oct. 22	00.8	941.53	9.03	3.56
19222-A	Oct. 25	05.0	944.71	12.21	4.70
19225-A	Oct. 26	00.7	945.52	13.02	4.98
19229-A	Oct. 28	01.6	947.56	15.06	5.71
19258-B	Oct. 30	00.7	949.53	17.03	6.41
19239-A	Oct. 31	00.9	950.54	18.04	6.77
19245-B	Nov. 6	00.9	956.54	24.04	8.91
19253-B	Nov. 10	01.4	960.56	28.08	10.34
19259-A	Nov. 11	00.5	961.52	29.02	10.68
19278-B	Nov. 21	00.2	962.50	30.00	11.03

The observed curve of growth: The observational material collected in Table I enables us to make a systematic study of the atmosphere of 31 Cygni. One way of obtaining physical information concerning the structure of stellar atmospheres is to study the curve of growth constructed for different layers of the atmosphere.

Suppose that the lines are formed according to the mechanism of extinction, i. e. that at each point in the line

$$I = I_c e^{-\overline{NL}.\alpha}$$

where $\overline{NL}$ is the total number of atoms in a column taken through the whole atmosphere while α_ν is the absorption coefficient.

The equivalent-width of the line is

$$W_\nu = \left(1 - \frac{I}{I_c}\right) d\lambda$$

Using the following substitutions

$$\Delta\lambda = \frac{c}{2}\,\Delta\nu \qquad \frac{\nu - \nu_0}{\Delta\nu} = u \qquad \Delta\lambda = \frac{v}{c}\,\lambda$$

We obtain

$$\frac{W}{\lambda}\,\frac{c}{v} = (1 - r)\,du$$

Following the method of Wrubel for constructing the curve of growth for different levels we obtain different curves of growth which for these levels gives the relation between

$$\frac{W}{\lambda}\,\frac{c}{v} \quad \text{and} \quad \eta_0 = \frac{N\alpha_0}{k_\nu}$$

Here k_ν is the continuous absorption coefficient which depends on wavelength, v the velocity of the atoms while α_0 is the absorption coefficient at the center of the line per atom and is given by

$$\alpha_0 = \frac{\pi\varepsilon^2}{mc}\,\frac{c}{v\,\nu\sqrt{\pi}}\,f$$

where the symbols have the usual meaning. The f-values = oscilator strengths for iron used in this paper are King's absolute f-values multiplied by the factor 3·2 in order to get agreement with the values obtained by H. Kopfermann and G. Wessel at Göttingen. Prof. Minnaert and Dr. Koelbloed kindly communicate to me the f-values of several other elements, for which we had as yet no data available in our library.

In the construction of the curve of growth, all lines of the same transition array of a given element are treated together. Since different terms have slightly different excitation potentials, their population depend on temperatnre. To account for this the Boltzmann correction term is applied for reducing a group of lines of a multiplet to the lowest state. The correcrection term is in the form of :

$$\frac{5040}{T_1}\Delta x$$

where T_1 is the excitation temperature in the chromosphere of 31 Cygni and is the difference between the excitation potential of the multiplet which is considered and that of the multiplet which has the lowest excitation potential. Possibly, T_1 is a function of the height, but for the present T_1 is taken to be a constant. Throughout this reduction for T_1 we used the value $T_1 = 4000°$ K. The only effect is a slight scattering of the points in horizontal direction along the curve of growth Now the correction term is easily computed and the result are given in Table III. For the three states of iron a^5D, a^5F, and a^3F the excitation potentials which were used are 0.06, 0.95 and 1.54 e. v. respectively.

Since a single theoretical curve of growth is used for all lines distributed over a considerable range of wave-length, it is necessary to apply to log η_0 a correction term $\Delta \log \eta_0$. The correction $\Delta \log \eta_0$ depends on the value of $B^{(0)}/B^{(1)}$ at the wave-length which is considered.

$$\frac{B^{(0)}}{B^{(1)}} = \frac{8}{3}\ \frac{k}{h}\ \frac{T}{\nu}\ \frac{k_\nu}{\bar{k}}$$

For the middle part of our spectral region (λ 3800) the ratio of the continuous absorption coefficient to the mean absorption coefficient can be taken equal to $k_\nu/\bar{k} = 1$. With this value of $k_\nu/\bar{k}$, $T = 4000°K$, $B^{(0)}/B^{(1)}$ is nearly 1/3. When we take into account the remaining uncertainties in the determination of the ratio $k_\nu/\bar{k}$ for the sun and for the K stars, we may take $B^{(0)}/B^{(1)}$ to be exactly 1/3.

