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On $|\overline{N}, p_n|_k$ Summability Factors Of Infinite Series

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SUMMARY

In this paper a theorem on $|\bar{N}, p_n|_k$ summability factors which generalizes the theorems in [1], [3], and [4], has been proved.

1. Let Σa_n be a given infinite series with partial sums s_n and (p_n) be a sequence of positive real constants such that

$P_n = po + p_1 + p_2 + \dots + p_n \rightarrow \infty$, as $n \rightarrow \infty$, $(P_{-1} = p_{-1} = o)$.
A series $\Sigma\, a_n$ is said to be summable $|\bar{N}, p_n|_k$, $k \geqslant 1$, if

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n}\right)^{k-1} |t_n - t_{n-1}|^k < \infty$$

where $t_n = \frac{1}{P_n} \sum_{v=o}^{n} p_v s_v$ [1].

It should be noted that $|\bar{N}, p_n|_k$ summability is identical with $|C, 1|_k$ summability [2] for $p_n = 1$.

2. The purpose of this paper is to prove the following theorem, which generalizes the result in [1], [3], and [4].

Theorem. Let (λ_n) be a sequence such that

(i) $\sum_{2}^{m} \frac{p_n}{P_n} |\lambda_n| = 0\,(1)$, (ii) $\frac{P_n}{p_n} \triangle\lambda_n = 0\,(|\lambda_n|)$.

If

$$\sum_{v=1}^{n} p_v \, |s_v|^k = 0\,(P_n \, \gamma_n),\ n \to \infty,$$

where (γ_n) is a positive non-decreasing sequence such that

$$\frac{P_{n+1}}{p_{n+1}} \gamma_n \ \Delta \left(\frac{1}{\gamma_n} \right) = 0\,(1),\ n \to \infty,$$

then $\Sigma \dfrac{a_n \lambda_n}{\gamma_n}$ is summable $|\bar{N}, p_n|_k$, $(k \geqslant 1)$.

3. Proof of the theorem Let. T_n denote the $(\bar{N}, p_n)$ mean of the series $\Sigma \dfrac{a_n \lambda_n}{\gamma_n}$.

Then

$$T_n = \frac{1}{P_n} \sum_{v=o}^{n} p_v \sum_{r=o}^{v} \frac{a_r \lambda_r}{\gamma_r} = \frac{1}{P_n} \sum_{v=o}^{n} (P_n - P_{v-1}) \frac{a_v \lambda_v}{\gamma_v}.$$

$$T_n - T_{n-1} = \frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n} P_{v-1} \frac{a_v \lambda_v}{\gamma_v} .$$

Using Abel's transformation, we get

$$T_n - T_{n-1} = - \frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n-1} \frac{p_v s_v \lambda_v}{\gamma_v} + \frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n-1} \frac{s_v}{\gamma_v} P_v \, \Delta\lambda_v$$

$$+ \frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n-1} s_v P_v \lambda_{v+1} \, \Delta \left(\frac{1}{\gamma_v} \right) + \frac{s_n p_n \lambda_n}{P_n \gamma_n}$$

$$= T_{n,1} + T_{n,2} + T_{n,3} + T_{n,4} \text{ say.}$$

To prove the theorem, by Minkowski's inequality, it is sufficient to show that

$$\sum_{n=1}^{\infty} \left(\frac{P_n}{p_n} \right)^{k-1} |T_{n,r}|^k < \infty, \text{ for } r = 1,2,3,4.$$

Now, applying Hölder's inequality, we have

$$\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} |T_{n,1}|^k \leq \sum_{n=2}^{m+1} \frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n-1} \frac{p_v |s_v|^k |\lambda_v|}{\gamma_v} \times \left\{ \frac{1}{P_{n-1}} \sum_{v=1}^{n-1} p_v \frac{|\lambda_v|}{\gamma_v} \right\}^{k-1}$$

$$= 0(1) \sum_{n=2}^{m+1} \frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n-1} \frac{p_v |s_v|^k |\lambda_v|}{\gamma_v} = 0(1) \sum_{v=1}^{m} \frac{p_v |s_v|^k |\lambda_v}{P_v \gamma_v}$$

$$= 0(1) \sum_{v=1}^{m-1} \triangle \left(\frac{|\lambda_v|}{P_v \gamma_v}\right) P_v \gamma_v + 0(1) |\lambda_m| = 0(1) \sum_{v=1}^{m-1} |\triangle \lambda_v|$$

