

## Fuzzy inverse logic: part-1. introduction and bases

### *Bulanık ters mantık: kısım-1. giriş ve temeller*

Ertekin ÖZTEKİN<sup>\*1,a</sup>

<sup>1</sup>Gümüşhane University, Faculty of Engineering and Natural Sciences, Civil Engineering Department, 29100, Gümüşhane

• Geliş tarihi / Received: 11.03.2021

• Düzeltilerek geliş tarihi / Received in revised form: 09.04.2021

• Kabul tarihi / Accepted: 13.04.2021

#### Abstract

In almost all deterministic and artificial intelligence techniques, for the solution of the scientific problems such as design and control problems, the output estimations are performed depending on manipulations on the values of input variables. With the other words, lots of different values derived from input parameters are tried in order to obtain desired output(s). Contrary to these conventional estimation methods, this study consists of two parts in which a new artificial intelligence method called fuzzy inverse logic (FIL) is developed to determine or estimate the value of the input parameters that give the targeted problem output. In the first part of this study, after providing a brief overview about the method of classical fuzzy logic (FL), the solution approaches and calculation details about FIL are given. In the second part of the study, fuzzy inverse logic method was used to solve one simple mathematical problem and one simple civil engineering problem. After the validity of the developed method was demonstrated by graphics and tables. some evaluations were made about the method.

**Keywords:** Artificial intelligence, Fuzzy inverse logic, Fuzzy logic, Logic, Inverse logic

#### Öz

Hemen hemen tüm analitik ve yapay zeka tekniklerinde, tasarım ve kontrol problemleri gibi bilimsel problemlerin çözümü için, arzu edilen sonucu elde edebilmek için girdi parametrelerinin değerleri üzerinde manuplasyonlar yapılır. Başka bir deyişle girdi parametrelerinin herbiri için birçok farklı değer arzu edilen problem sonucu elde edilene kadar kullanılan çözüm yöntemi üzerinde denenir. Bu alışlagelmiş tahmin yöntemlerinin tersine, hedeflenen problem çıktısını veren girdi parametrelerinin değerinin ne olması gerektiğini belirlemek veya tahmin etmek amacıyla bulanık ters mantık adıyla yeni bir yapay zeka yönteminin geliştirildiği bu çalışma iki kısımdan oluşmaktadır. İlk kısım olan bu makalede, klasik bulanık mantık hakkında kısa öz bilgi sunulduktan sonra, bulanık ters mantıktaki çözüm yaklaşımı ve hesaplama detayları verilmiştir. Çalışmanın ikinci kısmını oluşturan diğer makalede ise bir adet matematik problemi ve bir adet inşaat mühendisliği probleminin çözümü için bulanık ters mantık yöntemi kullanılmıştır. Geliştirilen yöntemin geçerliliği tablo ve grafiklerle ortaya konulduktan sonra yöntem hakkında bazı değerlendirmeler yapılmıştır.

**Anahtar kelimeler:** Yapay zeka, Bulanık ters mantık, Bulanık mantık, Mantık, Ters mantık

\*a Ertekin ÖZTEKİN; ertekinoztekin@hotmail.com, Tel: (0532) 777 26 67, orcid.org/0000-0002-4229-0953

## 1. Introduction

The human beings created as the most perfect and honorable of all creatures, by developing many simple mechanical devices with the knowledge, intelligence, talent and skill granted to them, they are now able to manufacture machines that behave like themselves or that can imitate their own behavior and use them in many fields since their creation. The human beings have succeeded in turning not only their own characteristics, but also the behavior and creation characteristics of many other living things, even some natural phenomena into machine behavior and / or machine intelligence sometimes as parts, sometimes as a whole. The most well-known algorithms that human beings transfer the features and behaviors of other creatures to artificial intelligence are artificial bee colony (Karaboga and Akay 2009), ant colony optimization algorithm (Parpinelli et al. 2002), cuckoo bird algorithm (Rajabioun 2011), bat algorithm (Yang and Gandomi, 2012), particle swarms (Kennedy and Eberhart 1995) etc. Some of the algorithms that human beings transfer some of their own biological and behavioral features to artificial intelligence technology are artificial neural networks (Jain et al., 1996), genetic algorithm (Whitley 1994), memetic algorithms (Moscato et al., 2004), cellular automata (Chopard and Droz 1998), Fuzzy logic (FL) (Zadeh, 1965; Zadeh, 1973; Zadeh, 1975) etc. Apart from these listed artificial intelligence methods, other algorithms and methods such as simulated annealing (Van Laarhoven, and Aarts, 1987), taboo search algorithm (Cvijović and Klinowski, 1995) etc. are also available in the technical literature.

By using of artificial intelligence algorithms in machine intelligence, advanced technological machines, called smart machines, which can decide on their own, communicate and work synchronously with each other, and learn themselves, have been manufactured. While these technological developments have facilitated human life in many areas, they have also started to cause some problems in sociological, health, psychological fields such as unemployment, anti-socialization and obesity. Even the concern has already emerged that these smart machines may someday be out of control and become one of the enemies of mankind in the future. Rapid and intense developments in the technological hardware and software fields in the last few decades have been effective in the invention and manufacturing of these advanced technological products. Particularly, discovered methods on artificial intelligence and the developed computer

and / or electronic technologies have a great contribution on these inventions and manufacturing of the smart products. Finally, it is obvious that the ongoing studies in these areas will lead to many new technological developments and progress.

### *1.1. Purposes and methods in a scientific problem solution*

Although the purpose in most scientific problems is to determine the values of the variables to achieve the desired results as accurately as possible with an acceptable error, computation flow in the methods used widely is based changing of variables values in the problem under consideration by trial and error algorithm or by another algorithm. To be more clearly expressed, after the computation flow starts with determining of initial values of the variables by a researcher, first result(s) is(are) calculated according to the first certain values of these input variables. Then, new values of variables are computed and assigned according to the error amount of the calculated results. This computation process is repeated according to these new values. In the new value assignments many different approaches can be used. Especially in control problems and in engineering designs, various approaches, algorithms and functions can be used for optimization of input values. The revision of the input values continues until the output with the acceptable error level is achieved or until the exact solution is reached. This is the same in most traditional methods and in most traditional artificial intelligence methods. In the FL approach, which is the one of the widely known of artificial intelligence method and used successfully in many fields, the situation is not different. In the classical FL approach, which is based on the ability of the human being to make new inferences in the light of what is known, the inferences are performed for the determination or prediction of problem outputs according to the values of the known input parameters. However, the human beings have also the ability to make backward inferences or reverse inferences against the events and situations according to they experienced. In the other words, taking into account the events or situations experienced, a person can predict the input parameters that can produce similar results. Thanks to this vital feature, human beings have managed to be the ruler of the earth. Because they did not repeat the mistakes they made by making backward inferences, they predicted situations that could give possible bad results and thus they took precautions. Similarly, they were able to manage many

situations, events and production that would give the results they wanted by making inferences in reverse.

The basics of this study, which was inspired by the ability of the human being to backward inference, are similar with the basics of the classical FL approach. Therefore, it is going to be useful to present here some essential information about FL before beginning to introduce FIL in detail.

### 1.2 A short overview on FL

As known FL is one of the powerful tool among artificial intelligent methods. FL theory first started to appear in 1965 with the introduction of fuzzy sets theory by Zadeh (Zadeh, 1965; Zadeh, 1973; Zadeh, 1975) and then it has been used in many scientific and engineering fields successfully. Some lately performed examples about FL can be found in the technical literature (Altaş, 1999a; Altaş, 1999b; Yager and Zadeh, 2012; Öztekin and Filiz, 2015; Altaş, 2017; Pörge, 2019).

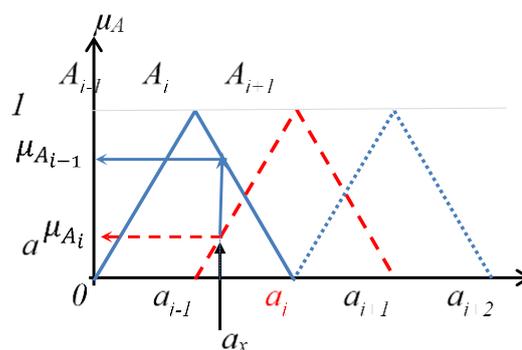
Computation flow in the FL can be divided into four main steps. First step is definition of membership functions and fuzzyfication of input and output parameters. The second is constitution of rule table or matrice according to the data obtained from previous experiences or computations and the third is computation of fuzzy outputs. Finally, de-fuzzyfication of fuzzy outputs is last step of FL computations. Summary information about these four steps is given following.