The final equation from which the values of η_0 were calculeted is

$$\log \frac{\eta_0}{N} = \log \alpha_0 - \frac{5040}{T_1}\Delta x + \Delta \log \eta_0 + \text{const.}$$

$$\alpha'_0 = \frac{\sqrt{\pi}\,\varepsilon^2}{mcv}\ \left(\frac{\lambda}{3800}\right) f$$

where λ is expressed in Angström units.

Apparently the K-component of the system 31 Cygni has a turbulent atmosphere. The velocities of the atoms are not only due to thermal motions of the atoms but also to turbulent motions of the masses of gas. In the present investigation the following cases are considered :

a — The atmosphere of 31 Cygni is governed by large turbulent eddies. So in the layers concerned, the turbulent velocity is $\zeta = 0$ and the curve of growth is determined by the gas kinetic velocity and the classical damping constant. In this case v is taken as $v^2 = \frac{2kT}{M}$.

b — The turbulent elements are tiny in the atmosphere of 31 Cygni. The curve of growth will be determined by $v^2 = \frac{2kT}{M} + \zeta^2$.

For the turbulent velocity ζ, Dr. Aller has obtained a value 20 km/sec. from the chromospheric K-line profile. In this paper we have used his value for the turbulent velocity.

The values of log η_0 corresponding to $\zeta = 0$ and $\zeta = 20$ km/sec. are given in the Table III in the columns under log $\frac{\eta'_0}{N}$ and log $\frac{\eta''_0}{N}$ respectively. For each separate day of observation and both for $\zeta = 0$ and $\zeta = 20$ km/sec. the values of log $\frac{-W}{\lambda} \frac{c}{v}$ corresponding the different atmospheric levels appear in the Tables IV and V.

For each observed atmospheric level separately, the curve of growth was constructed in the following way. On sheets of tracing paper the values of log $\frac{W}{\lambda} \frac{c}{v}$ for all lines of a multiplet are shifted horizontally by a constant amount so as to have the best fit with the theoretical curve. The theoretical curve of growth which is used is the one which Wrubel calculated for $B^{(0)}/B^{(1)} = 1/3$ and log $a = -2.6$.

The final results were plotted for $\zeta = 0$ and $\zeta = 20$ km/sec while also the theoretical curve of growth was indicated.

Table III

Element Transition	Wave-length	Excitation Poten. (in volt)	log f	Boltzmann Corr.	$-\log \frac{\eta_0'}{N}$	$-\log \frac{\eta_0''}{N}$
Fe I						
$a^5D-z^5D^0$	3930.30	.09	7.98-10	.00	13.18	14.45
	3927.93	.11	8.14	»	13.03	14.30
	3922.91	.05	7.68	»	13.49	14.76
	3920.26	.12	8.01	»	13.16	14.43
	3895.66	.11	7.96	»	13.21	14.48
	3859.91	.00	8.35	»	12.91	14.08
	3856.37	.05	7.98	»	13.19	14.46
	3824.44	.00	7,76	»	13.41	14.68
$a^5F-y^5D^0$	3865.53	1.01	8.92	—1.22	13.47	14.74
	3849.53	1.01	9.00	»	13.39	14.66
	3825.88	.91	9.17	»	13.23	14.50
	3820.43	.86	9.22	»	13.18	14.45
$a^5F-z^3P^0$	3812.97	.95	8.44	»	13.96	15.23
$a^5F-y^5F^0$	3763.79	.99	9.33	»	13.03	14.35
	3758.24	.95	9.37	»	13.03	14.30
$a^3F-y^3F^0$	4045.82	1.48	9.60	—1.86	13.41	14.68
	4005.25	1.55	9.00	»	14.02	15.29
$a^3F-y^3D^0$	3827.83	1.55	9.60	»	13.44	14.71
	3815.84	1.48	9.54	»	13.50	14.77
Mn I						
a^6S-z^6P	4034.49	3.06	9.48	.00	11.68	12.95
	4033.07	3.06	9.67	»	11.49	12.76
	4030.76	3.06	9.79	»	11.36	12.63
Ti I						
$a^3F-y^3F^0$	3998.64	.05	8.95	—0.06	12.30	13.54
	3989.76	.02	8.81	—0.03	12.38	13.62
$a^3F-z^5S^0$	3982.54	.06	7.84	.00	13.35	13.59
Ti II						
$a^2G-z^2G^0$	3913.46	1.11	8.30	—0.68	13.58	14.82
	3900.55	1.13	8.30	—0.69	13.59	14.83
$a^2F-z^2F^0$	3761.32	.57	9.54	0.00	11.64	12.88
	3759.29	.60	9.50	—0.04	11.65	12.89
Mg I						
$3^3P^0-3^3D$	3838.29	2.70	9.75	.00	11.61	12.70
	3832.30	2.70	9.70	»	11.66	12.65
	3829.35	2.70	9.83	»	11.53	12.62
Al I						
$3^2P^0-4^2S$	3961.52	.01	9.12	»	12.20	13.31
	3944.01	.00	9.12	»	12.20	13.31

