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On Absolute φ - Summability Factors

by

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ON ABSOLUTE φ SUMMABILITY FACTORS

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SUMMARY

In this paper we give sufficient conditions for the series $\Sigma\lambda_k x_k$ to be $\varphi-|C,\alpha|_p$ summable. In addition, if

$$\sum_{k=1}^{\infty} \left| \frac{\varphi_k s_k}{k^{\alpha}} \right|^p < \infty \quad (0 \leq \alpha \leq 1)$$

then we observe that the series Σx_k is $\varphi - |C,\alpha|_p$ summable, where s_k is the partial sums of the series Σx_k. Using this result we prove that the Jacobi series of which generating function belongs to a certain Lipschitz class is $|C,1|_p$ summable at the point $x = 1$. Furthermore we are giving the $\varphi - |C,\alpha|_p$ summablity factors for this Jacobi series.

1. INTRODUCTION

Let $A = (a_{nk})$ be an infinite matrix of complex numbers a_{nk} $(n, k = 0,1,\ldots)$ and (φ_n) be a sequence of complex numbers. Let Σx_k be a given infinite series of complex numbers with partial sums s_n. We denote the A transform of the sequence $s = (s_k)$ by $A_n(s)$ which is given by

$$A_n(s) = \sum_{k=0}^{\infty} a_{nk} s_k$$

If

$$\sum_{n=1}^{\infty} |\varphi_n \overline{\triangle} A_n(s)|^p < \infty \tag{1}$$

for $p \geq 1$ then the series Σx_k is called $\varphi-|A|_p$ summable where

$$\overline{\triangle} A_n(s) = A_n(s) - A_{n-1}(s).$$

If we take $\varphi_n = n^{1-p^{-1}}$ and $\varphi_n = n^{\gamma+1-p^{-1}}$ then $\varphi - |A|_p$ summability is identical with $|A|_p$ and $|A,\gamma|_p$ summability, respectively, [6], [2]. Moreover, if we take $\varphi_n = (\log n)^{1-p^{-1}}$, we obtain a summability method which is stronger than the summability method $|A|_p$

The series Σx_k is called $[B,r,\varphi,\alpha]_p$ –bounded if

$$\sum_{\nu=1}^{n} \left| \frac{\varphi_\nu s_\nu}{\nu^\alpha} \right|^p = O(r_n) \quad (n \to \infty), \tag{2}$$

where (r_n) is a non–decreasing sequence of positive numbers and α is a real number. In particular, it is easy to show that if $r_n = \log n$, $\varphi_n = n^{1-p^{-1}}$ and $\alpha = 1$ then the $[B,r,\varphi,1]_p$ –boundedness is equivalent to $[R, \log n, 1]_p$ –boundedness.

For any sequence (λ_n) we use the following notation:

$$\triangle^s \lambda_n = \sum_{k=o}^{\infty} (-1)^k \binom{s}{k} \lambda_{n+k}$$

As we know a sequence (λ_n) is said to be soncex if $\triangle^2\lambda_n \geq 0$ for every positive integer n.

Let s^α_n and t^α_n denote the n–th Cesaro means of order α $(\alpha > -1)$ of the sequences (s_n) and $(n\, x_n)$ respectively, i.e.

$$s^\alpha_n = \frac{1}{E^\alpha_n} \sum_{\nu=o}^{n} E^{\alpha-1}_{n-\nu} s_\nu$$

$$t^\alpha_n = \frac{1}{E^\alpha_n} \sum_{\nu=o}^{n} E^{\alpha-1}_{n-\nu} \nu x_\nu$$

where

$$E^\alpha_n = \binom{n+\alpha}{n} = \frac{\Gamma(n+\alpha+1)}{\Gamma(n+1)\Gamma(\alpha+1)} \sim \frac{n^\alpha}{\Gamma(\alpha+1)}, \quad (\alpha \neq -1,-2,\ldots).$$

Since

$$t^\alpha_n = n\left(s^\alpha_n - s^\alpha_{n-1}\right) = \alpha\left(s^{\alpha-1}_n - s^\alpha_n\right) \tag{3}$$

