COMMUNICATIONS

DE LA FACULTÉ DES SCIENCES
DE L'UNIVERSITÉ D'ANKARA

Série A_1 : Mathématiques

TOME : 33 ANNÉE : 1984

A Note On Matrix Transformations Of Some Generalized Sequence Spaces Into Semiperiodic Sequence Space

by

S. M. SIRAJUDEEN and D. SOMASUNDARAM

Faculté des Sciences de l'Université d'Ankara
Ankara, Turquie

Communications de la Faculté des Sciences de l'Université d'Ankara

---

La Revue "Communications de la Faculté des Sciences de l'Université d'Ankara" est un organe de publication englobant toutes les diciplines scientifique représentées à la Faculté des Sciences de l'Université d'Ankara.

La Revue, jusqu'à 1975 à l'exception des tomes I, II, III etait composé de trois séries

Série A: Mathématiques, Physique et Astronomie,
Série B: Chimie,
Série C: Sciences Naturelles.

A partir de 1975 la Revue comprend sept séries:

Série A_1: Mathématiques,
Série A_2: Physique,
Série A_3: Astronomie,
Série B: Chimie,
Série C_1: Géologie,
Série C_2: Botanique,
Série C_3: Zoologie.

A partir de 1983 les séries de C_2 Botanique et C_3 Zoologie ont été réunies sous la seule série Biologie C et les numéros de Tome commencerons par le numéro 1.

En principe, la Revue est réservée aux mémoires originaux des membres de la Faculté des Sciences de l'Université d'Ankara. Elle accepte cependant, dans la mesure de la place disponible les communications des auteurs étrangers. Les langues Allemande, Anglaise et Française seront acceptées indifféremment. Tout article doit être accompagnés d'un resume.

Les articles soumis pour publications doivent être remis en trois exemplaires dactylographiés et ne pas dépasser 25 pages des Communications, les dessins et figures portes sur les feuilles séparées devant pouvoir être reproduits sans modifications.

Les auteurs reçoivent 25 extraits sans couverture.

l'Adresse : Dergi Yayın Sekreteri,
Ankara Üniversitesi,
Fen Fakültesi,
Beşevler-Ankara
TURQUIE

A Note On Matrix Transformations Of Some Generalized Sequence Spacee Into Semiperiodie Sequence Space

S. M. SIRAJUDEEN and D. SOMASUNDARAM

(Received November 24, 1983, accepted January 30, 1984)

ABSTRACT

By a counter example a minor errer in the principle of the preoofs of the inclusion theorems on matix transfermations of some generalized suquence spaces into semiperiodic sequence space in [12] is pointed and corrected with suitable changes in the statements. Our conclysion in one of the tehorems is strengthened by anexanple due to Prof. B. Kuttner.

INTRODUCTION

The generalized sequence spaces l (p), c_0(p) and l_∞ (p) introduced by I.J. Maddox [5] and Q, the space of semiperiodic sequences are defined in § 2. Let $x = (x_k)$ and $\Sigma\, a_k$ denote an infinite sequence $(x_1, x_2, \ldots, x_k, \ldots)$ and $\sum_{k=1}^{\infty} a_k$ respectively. If X and Y are any two sequence spaces, let (X,Y) represent the class of all matrices $A = (a_{nk})$ n, k = 1,2,... that transform a sequence $x = (x_k) \in X$ into a sequence $Ax = y = (y_n) \varepsilon Y$ defined by

$$y_n = \Sigma\, a_{nk}\, x_k \; ; \qquad n = 1,2,\ldots..$$

In [12], the principle involved in proving the theorems on the classes of matrices (l (p),), Q (c_0(p), Q) and (l_∞ (p), Q) is that "if $\sup_n \Sigma \mid a_{nk} \mid < \infty$, then given $\varepsilon > 0$, there exists $P = P(\varepsilon)$ such that $\sum_{k=P+1}^{\infty} |a_{nk}| < \varepsilon$ for every n."

But this is not true in general. For example take (a_{nk}) to be the upper triangular matrix

$$a_{nk} = 2^{n-k} \ (k \geq n); \ a_{nk} = 0 \ (k < n),$$

so that $\Sigma \mid a_{nk} \mid = 1 + 2^{-1} + 2^{-2} + \ldots\ldots = 2$ for every n.