#### 1.2.1. Membership functions and fuzzyfication

Fuzzyfication of values of a variable is done according to the fuzzy sets theory (Zadeh, 1965) in FL. Value range of variables are divided into parts in this step. Each part is called as a fuzzy set. Constitution of fuzzy sets or dividing into parts of value ranges of variables are made depending upon some parameters such as computation sensitivity, properties of variables, properties of experimental or available data obtained from scientific experiments, solutions, previous experiences etc. If it is need to be expressed with another way, value range of a variable can be represented by fuzzy sets in FL theory. Although the fuzzy sets can be expressed or named differently from each other in a problem, in some cases some parts of their range values can be common. That is to say, there may not be an exact distinction between different fuzzy

sets. So, an exact value of a variable can be a member of different fuzzy sets. The belonging of an exact value to a fuzzy set is expressed by its' degree of membership between 0 and 1. In order to determine the membership degree, functions such as triangle, trapeze, sigmoid etc. are used (Terano etc., 1992; Tanaka, 1997; Ross, 2004; Harris, 2005; Altaş, 2017).

In the Figure 1, fuzzy sets ( $A_{i-1}$ ,  $A_i$ ,  $A_{i+1}$ ), value range of variable A ( $0 - a_{i+2}$ ), value ranges of fuzzy sets ( $0 - a_i$ ,  $a_{i-1} - a_{i+1}$ ,  $a_i - a_{i+2}$ ), triangle membership functions, membership degrees ( $\mu_{A_{i-1}}$ ,  $\mu_{A_i}$ ) of value  $a_x$  for  $A_{i-1}$  and  $A_i$  fuzzy sets are presented with triangle memberships functions for example.



**Figure 1.** Value Ranges of fuzzy sets belong to variable A and memberships degrees of value  $a_x$

#### 1.2.2. Constitution of rule table (matrice)

The aim in this step is constitution of the table include rules which will be used in FL solutions. In the constitution of rules, the data obtained from previous experiences or from previous solutions is used. The terms such as “if”, “else”, “then”, “and”, “or” etc. are used in the rule constitution in FL as in the common logic. Some simple rule examples used in FL are given in Equation 1 and Equation 2.

$$\text{if } A = a \text{ and } B = b \text{ then } C = c \quad (1)$$

$$\text{if } A = a \text{ or } B = b \text{ then } C = c \quad (2)$$

Especially “and” and “or” terms are used for the calculations of intersection of sets and union of sets and they correspond to numerically minimum value and numerically maximum value in FL computations respectively. These situations are expressed following Equation 3 and Equation 4 for a problem with two input variables (A and B) and one output (C) (Mamdani, 1975; Mamdani, 1976).

If  $A=A_i$  and  $B=B_j$  then  $C=C_{i,j} \Rightarrow A \cap B \Rightarrow \mu_{C_{i,j}} = \min(\mu_{A_i}; \mu_{B_j})$  (3)

If  $A=A_i$  or  $B=B_j$  then  $C=C_{i,j} \Rightarrow A \cup B \Rightarrow \mu_{C_{i,j}} = \max(\mu_{A_i}; \mu_{B_j})$  (4)

Constituted rules for a FL problem are gathered in a table called as rule matrix or rule table. All computations are made depending upon this table.

1.2.3. Computation of fuzzy outputs

In this step, after membership degrees of input variables in fuzzified space are computed, they are used in the determination of membership degrees of fuzzy output sets according to rules in the rule table (see Equation 3 and Equation 4). The area(s) under the line corresponding to the membership degree in a fuzzy output set is a fuzzy output of a rule. If there exist, other fuzzy outputs are determined for other rules in this step. The existence of a fuzzy outputs or the number of a fuzzy outputs depends on the numbers of the rules corresponding to the values of input variables.

1.2.4. Defuzzification of fuzzy outputs

After all areas under membership functions of output parameters for valid rules are determined according to the previous steps given above, they are drawn on the same axes and net values of output variables are computed according to the some defuzzification methods or equations. Some widely used defuzzification methods are Center of Sums Method (COS), Center of Gravity (COG)/Center of Area (COA) Method, Center of Area/Bisector of Area Method (BOA), Weighted Average Method (WAM, First of Maxima Method (FOM), Last of Maxima Method (LOM), Mean of Maxima Method (MOM) etc. (Erdun, 2020).

All FL computation steps summarized in the 4 subheadings above are summarized in the Figure 2 given below.

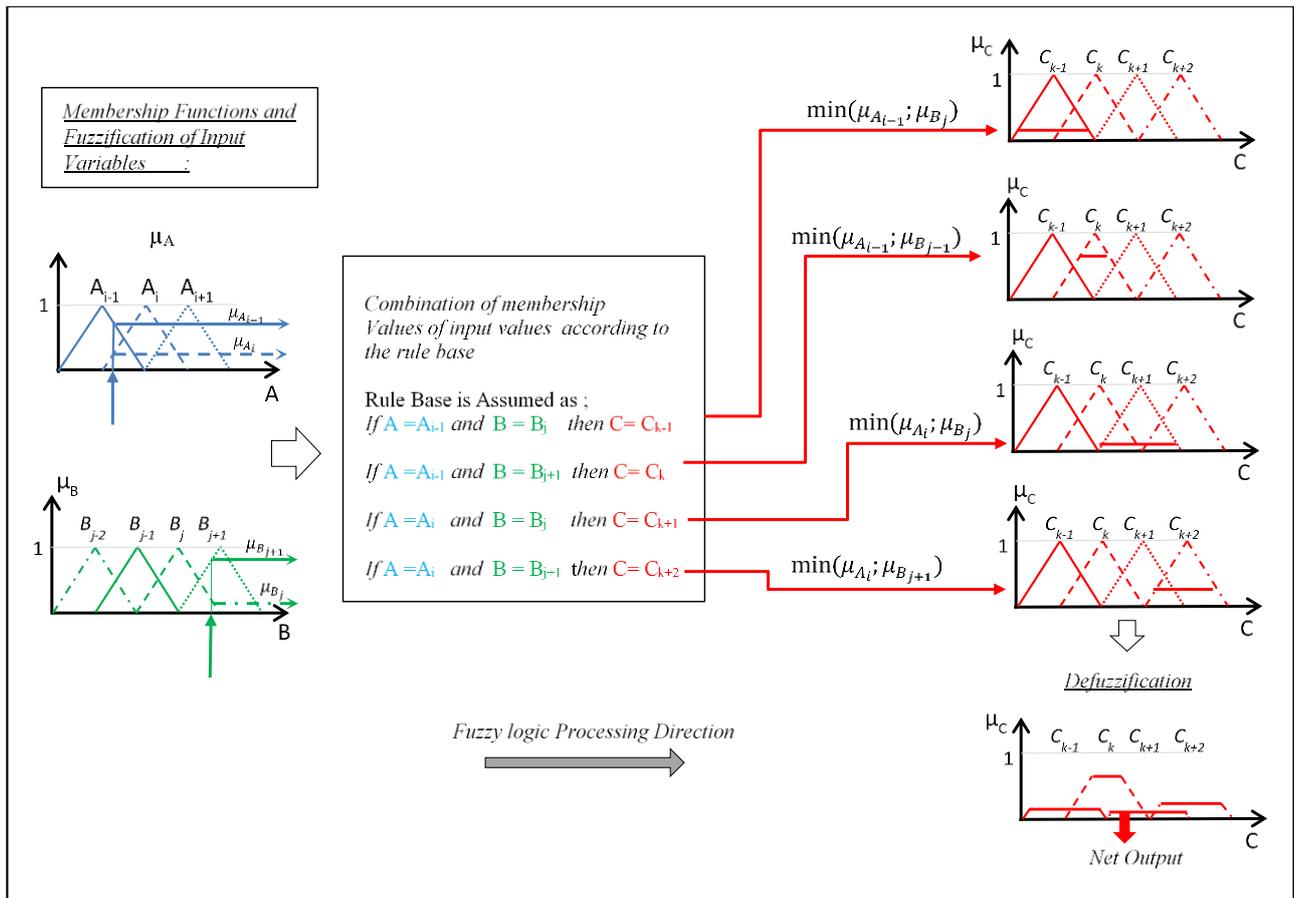


Figure 2. Summarized FL processing steps for a problem with 2 variables.