[3], the series Σx_k is $\varphi - |C,\alpha|_p$ summable whenever

$$\sum_{n=1}^{\infty} \left| \frac{\varphi_n}{n} t^\alpha_n \right|^p < \infty . \tag{4}$$

The Jacobi polynomials $P_n^{(r,s)}(x)$, $(r>-1, s>-1)$ are defined by the following expansion:

$$2^{r+s}(1-2xt+t^2)^{-1/2}\,[1-t+(1-2xt+t^2)^{1/2}]^{-r}\,[1+t+(1-2xt+t^2)^{1/2}]^{-s} = \sum_{n=o}^{\infty} P_n^{(r,s)}(x)\; t^n. \tag{5}$$

Let f be a function defined on the interval [-1, 1], such that the integral

$$\int_{-1}^{1} (1-x)^r(1+x)^s\, f(x)\, dx$$

exist in the sense of Lebesgue. The Jacobi series corresponding to the function f is given by

$$f(x) \sim \sum_{n=o}^{\infty} a_n\, P_n^{(r,s)}(x) \tag{6}$$

where

$$a_n = \frac{2n+r+s+1}{2^{r+s+1}}\;\frac{\Gamma(n+1)\,\Gamma\,(n+r+s+1)}{\Gamma(n+r+1)\,\Gamma\,(n+s+1)}\;X$$

$$\int_{-1}^{1} (1-t)^r(1+t)^s\, P_n^{(r,s)}(t)\, f(t)\, dt. \tag{7}$$

Legendre and Ultraspherical series are particular cases of the series (6) when $r = s = o$ and $r = s = \lambda-1/2$ respectively.

If f has domin D contained in R^m and range in R^n we say that f satisfies the Lipschitz condition of degre δ if there exists a $K>0$ and a $\delta>0$ such that

$$\| f(x) - f(u) \| \leq K \, \| x-u \|^{\delta}$$

for all points x, u in D. If the function f satisfies the Lipschitz condition of degre δ then we shall write $f \in \mathrm{Lip}\ \delta$.

We write

$$\Phi(w) = f(\cos w) - A$$

where A is a fixed constant.

In this paper we shall generalize a theorem which is given by Szalay [5] for the $|C,\alpha|_p$ summability to the $\varphi-|C,\alpha|_p$ summability and furthermore we shall investigate the $\varphi-|C,\alpha|_p$ summability factors of the Jacobi series at the point $x = 1$.

2. LEMMAS

Following lemmas will be used to prove the theorems.

LEMMA 1. Let $d_n \geqslant 0$ an d $l_n > 0$ $(n = 1,2,\ldots)$ be given sequences. If the triangular matrix $C = (c_{nk})$ satisfies the condition

$$0 \leq c_{mk} \leq Q.c_{nk} \qquad (0 < k \leq n \leq m),$$

where Q denotes a positive absolute constant, then for any $p \geqslant 1$

$$\sum_{n=1}^{\infty} l_n \left(\sum_{k=1}^{n} c_{nk} d_k \right)^p \leq Q^{p(p-1)} \, p^p \sum_{n=1}^{\infty} l_n^{1-p} \left(\sum_{k=n}^{\infty} l_k c_{kn} \right)^p d_n^p \tag{8}$$

[4].

LEMMA 2. Let $p \geqslant 1$ and $0 \leq \alpha \leq 1$. Let (φ_n) be a sequence of complex numbers for which there exists any $c > 0$ such that $(n^{c-p-\alpha+1} |\varphi_n|^p)$ is non-increasing. If (λ_n) a positive sequence such that the sequence $(|\triangle \lambda_n|)$ is non–increasing,

$$\sum_{n=1}^{\infty} \left| \frac{\varphi_n}{n} \lambda_n t_n^{\alpha} \right|^p < \infty \tag{9}$$

and

$$\sum_{n=1}^{\infty} \left| \varphi_n \triangle \lambda_n t_n^{\alpha} \right|^p < \infty \tag{10}$$

then the series $\Sigma \lambda_k x_k$ is $\varphi-|C,\alpha|_p$ summable.