Then whatever P be chosen, we have

$$\sum_{k=P+1}^{\infty} \mid a_{nk} \mid = \sum_{k=n}^{\infty} \mid a_{nk} \mid = 2 \text{ whenever } n > P.$$

Hence in this note, we have established the theorems in [12], using the principle which will hold always, with suitable changes in the statements.

2. If $p = (p_k)$ is a sequence of strictly positive real numbers (not necessarily bounded in general), let us define the required sequence spaces as follows (see [1], [2], [3], [4], [5], [6], [7], [8], [9], [11] and [12]).

$l\,(p) = \{x = (x_k): \Sigma \mid x_k \mid^{p_k} < \infty \}$,

$l_\infty(p) = \{ x = (x_k): \sup_k \mid x_k \mid^{p_k} < \infty\}$,

$c_0(p) = \{ x = (x_k): \mid x_k \mid^{p_k} \to 0$ as $k \to \infty\}$. $c_0\,(p)$ is a topological linear space with paranorm $g\,(x) = \sup_k \mid x_k \mid^{p_k/H}$ where $H = \max(1, \sup p_k)$ (see [7]) and $c_0\,(p)$ (is a proper subset of m, the space of bounded sequences.

Q $= \{ x = (x_k)$: (x_k) is semiperiodic$\}$. A sequence $x = (x_k)$ is said to be semiperiodic, if to each $\varepsilon > 0$, there exists a a positive integer i such that $\mid x_k - x_{k+ri} \mid < \varepsilon$ for all r and k. The sequence Q is a separable subpace of m, the space of bounded sequences.

When $p_k = 1$ for all k, we write $l\,(p)$, $l_\infty\,(p)$, $c_0\,(p)$ as l, m and c_0 respectively. When $p_k = 1/k$, $c_0\,(p)$ and $l_\infty\,(p)$ become respectively Γ and Γ^*, the spaces introduced by V. Ganapathy Iyer [2]. When $p_k = p > 1$ for all k, we write $l\,(p)$ as $l\,p$.

Now let us quote some required known results as follows.

LEMMA A (Theorem 1 [4]), (i) Let $0 < p_k \leq 1$ for every k. Then $A \in (l\,(p), m)$ if and only if

$$\sup_{n,k} \mid a_{nk} \mid^{p_k} < \infty$$

and

(ii) Let $1 < p_k \leq \sup_k p_k < \infty$ and $p_k^{-1} + q_k^{-1} = 1$ for every k. Then $A \in (l(p), m)$ if and only if there exists an integer $M > 1$ such that

$$\sup_n \Sigma \mid a_{nk} \mid^{q_k} M^{-q_k} < \infty$$

LEMMA B (Theorem 10 and 11 [3]). Let $p = (p_k)$, $q = (q_n) \in m$. Then $A \in (c_0(p), l_\infty(p))$ if and only if there exists an absolute constant $M > 1$ such that

$$\sup_n (\Sigma \mid a_{nk} \mid M^{-1/p_k})^{q_n} < \infty$$

which is equivalent to

$$\sup_{n,k} \mid a_{nk} \mid^{1/(r_k+s_n)} < \infty$$

when for the set of all $p = (p_k)$, there exists $N > 1$ such that $\Sigma N^{-p_k} < \infty$ where $r_k = p_k^{-1}$ and $s_n = q_n^{-1}$.

LEMMA C (Theorem 3 [4] and Corollary 2 of theorem 2 [11]). Let $p_k > 0$ for every k. Then $A \in (l_\infty(p), m)$ if and only if

$$\sup_n \Sigma \mid a_{nk} \mid M^{1/p_k} < \infty \text{ for every integer } M > 1.$$

3. Here and henceforth let e and e_k represent respectively the sequences (1, 1, 1,....) and (0, 0, ...,0, 1, 0,....) the 1 in the kth place. Now in the following theorems let us establish the necessary and sufficient conditions for an infinite matrix $A = (a_{nk})$ to transform the spaces $l(p)$, $c_0(p)$ and $l_\infty(p)$ into Q and derive some known results as corollaries.

