<u>*Araştırma Makalesi*</u>

<u>*Research Article*</u>

Feasible Waveguide Method for Obtaining Electromagnetic Properties of Biaxial Bianisotropic Materials with Strong Magneto-Electric Coupling

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Abstract

In this study, an easy-to-apply waveguide method is proposed for material characterization of biaxial bianisotropic samples with strong magneto-electric coupling. Its underlying expressions are derived considering scattering (S-) parameters of the sample at one fixed orientation in two waveguides at their dominant modes with different cross sections. The method is validated by synthesized and retrieved electromagnetic properties of a sample when there is no/some noise.

Keywords: Biaxial Biansiotropic Materials, Strong Magneto-Electric Coupling, Parameter Extraction.

Güçlü Manyeto-Elektrik Kuplajlı Biaksiyal Bianizotropik Malzemelerin Elektromanyetik Özelliklerini Elde Etmek İçin Uygulanabilir Dalga Kılavuzu Yöntemi

Öz

Bu çalışmada, güçlü manyeto-elektrik kuplajlı biaksiyal bianizotropik numunelerin malzeme karakterizasyonu için uygulanması kolay bir dalga kılavuzu yöntemi önerilmiştir. Temel ifadeleri, farklı kesitlere sahip baskın modlarında iki dalga kılavuzunda sabit bir yönelimde numunenin saçılma (S-) parametreleri dikkate alınarak türetilmiştir. Yöntem, gürültü olmadığında/bir miktar gürültü olduğunda bir numunenin sentezlenen ve yeniden elde edilen elektromanyetik özellikleriyle doğrulanmıştır.

Anahtar Kelimeler: Biaksiyal Bianizotropik Malzemeler, Güçlü Manyeto-Elektrik Kuplaj, Parametre Çıkarma.

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1. Introduction

Electromagnetic characterization of materials could be used in many applications areas such as medicine, food industry, and civil engineering since physical, chemical, and mechanical properties of these materials could be correlated to their electromagnetic parameters (Chen et al., 2004). There are many microwave methods available in the literature for electromagnetic characterization such as resonant-method (Jha & Akhtar, 2014), free-space method (Akhter & Akhtar, 2016), coaxial line method (Jablonskas et al., 2017), andwaveguide method (Damaskos et al., 1984; Baker-Jarvis et al., 1990; Akhtar et al., 2006; Chen et al., 2006; Allen et al., 2016; Hasar et al., 2017; Hasar & Ozturk, 2018; Xu, 2018). Considering the criteria of broadband measurements, repeatability in measurements, simplicity in measurements and easiness of sample preparation/machining, classical waveguide methods are extensively preferred measurement environment for electromagnetic parameter characterization of materials (Baker-Jarvis et al., 1990). Most of the waveguide methods proposed in the literature assume that the material under analysis is isotropic (Baker-Jarvis et al., 1990; Hasar et al., 2017; Hasar & Ozturk, 2018). For electromagnetic characterization of biaxial materials, the method proposed used in Damaskos et al. (1984) could be applied. However, this method requires the measurement of two-different modes (TE$_{10}$ and TE$_{20}$) and thus mode conversion step so as to extract orthogonal components of the complex permittivity tensor. In order to eliminate this need, the methods in Akhtar et al. (2006), Chen et al. (2006), and Hasar et al. (2017), which use measurements of two (orthogonal) different configurations of the sample inside one guide operating at one dominant mode, can be applied for uniaxial/biaxial material characterization. However, rotating the sample and locating it into the waveguide in some instances are not a convenient approach for characterization of these materials (Hasar et al., 2017). Recently, a waveguide method (Xu, 2018) based upon one biaxial sample configuration inside two different waveguides operating at their dominant modes (TE$_{10}$) is proposed as an alternative approach for alleviating the requirement of orthogonal arrangement of uniaxial/biaxial samples as well as eliminating the necessity of using mode conversion. Nonetheless, this method in this present form is not applicable for biaxial bianisotropic samples having coupling between electric and magnetic fields. In this study, we extend the method in Xu (2018) and apply it for electromagnetic characterization of biaxial bianisotropic samples with strong magneto-electric coupling.
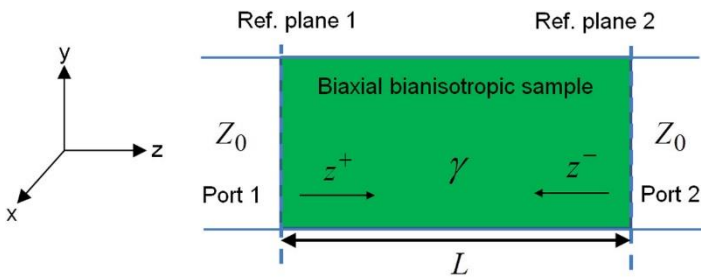

Figure 1. Biaxial bianisotropic sample with length L and magneto-electric coupling located inside a waveguide.