Table IV

The values of $\log \frac{W}{\lambda} \frac{c}{v}$ for $\xi = 0$

Element Transition	Wave-length	J. D. 2433 937.5	J. D. 2433 939.5	J. D 2433 940 5	J. D. 2433 941 5	J D. 2433 944.7	J. D. 2433 945.5	J. D. 2433 947.5
eI								
^{5}D-z^5D^0	3930.30	1.15	1.14	0.89	0.16			0.32
	3927.93	.54	.89		.75	.15	—.16	
	3922.91	.92	.89		.75	.75	—.56	.62
	3920.26	.62	.99		.92	.86		.53
	3895.66	1.28	1.26	1.13	1.13	.70	.80	.96
	3859.91	1.24	1.07	1.00	1.06	.55	.63	.76
	3856.37	1.40	1.28	1.18	1.03	1.03	1.30	1.20
	3824.44	1.25	.94	.90	.46	.46		
^{5}F-y^5D^0	3865.53	.33	.45	—.15	.45	.33	.81	.81
	3849.97	.55	.33		—.56		.33	.93
	3825.80	1.09	1.25	1.09	.97	.70	1.00	.97
	3820.43	1.63	1.48	.90	1.16	.86	.34	
^{5}F-z^3P^0	3812.97	1.22	.81	.34	1.03	.64	.86	.84
^{5}F-y^5F^0	3763.79	1.63	1.38	.98	1.31	1.26	1.24	1.01
	3758.24	1.56	1.48		1.58	.86	1.62	.56
^{3}F-y^3F^0	4045.82	2.05						
	4005.82	1.59	1.54	1.69	1.67	1.51	1.55	.63
^{3}F-y^3D^0	3827.83	1.09	1.27	.63	.16	1.03	.94	.94
	3815.84	1.09	1.09	.70	—.15	.70	.86	.86
n I								
S-z^6P	4034.49	.31		—.56				1.01
	4033.07	1.26	1.26	1.38	1.09	—.56	.83	1.01
	4030.76	1.20	1.48	1.20	1.57	1.01	1.26	1.46
I								
F-y^3F^0	3998.64	1.04	.92	1.22	1.01	1.11	1.01	1.25
	3989.76	1.30	1.27	1.33	1.21	.76	.76	1.04
F-z^5S^0	3982 54	—.59	.41		.81	.41	.59	
II								
G-z^2G^0	3913.46	1.25	1.12	.86	.86	.86	.42	
	3800.55	1.38	1.20	1.20	.90	.12	.12	
F-z^2F^0	3761.32	1.67	1.54	1.49	1.54	1.59	1.59	1.43
	3759.29	1.74	1.71	1.30	1.61	1.18	1.42	1.45
r I								
^{0}-3^{3}D	3838.29	1.58	1.36	1.21	1.11	.58	.93	—.37
	3832.30	1.57	1.18	1.04	1.04	—.74		
	3829.55	1.08	1.04	.79	.76	.46		.68
I								
^{0}-4^{2}S	3961.52	.84	.89	.95	.73	.39	.29	.84
	3944.01	.92	.89	.47	.81	.00		—.31

Table IV (continued)

Element Transition	Wave-length	J. D. 2433 949.5	J. D. 2433 950 5	J. D. 2433 956.5	J. D. 2433 960.5	J. D. 2433 861.5	J. D. 2433 962 5
Fe I							
a^5D-z^5D^0	3930.30						
	3927.93	.15	1.05	.75		—.56	
	3922.91	— 56	.90	.45			
	3920.26	.15		.45	.45	—.56	
	3895.66	.96	—.56	.33	.15	.55	.14
	3859.91	.70					
	3856.37	1.23	1.08	1.03	1.00	.55	1.06
	3824.44	.86			.76		
a^5F-y^5D^0	3865.53	.70	.81	.70	.85	.85	.45
	3849.97	.63	.33				
	3826.88	—.56	.56	—.56	.46	—.56	—.56
	3820.43	.70		.46			
a^5F-z^3P^0	3812.97	—.56	.81	—.56	.34	.81	.33
a^5F-y^5F^0	3763.79	1.37	1.19	1.21	1.18	1.01	.86
	3758.24	1.04	1.01	—.13	1.01	.77	
a^3F-y^3F^0	4045.82						
	4005.25	1.61	1.56	1.34	1.43	1.51	1.50
a^3F-y^3D^0	3827.83	.16	.85	.16		.33	
	3815.84	.56	.85	—.15			—.15
Mn I							
a^6S-z^6P	4034.49	1.21	.31		.31	.74	.53
	4033.07	1.01	1.38	1.31	1.01	1.01	1.06
	4030.76	1.35	1.46		1.31	.68	
Ti I							
a^3F-z^3F^0	3998.64	1.31	1.09	1.13	1.09	1.04	.92
	3989.76		.99	.85	.92	.76	.81
a^3F-z^5S^0	3982.54	.41			—.08		
Ti II							
a^2G-z^2G^0	3913.46		—.59				
	3900.55	—.18	.12	—.18		—.18	
a^2F-z^2F^0	3761.32	1.06	1.30	.43			.43
	3759 29	1.09	1.20		.61	.68	
Mg I							
3^3P^0 3^3D	3838.29	1.11	.91	—.74	—.33		.28
	3832.30						
	3829.35	.38					
Al I							
3^2F^0-4^2S	3961.52	—.71	—.71	.29	.39		

Table V

The values of $\log \frac{W}{\lambda} \frac{c}{v}$ for $\xi = 20$ km/sec.

Element Transition	Wave-length	J. D. 2433 937.5	J. D. 2433 938.5	J. D 2433 940.5	J. D. 2433 941.5	J. D. 2433 944.5	J. D. 2433 945.5	J. D. 2433 947.5
Fe I								
a^5D-z^5D^0	3930.30	—.10	—.11	—.37	—.10			—.94
	3927.93	—.72	—.37		—.51	—1.11	—1.42	
	3922.91	—.33	—.37		—.51	—.54	—1.82	—.64
	3920.26	—.64	—.27		—.34	—.40		—.73
	3895.66	.02	.00	—.13	—.23	—.56	—.46	—.30
	3859.91	—.01	—.19	—.26	—.20	—.71	—.63	—.50
	3856.37	.14	.02	—.08	—.23	—.23	.04	—.06
	3824.44	.00	—.33	—.36	—.80	—.80		
a^5F-y^5D^0	3865.53	—.93	—.81	—1.41	—.81	—.93	—.45	—.45
	3849.97	—.71	—.93		—1.82		—.93	—.33
	3825.88	—.17	—.01	—.17	—.29	—.56	—.26	—.29
	3820.43	.37	.22	—.36	—.10	—.40	—.92	
a^5F-z^3P^0	3812.97	—.04	—.45	—.92	—.23	—.62	—.40	—.42
a^5F-y^5F^0	3763.79	.37	.12	—.28	.05	.00	—.02	—.25
	3758.24	.30	.21		.32	—.40	—.64	—.70
a^3F-y^3F^0	4045.82	.80						
	4005.25	.34	.29	.43	.41	.25	.29	—.63
a^3F-y^3D^0	3827.83	—.17	.01	—.63	—1.10	—.23	—.32	—.38
	3815.84	—.17	—.17	—.56	—1.41	—.56	—.40	—.40
Mn I								
a^6S-z^6P	4034.49	—.95		—1.82				—.25
	4033.07	.00	.00	.12	—.17	—1.82	—.43	—.25
	4030.76	—.06	.22	—.06	.31	—.25	.00	.20
Ti I								
a^3F-y^3F^0	3998.46	—.19	—.31	—.01	—.22	—.12	—.22	.02
	3989.76	.07	.04	.10	—.02	—.47	—.47	—.19
a^3F-z^5S^0	3982.54	—1.82	—.82		—.42	—.82	—.64	
Ti II								
a^2G-z^2G^0	3913.46	.02	—.11	—.37	—.37	—.36	—.81	
	3900.55	.15	—.03	—.03	—.33	—1.11	—1.11	—1.11
a^2F-z^2F^0	3761.32	.44	.31	.26	.31	.36	.36	—.06
	3759.29	.51	.48	.07	.38	—0.5	.26	—.02
Mg I								
3^3P^0-3^3D	3838.29	.50	.28	.13	.03	—.50	—.15	—.41
	3832.30	.49	.10	—.04	—.04	—1.82		
	3829.35	.00	—.04	—.29	—.32	.62		—.40
Al I								
3^2P^0-4^2S	3961.52	—.27	—.22	—.38	—.38	—.72	—.82	—.27
	3944.01	—.9	—.22	—.64	—.80	—1.11		—1.42

Table V (continued)

Element Transition	Wave-length	J. D. 2433 949.5	J. D. 2433 950.5	J. D. 2433 956.5	J. D. 2433 960.5	J. D. 2433 961.5	J. D. 2433 962.5
Fe I							
a^5D-z^5D^0	3930.30						
	3927·93	—1.11	— .21	— .51		—1.82	
	3922.91	—1.82	— .36	— .81			
	3920.26	—1.11		— .81	— .81	—1.82	
	3895.66	— .30	—1.82	— .93	—1.11	— .71	—1.12
	3859,91	— .56					
	3856.37	— .03	— .18	— .23	— .26	— .71	— .20
	3824.44	— .40			— .50		
a^5F-y^5D^0	3865.53	— .56	— .45	— .56	— .41	— .41	— .81
	3849.97	— .63	— .93				
	3825.88	—1.82	— .70	—1.82	— .80	—1.82	—1.82
	3820.43	— .56		— .80			
a^5F-z^3P^0	3812.97	—1.82	— .45	—1.82	— .92	— .45	— .93
a^5F-y^5F^0	3763.79		— .07	— .05	— .03	— .25	— .40
	3758.24	— .22	— .25	—1.39	— .25	— .49	
a^3F-y^3F^0	4045.82						
	4005.25	.35	.30	.08	.17	.25	.24
a^3F-y^3D^0	3827.83	—1.10	— .41	—1.10		— .93	
	3815.84	— .70	— .41	—1.41			—1.41
Mn I							
a^6S-z^6P	4034.49	— .05	— .95		— .95	— .52	— .73
	4033 07	— .25	.12	.03	— .25	— .25	— .20
	4030.76	.10	.20		— .95	— .58	
Ti I							
a^3F-y^3F^0	3998.64	.08	— .14	— .10	— .14	— .19	— .31
	3989.76		— .24	— .38	— .31	— .47	— .42
a^3F-z^5S^0	3982.54	— .82					
Ti II							
a^2G-z^2G^0	3913.46		—1.82				
	3900.55	—1.41	—1.11	—1.41		—1.41	
a^2F-z^2F^0	3761.32	— .17	.07	— .80			— .80
	3759.29	— .14	— .03		— .62	— .55	
Mg I							
3^3P^0-3^3D	3838.29	.03	— .17	—1.82	—1.41		— .80
	3832.30						
	3829.55	— .07					
Al I							
3^2P^0-4^2S	3961.52	—1.82	—1.82	— .82	— .72		
	3944.01		—1.42				

Table VI

log $\overline{N f L}$. Relative number of atoms for $\xi = 0$ km/sec.

Atomic state		heights × 10^{-7} km.						
		2.14	2.86	3.26	3.56	4.70	4.98	5.71
Fe I	a^5D	18.35	18.38	18.08	18.02	17.07	17.25	16.79
	a^5F	18.70	18.53	17.51	18.03	17.34		17.31
	a^3F	18.32	18.24	16.91		17.04	17.64	17.61
Ti I	a^3F	19.05	18.80	18.99	18.90	17.77	17.85	18.63
Ti II	a^2G	18.80	18.52	18.07	17.85	16.57	14.52	14.20
		17.50	17.26	17.05	17.23	16.98	17.18	16.63
Mn I	a^6S	16.60	17.12	16.60	17.05	16.26	16.33	16.82
Mg I	3^3P^0	17.55	17.03	16.80	16.68	14.83	16.70	15.25
Al I	3^2P^0	16.68	17.05	16.78	16.50	13.85	13.60	16.27

Table VI (continued)

Atomic state		heights × 10^{-7} km.					
		6.41	6.77	8.71	10.34	10.68	11.03
Fe I	a^5D						
	a^5F	16.89					
	a^3F	16.65					
Ti I	a^3F	18.65	18.70	18.57	18.20	18.13	
Ti II	a^2G	13.57	14.05	13.56		13.50	
	a^2F	16.56	16.70	13.10	14.83	14.90	14.34
Mn I	a^6S	16.83	16.90		16.42	16.35	16.30
Mg I	3^3P^0	16.67	16.16		11.60		10.83
Al I	3^2P^0	11.60	12.26	13.28	13.48		

Table VII

log $\overline{N/L}$. Relative number of atoms for $\xi = 20$ km/sec.

Atomic state		heights × 10^{-7} km.						
		2.14	2.86	3.26	3.56	4.70	4.98	5.71
Fe I	a^5D	14.90	14.75	14.60	14.28	13.82	14.04	13.82
	a^5F	15.61	15.16	14.11	14.56	14.18	14.06	19.04
	a^3F	14.79	14.76	14.08		14.19	14.40	14.37
Ti I	a^3F	15.53	15.40	15.56	15.38	14.75	14.67	15.12
Ti II	a^2G	15.25	15.07	14.77	14.53	14.02	13.98	13.55
	a^2F	14.80	14.12	13.68	14.00	13.62	13.53	12.98
Mn I	a^6S	13.05	13.49	13.27	13.53	12.39	13.20	13.43
Mg I	3^3P^0	14.37	13.68	13.48	13.18	12.42	12.55	12.50
Al I	3^2P^0	13.45	13.47	13.51	13.15	12.67	12.63	13.28

Table VII (continued)

Atomic state		heights × 10^{-7} km.					
		6.41	6.77	8.71	10.34	10.68	11.03
Fe I	a^5D	13.72					
	a^5F	14.02					
	a^3F	14.57					
Ti I	a^3F	15.45	15.05	15.01	14.90	14.87	
Ti II	a^2G	12.98	13.41	12.80		12.92	
	a^2F	13.03	13.48	11.96	12.23	12.43	13.13
Mg I	a^6S	13.40	13.62		13.27	12.80	12.80
Mg I	3^3P^0	13.33	13.02				
Al I	3^2P^0			12.50	12.67		

The plots were in good agreement with the theoretical curve of growth. However, with a few of the iron lines the agreement is poor and consequently, the plots of these particular lines had to be rejected. For the higher levels of the atmosphere for some of the elements our date are not complete and no plot of the observed points could be made. Therefore, in those cases, it also impossible to determine the horizontal shift which is needed in order to have the observed curve fit into the theoretical curve of growth. The plots for $\zeta = 0$ fall on the damping part of the curve of growth.

The amount of the horizontal shift which is needed, determines the relative number of atoms for each element. The results are given in the tables VI and VII.

Mean Apparent Gradients : The values of log NfL for each multiplet were plotted against the height of each atmospheric level. In order to derive the mean density gradient, a straight line was drawn through these points. The resulting apparent gradients are shown in Table VIII.

Table VIII

Mean Apparent Gradients

Elements		$\xi = 0$	$\xi = 20$	The numbers of lines used heights $\times 10^{-7}$ km.			
				2.14	4.70	6.41	10.34
Fe I	a^5D	$.45 \times 10^{-12}$	$.30 \times 10^{-12}$	8	7		
	a^5F	.44	.33	7	6		
	a^3F	.38	.32	3	3		
Ti I	a^3F	.14	.11	3	3	2	2
Ti II	a^2F	.24	.30	2	2	2	1
	a^2G	.55	.43	2	2	2	1
Mn I	a^6S	.10	.13	3	2	3	3
Mg I	3^3P	.52	.40	3	3	2	1
Al I	3^2P^0	.49	.33	2	2	1	1

One of the interesting conlusions which we can derive from this table is that the apparent gradient does not greatly depend on the magnitude of the turbulent velocity which is adopted.

The gradients corresponding to the turbulent velocity $\zeta = 0$ are only slightly steeper than the ones corresponding to $\zeta = 20$ km/sec. The gradients obtained for the different elements are consistent with each other except for Ti I and Mn I. Of this, a possible explanation could be that the distribution of these elements is such that they extent throughout the whole stellar atmosphere.

Comparison of Observed and Theoretical Gradients: From table VIII and figure 2 we conclude that for most atoms the apparent gradients are consistent with each other. For comparison with theory we may take the mean apparent gradient to be around

$$\frac{\log N}{h} = 0.35 \times 10^{-12}\ \mathrm{cm}^{-1}$$

We assume that in the atmosphere of the K-star, the density of the atoms varies radially as the negative exponential:

$$n = n_0\, e^{-ah}$$

where a is a parameter describing the atmospheric density distribution. Then the total number of atoms in the line of sight is given by

$$N = n_0 \left(\frac{2R}{a}\right)^{1/2} e^{-ah}$$

For this case the apparent gradient is

$$\frac{\Delta \mathrm{L}n\, N}{\Delta h} = a \qquad \text{(natural logarithm)}$$

If we assume the atmosphere of the K-star to be in hydrostatic equilibrium, a is given by

$$a = \frac{m\,g}{k\mathrm{T}}$$

The surface gravity g of the K-star is

$$g = g_E \frac{M_K\, R^2_E}{M_E\, R^2_K}$$

For M and R we adopt the values $M_K = 12\ M_\odot$ and $R_K = 15\ 0R_\odot$ which for a K-star are reasonable values. For the surface gravity we find

$$g = 14.6 \text{ cm/sec}^{-2}$$

Hence, for iron atoms at a temperature $T = 4000°$

$$a = 2.43 \times 10^{-9} \text{ cm}^{-1}$$

We had the mean observed values $\frac{\Delta \log N}{\Delta h} = 0.35 \times 10^{-12} \text{cm}^{-1}$. This corresponds to $a = 0.81 \times 10^{-11}$.

Thus for iron atoms the gradient which follows from the hypothesis of hydrostatic equilibrium is about three thousand times greater than the observed one.

For each element the theoretical gradient would be proportional to its atomic weight.

In his theory which he first suggested for the solar chromosphere, Mc Crea supposes the atmosphere to be supported by the turbulent motions. According to this theory for atoms for which the temperature kinetic velocities are less than the turbulent velocities, the quantity a is given by

$$a = \frac{2g}{V^2_{Tur.}}$$

If for 31 Cygni the turbulent velocity is taken to be 20 km/sec., this leads to a value

$$a = 7.3 \times 10^{-12} \text{ cm}^{-1}$$

that it to say that from the hypothesis of Mc Crea follows a gradient about nine times greater that the observed one.

So the assumed turbulent velocity is inadequate fully to account for the support of the chromosphere of the K-star.

But it is obvious that it yields results which are far closer to the observed facts.

Possibly, if a number of errors and approximations could be eliminated from the results, Mc Crea's theory might be completely successful.

This confirms that we must discard the picture of an atmosphere in static equilibrium and instead adopt some kind of dynamical picture for the atmosphere of the giant K-stars.

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