## 2. Bases of fuzzy inverse logic (FIL)

Human beings gain experiences in the face of physical problems and use these experiences to reach the desired goal in the face of new problems. They usually use these experiences by making two different types of inference. The first of these is the prediction of what the result might be by changing the values of the variable parameters in the problem. This type of inference constitutes the logic of classical FL. The second type of inference is the prediction of what the values of the input parameters should be in order to achieve the desired result, which does not exist among the existing results. This second inference constitutes the logic of the FIL method that is tried to be presented in this study. The data used in both inferences are the same. However, the inferences made are in the opposite direction.

Considering all these, the main purpose of FIL is to determine which values the problem variables should take by using the data gathered from experienced samples in order to achieve the desired outputs. On the other hand, in many techniques with/without artificial intelligence, the aim is to predict what the problem output might be by using new input values in the models constituted. As an example constitution of a FL model to estimate the strength of concrete to be produced in a concrete plant based on the amount of materials in the concrete mix can be given. On the other hand, in order to determine the amount of materials in the mixture of a concrete production with a certain targeted compressive strength, it requires to search for solutions with many trial and error on the current FL model or it requires to constitute a new FL model. Instead of using these two options, using the FIL method on the same data and the same fuzzy model will be much more practical and effective.

To give another example, the purpose of engineering designs is to design elements with dimensions and material properties that will bear external effects safely. In FL or other classical methods, in the light of the previously obtained information and data, it is determined whether the external load is carried for the previously selected material quality and element sizes. Then, in the light of the results obtained, the material quality and / or element dimensions are revised depending on the error in the results obtained and the operations are repeated until the result is obtained with acceptable error limits. If the FIL method is used in this problem, depending on the size of the external effects to be carried, the element sizes and

material quality can be determined directly by performing the process flow in the opposite direction of FL calculations.

## 3. Fuzzy inverse logic

In order to apply FIL method on a problem, a suitable, accurate and precise FL model must be successfully constituted and validated for this problem. To ensure that the FIL method can produce accurate and precise results, it should be applied on an accurate and precisely modeled FL model. Because as stated before, FIL calculations are based on FL completely and in FIL calculations, the data and rules prepared for FL are used exactly. In addition, the defuzzification method used in the testing and validation stage of the FL model, should be used in FIL method. Otherwise, calculations performed with FIL may produce incorrect results. In addition, while FL and classical methods are applied directly, only one solution is produced at a time, depending on the content and size of the rule matrix used in the FIL method, it may be possible to determine a large number of solutions at once. Although this may seem like a disadvantage for FIL on the one hand, it is actually one of the most important and distinctive features of FIL. This important feature indicates the ability to produce alternative solutions to FIL.

### 3.1. FIL computations

With the rule matrix constituted during the development of the FL model, the digital map of the output parameter(s) is/are actually constituted depending on the input parameters. Each rule output is the known landmarks of all the coordinates of this digital map for that output parameter. Rule outputs are landmarks with known coordinates of this digital map for those output parameters. The first step in FIL studies is to scan this numerical map completely to reveal among which landmarks (valid rule outputs) the target value of an output parameter is located. These regions, where the target values of the output parameters are located, are defined as sub-solution spaces in this study. One or more solutions can be obtained in each sub-solution space. After determining the sub-solution spaces where the desired output(s) can be located, the coordinates defining each of these regions are obtained (Step 2). That is, the fuzzy sets of input parameters in the rules that point to the landmarks in the digital map are determined. These fuzzy sets will hereafter be referred to as valid fuzzy sets in this study. In other words, the axes that make up the digital map are the

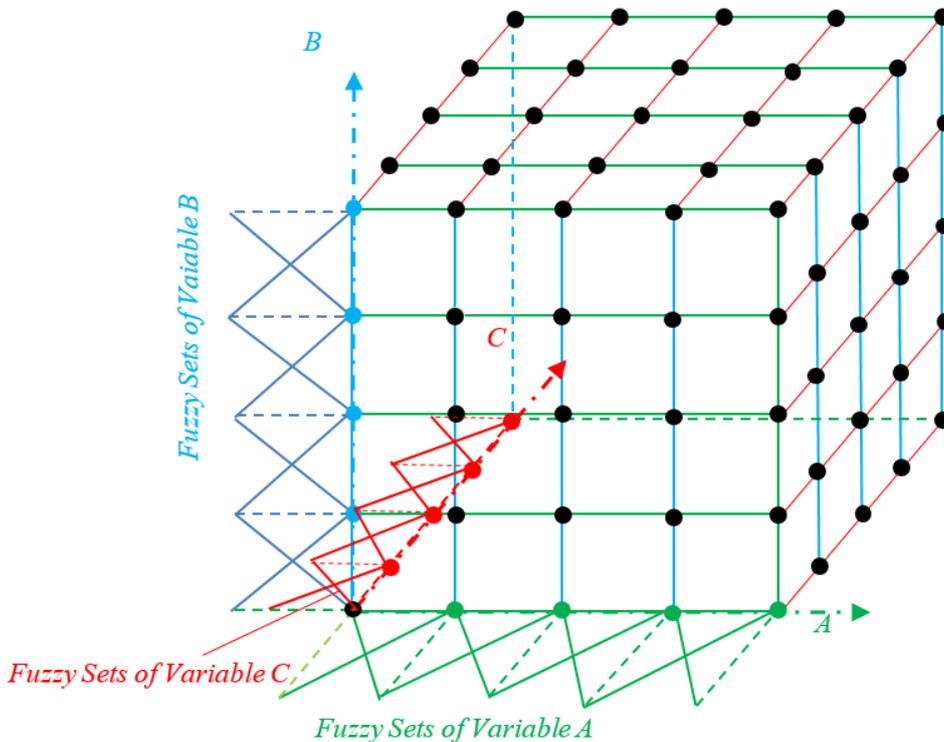
axes on which fuzzy sets of input parameters take place and those valid fuzzy sets for each input parameter are consecutive fuzzy sets. Determining consecutive valid fuzzy sets for each input parameter is the second step of FIL.

In the third step, membership values of valid fuzzy sets determined for input parameters are searched. The computations made at this stage are similar to those in FL. For these computations, the membership values of the valid fuzzy sets are changed by iteration for the desired processing precision, and it is checked whether the solution obtained in each sub-solution space is equal to the desired output or if it is obtained with an acceptable error. If a result equal to the desired result or an approximate result with an acceptable error is not found in this step, the operations can be repeated by increasing the sensitivity in iterative operations. In the 4<sup>th</sup> step, if the result(s) is obtained with an acceptable error or equal to the desired output as a result of any iterative operation performed in the third step, the membership values of the valid fuzzy sets of the variable parameters and the net input values of those parameters are computed. The computations made at this stage are the opposite of fuzzyfication of the net input values in FL. Membership values in two consecutive valid fuzzy input sets are processed in membership functions representing these sets, and net values of the input

parameter are calculated. In these calculations, relationships between membership functions of consecutive fuzzy sets are used.

The operations performed in the third step of the method given in the 4 steps above are similar to the search for the desired output by trial and error in the FL method. Although this situation seems like a contradiction, the operations performed in the first two steps allow the search for a solution in a very small subspace compared to a very large global solution space. In this way, the transaction volume that may be very large is reduced and a solution is found in a reasonable time. The operations performed in these four steps of the FIL method are explained in detail below on a parametric problem with three input parameters and one output parameter. It is worth mentioning here that choosing a problem with 3 input parameters below was chosen to visually increase the understandability of the method. It is possible to easily apply the FIL method to problems with many input and output parameters.

It is supposed that, there is a fuzzy model that is prepared and validated for the solution of a problem with three input variables such as A, B and C and an output such as O. This fuzzy model can be schematized in a three-dimensional space as shown in Figure 3.



**Figure 3.** A fuzzy model schematized in 3D space

As can be seen from this figure, since the number of input parameters is 3, the dimension of this parametric problem is 3. Fuzzy sets of input parameters are shown on their respective axes with triangular membership functions in Figure 3. In this problem, if the input parameters A, B and C have the number of  $N_A$ ,  $N_B$  and  $N_C$  fuzzy sets respectively, then  $N_R = N_A \times N_B \times N_C$  rules must be used while constituting the fuzzy model. The fuzzy outputs of all these fuzzy rules are represented by solid dots in Figure 3. These points of which the net values and fuzzy sets of the input and output parameters are known, are landmarks in fuzzy output map. These points are vital points that guide and determinative in FIL computations.

The FIL calculation steps to be performed on a FL model constituted as described above are given below.