2. Method

Fig. 1 illustrates the schematic view of a biaxial bianisotropic sample with length L positioned into a waveguide with cross section $a \times b (a > b)$. Assuming that longitudinal components (z) of electric and magnetic fields do not change with x and y (over the cross section), then the coupling between these fields for biaxial samples cancel for the wave propagation in z direction with dominant mode (TE$_{10}$) (Damaskos, et al., 1984; Hasar et al., 2017). However, a biaxial bianisotropic sample has a magneto-electric coupling due to their internal property (Hasar et al., 2017) and the relations between fields can be written for such as coupling in x and y directions for $\exp(-i\omega t)$ time reference as

$$\overline{D} = \overline{\overline{\varepsilon}} \cdot \overline{E} + \overline{\overline{\varsigma}} \cdot \overline{H}, \quad \overline{B} = \overline{\overline{\mu}} \cdot \overline{H} + \overline{\overline{\zeta}} \cdot \overline{E}, \quad (1)$$

$$\overline{\overline{\varepsilon}} = \varepsilon_0 \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}, \quad \overline{\overline{\mu}} = \mu_0 \begin{bmatrix} \mu_x & 0 & 0 \\ 0 & \mu_y & 0 \\ 0 & 0 & \mu_z \end{bmatrix}, \quad (2)$$

$$\overline{\overline{\varsigma}} = \frac{1}{c} \begin{bmatrix} 0 & 0 & 0 \\ -i\xi_0 & 0 & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}, \quad \overline{\overline{\zeta}} = \frac{1}{c} \begin{bmatrix} 0 & +i\xi_0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \mu_z \end{bmatrix}. \quad (3)$$

Here, $\varepsilon_x (\mu_x)$, $\varepsilon_y (\mu_y)$, and $\varepsilon_z (\mu_z)$ are the relative complex permittivities (complex permeabilities) along x, y, and z directions; ε_0 and μ_0 are the permittivity and permeability of vacuum; c is the velocity of light in vacuum; $\overline{\overline{\varsigma}}$ and $\overline{\overline{\zeta}}$ are the two tensors describing the coupling; and ξ_0 is the coefficient of magneto-electric coupling. It is seen from Eq. (3) that x and y components of fields are coupled. Thus, of the constitutive parameters, μ_x, μ_y, and μ_z are unknown parameters for the biaxial bianisotropic sample under analysis with magneto-electric coupling. It was demonstrated that for such a sample, the forward and backward reflection and transmission S-parameters can be derived as (Hasar et al., 2017)

$$S_{11} = \frac{\Gamma_1 (1 - T^2)}{1 - \Gamma_1 \Gamma_2 T^2}, \quad S_{22} = \frac{\Gamma_2 (1 - T^2)}{1 - \Gamma_1 \Gamma_2 T^2}, \quad (4)$$

$$S_{21} = S_{12} = \frac{T (1 - \Gamma_1 \Gamma_2)}{1 - \Gamma_1 \Gamma_2 T^2}, \quad (5)$$

$$\Gamma_1 = \frac{z^+ - 1}{z^+ + 1}, \quad \Gamma_2 = \frac{z^- - 1}{z^- + 1}, \quad z^+ = \frac{\mu_x \beta_{0z}}{\beta_{sz} + ik_0 \xi_0}, \quad (6)$$

$$z^- = \frac{\mu_x \beta_{0z}}{\beta_{sz} - ik_0 \xi_0}, \quad T = e^{+i\beta_{sz} L}, \quad B_{0x} = \frac{\pi}{a}, \quad (7)$$

$$\beta_{sz} = \sqrt{k_0^2 (\varepsilon_y \mu_x - \xi_0^2) - \frac{\mu_x}{\mu_z} \beta_{0x}^2}, \quad \beta_{0z} = \sqrt{k_0^2 - \beta_{0x}^2}. \quad (8)$$

Here, Γ_1, Γ_2, and T are the semi-infinite reflection coefficients and the propagation factor, and z^+ and z^- are the forward and backward normalized wave impedances; β_{sz} and

β_{0z} are the phase constants in the z direction of the sample-loaded and air-loaded waveguide sections; β_{0x} is the phase constant of the air-loaded waveguide section in the x direction; and k_0 is the wavenumber of air.