Proof: Let T^{α}_n be the n–th Cesaro means of order α of the sequence $(n \lambda_n x_n)$. Obviously it will be sufficient to show that

$$\sum_{n=1}^{\infty} \left| \frac{\varphi_n}{n} T^{\alpha}_n \right|^p < \infty \tag{11}$$

to prove the lemma. Since

$$T_n^{\alpha} = t_n \lambda_n + \frac{1}{E^{\alpha}_n} \sum_{k=1}^{n-1} E_k^{\alpha} t_k^{\alpha} \sum_{\nu=0}^{n-k} E_{n-k-\nu}^{\alpha-1} E_{\nu}^{-\alpha-1} \lambda_{k+\nu}$$

we have that

$$| T_n^{\alpha} | \leq | t_n^{\alpha} \lambda_n | + \frac{1}{E_n^{\alpha}} \sum_{k=1}^{n-1} E_k^{\alpha} | t_k^{\alpha} | | \sum_{\nu=0}^{n-k} E_{n-k-\nu}^{\alpha-1} E_{\nu}^{-\alpha-1} \lambda_{k+\nu} | .$$

Thus

$$| T_n^{\alpha} | \leq | t_n^{\alpha} \lambda_n | + \frac{1}{E_n^{\alpha}} \sum_{k=1}^{n-1} E_k^{\alpha} | t_k^{\alpha} | | \Delta \lambda_k |$$

since $(| \Delta \lambda_k |)$ is non–increasing. Hence

$$\sum_{n=1}^{\infty} \left| \frac{\varphi_n}{n} T_n^{\alpha} \right|^p \leq K \sum_{n=1}^{\infty} \left| \frac{\varphi_n}{n} t_n^{\alpha} \lambda_n \right|^p +$$

$$K \sum_{n=1}^{\infty} \left| \frac{\varphi_n}{n} \right|^p \left(\frac{1}{E_n^{\alpha}} \sum_{k=1}^{n-1} E_k^{\alpha} | t_k^{\alpha} \Delta \lambda_k | \right)^p$$

and the lemma will be proved if we can show that the second series on the right–hand side converges. By Lemma 1, we have that

$$\sum_{n=1}^{\infty} \left| \frac{\varphi_n}{n} \right|^p \left(\frac{1}{E_n^{\alpha}} \sum_{k=1}^{n-1} E_k^{\alpha} \left| t_k^{\alpha} \Delta \lambda_k \right| \right)^p$$

$$\leq K \sum_{n=1}^{\infty} \left| \frac{\varphi_n}{n} \right|^{p(1-p)} n^{\alpha p} \left(\sum_{k=n}^{\infty} \frac{| \varphi_k |^p}{k^{p+\alpha-1-c}} \cdot \frac{1}{k^{1+c}} \right)^p \left| t_n^{\alpha} \Delta \lambda_n \right|^p$$

$$\leq K \sum_{n=1}^{\infty} \left| \varphi_n t_n^{\alpha} \Delta \lambda_n \right|^p . \qquad (*)$$

So the proof is completed.

LEMMA 3: Let $p \geq 1$ and $0 \leq \alpha \leq 1$. If there exists any $c > 0$ such that the sequence $(n^{c-p-\alpha+1} | \varphi_n |^p)$ is non–increasing then

$$\sum_{\nu=1}^{n} \left| \varphi_{\nu} \bar{\Delta} s_{\nu}^{\alpha} \right|^p = O \left(\sum_{\nu=1}^{n} \left| \frac{\varphi_{\nu} s_{\nu}}{\nu^{\alpha}} \right|^p \right) \qquad (12)$$

i.e, if

$$\sum_{\nu=1}^{\infty} \left| \frac{\varphi_{\nu} s_{\nu}}{\nu^{\alpha}} \right|^p < \infty$$

then Σx_k is $\varphi - |C,\alpha|_p$ summable, where $s_{\nu} = \sum_{k=1}^{\nu} x_k$.