$$+ 0(1) \sum_{v=1}^{m-1} |\lambda_{v+1}| \gamma_v \triangle \left(\frac{1}{\gamma_v}\right) + 0(1) \sum_{v=1}^{m-1} \frac{|\lambda_{v+1}| p_{v+1} \gamma_v P_v}{P_{v+1} P_v \gamma_{v+1}} + 0(1)$$

$$= 0(1) \sum_{v=1}^{m-1} \frac{p_v}{P_v} |\lambda_v| + 0(1) \sum_{v=1}^{m-1} \frac{p_{v+1}}{P_{v+1}} |\lambda_{v+1}| + 0(1) \sum_{v=1}^{m+1} \frac{p_{v+1}}{P_{v+1}} |\lambda_{v+1}| + 0(1)$$

$$= 0(1), \text{ as } m \rightarrow \infty,$$

by the hypothesis of the theorem.

Also we obtain

$$\sum_{n=2}^{m+1} \left(\frac{P_n}{p_n}\right)^{k-1} |T_{n,2}|^k \leq \sum_{n=2}^{m+1} \frac{p_n}{P_n P^k_{n-1}} \left\{ \sum_{v=1}^{n-1} \frac{|s_v|}{\gamma_v} P_v |\triangle \lambda_v| \right\}^k$$

$$\leq K \sum_{n=2}^{m+1} \frac{p_n}{P_n P_n{}^k{}_{-1}} \left\{ \sum_{v=1}^{n-1} \frac{|s_v|}{\gamma_v} p_v |\lambda_v| \right\}^k$$

$$= 0(1) \sum_{n=2}^{m+1} \frac{p_n}{P_n P_{n-1}} \sum_{v=1}^{n-1} \frac{p_v |s_v|^k |\lambda_v|}{\gamma_v}$$

$$= 0\,(1) \sum_{v=1}^{m} \frac{p_v |s_v|^k |\lambda_v|}{P_v \gamma_v} = 0\,(1),$$

as $m \to \infty$ where K is a positive constant

Again,

$$\sum_{n=2}^{m+1} \left(\frac{P_n}{P_n}\right)^{k-1} |T_{n,3}|^k = \sum_{n=2}^{m+1} \frac{p_n}{P_n P^k_{n-1}} \Big| \sum_{v=1}^{n-1} s_v p_v \lambda_{v+1} \triangle \left(\frac{1}{\gamma_v}\right) \Big|^k$$

$$= 0\,(1) \sum_{n=2}^{m+1} \frac{p_n}{P_n P^k_{n-1}} \Big| \sum_{v=1}^{n-1} \frac{p_v s_v \lambda_v}{\gamma_v} \Big|^k$$

$$0\,(1) \sum_{v=1}^{m} \frac{p_v |s_v|^k |\lambda_v|}{P_v \gamma_v} = 0\,(1),$$

as $m \to \infty$.

Finally, we have

$$\sum_{n=1}^{m} \left(\frac{P_n}{p_n}\right)^{k-1} |T_{n,4}|^k = \sum_{n=1}^{m} \frac{p_n |s_n|^k |\lambda_n|^k}{P_n (\gamma_n)^k}$$

$$= \sum_{n=1}^{m} \frac{p_n |s_n|^k |\lambda_n|}{P_n \gamma_n} \left(\frac{|\lambda_n|}{\gamma_n}\right)^{k-1}$$

$$= 0\,(1) \sum_{n=1}^{m} \frac{p_n |s_n|^k |\lambda_n}{P_n \gamma_n} = 0\,(1), \text{ as } m \to \infty.$$

Therefore, we get

$$\sum_{n=1}^{\infty} \left(\frac{p_n}{P_n}\right)^{k-1} |T_{n,r}|^k < \quad , \text{ for } r = 1, 2, 3, 4.$$

This completes the proof of the theorem.

Remark. If we take $\gamma_n = 1$, $k = 1$, and $p_n = 1$ in the theorem, then we get the results in [1], [3], and [4]. respectively.

ÖZET

Bu çalışmada [1], [3] ve [4]'ın teoremlerini genelleştiren $|\bar{N}, p_n|_k$ toplanabilme çarpanlarıyla ilgili bir teorem ispat edilmiştir.

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