For known S-parameters in Eqs. (4) and (5), z^+ and z^- can be first extracted from (Hasar et al., 2018)

$$z^+ = \frac{-\Lambda_2 \pm \sqrt{\Lambda_2^2 - 4\Lambda_1\Lambda_3}}{2\Lambda_1}, \quad z^- = \frac{z^+ + \Lambda_4}{1 + z^+\Lambda_4}, \qquad (9)$$

$$\Lambda_1 = S_{21}^2 - (1 - S_{11})(1 - S_{22}), \quad \Lambda_2 = 2(S_{11} - S_{22}), \qquad (10)$$

$$\Lambda_3 = (1 + S_{11})(1 + S_{22}) - S_{21}^2, \quad \Lambda_4 = \frac{S_{11} - S_{22}}{S_{11}S_{22}}. \qquad (11)$$

The correct sign for z^+ in (9) can be ascertained so that $\Re\{z^+\} \geq 0$. Next, β_{sz} can be retrieved from

$$\beta_{sz} = \frac{\ln\left(\frac{(z^- + 1)S_{21}}{(z^- + 1) - S_{11}(z^- - 1)}\right) \mp i2\pi m}{iL}, \qquad (12)$$

where m designates a branch index $(m = 0, 1, 2, \cdots)$ whose correct value can be ascertained by various methods (Siddiqui et al., 2003; Barroso & Hasar, 2012; Hasar et al., 2013, 2015). Then, ξ_0 and μ_x can be obtained from

$$\xi_0 = \frac{i\beta_{sz}}{k_0}\left(\frac{z^+ - z^-}{z^+ + z^-}\right), \quad \mu_x = z^+\left(\frac{\beta_{sz} + ik_0\xi_0}{\beta_{0z}}\right). \qquad (13)$$

The parameters μ_x, ξ_0, and β_{sz} are determined using expressions of one guide with its dominant mode. However, knowing them is not sufficient to determine the remaining parameters ε_y and μ_z from Eq. (8). Therefore, we need an additional equation to calculate their values. Following the study Xu (2018) and assuming that S-parameters of another waveguide operating at its dominant mode with different cross section are present, ε_y and μ_z can be derived from Eq. (8) as

$$\mu_z = \mu_x \frac{\beta_{0x}^{(2)^2} - \beta_{0x}^{(1)^2}}{\beta_{sz}^{(1)^2} - \beta_{sz}^{(2)^2}}, \qquad (14)$$

$$\varepsilon_y = \frac{1}{\mu_x}\left[\frac{1}{k_0^2}\frac{\mu_x}{\mu_z}\left(\beta_{0x}^{(1)^2} + \beta_{sz}^{(1)^2}\right) + \xi_0^2\right], \qquad (15)$$

where $\beta_{0x}^{(1)}$, $\beta_{0x}^{(2)}$, $\beta_{sz}^{(1)}$, and $\beta_{sz}^{(2)}$ are the phase constants in the x and z directions of these two waveguides with different cross sections ($a_1 \times b_1$ and $a_2 \times b_2$). It is assumed that $\beta_{sz}^{(1)}$ and $\beta_{sz}^{(2)}$ are calculated from Eq. (12) and $\beta_{0x}^{(1)}$ and $\beta_{0x}^{(2)}$ are computed from Eq. (7). We also note that z^+, z^-, ξ_0, and μ_x can be evaluated from Eqs. (9)-(11) and (13) for either waveguide at its dominant mode.

For validation of our proposed algorithm, a biaxial bianisotropic sample with corresponding constitutive parameters was synthesized and then its S-parameters were computed for two different guide cross sections. Finally, its parameters were

retrieved using computed S-parameters. In our analysis, a biaxial bianisotropic sample with the following synthesized constitutive parameters changing with frequency was considered as

$$\varepsilon_y = 1 - \frac{2}{f_G^2 + i0.2f_G - 4}, \qquad (16)$$

$$\mu_x = 1 - \frac{10}{f_G^2 + if_G - 5}, \quad \mu_z = 2 + i0.5, \qquad (17)$$

$$\xi_0 = 1 - \frac{1}{f_G^2 + i0.5f_G - 6}, \qquad (18)$$

where f_G is the frequency value at GHz. It is noted from (16)-(18) that the sample has both dispersion (frequency-dependent) and loss behaviors. Next, S-parameters of this sample with length $L = 10$ mm corresponding to two different cross sections $a_1 = 150$ mm, $b_1 = 75$ mm, $a_2 = 120$ mm, and $b_2 = 60$ mm were computed from (4)-(8) for a frequency range 1.0-2.8 GHz. Then, ξ_0 and μ_x were retrieved from (9)-(13) for the first (or second) waveguide cross section. After, ε_x and μ_z were computed from (14) and (15).