* K denotes a positive constant, not always the same

Proof: First, we assume that $0 \leq \alpha < 1$. It will be enough to show that

$$\sum_{n=1}^{\infty} \left| \frac{\varphi_n}{n} t_n^\alpha \right|^p < \infty .$$

Using (3) we have

$$\left| t_n^\alpha \right| \leq \frac{1}{E_n^{\alpha-1}} \sum_{\nu=0}^{n} \left| E_{n-\nu}^{\alpha-2} \right| |s_\nu| + \frac{1}{E_n^\alpha} \sum_{\nu=0}^{n} E_{n-\nu}^{\alpha-1} |s_\nu| .$$

By Lemma 1 we write

$$\sum_{n=1}^{\infty} \left(\frac{|\varphi_n|}{nE_n^{\alpha-1}} \sum_{\nu=0}^{n} \left| E_{n-\nu}^{\alpha-2} \| s_\nu \right| \right)^p$$

$$\leq K \sum_{n=1}^{\infty} \left| \frac{\varphi_n s_n}{n^\alpha} \right|^p \left(\sum_{n=1}^{\infty} \left| E_{n-k}^{\alpha-2} \right| \right)$$

$$\leq K \sum_{n=1}^{\infty} \left| \frac{\varphi_n s_n}{n^\alpha} \right|^p < \infty.$$

Similarly using Lemma 1 we get

$$\sum_{n=1}^{\infty} \left(\frac{|\varphi_n|}{nE_n^\alpha} \sum_{\nu=0}^{u} E_{n-\nu}^{\alpha-1} |s_\nu| \right)$$

$$\leq K \sum_{n=1}^{\infty} \left| \frac{\varphi_n}{n} \right|^{p(1-p)} \left(\sum_{k=n}^{\infty} \left| \frac{\varphi_k}{k} \right|^p \frac{E_{k-n}^{\alpha-1}}{E_k^\alpha} \right) | s_n |^p$$

$$\leq K \sum_{n=1}^{\infty} \left| \frac{\varphi_n}{n} \right|^{p(1-p)} \left(\sum_{k=n}^{\infty} \frac{|\varphi_k|^p}{k^{p+\alpha-\bullet-1}} \frac{(k-n)^{\alpha-1}}{k^{1+c}} \right)^p$$

$$\leq K \sum_{n=1}^{\infty} \left| \frac{\varphi_n s_n}{n^\alpha} \right|^p$$

Now let $\alpha = 1$. Then we must show that

$$\sum_{n=1}^{\infty} \left| \frac{\varphi_n t_n^1}{n} \right|^p < \infty.$$

By (3) we get

$$\overset{1}{t}_n = \frac{-1}{n+1}\sum_{\nu=0}^{n-1} s_\nu + \frac{n}{n+1} s_n .$$

So the problem reduced to the convergence of

$$\sum_{n=1}^{\infty}\left(\frac{|\varphi_n|}{n}\cdot\frac{1}{n+1}\sum_{\nu=0}^{n-1}|s_\nu|\right)^p .$$

Then we write

$$\sum_{n=1}^{\infty}\left(\frac{|\varphi_n|}{n}\,\frac{1}{n+1}\sum_{\nu=0}^{n-1}|s_\nu|\right)^p \leq K.\sum_{n=1}^{\infty}\left|\frac{\varphi_n}{n^{1-c}}\right|^p |s_n|^p\left(\int_n^{\infty}\frac{dx}{x^{1+c}}\right)^p$$

$$\leq K\sum_{n=1}^{\infty}\left|\frac{\varphi_n s_n}{n}\right|^p .$$

which completes the proof.

3. MAIN THEOREMS

THEOREM 1. Let $p \geqslant 1$, $0 \leq \alpha \leq 1$ and let (r_n) be a positive, non–decreasing sequence such that $\triangle r_\mu = O(n^{-1}r_n)$ $(n = 1,2,3,\ldots,$ $\mu = n, n+1,\ldots,2n)$, and (φ_n) be a sequence of complex numbers for which there exists any $c > 0$ such that $(n^{c-p-\alpha+1}\,|\varphi_n|^p)$ is non–increasing. Let (λ_n) be a positive, non–increasing and convex sequence such that

$$\sum_{n=1}^{\infty}\lambda_n^p\,|\triangle r_n| < \infty . \tag{13}$$

If

$$\sum_{\nu=1}^{n}\left|\frac{\varphi_\nu t_\nu^\alpha}{\nu}\right|^p = O(r_n) \tag{14}$$

then the series $\Sigma\lambda_k x_k$ is $\varphi - |C, \alpha|_p$ summable.