THEOREM 1. $A \in (l(p), Q)$ if and only if

(1.1) each column of the matrix $A = (a_{nk})$ belongs to A, and

(1.2) $\sup_{n,k} \mid a_{nk} \mid^{p_k} < \infty$ when $0 < p_k \leq 1$

or

there exists an integer $M > 1$ such that

$\sup_n \Sigma \mid a_{nk} \mid^{q_k} M^{-q_k} < \infty$ when $1 < p_k \leq \sup p_k < \infty$ and

$p_k^{-1} + q_k^{-1} = 1$.

PROOF: Let $A \in (l(p), Q)$. Since $e_k \in l(p)$, the necessity of (1.1) is obvious. Since $Q \subset m$, the necessity of (1.2) follows from Lemma A.

Conversely , let (1.1) and (1.2) hold, $(x_k) \in l\,(p)$ and $H = \max(1, \sup p_k)$. From (1.2), we have

$$(1.3)\begin{cases} \text{(i)} \ |a_{nk}|^{p_k} \leq L \text{ independent of } n \text{ when } 0 < p_k \leq 1, \\ \text{or} \\ \text{(ii)} \ \Sigma\, |a_{nk}|^{q_k} M^{-q_k} \leq L \text{ independent of } n \text{ for some integer} \\ \qquad M > 1, \text{ when } 1 < p_k \leq H < \infty \end{cases}$$

Since $l\,(p) \subset c_0\,(p) \subset m$, there exists a R such that

$$(1.4) \quad |x_k| \leq R \text{ for all } k,$$

and for a given $\varepsilon > 0$, there exists a $P \geq 1$ such that

$$(1.5)\begin{cases} \sum\limits_{k=P+1}^{\infty} |x_k|^{p_k} < \dfrac{\varepsilon}{4L} \text{ when } 0 < p_k \leq 1, \text{ and} \\ \left(\sum\limits_{k=P+1}^{\infty} |x_k|^{p_k}\right)^{1/H} < \dfrac{\varepsilon}{4M(L+1)} \text{ when } 1 < p_k \leq H < \infty \end{cases}$$

When P is fixed ,by (1.1), for $\varepsilon > 0$ and for all n and r, there exists i_k, $k = 1,2,\ldots,P$ such that $|a_{nk}-a_{n+ri_k,\,k}| < \dfrac{\varepsilon}{2PR}$ If i is the least common multiple of i_k; $k = 1, 2\ldots, P$, then

$$(1.6) \quad \sum_{k=1}^{P} |a_{nk} - a_{n+ri,k}| < \frac{\varepsilon}{2R}\,.$$

Now

$$(1.7) \quad |y_n - y_{n+ri}| \leq S_1 + S_2 \text{ where } S_1 = \sum_{k=1}^{p} |(a_{nk}-a_{n+ri,k})\, x_k| \text{ and}$$

$$S_2 = \sum_{k=P+1}^{\infty} |(a_{nk}-a_{n+ri,k})\, x_k|$$

Case (i): When $0 < p_k \leq 1$, Since $(x_k) \in l\,(p)$, $\Sigma\, |x_k|^{p_k} \leq 1/L$ where we can, without loss of generality use the same L as in (1.3), so that $|x_k|\, L^{1/p_k} < 1$. Hence.

$$S_2 \leq 2 \sum_{k=P+1}^{\infty} L^{1/p_k}\, |x_k| \text{ using (1.3)}$$

$$\leq 2 \sum_{k=P+1}^{\infty} L \mid x_k \mid^{p_k}$$

$< \varepsilon / 2$ using (1.5)

and $S_1 < \varepsilon / 2$ using (1.4) and (1.6).

Then form (1.7), we have $\mid y_n - y_{n+ri} \mid < \varepsilon$. Hence $(y_n) \in A$.