3. Results

Figs. 2-5 illustrate the synthesized and retrieved real and imaginary parts of ξ_0, μ_x, ε_y, and μ_z. It is seen from Figs. 2-5 that synthesized and retrieved parameters are in good agreement with each other over the whole frequency band.
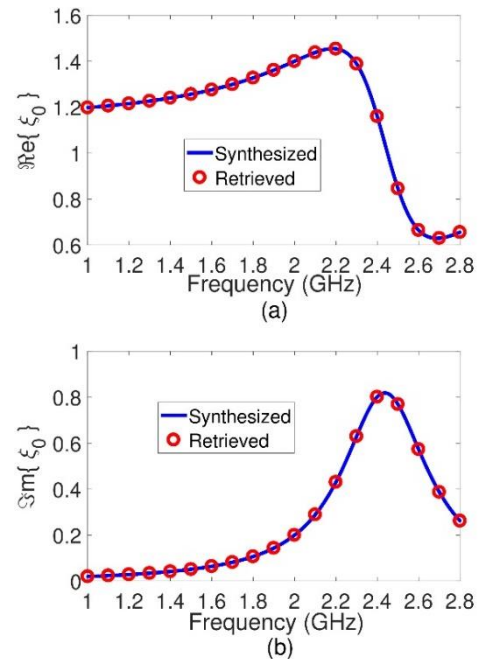

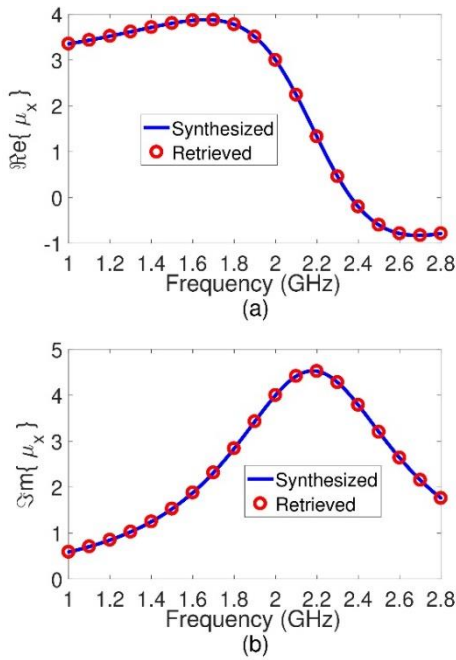*Figure 2. (a) Real and (b) imaginary parts of the synthesized and retrieved ξ_0 versus frequency.*

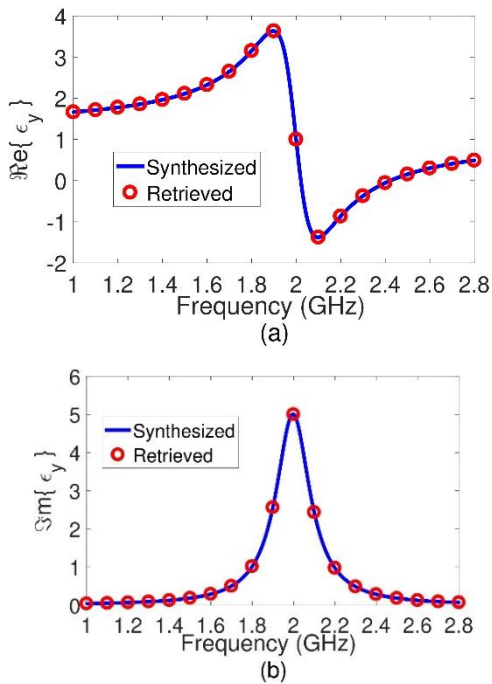*Figure 3. (a) Real and (b) imaginary parts of the synthesized and retrieved μ_x versus frequency.*

Figure 4. (a) Real and (b) imaginary parts of the synthesized and retrieved ε_y versus frequency.