Proof: Under the hypothesis of the theorem we will show that (9) and (10) are satisfied. By (13) and (14) we have that

$$\sum_{\nu=1}^{n}\left|\frac{\varphi_\nu}{\upsilon}\lambda_\nu t_\nu^\alpha\right|^p = \sum_{\nu=1}^{n}\lambda_\nu^p\;|\,v^{-1}\varphi_\nu t_\nu^\alpha\,|^p$$

$$= \sum_{\nu=1}^{n-1} \triangle(\lambda_\nu^p) \left(\sum_{k=1}^{\nu} |k^{-1} \varphi_k t_k^\alpha|^p \right) + \lambda_n^p \sum_{k=1}^{n} |\varphi_k k^{-1} t_k^\alpha|^p$$

$$= O(1) + O(1) \sum_{\nu=1}^{n} \lambda_{\nu+1}^p |\triangle r_\nu| = O(1) .$$

Using the Abel transformation we get

$$\sum_{\nu=1}^{n} |\varphi_\nu \triangle \lambda_\nu t_\nu^\alpha|^p = \sum_{\nu=1}^{n-1} \triangle(\triangle\lambda_\nu^p) \sum_{k=1}^{\nu} |\varphi_k t_k^\alpha|^p + \triangle\lambda_n^p \sum_{k=1}^{n} |\varphi_k t_k^\alpha|^p$$

$$= O(1) \left\{ \sum_{\nu=1}^{n-1} \triangle(\triangle\lambda_\nu^p) \triangle \upsilon^p r_\nu + (\triangle\lambda_n^p) n^p r_n \right\}$$

$$= O(1) + O(1) \sum_{\nu=1}^{n} \triangle(\lambda_{\nu+1}^p) \; |\triangle(\upsilon^p r_\nu)| .$$

Therefore we know that

$$\sum_{n=1}^{\infty} (\triangle\lambda_{\nu+1}^p) |\triangle(\upsilon^p r_\nu \quad)| < \infty$$

[5]. So our proof is complete.

Using Lemma 3 and above theorem we can state the following corollaries about the $|C,\alpha;\gamma|_p$ summability.

COROLLARY I. Let $p \geqslant 1$, $0 < \alpha \leq 1$, $\gamma \geqslant 0$ and $\gamma p < \alpha$, and let (λ_n) be a positive non–increasing and convex sequence, (r_n) be a positive non–decreasing sequence such that $\triangle r_\mu = O(n^{-1} r_n)$ $(n = 1,2,\ldots, \mu = n, n+1, \ldots, 2n)$. If

$$\sum_{n=1}^{\infty} \lambda_n^p |\triangle r_n| < \infty$$

and

$$\sum_{n=1}^{m} n^{\gamma p + p - 1} |s_n^\alpha - s_{n-1}^\alpha|^p = O(r_m)$$

then the sequence (λ_n) is a $|C, \alpha; \gamma|_p$ summability factor of the series Σx_k.

COROLLARY 2. Let (λ_n) and (r_n) be sequences satisfying the conditions of Corollary 1 and let

$$\sum_{\nu=1}^{n} \frac{|s_\nu|^p}{\nu^{p(\alpha-\gamma-1)+1}} = O(r_n)$$

then the series $\Sigma\lambda_k x_k$ is $|C, \alpha;\gamma|_p$ summable.

It can be easily seen that the teorem given [1] is the special case of Corollary 2 for $\alpha = 1$ and $r_n = \log n$.

THEOREM 2. Let $p \geqslant 1$, $0 < \alpha \leq 1$, $-\frac{1}{2} < r < \frac{1}{2}$, $s > -1$ and let (λ_n) be a positive, non–increasing convex sequence, (φ_n) be a sequence of complex numbers. If there exists a c belong to interval $(0, \alpha]$ such that the sequence $(n^{c-p-\alpha+1}|\varphi_n|^p)$ is non–increasing then (λ_n) is a $\varphi - |C, \alpha|_p$ summability factor of the Jacobi series $\sum_{n=1}^{\infty} a_n P_n^{(r,s)}(x)$ at the point $x=1$, provided that

$$\sum_{n=1}^{\infty} \frac{\lambda_n^p}{n^{(p-1)(\alpha-1)+c}} < \infty \quad , \tag{15}$$

$$\Phi(w) \in \mathrm{Lip}\,\delta \quad (\delta > r + \tfrac{1}{2}) \tag{16}$$

and the antipole condition that

$$\int_0^t w^{s-\frac{1}{2}} \, |\Phi(\pi-\omega)| \, dw < \infty, \ (t \to 0) \tag{17}$$

is satisfied. If $s \geqslant r$ no antipole condition is required.