Step1- Determination of sub-solution spaces (valid rule groups)

- a) 0 dimensional output Searching
- b) 1 dimensional output Searching
- c) 2 dimensional output Searching
- d) 3 dimensional output Searching

Step 2- Determination of fuzzy koordinates of sub-solution spaces (determination of valid fuzzy sets of input parameters)

- a) For 1 dimensional output searching (for 1 dimensional su-solution space)
- b) For 1 dimensional output searching (for 1 dimensional su-solution space)
- c) For 1 dimensional output searching (for 1 dimensional su-solution space)

Step 3- Determination of memberships values of valid fuzzy sets of input parameters

Step-4 Determination of net values of input parameters

*3.1.1 Determination of sub-solution spaces (valid rule groups)*

The first step in FIL calculations is to determine in which region(s) the desired  $O_{\text{desired}}$  value can be in the 3-dimensional space given in Figure 3. In general, if the  $O_{\text{desired}}$  value is between the smallest net output value and the largest net output value in the net output space, it can be said that there is at least one  $O_{\text{desired}}$  value in this space. This also means that it is possible to have the  $O_{\text{desired}}$  value at more than one point in this space. Since the  $O_{\text{desired}}$

value is a net value and consists of fuzzy sets in the output space, for a precise and accurate search, the net values or value ranges corresponding to the membership 1 of the fuzzy output sets in the output space should be used. In order to determine the location of the  $O_{\text{desired}}$  value, it is necessary to perform a combined search in all sub-dimensions of the global output space. In this 3D example, the solution searches to determine the valid solution space can be done as follows depending on the number of dimensions.

*3.1.1.1 0 dimensional output searching*

0 dimensional output searching means that the  $O_{\text{desired}}$  value is searched in the fuzzy output space without looking at any dimension. In other words, it means that the solution is searched at solid points in Figure 3 where none of the input parameters are variable. Another means of this is to investigate whether the  $O_{\text{desired}}$  value is equal to the net output of any  $2^0=1$  of the SR rules. If it is detected that there is an  $O_{\text{desired}}$  value at the end of this research, Fuzzy rule(s) is/are determined, which output the fuzzy output set at the detected point(s).

Each of these rules is defined as a group of valid rules in 0 dimensional output searching searches. In 0 dimensional output searching operations, a rule group consists of 1 valid rule.

*3.1.1.2. 1 dimensional output searching*

In 1 dimensional output searching operations, one parameter of the problem is considered as variable and other parameters are considered as constant, and sub-solution space(s) where  $O_{\text{desired}}$  value may exist is searched. The searched solution space(s) is/ are actually one dimensional. This means that a solution will be searched between  $2^1=2$  points.

Mathematically, this is the same as searching for a third point between two points whose coordinates are known. For this parametric problem, when the input parameter A is taken as variable and the other parameters are constant, whether there is an  $O_{\text{desired}}$  value on all the thick green lines that can be drawn between two solid points inside and on the surfaces of this cube is investigated. Two thick green lines drawn on the upper and front surfaces of the cube in Figure 4 can be given as two examples. The value of A parameter between the end points of this green thick lines are the same. Since both ends of a green lines correspond to the net values of two consecutive fuzzy output sets,

such as  $O_{X1}$  and  $O_{X2}$ , whose membership value corresponds to 1 at that both ends, the search is performed by checking whether the condition given by Eq 5 is met.

$$O_{X1-net} \leq O_{desired} \leq O_{X2-net} \tag{5}$$

After all of those processes described above are done for other parameters (B and C for blue and red lines respectively) in the problem, 1 dimensional output searching process is completed. The number of searching combinations to be performed in the 1-dimensional output searching process can be determined by the following formula.

$$Sn_{1D} = \frac{N!}{R!(N-R)!} = \frac{3!}{1!(3-1)!} = 3 \tag{6}$$

In this formula, N is the numbers of parameters in the problem, R is searching dimension and  $Sn_{1D}$  is the number of searching combinations.

At the end of this search, if it is revealed that there may be an  $O_{desired}$  value on the line joining  $2^1 = 2$  consecutive points, all the rules whose output corresponds to any of the fuzzy sets in these two points are valid rules for the solution. Rule groups that define all 1-dimensional sub-solution spaces by scanning the entire global solution space are determined at this stage. In 1 dimensional output searching, 1 rule group consists of 2 validrules.

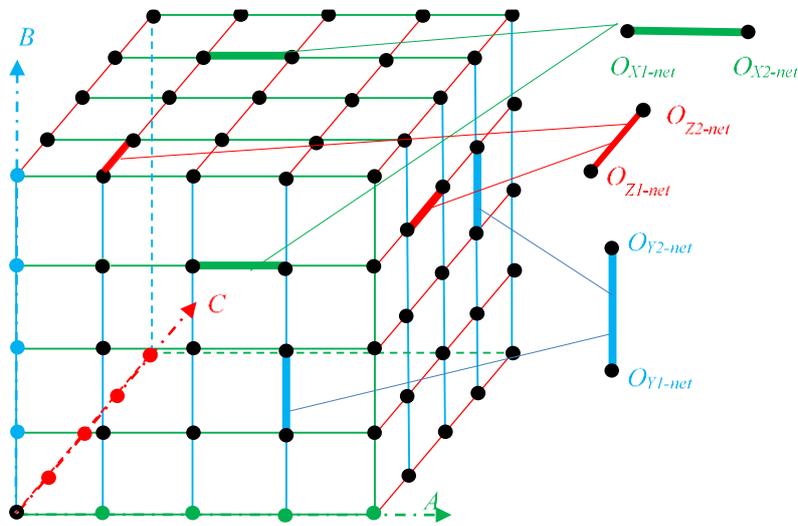


Figure 4. 1 dimensional Output Searching

### 3.1.1.3. 2 dimensional output searching

In 2 dimensional output searching, two parameters of the problem are taken into account as variables and the other parameter(s) are considered as constant, and sub-solution space(s) where the  $O_{desired}$  value may exist is searched. Searched space(s) is/are two-dimensional surfaces. This means that a solution will be searched on a solution surface located between  $2^2 = 4$  points. Mathematically, this is the same as investigating whether a desired point or points exist within a

surface whose coordinates are known at all four corners. The solutions searched on the solution surface can not be in point form, but can also be on a line or a curve in this surface (Figure 5a, 5b, 5c). If the searched  $O_{desired}$  value is on a line or curve as shown in Figure 5b or Figure 5c, it means that there are infinite number of solutions. However, infinite number of solutions are not obtained in FIL computations. The number of solutions can be obtained in different numbers depending on the sensitivity determined in membership iterations and the acceptable error level.

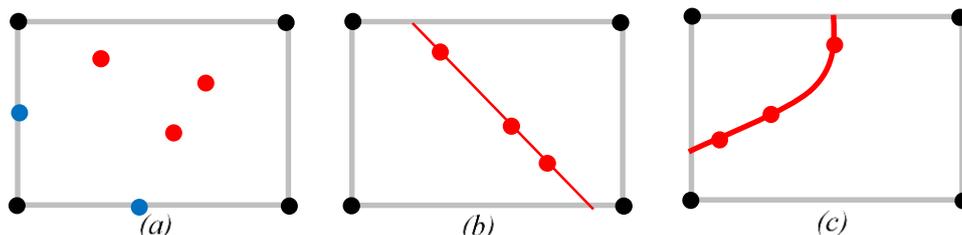
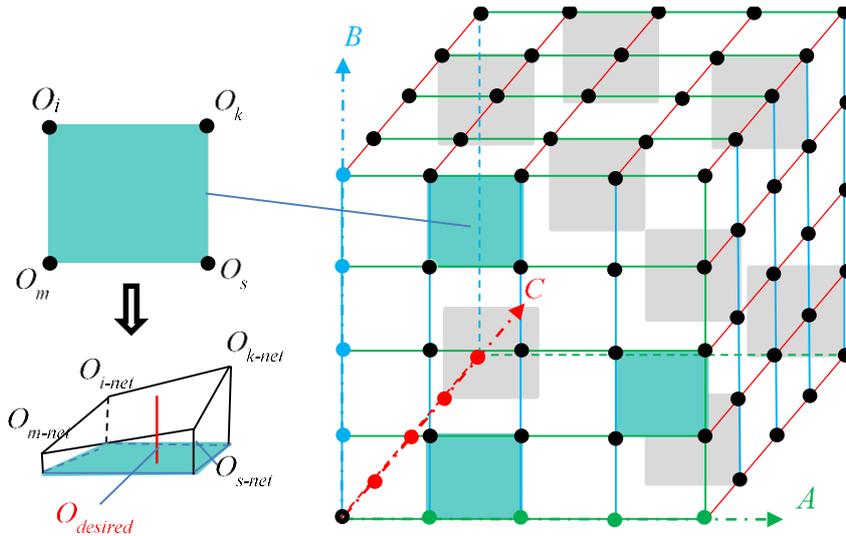


Figure 5. The existence of the  $O_{desired}$  value searched in the 2-dimensional Output Searching operation in the sub-solution space (a) on a point, (b) on a line, (c) on a curve.

Some of the searched solutions (blue points) can be between any two of the horizontal or vertical black points shown in Figure 5 and defining the two-dimensional solution space. Such solutions, shown with blue points in Figure 5-a, are points that can also be obtained in a 1 dimensional output searching computations. The points shown in red in Figures 5a, 5b and 5c are solution points that cannot be obtained by the one-dimensional output searching computations. These red solution points

can only be obtained in 2 or more dimensional searching computations.

For the example problem, when input parameters A and B are taken as variables and the other parameter C is constant, on the front surface of the cube shown in Figure 6 and on the other layers parallel to this surface, (inside and on the surfaces of the cube) it is investigated whether there is an  $O_{desired}$  value on all two-dimensional sub-solution spaces that can be created with 4 adjacent points.



**Figure 6.** 2 dimensional output searching

As can be seen from Figure 6 that the  $O_i$ ,  $O_k$ ,  $O_m$  and  $O_s$  are fuzzy output sets and they correspond to the points i, k, m and n respectively. In order to prove the existence of the solution mathematically,  $O_{desired}$  is compared with the net output values of  $O_i$ ,

$O_k$ ,  $O_m$  and  $O_s$  with 1 membership value. In comparison, if the condition given in the Equation-7 is satisfied, the solution for  $O_{desired}$  is available on the plane constituted by the points i, k, m and n.

$$\min(O_{i-net}; O_{k-net}; O_{m-net}; O_{s-net}) \leq O_{desired} \leq \max(O_{i-net}; O_{k-net}; O_{m-net}; O_{s-net}) \quad (7)$$

As stated above, when parameters A and B are taken as variables, sub-solution spaces are investigated in all other planes parallel to the plane created by these parameters.

In Equation 8, N is the numbers of parameters in the problem, R is searching dimension and  $Sn_{2D}$  is the number of binary searching combinations.

2 dimensional output Searching operations do not end with considering only A and B parameters as variables. 2 dimensional output searching is completed after all the remaining binary combinations of all input parameters (A with C and B with C) are also performed. The number of binary searching combinations to be performed in the 2-dimensional output searching process can be determined by the Equation-8.

At the end of this search, if it is revealed that there may be an  $O_{desired}$  value on the surface constituted by  $t^2 = 4$  adjacent points, all the rules whose output corresponds to any of the fuzzy sets in these four points are valid rules for the solution. Rule groups that define all 2-dimensional sub-solution spaces by scanning the entire global solution space are determined at this stage. In 2 dimensional output searching, 1 rule group consists of 4 valid rules.

$$Sn_{2D} = \frac{N!}{R!(N-R)!} = \frac{3!}{2!(3-2)!} = 3 \quad (8)$$

3.1.1.4. 3 dimensional output searching

In 3 dimensional output searching, three parameters of the problem are taken into account as variables and the other parameter(s) is/are considered as constant, and sub-solution space(s) where the  $O_{desired}$  value may exist is searched. Searched space(s) is/are like three-dimensional volumes. This means that a solution will be searched on a solution volume located between  $2^3 = 8$  points. The solutions searched in the three-

dimensional sub-solution space can be not only as a point, but also on a line, on a curve or on a surface, as in Figures 7a, 7b, 7c and 7d. If the  $O_{desired}$  value(s) is/are located on a line, a curve or a surface as shown in Figure 7b, Figure 7c or Figure 7d, then an infinite number of solutions may be available. However, an infinite number of solutions are not obtained in FIL calculations. Different number of solutions can be obtained depending on the sensitivity determined in membership iterations.

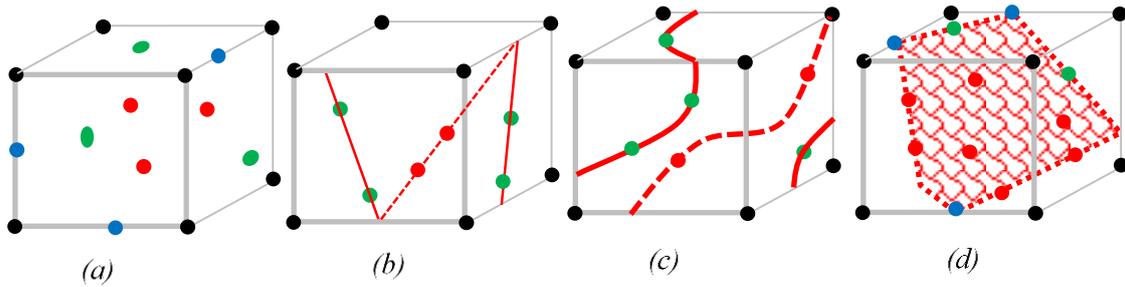


Figure 7. The presence of the  $O_{desired}$  value searched in the 3-dimensional output searching operation in the sub-solution space (a) on a point, (b) on a line, (c) on a curve, (d) on a surface

The searched solutions (blue points) can be found between any two of the horizontal or vertical black filled points shown in Figure 7 that define a three-dimensional sub-solution space. Such solutions, shown with blue solid points in Figure 7- a, are the solutions that can also be found in a one-dimensional output search process. The searched solutions (green solid points), can be found between any 4 of the black solid points in the same

space defining two-dimensional subspaces of space. Such solutions, shown with green dots in Figure 7-b, are solutions that can also be found in two-dimensional search operations. Since the solid points shown in red in Figures 7a, 7b, 7c and 7d are located in 3-dimensional volumetric space, they cannot be found by one-dimensional and two-dimensional output search operations. For this, 3-dimensional searching are required.

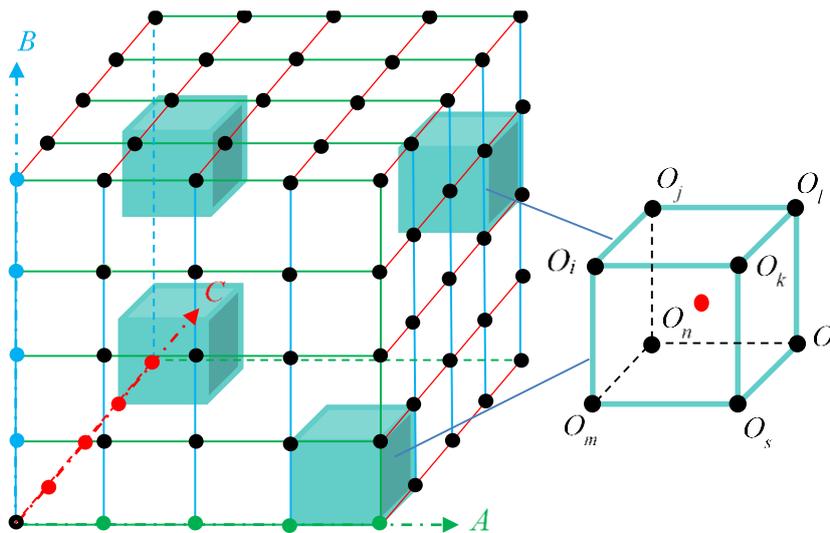


Figure 8. 3 dimensional output searching

For the explanatory problem, considering all input parameters A, B and C as constant, it is investigated whether the  $O_{desired}$  value is available in all three-

dimensional subspaces constituted by adjacent  $2^3 = 8$  points within small cubes in the big cube shown in Figure-8.

As can be seen from Figure 8 that the  $O_i, O_j, O_k, O_l, O_m, O_n, O_s$  and  $O_t$  are fuzzy output sets and they correspond to the points  $i, j, k, l, m, n, s$  and  $t$  respectively. In order to prove the existence of the solution mathematically,  $O_{desired}$  is compared with the net output values of  $O_i, O_j, O_k, O_l, O_m, O_n, O_s$

$$O_{min} = \min (O_{i-net}; O_{j-net}, O_{k-net}; O_{l-net}; O_{m-net}; O_{n-net}; O_{s-net}; O_{t-net}) \tag{9}$$

$$O_{max} = \max (O_{i-net}; O_{j-net}, O_{k-net}; O_{l-net}; O_{m-net}; O_{n-net}; O_{s-net}; O_{t-net}) \tag{10}$$

$$O_{min} \leq O_{desired} \leq O_{max} \tag{11}$$

3 dimensional output searching operations end because there are 3 variables in this explanatory problem. However, if the number of variables (problem dimension) in the problem were greater than 3, just like in 2 dimensional searching, then 3 dimensional output searching should be performed for all triple combinations of variables.

The number of triple searching combinations to be performed in the 3-dimensional output searching process can be determined by the Equation-12.

$$Sn_{3D} = \frac{N!}{R!(N-R)!} = \frac{3!}{3!(3-3)!} = 1 \tag{12}$$

In Equation 12, N is the numbers of parameters in the problem, R is searching dimension and  $Sn_{3D}$  is the number of triple searching combinations.

At the end of this search, if it is revealed that there may be an  $O_{desired}$  value on the sub-space constituted by  $2^3 = 8$  adjacent points, all the rules whose output corresponds to any of the fuzzy sets in these eighth points are valid rules for the solution. Rule groups that define all 3-dimensional sub-solution spaces by scanning the entire global solution space are determined at this stage. In 3 dimensional output searching, 1 rule group consists of 8 valid rules.

### 3.1.1.5. R dimensional output searching

In an R dimensional output searching, similar operations those of 1D, 2D and 3D is performed. The only difference is the number of variable parameters (dimension) in the searching operations. The solution is searched among all different  $2^R$  adjacent points within the global solution space. In R dimensional output searching, 1 rule group consists of  $2^R$  valid rules.

The number of searching combinations with R parameters to be performed in the R-dimensional

and  $O_t$  with 1 membership value. In comparison, if the conditions defined by Eq-9, Eq-10 and Eq-11 is satisfied, the solution for  $O_{desired}$  is available in the space constituted by the points  $i, j, k, l, m, n, s$  and  $t$ .

output searching process can be determined by the Equation-13.

$$Sn_{3D} = \frac{N!}{R!(N-R)!} \tag{13}$$

In Equation 13, N is the numbers of parameters in the problem, R is searching dimension and  $Sn_{3D}$  is the number of searching combinations with R variable parameters. As understand from Eq-13 that R can not be greater than N. Thats mean, the biggest value of searching dimension (R) can equal to N.

### 3.2. Determination of fuzzy coordinates (valid fuzzy sets of input parameters) of sub solution spaces

At the end of sub-solution space searches in different dimensions, the points defining the region(s) where the  $O_{desired}$  value can exist are determined. In other words, after all valid rule groups are determined, fuzzy input sets constitutes each rule in each rule group are determined. The most important feature of a sub-solution space is that they are defined by a single fuzzy input set with a membership value of 1 for constant input parameters, and by two consecutive fuzzy input sets for each of the variable input parameters. Since the fuzzy sets of the constant parameters in the problem are known and the value of their membership is equal to 1, their net values are also known. Therefore, there is no need to do anything in FIL for constant parameters.

However, the consecutive fuzzy sets of variable parameter(s) and their memberships must be determined. Since the net value of a variable parameter is searched in 2 consecutive fuzzy sets in FIL calculations, these consecutive fuzzy sets are named as valid fuzzy sets of the parameter in this study. Valid fuzzy sets of input parameters are fuzzy sets of input parameters in the rules that give the points as output that constitute the sub-solution



3.2.3. Determination of valid fuzzy sets in 3 dimensional sub solution space)

After 3-dimensional output searching operations, if the existence of the solution is proved by satisfying the condition in Equation 9, Equation 10 and Equation 11, there are the two consecutive fuzzy sets such as  $A_{X1}$  and  $A_{X2}$  called valid fuzzy sets of variable A, there are two consecutive fuzzy sets such as  $B_{Y1}$  and  $B_{Y2}$  called valid fuzzy sets of variable B and also there are two consecutive fuzzy sets such as  $C_{Z1}$  and  $C_{Z2}$  called valid fuzzy sets of variable C (See Figure 11). If all parameters are variable in a 3 dimensional sub solution space then, two valid fuzzy sets belong to each of the A, B and C variables can be determined for an existing solution. If the number of parameters are bigger than solution dimension, which is 3, similar

expressions can be written here for other triple combinations of variables.

If it needs to be expressed here in general, when R parameters vary in N dimensional sub solution space, the consecutive two consecutive fuzzy sets of each R variable parameters can be valid fuzzy sets. If there exist  $(R < N)$ , other parameter(s) of the problem is/are constant in this N dimensional sub solution space. When the rules which constitute N dimensional sub solution space are examined for an existing solution, it is understand that the only two consecutive fuzzy sets belong to the each variable are different. Each fuzzy sets of other parameter(s) is/are constant and do not vary. Selection of the two consecutive fuzzy sets belong to the each variable are done in this step.

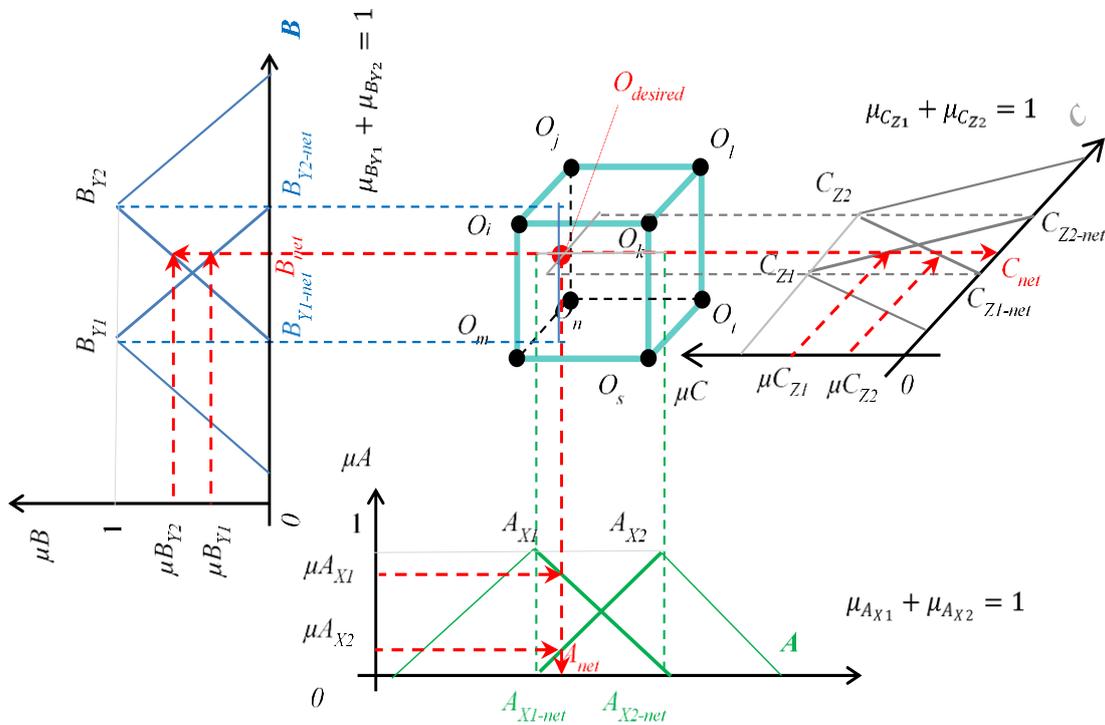
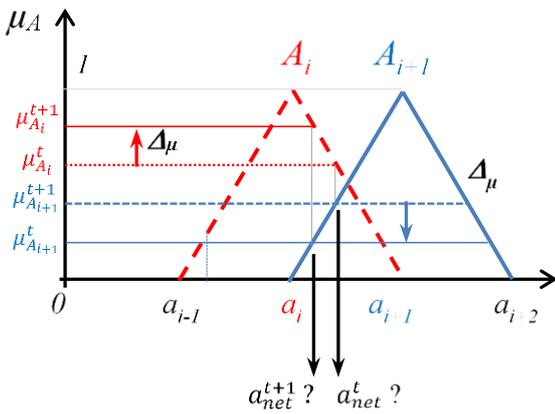


Figure 11. Valid Fuzzy input sets for variable A and B parameters, membersip values and  $A_{net}$  and  $B_{net}$  input values

3.3. Determination of memberships values of valid fuzzy sets

Memberships values of valid fuzzy sets of input parameters must be known to determine the net values of the input parameters that give the desired  $O_{desired}$  value. In 0 dimensional output searching, membership values of valid fuzzy sets for each input parameter are constant and value is 1. For this reason, there is no need to make an extra computation for determining the membership value.

In output searching operations with a dimension greater than 0, the memberships of consecutive fuzzy input sets belonging to the input parameter of a variable are tried to be determined by iteration operations performed with the desired sensitivity between 0 and 1. Relationships between membership functions of consecutive fuzzy input sets are used in iterative determination of membership value.



**Figure 12.** Membership iterations and membership sensitivity

As seen from Figure 12 that, the difference between membership values between two consecutive iterations such as t and t+1 is shown with  $\Delta\mu$ . The values of  $\Delta\mu$  is increased, the membership sensitivity decreases. In order to perform sensitive computations in FIL method, value of  $\Delta\mu$  may be chosen as small as possible. However, if the value of  $\mu$  is chosen too small, too much iteration may be required in FIL calculations. Therefore, one should be careful in the choice of  $\Delta\mu$ . In addition, there is no need to iterate separately for both  $A_i$  and  $A_{i+1}$  fuzzy sets given in Figure 12 to determine membership values. The memberships of both  $A_i$  and  $A_{i+1}$  fuzzy sets can be determined with a single iteration by using the relationships between the membership functions of adjacent fuzzy sets. Since membership functions belonging to  $A_i$  and  $A_{i+1}$  fuzzy sets in Figure 12 are triangular membership functions, the relationship between membership functions of these two fuzzy sets is as given in equation 14. In addition, the expressions given by Equation-15 and Equation-16 are other relations that can be used for  $A_i$  and  $A_{i+1}$  fuzzy sets. Similarly, Equation 16- Equation 17 and Equation 18 are used for parameter B and Equation 19, Equation 20 and Equation 21 are used for parameter C in membership iterations in this explanatory example.

$$\mu_{A_{X_i}} + \mu_{A_{X_{i+1}}} = 1 \tag{13}$$

$$0 \leq \mu_{A_{X_i}} \leq 1 \tag{14}$$

$$0 \leq \mu_{A_{X_{i+1}}} \leq 1 \tag{15}$$

$$\mu_{B_{Y_j}} + \mu_{B_{Y_{j+1}}} = 1 \tag{16}$$

$$0 \leq \mu_{B_{Y_j}} \leq 1 \tag{17}$$

$$0 \leq \mu_{B_{Y_{j+1}}} \leq 1 \tag{18}$$

$$\mu_{C_{Z_k}} + \mu_{C_{Z_{k+1}}} = 1 \tag{19}$$

$$0 \leq \mu_{C_{Z_k}} \leq 1 \tag{20}$$

$$0 \leq \mu_{C_{Z_{k+1}}} \leq 1 \tag{21}$$

In FIL computations, iterations are performed for each parameter depending on the solution dimension. For solutions, these iterations are carried out not separately but in combination with each other. Therefore, as the number of parameters increases, an increasing number of iterations may be required. So, the extremely sensitive membership iteration selection may cause unnecessary transaction volume. However, membership iteration, which is not sensitive enough, may cause the solution not to be obtained.

It is not necessary to use only ordinary iterative methods to determine the membership values of the problem's fuzzy input sets. These membership values can be determined using many different methods that are available and widely used in the literature. With any of these methods, FIL analysis can be achieved much more effectively and in a shorter time or with less iteration. Which method or methods will be more effective to use for FIL calculations may be the subject of another study(ies).

### 3.4. Determination of net values of input parameters

The computations performed at this stage in the FIL method are the same as most computations made in classical FL. However, there are some differences between of them. However, there are some differences in the application of the FIL method. Valid fuzzy sets and membership values of these sets are used to determine the net values of the input parameters in FIL method. These memberships are determined by iterations as described previously, depending on the specific membership sensitivity. By taking into considering the solution dimension of the problem, all combinations of membership values are processed in fuzzy rules that constitute the fuzzy model. Not all fuzzy rules are used for this. Valid rules constituted with valid fuzzy sets of problem variables are used. The number of these valid rules is  $2^N$  in a problem with N variables. The fuzzy outputs obtained from the valid rules are passed through the de-fuzzicator used in the fuzzy model on which the FIL method is applied, and the net

output value ( $O_{net}$ ) is obtained. If the calculated  $O_{net}$  value is equal to the  $O_{desired}$  value or is close to the  $O_{desired}$  value with an acceptable error ( $e_a$ ) the net values of the input parameters used in the calculation of the  $O_{net}$  value are the values searched in this problem. Since variable memberships in FIL computations are not computed based on the net value of variables, net values must be computed for membership values that provide equations (22) and (23). In FIL method, the computations required to determine the net input values of problem variables are the opposite of the fuzzification computations of net values performed at the beginning of the FL computations.

$$O_{net} = O_{desired} \quad (22)$$

$$O_{net} - O_{desired} \leq e_a \quad (23)$$

#### 4. Summary and evaluations

The application of the FIL method, described in detail in the previous sections, on a FL model is schematically summarized in Figure 13. As can be understood from this figure, while valid fuzzy set memberships in classical FL calculations are obtained depending on the net values of the input parameters, in the case of applying the FIL method, the memberships of the valid fuzzy sets corresponding to the determined sub-solution spaces are determined by iterations between 0 and 1 values. After determining the membership values, the calculations performed in FL and FIL are the same until the net output values are obtained. After this stage, it is checked whether the desired output is achieved with the memberships determined by iterations in the FIL. These computations continue until solution (s) is/are found and/or membership iterations are completed. Net input values are obtained by the reverse application of the fuzzyfication in FL with the membership values for which the desired output is computed. Determination of sub-solution spaces in FIL method, finding valid rule and valid fuzzy sets, using relations between membership functions of valid consecutive sets, application of fuzzyfication in FL in reverse are the most important features that reduce the amount of computation volume.

Although the solution can be found in a very short time with FL computations, the solution time in FIL computations can be longer depending on the size and number of dimensions of the sub-space searched and membership iterations and membership sensitivity. This is an indication that humanbeings have more difficulty and need more thinking while inferring backwards. However, this

time is quite small besides the time taken to obtain the solution (s) by scanning the entire solution space by applying methods such as iterative or trial and error directly on a FL model without finding sub-solution spaces. On the other hand, while only one solution can be obtained in FL computations, many solutions can be obtained depending on the membership sensitivity and the dimension(s) and number of the space (s) that are searched in FIL computations. This situation, on the one hand, compensates for the time spent in solution, on the other hand, reveals the importance of the FIL method for problems where alternative solutions are very valuable. These general and brief evaluations, which are made here, are given together with sample applications in the second part of this study in detail. Thus, the validity and usability of the method has also been proven.

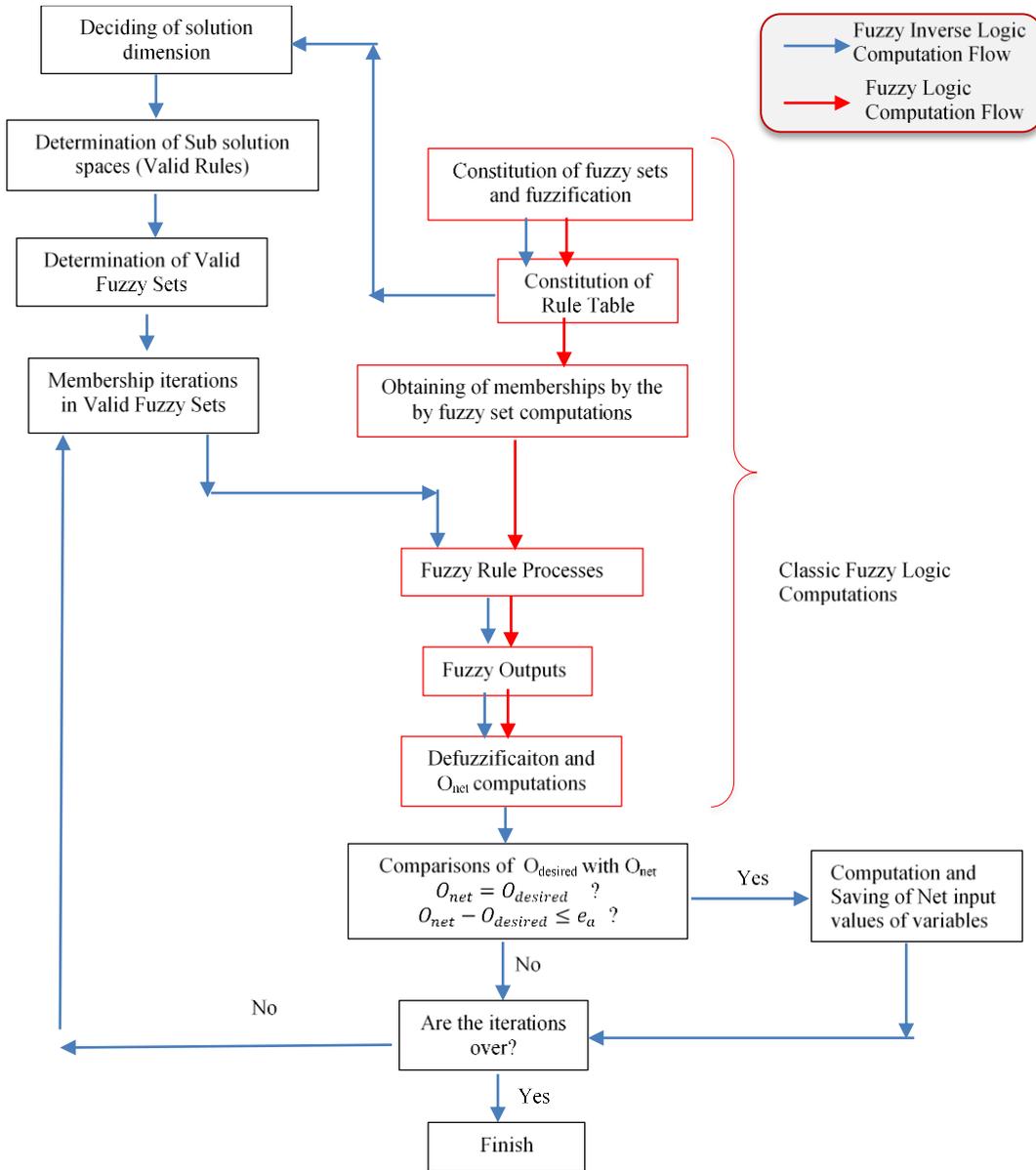
#### 5. Conclusions

In this study, contrary to the FL method, which is based on the forward inference of human beings (output estimation from input values), the FIL method, which converts human beings' ability to make backward inferences (estimation of inputs that can give an output) into an algorithm on the same basis, is tried to be given. Some conclusions that can be drawn from this first part of the study are given below.

- The basis of the FIL method is the same as that of FL.
- The data used in the FL method can be used in the FIL method without any changes.
- The FIL method can be applied on a previously developed FL model.
- Unlike most methods, it is possible to make reverse inference and prediction with the FIL method.
- A lot of alternative solutions can be produced with FIL analysis.
- FIL computations can take more time than FL.
- FIL and FL together fully represent human inference behaviors.
- While FL allows one-way inference, its usage with FIL is much more effective and allows two-way inferences.

➤ The sensitivity of the FIL method and its ability to produce accurate results depend on the sensitivity of iterative membership computation and the accuracy and sensitivity of the FL model on which it is applied.

It is thought that the FIL method alone and with FL can be widely used in many areas.



**Figure 13.** Schematic presentation of the application of FIL method on a fuzzy model

**References**

Altaş, İ. H. (1999a). Bulanık Mantık: Bulanıklılık Kavramı. Enerji, Elektrik, Elektromekanik-3e,62, 80-85.

Altaş, İ. H. (1999b). Bulanık mantık: Bulanık denetim. Enerji, Elektrik, Elektromekanik-3e, 64(1999), 76-81.

Altaş, I. H. (2017). Fuzzy Logic Control in Energy Systems with Design Applications in MATLAB®/Simulink® (91). IET.

Chopard, B. and Droz, M. (1998). Cellular automata (Vol. 1). Berlin, Germany: Springer.

Cvijović, D. and Klinowski, J. (1995). Taboo search: an approach to the multiple minima problem. Science, 267 (5198), 664-666. <https://doi.org/10.1126/science.267.5198.664>

Erdun, H. (2020). Fuzzy Logic Defuzzification (Bulanıklaştırma) Methods with Examples: Erişim adresi [https://www.researchgate.net/publication/344196954\\_Fuzzy\\_Logic\\_Defuzzification\\_Bulaniklastirma\\_Methods\\_with\\_Examples](https://www.researchgate.net/publication/344196954_Fuzzy_Logic_Defuzzification_Bulaniklastirma_Methods_with_Examples)

- Harris, J. (2005). Fuzzy logic applications in engineering science (Vol. 29). Springer Science & Business Media.
- Jain, A. K., Mao, J. and Mohiuddin, K. M. (1996). Artificial neural networks: A tutorial. *Computer*, 29(3), 31-44. <https://doi.org/10.1109/2.485891>
- Kennedy, J. and Eberhart, R. (1995). Particle swarm optimization. In *Proceedings of ICNN'95-international conference on neural networks*, 4, 1942-1948.
- Karaboga, D. and Akay, B. (2009). A comparative study of artificial bee colony algorithm. *Applied mathematics and computation*, 214(1), 108-132. <https://doi.org/10.1016/j.amc.2009.03.090>
- Mamdani, E. H. and Assilian, S. (1975). An experiment in linguistic synthesis with a fuzzy logic controller. *International Journal of Man-Machine Studies*, 7(1), 1-13. [https://doi.org/10.1016/S0020-7373\(75\)80002-2](https://doi.org/10.1016/S0020-7373(75)80002-2)
- Mamdani, E. H. (1976). Advances in the linguistic synthesis of fuzzy controllers. *International Journal of Man-Machine Studies*, 8(6), 669-678. [https://doi.org/10.1016/S0020-7373\(76\)80028-4](https://doi.org/10.1016/S0020-7373(76)80028-4)
- Moscato, P., Cotta, C. and Mendes, A. (2004). Memetic algorithms. In *new optimization techniques in engineering* (pp. 53-85). Springer, Berlin, Heidelberg.
- Öztekin, E. ve Filiz, K. (2015). Beton gerilme şekildeğiştirme eğrilerinin bulanık mantık yaklaşımıyla elde edilmesi. *Mühendislikte Yeni Teknolojiler Sempozyumu*, Bayburt.
- Parpinelli, R. S., Lopes, H. S. and Freitas, A. A. (2002). Data mining with an ant colony optimization algorithm. *IEEE Transactions On Evolutionary Computation*, 6(4), 321-332. <https://doi.org/10.1109/TEVC.2002.802452>
- Pörge, B. (2019). Investigation of reliabilities of the triaxial concrete compressive strength models by fuzzy logic approach, *Yüksek Lisans Tezi*, Bayburt Üniversitesi Fen Bilimleri Enstitüsü, Bayburt.
- Rajabioun, R. (2011). Cuckoo optimization algorithm. *Applied Soft Computing*, 11(8), 5508-5518.
- Ross, T. J. (2004). Fuzzy logic with engineering applications (Vol. 2). New York: Wiley.
- Terano, T., Asai, K. and Sugeno, M. (1992). Fuzzy systems theory and its applications. Academic Press Professional, Inc.
- Tanaka, K. (1997). An introduction to fuzzy logic for practical applications.
- Van Laarhoven, P. J. and Aarts, E. H. (1987). Simulated annealing. In *simulated annealing: Theory and applications* (pp. 7-15). Springer.
- Whitley, D. (1994). A genetic algorithm tutorial. *Statistics and Computing*, 4(2), 65-85. <https://doi.org/10.1007/BF00175354>
- Yager, R. R. and Zadeh, L. A. (Eds.). (2012). An introduction to fuzzy logic applications in intelligent systems (Vol. 165). Springer Science & Business Media.
- Yang, X. S. and Gandomi, A. H. (2012). Bat algorithm: a novel approach for global engineering optimization. *Engineering Computations*, 29(5), 464-483. <https://doi.org/10.1108/02644401211235834>
- Zadeh, L. A. (1965). Information and control. *Fuzzy Sets*, 8(3), 338-353. [https://doi.org/10.1016/S0019-9958\(65\)90241-X](https://doi.org/10.1016/S0019-9958(65)90241-X)
- Zadeh, L. A. (1973). Outline of a new approach to the analysis of complex systems and decision processes. *IEEE Transactions on systems, Man and Cybernetics*, (1), 28-44. <https://doi.org/10.1109/TSMC.1973.5408575>
- Zadeh, L. A. (1975). The concept of a linguistic variable and its application to approximate reasoning-III. *Information Sciences*, 9(1), 43-80. [https://doi.org/10.1016/0020-0255\(75\)90036-5](https://doi.org/10.1016/0020-0255(75)90036-5)