After validation of our proposed method, we also examined its accuracy when there is some noise present in S-parameters. To reflect the effect of noise, a random uncorrelated noise was added to the real and imaginary parts of S-parameters (Hasar, 2018). Randomness is assumed to be normally distributed with the different mean values ρ and standard deviations σ. Fig. 6 shows the extracted ε_y (the last parameter in the extraction process) of the analyzed sample when noise with $\rho = 0$ and $\sigma = 0.015$ (a large value in many S-parameter measurements (Hasar, 2018)) is added all S-parameters of the first guide. It is seen from Fig. 6 that the retrieved ε_y of the sample follows its

synthesized ε_y for even high amount of noise is present, demonstrating the resistance of the proposed method for a relatively high noise in S-parameters.
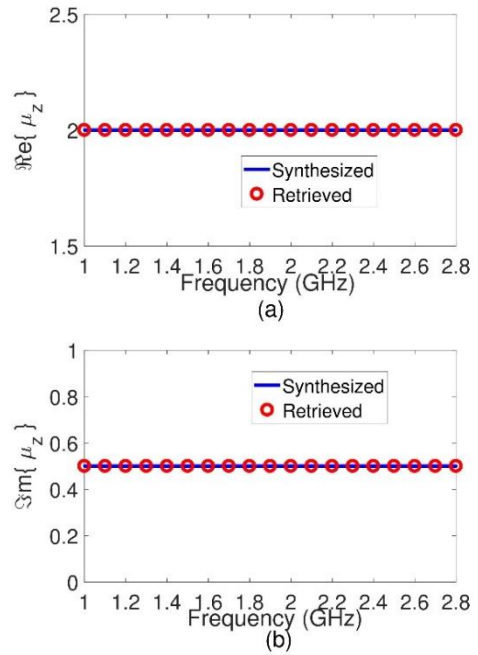

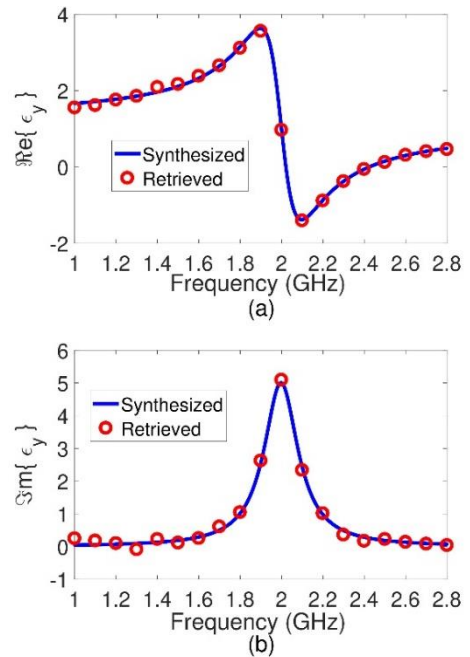*Figure 5. (a) Real and (b) imaginary parts of the synthesized and retrieved μ_z versus frequency.*

Figure 6. (a) Real and (b) imaginary parts of the synthesized and retrieved ε_y versus frequency when noise with $\rho = 0$ and $\sigma = 0.015$ is added to all S-parameters of the first guide.

For the quantitative effect of noise on the extracted parameters, we can perform some numerical analysis using the mean percentage error (MPE) (Hawro et al., 2019).

$$MPE = \frac{1}{n} \sum_{j=1}^{n} \left| \frac{S_j - R_j}{S_j} \right| \cdot 100\% \qquad (19)$$

Here, n is the number of frequency points, j is the frequency point, and S_j and R_j are the values synthesized and retrieved at the frequency point j, respectively. Accordingly, Table 1 shows the MPE values among the synthesized and retrieved ε_y values of a sample when there is no noise (Fig. 4) and there is some noise (Fig. 6).

Table 1. MPE values among the synthesized and retrieved ε_y values of a sample when there is no noise (Fig. 4) and there is some noise (Fig. 6).

Figure	MPE (%)	
	Real part of ε_y	Imaginary part of ε_y
Fig. 4 (no noise)	1.202×10^{-15}	4.229×10^{-15}
Fig. 6 (some noise)	0.222	0.404

4. Conclusions

A double waveguide method is proposed for extraction of electromagnetic properties of biaxial bianisotropic samples with strong coupling between electric and magnetic fields. The method relies on application of S-parameters of two waveguides with different cross sections operating at their dominant modes. The effect of noise in the extraction process of the proposed method is also considered to examine its accuracy against noise.

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