Case (ii): When $1 < p_k \leq H < \infty$, by the proof of Theorem 2 [7] and the inequality

$\mid ax \mid \leq B(\mid a \mid^q B^{-q} + \mid x \mid^p)$ where $p^{-1} + q^{-1} = 1$,
we have,

$$\sum_{k=P+1}^{\infty} \mid a_{nk} x_k \mid \leq M \left(\sum_{k=P+1}^{\infty} \mid a_{n+} \mid^{q_k} M^{-q_k} + 1 \right) \left(\sum_{k=P+1}^{\infty} \mid x_k \mid^{p_k} \right)^{1/H}$$

$< \varepsilon / 4$ using (1.3) and (1.5)

Similarly $\sum_{k=P+1}^{\infty} \mid a_{n+ri,k} x_k \mid < \varepsilon / 4$ so that $S_2 < \varepsilon / 2$ and

$S_1 < \varepsilon / 2$ using (1.4) and(1.6).

Hence $(y_n) \in Q$ so that $A \in (l(p), Q)$.

COROLLARY 1a (Theorem 3 [1]). $A \in (l, Q)$ if and only if

(i) each column of the matrix $A = (a_{nk})$ belongs to Q, and

(ii) $\mid a_{nk} \mid \leq M$ independently of n and k.

PROOF: Take $p_k = 1$ for all k.

COROLLARY 1b (Theorem 4 [13]). Let $p > 1$ and $p^{-1} + q^{-1} = 1$. Then $A \in (l_p, Q)$ if and only if

(i) each column of the matrix $A = (a_{nk})$ belongs to Q, and

(ii) $\sup_n \Sigma \mid a_{nk} \mid^q < \infty$

PROOF: Take $p_k = p > 1$ for all k so that $q_k = q$ for all k and $p_k^{-1} + q_k^{-1} = 1$ becomes $p^{-1} + q^{-1} = 1$.

THEOREM 2. Let $p = (p_k) \in m$. Then $A \in (c_0(p), Q)$ if and only if
(2.1) each column of the matrix $A = (a_{nk})$ belongs to Q, and

(2.2) there exists an absolute constant $M > 1$ such that

$$\sup_n \Sigma \mid a_{nk} \mid M^{\frac{-1}{p_k}} < \infty$$

which is equivalent to

$$\sup_{n,k} \mid a_{nk} \mid^{p_k} < \infty$$

when the set of all $p = (p_k)$ is such that there exists $N > 1$ such that $\Sigma N^{-1/p_k} < \infty$.

PROOF: Let $A \in (c_0(p), Q)$. Since $e_k \in c_0(p)$, the necessity of (2.1) is trivial.

Since $Q \subset m$, the necessity of (2.2) follows from Lemma B.

Conversely, let (2.1) and (2.2) hold and $(x_k) \in c_0(p)$. Then

(2.3) $\Sigma \mid a_{nk} \mid M^{\frac{-1}{p_k}} \leq L$ independent of n.

Since $c_0(p) \subset m$,

(2.4) $\mid x_k \mid \leq R$ for all k.

Since $(p_k) \in m$, we can take on c_0 (p), the paranorm $g(x) = \sup_k \mid x_k \mid^{p_k/H}$ where $H = \max(1, \sup p_k)$. Then

$g(x - \sum_{k=1}^{n} x_k e_k) = \sup_{k \geq P+1} \mid x_k \mid^{p_k/H} \to 0$ as $P \to \infty$ so that $x = \Sigma x_k e_k$ with this topology on $c_0(p)$.

Hence given an $\varepsilon > 0$, there exists $p \geq 1$ such that

(2.5) $|x_k|^{p_k/H} < \frac{\varepsilon}{4LM^{1/H}}$ for $k \geq P$

When P is fixed ,by (2.1) , for $\varepsilon > 0$ and for all n and r, there exists i_k; $k = 1, 2 \ldots P$ such that $\mid a_{nk} - _{n+ri_k,k} \mid < \frac{\varepsilon}{2PR}$. If i is the least common multiple of i_k; $k = 1, 2, \ldots, P$, we have

(2.6) $\sum_{k=1}^{p} \mid a_{nk} - a_{n+ri,k} \mid < \frac{\varepsilon}{2R}$

As in Theorem 1, we have

(2.7) $\mid y_n - y_{n+ri} \mid \leq S_1 + S_2$

Now $S_1 < \varepsilon / P\ 2$ using (2.4) and (2.6). