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Left And Right Spectra

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Left And Right Spectra

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ABSTRACT.

The left spectrum $\sigma^l(a)$ and tbe right spectrum $\sigma^r(a)$ of an element in a Banach algebra A are considered and some properties are proved. Operator algebras in which, for every element T, $\sigma^l(T) = \sigma^r(T)$ are investigated, and a characterization of $\sigma^l(T)$ and $\sigma^r(T)$ is given.

INTRODUCTION

The left spectrum σ^l (a) and the right spectrum $\sigma^r(a)$ of an element a in a Banach algebra A with identity are defined to be the following subsets of the field C of complex numbers:

$$\sigma^l(a) = \{\lambda \varepsilon C: a-\lambda e \text{ is not left invertible}\}$$

$$\sigma^r(a) = \{\lambda \varepsilon C: a-\lambda e \text{ is not right invertible}\}.$$

Equivalently, $\lambda \varepsilon \sigma^l$ (a) ($\lambda \varepsilon \sigma^r(a)$) if and only if a-λe generates a proper left (right) ideal in A. If the algebra A is commutative then

$$\sigma^l(a) = \sigma^r(a) = \{\psi(a): \psi \varepsilon \Phi\}$$

where Φ is the maximal ideal space of A [1, p. 320]. For an element a in a noncommutative algerra A, $\sigma^l(a) = \sigma^r(a)$ is not true in general.

The notion was first introduced by Robin Harte ([2] [3]) to prove spectral mapping theorrems for the joint spectrum of an n-tuple $a = (a_1, a_2, \ldots, a_n)$ in A. In the present paper we shall prove some properties of $\sigma^l(a)$ and $\sigma^r(a)$, and we shall give a characterization of $\sigma^l(T)$ and $\sigma^r(T)$ for an element T in the Banach algebra A of operators on a Banach space.

II. PROPERTIES OF $\sigma^l(a)$ AND $\sigma^r(a)$

Let A be a Banach algebra with identity e, and $a \varepsilon A$. It is well known that $\sigma(a) = \sigma^l(a) \cup \sigma^r(a)$ is a non-empty compact subset of C contained in the disk $\{z \varepsilon C: |z| \leq \|a\|\}$. Now we note that $\sigma^l(a)$ or $\sigma^r(a)$ can be proper subsets of $\sigma(a)$. This is demonstrated by the following example.

Example. Let $H = l^2$ and A be the Banach algebra of all bounded linear operators on H. Then for any $T \varepsilon A$,

$$\sigma^l(T) = \{\lambda \varepsilon C : \inf_{\|x\| = 1} \|(T-\lambda)x\| = 0\},$$

$$\sigma^r(T) = \{\lambda \varepsilon C : (T-\lambda) H \neq H\}$$

[3, pp. 95-97]. Therefore if we take an operator $T \varepsilon A$ which is not one-to-one but onto, then $0 \varepsilon \sigma^l(T)$ but $0 \notin \sigma^r(T)$. For instance define T by

$$T(x) = (x_1, x_3, x_5, \ldots) \text{ for } x = (x_1, x_2, x_3, \ldots).$$

It is easy to see that T is linear, and bounded since

$$\| T(x) \|^2 = \sum_{n=1}^{\infty} | x_{2n-1} |^2 \leq \| x \|^2.$$

We observe that T is onto. If $y = (y_1, y_2, y_3, \ldots)$ is in H, then $T(x) = y$ for $x = (y_1, 0, y_2, 0, y_3, \ldots)$. We note that

$\operatorname{Ker} T \neq \{0\}$, since Ker T consists of all vectors x of the form $x = (0, x_2, 0, x_4, 0, x_6, \ldots)$.

Since $\sigma^l(a)$ or $\sigma^r(a)$ could be proper subsets of $\sigma(a)$ it is natural to ask whether either of them can be empty. We shall prove that neither $\sigma^l(a)$ nor $\sigma^r(a)$ can be empty.

An element a in A is said to be a left (right) topological zero divisor if there exists a sequence $\{b_n\}$ in A such that $\| b_n \| = 1$, $n = 1,2,3, \ldots$, and

$$\lim_{n\to\infty} \| ab_n \| = 0 \quad (\lim_{n\to\infty} \| b_n a \| = 0),$$

and a is said to be a two-sided topological zero divisor if there exists a sequence $\{b_n\}$ in A for which $\| b_n \| = 1$, $n=1,2,3, \ldots$, and

$$\lim_{n\to\infty} \| ab_n \| = 0 = \lim_{n\to\infty} \| b_n a \|.$$

Theorem 1. $\sigma^l(a)$ and $\sigma^r(a)$ are both non-void compact subsets of C. Furthermore the boundary of σ (a) (bdyσ (a)) is included in both σ^l (a) and $\sigma^r(a)$.

Proof. We give the proof for the left spectrum. The prood for the right spectrum is similar. Let $\lambda \varepsilon$ bdy σ (a). Then a-λ e is a boundary point of the group G of regular elements, therefore a–λ e is a two-sided topological zero divisor [4, p. 862]. We claim that $\lambda \varepsilon \sigma^l(a)$. If b ε A is a left inverse for a-λ e, then b (a-λ e) = e implies that b_n= b (a-λ e) b_n and hence there is inequality

$$\| b_n\| \leq \| b \| \quad \| (a\text{-}\lambda e)\ b_n \|.$$

which rules out the possibility that a-λ e is a left topological zero divisor. So, $\lambda \varepsilon \sigma^l(a)$. Similarly, a-$\lambda$ e is a right topological zero divisor implies that λ is in $\sigma^r(a)$, and the proof is complete.

Definition. A complex linear algebra A with identity e will be called semi-commutative if $\sigma^l(a) = \sigma^r(a)$ for every element a in A.

Of course every commutative algebra is semi-commutative. It is interesting to investigate semi-commutative algebras which are not commutative. An example of such an algebra which comes first to the mind is the algebra A of nxn complex matrices. If a ε A then $\lambda \varepsilon \sigma^l(a)$ if and only if a-λ e is not left invertible but a square matrix is left invertible in and only if it is right invertible. Therefore, $\sigma^l(a) = \sigma^r(a) = \sigma$ (a). In this case σ (a) is the set of eigenvalues of the nth order complex matrix a.

A semi-commutative algebra can easily be characterized as follows:

Proposition. A Banach algebra A with identity e is semi-commutative if and only if for any two elements a,b in A

ab = e if and only if ba=e

that is, an element a is left invertible if and only if it is right invertible.

In a Banach algebra A it is possinle to have ab=e$\neq$ba. For example, let A be the Banach algebra of all bounded linear operators on the Hilbert spacel^2 Consider the right and left shifts S_R and S_L defined by

$S_R\ (x_1, x_2, x_3, \ldots) = (0, x_1, x_2, x_3, \ldots)$,

$S_L\ (x_1, x_2, x_3, \ldots) = (x_2, x_3, x_4\ \ldots)$,

If is easy to see that $S_L\ S_R = I \neq S_R\ S_L$. Of course this algebra cannot be semi-commutative according to our preceding proposition, for instance one can show that $\sigma^l(S_R) \neq \sigma^r(S_R)$. We note that $\sigma^r(S_R) = \{0\}$, but $0 \notin \sigma^l\ (S_R)$. To see this we recall that $\lambda \varepsilon \sigma^r(S_R)$ if and only if $S_R - \lambda I$ is not onto. But $S_R - \lambda I$ is onto for any $\lambda \neq 0$, since if $y = (y_1, y_2, y_3, \ldots)$ is in l^2then $(S_R - \lambda I)(x) = y$

for $x = (x_1, x_2, x_3, \ldots)$ where. $x_1 = \frac{y_1}{\lambda}$, $x_2 = \frac{x_1 - y_2}{\lambda}$

$x_3 = \frac{x_2 - y_3}{\lambda}$, ... , $x_n = \frac{x_{n-1} - y_n}{\lambda}$, for any $n=2,3,4, \ldots$. Hence

$\sigma^r(S_R) = \{0\}$. Now we show that $0 \notin \sigma^l(S_R)$. Again we recall that $\lambda \varepsilon \sigma^l(S_R)$ if and only $\inf_{\|x\|=1} \|(S_R - \lambda I)\ (x)\| = 0.$

But $\|S_R\ (x)\|^2 = \sum_{i=1}^{\infty} |x_i|^2 = \|x\|^2$. Therefore $\inf_{\|x\|=1} \|S_R\ (x)\| = 1,$ and hence $0 \notin \sigma^l(S_R)$.

Theorem 2. Every finite dimensional Banach algebra with identity is semi-commutative.

Proof. Let A be a Banach algebra with identity e, and let L (A) be the Banach algebra of all bounded linear operators on A. We identify A with the subalgebra of L (A) consisting of the operators T_a, $a \varepsilon A$. where $T_a(b) = ab$ If the dimension of A is n, then L (A) is isomorphic to $C^{n x n}$, nxn matrices. Therefore A is isomorphic to an n-dimensional subspace of $C^{n x n}$. Let $e_1, e_2, \ldots , e_n$ be the the standard basis of C^n and M_a be the matrix of $T_a$relative to this basis. Then for any $a \varepsilon A$ we have

$$\sigma_A(a) = \sigma_{L(A)}\ (T_a) = \sigma_{C^{nxn}}\ (M_a)$$

where σ denotes the spectrum of any sort left or right. But we have already observed that for any nth order complex matrix M_a, $\sigma^l(M_a) = \sigma^r(M_a)$. Therefore for any $a \varepsilon A$ we have $\sigma^l(a) = \sigma^r(a)$, and A is a semi-commutative algebra.

In our previous discussions, we proved that in a non-commutative Banach algebra A it is not always true that $\sigma^l(a) = \sigma^r(a)$ for every $a \varepsilon A$. It is interesting to know for which elements $\sigma^l(a) = \sigma^r(a)$, in case of an algebra whose structure is familiar to us. We shall answer this question in case of the Banach algebra of all bounded linear operators on a Hilbert space H. We know that in an algebra of linear operators on a finite dimesional space, it is always true that $\sigma^l(T) = \sigma^r(T)$ for every operatcr T. Many of the results that hold for linear transformations on finite dimensional space also hold in the infinite-dimensional case, provided the additional hypothesis of compactness is imposed.

Theorem 3. Let H be a Hilbert space, and A=L (H) be the Banach algebra of all bounded linear operators on H. If T is a compact operator and $\lambda \neq 0$ is a complex number, then $\lambda \varepsilon \sigma^l{}_A(T)$ if and only if $\lambda \varepsilon \sigma^r{}_A(T)$.

Proof. We recall once more that for any $T \varepsilon A$ we have

$$\sigma^l(T) = \{\lambda \varepsilon C : \inf_{\|x\|=1} \| (T-\lambda I)(x) \| = 0\}$$

$$\sigma^r(T) = \{\lambda \varepsilon C : (T-\lambda I) H \neq H\}.$$

If T is a compact operator and $\lambda \notin \sigma^l(T)$ for $\lambda \neq 0$, then $\inf_{\|x\|=1} \| (T-\lambda I)(x) \| > 0$, i.e., $T-\lambda I$ is one-to-one. But this is true if and only if $T-\lambda I$ is onto [5, pp. 393-393]. So $\lambda \notin \sigma^r(T)$. Similarly, if $\lambda \notin \sigma^r(T)$ then $(T-\lambda I) H=H$, i.e., $T-\lambda I$ is onto. But this is true if and only if $T-\lambda I$ is one to one. Thus, clearly $\inf_{\|x\|=1} \| (T-\lambda I)(x) \| > 0$, and $\lambda \notin \sigma^l(T)$.

We can not sharpen the statement of theorem 3 to conclude that $\sigma^l(T) = \sigma^r(T)$ for every compact operator T in A=L (H). The point $\lambda = 0$ has a status different from other points in relation to T if T is compact and H is infinite dimensional. In this case 0 is always in the spectrum $\sigma(T) = \sigma^l(T) \cup \sigma^r(T)$, because the Banach subalgebra of all compact operators in A is a two-sided ideal in A which is not inverse closed [6, pp. 98-991}.

Corollary 1. Let A be the Banach algebra of all bounded linear operators on a Hilbert space H. Then $\sigma^l(T) = \sigma^r(T)$ for every finite rank operator T.

Proof. If H is finite dimensional then $\sigma^l(T) = \sigma^r(T)$ for every T. Suppose that H is infinite dimensional. If T is a finite rank operator then it is compact, and furthermore $0 \varepsilon \sigma^l(T) \cap \sigma^r(T)$ because a finite rank operator can never be one-to-one, and it can never be onto if H is infinite dimensional. If $\lambda \neq 0$, then by theorem 3, $\lambda \varepsilon \sigma^l(T)$ if and only if $\lambda \varepsilon \sigma^r(T)$, and the prof is complete.

Corollary 2. Let A be the Banach algebra of all bounded linear operators on a Hilbert space H, and let T be a compact operator. Then every $\lambda \neq 0$ in σ (T) is an eigenvalue of T.

Proof. If $\lambda \neq 0$, $\lambda \varepsilon \sigma$ (T) then by theorem 3 λ is necessarily in $\sigma^l(T)$, therefore $\inf_{||x||=1} ||(T-\lambda I)(x)|| = 0$. Thus, $T-\lambda I$ is not one-to-one, and λ is an eigenvalue of T.

Although 0 is always in σ (T) for a compact operator T, 0 need not be an eigenvalue of T.

Example. Let $H=l^2$, and let $e_1 = (1, 0, 0, ...)$ $e_2 = (0, 1, 0, ...)$, $e_3 = (0, 0, 1, 0, ...)$ be the standard complete orthonormal set in H. For $x = (x_1, x_2, x_3, ...) \varepsilon H$ we define an operator T by

$$T(x) = (0, \frac{x_1}{2}, \frac{x_2}{3}, \frac{x_3}{4}, ...).$$

We show that T is a compact operator. If we define the sequence of operators $\{T_n\}$ by

$$T_n(x) = (0, \frac{x_1}{2}, \frac{x_2}{3}, ..., \frac{x_n}{n+1}, 0, 0, ...)$$

for n=1,2,3, ... then it is a Cauchy sequence in the norm topology of L (H), and therefore convergent. Clearly, $\lim_{n\to\infty} T_n = T$. Each T_n being a finite rank operator is compact and therefore, $\lim T_n =$ T is compact because the Banach subalgebra of all compact operators is the norm closure of the finite rank operators [7, pp. 124-125}

If T (x) = 0, then obviously x must be zero, therefore T is one-to-one. Thus 0 is not in $\sigma^l(T)$ but $0 \varepsilon \sigma^r(T)$ since T is not onto.

III. A CHARACTERIZATION OF σ^l (T) and σ^r (T)

Let X be a Banach space and A=L (X) be the Babach algebra of all bounded linear operators on X. We shall denote the set of all left (right) invertible elements in A by $G^l(G^r)$. We set $G=G^l \cap G^r$. We note that T ε G if and only if T is a topological isomorphism (i. e. a linear isomorphim which is also a homeomorphism) onto X.

Theorem 4. T ε G^l if and only if T is a topological isomorphism between X and the range of T, and there is a projection of X on the range of T.

Proof. If T ε G^l then T is not a left topological zero divisor and this implies that T is a topological isomorphism between X and the range of T. To prove the existence of a projection of X on the range of T we first show that ran T is a closed subpace. Since T is a topological isomorphism, T is bounded below, i.e., there exists an $\varepsilon > 0$ such that $\| T(x) \| > \varepsilon \| x \|$ for every x in X. Hence, if $\{T(x_n)\}_{n=1}^{\infty}$ is a Cauchy sequence in ran T, then the inequality

$$\| x_n - x_m \| < \frac{1}{\varepsilon} \| T(x_n) - T(x_m) \| ,$$

implies that $\{x_n\}$ is also a Cauchy sequence. If $x=\lim x_n$, then $T(x) = \lim_{n \to \infty} T(x_n)$ is in ran T. Thus ran T is closed.

Let S be the inverse mapping from Y=ran T to X. Then ST=I in A. By hypothesis there exists U in A such that UT=I in A. Consequently U=S on Y and U is an extension of S. Now we decompose X into cosets y+Ker U, y ε Y. By hypothesis each coset y+Ker U contains one and only one y ε Y, and every element of X is included in some coset since U is defined on all of X. Thus each x ε X has a unique decomposition x=y+(x-y) where y ε Y is the representative of the coset to which x belongs, so that x-y ε Ker U. Therefore Y and Ker U are complementary subspaces in X, and the transformation defined by P (x) = y is a projection on X to Y = ran T. Since both the range and the kernel of P are closed, P is bounded [8, p. 242}.

Conversely let T be a topological isomorphism between X and the range of T, and suppose that a bounded projection P of X on ran T exists. Let S be the inverse mapping between ran T and X. Then SP is a bounded operator with domain all of X. Furthermore (SP) T = I and thus $T \varepsilon G^l$

Corollary 1. If T is an operator on a Hilbert space H then $T \varepsilon G^l$ if and only if T is bounded below.

Proof. T is bounded below if and only if T is an isomorphism between H and the closed subspace ran T. Since H is a Hilbert space there exists a projection of H onto the closed linear subspace ran T and the corollary follows from theorem 4.

Corollary 2. If T is an operator on the Hilbert space H, then $\lambda \varepsilon \sigma^l(T)$ if and only if $\inf_{\|x\|=1} \| (T-\lambda I)(x) \| = 0$.

This is a restatement of Corollary 1 in terms of left spectrum.

Theorem 5. $T \varepsilon G^r$ if and only if T is onto and there exists a projection of X onto Ker T.

Proof. Suppose $T \varepsilon G^r$. Then T is not a right topological zero divisor. We know that ran T=X if T′ is a topological isomorphism [8, p. 234]. Assume the contrary that T′ is not an isomorphism. Then there exists a sequence $\{x_n'\} \subset X'$ with $\| x_n' \| = 1$ such that $\lim_{n\to\infty} \| T'(x'_n) \| = 0$, or $\lim_{n\to\infty} | x'_n(Tx) | = 0$ for every x in the closed unit ball of X. Let $u \varepsilon X$, $\| u \| = 1$; and let $U_n \varepsilon A$ be defined by $U_n(x) = x_n'(x)\, u$ for $n = 1,2,3,\ldots$ .It is eacy to show that $\| U_n \| = 1$, and also $\lim_{n\to\infty} \| U_n(T_x) \| = \lim_{n\to\infty} \| x'_n(T_x)u \| = \lim_{n\to\infty} | x'_n(T_x) | = 0$ for every x with $\| x \| \leq 1$, which contradicts the fact that T is not a right topological zero divisor. Consequently ran T=X.

To prove the existence of a projection of X on Ker T we show that X is the direct sum $X = \text{Ker } T \oplus \text{ran } U$ where U is a right inverse for T, i.e. TU=I. $\text{Ker } T \cap \text{ran } U = \{0\}$, for if $U(x) \neq 0$ and $U(x) \varepsilon \text{Ker } T$ then TU = I is violated.

We consider the quotient space $X/\mathrm{Ker}\ T$, and show that every coset $x + \mathrm{Ker}\ T$ contains one and only one element of ran U. Suppose that $x_0 + \mathrm{Ker}\ T$ contains two elements y_1 and y_2 of ran U. Let $y_1 = U(x_1)$ and $y_2 = U(x_2)$. Since $y_1 - y_2 \varepsilon \mathrm{Ker}\ T$ we have $TU(x_1) = TU(x_2)$ or $x_1 = x_2$, and hence $y_1 = y_2$. On the other hand $x_0 + \mathrm{Ker}\ T$ contains an element of ran U. For every $x \varepsilon x_0 + \mathrm{Ker}\ T$, $T(x)$ has the same value $T(x_0)$, moreover $T(x) = T(x_0)$ only if $x \varepsilon x_0 + \mathrm{Ker}\ T$. Now we note that $TU(Tx_0) = T(x_0)$. Then $z = UT(x_0)$ is in $(x_0 + \mathrm{Ker}\ T) \cap$ ran U Let $x \varepsilon X$, and let Y be a coset of $X/\mathrm{Ker}\ T$ which contains x. Let x_1 be the unique representative of Y in ran U. Then x has the representation $x = x_1 + (x - x_1)$ where $x_1 \varepsilon$ ran U and $x - x_1 \varepsilon \mathrm{Ker}\ T$ (since both x and x_1 are in Y). This representation is unique. For if also $x = x_2 + (x - x_2)$ where $x_2 \varepsilon$ ran U and $x_2 \neq x_1$ then $x_2 \notin Y$, because Y contains exactly one element of ran U. Since $x \varepsilon Y$, $x - x_2$ is not in Ker T. Consequently $X = \mathrm{Ker}\ T \oplus$ ran U. Since $TU = I$, $U \varepsilon G^l$ and by Theorem 4 U is a topological isomorphism, and thus ran U is closed. Therefore Ker T and ran U are closed complementary subspaces, and there exists a bounded projection of X on Ker T [8, p. 242].

Conversely, suppose that T is onto and there exists a bounded projection P_1 of X on Ker T. Then $X = \mathrm{ran}\ P_1 \oplus \mathrm{Ker}\ P_1 = \mathrm{Ker}\ T \oplus \mathrm{Ker}\ P_1$. If we let $P = I - P_1$, then ran $P = \mathrm{Ker}\ P_1$ and $X = \mathrm{Ker}\ T \oplus$ ran P. If we consider TP as a mapping with domain ran P and range in X, then TP is a topological isomorphism between ran P and all of X. Let x_1 and x_2 be in ran P. Then $P(x_1) = x_1$ and $P(x_2) = x_2$. If $TP(x_1) = TP(x_2)$, then $T(x_1 - x_2) = 0$ and $x_1 - x_2 \varepsilon \mathrm{Ker}\ T \cap \mathrm{ran}\ P = \{0\}$. Thus TP is a one-to-one mapping. To see that the range of TP is all of X, take any $y \varepsilon X$. Since ran $T = X$, there exists an element $x \varepsilon X$ such that $T(x) = y$. Let $x = x_1 + x_2$ be the decomposition of x where $x_1 \varepsilon \mathrm{Ker}\ T$ $x_2 \varepsilon$ ran P. Then $y = T(x_1) + T(x_2) = T(x_2) = TP(x_2)$. Then by the Open Mapping Theorem TP is a topological isomorphism. Let S be the inverse mapping from X to ran P. Then $(TP)S = I = T(PS)$ and $PS \varepsilon A = L(X)$, consequently $T \varepsilon G^r$.

Corollary 1. If T is an operator on a Hilbert space H then $T \varepsilon G^r$ if and only if T is onto.

Proof. By Theorem 5, $T \varepsilon G^r$ if and only if T is onto and there exists a projection of H on Ker T. Since H is a Hilbert space there always exists a projection on the closed linear subspace Ker T.

Corollary 2. If T is an operator on the Hilbert space H then $\lambda \varepsilon \sigma^r(T)$ if and only if $T-\lambda I$ is not onto.

This is a restatement of Corollary 1 in terms of the right spectrum of T.

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ÖZET

Sol ve Sağ Spektrumlar

Bir A Banach cebiri içindeki bir a elemanının $\sigma^l(a)$ sol spektrumu ve $\sigma^r(a)$ sağ spektrumu incelenmekte ve bazı özellikleri ıspatlanmaktadır. Her T elemanı için $\sigma^l(T) = \sigma^r(T)$ olan operatör cebirleri araştırılmakta ve $\sigma^l(T)$ ve $\sigma^r(T)$ cümlelerinin bir karekterizasyonu verilmektedir.

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