Proof: Let s_n be partial sums of Jacobi series $\sum_{n=0}^{\infty} a_n p_n^{(r,s)}(x)$ at the point $x=1$. Under the hypothesis of the theorem we have

$$s_n = O(n^{-q}) + O(n^{r-1/2}\log n)$$

and $\sum \frac{|s_n|}{n}$ is convergent [7], where

$$q = \begin{cases} \min(\delta,\ 1-r+s,\ \delta-r-1/2,\ \frac{1}{2}-r), & \text{if } s<r \\ \min(\delta,\ \delta-r-\frac{1}{2},\ \frac{1}{2}-r), & \text{if } s \geqslant r \end{cases}$$

Therefore

$$\sum \frac{|s_n|^p}{n} < \infty .$$

It is easy to show that

$$\sum_{\nu=1}^{n} \left| \varphi_\nu \bar{\triangle} s_\nu^\alpha \right|^p = O\left(n^{p(1-\alpha)+\alpha-c}\right)$$

and

$$\sum \lambda_n^p \left| \triangle n^{p(1-\alpha)+\alpha-c} \right| = O\left(\sum_{n=1}^{m} \frac{\lambda_n^p}{n^{(p-1)(\alpha-1)+c}} \right).$$

By Theorem 1, the series $\Sigma\ \lambda_n a_n P_n^{(r,s)}(x)$ is $\varphi - |C, \alpha|_p$ summable at the point $x = 1$.

If we take $\varphi_n = n^{\gamma+1-p^{-1}}$ we can choose $c = \alpha - \gamma p$, whenever $\gamma p < \alpha$. Now we can state the following corollary about the $|C, \alpha;\gamma|_p$ summability of Jacobi series.

COROLLARY 3. Let $p \geq 1$, $0 < \alpha \leq 1$, $\gamma \geq 0$, $\gamma p < \alpha$ and let (λ_n) be a positive non–increasing and convex sequence such that

$$\sum_{n=1}^{\infty} \frac{\lambda_n^p}{n^{p(\alpha-\gamma-1)+1}} < \infty .$$

If $\sum\limits_{n=1}^{\infty} a_n P_n^{(r,s)}(x)$ satisfies th conditions of Theorem 2 then (λ_n) is a

$|C,\alpha;\gamma|_p$ summability factor of the jacobi series $\Sigma\ a_n P_n^{(r,s)}(x)$ at the point $x=1$.

Using Lemma 3 we can state the following corollary about the $|C,1|_p$ summability of of Jacobi series.

COROLLARY 4. Let $p \geq 1$, $-\frac{1}{2} < r < \frac{1}{2}$, $s > -1$. The Jacobi series (6) is $|C,1|_p$ summable at the point $x=1$ provided that (16) and (17) is hold. If $s \geq r$ no antipole condition is required.

The special case of this corollary was given by Yadav [7] for $p = 1$.

ÖZET

Bu çalışmada $\Sigma \lambda_k x_k$ serisinin $\varphi-|C,\alpha|_p$ toplanabilmesi için yeter şartlar verilmiştir. Buna ilaveten, $0 \leq \alpha \leq 1$ için

$$\sum_{\nu=1}^{\infty} \left| \frac{\varphi_\nu s_\nu}{\nu^\alpha} \right|^p < \infty$$

olduğunda Σx_k serisinin $\varphi-|C,\alpha|_p$ toplanabilir olduğu gösterilmiştir. Bundan faydalanarak, ana fonksiyonu belli bir Lipschitz sınıfına ait olan fonksiyonların Jacobi serilerinin $\varphi-|C,\alpha|_p$ toplanabilme faktörleri verilmiştir. Ayrıca bu serilerin $|C,1|_p$ toplanabilmesi ile ilgili bir sonuç elde edilmiştir.

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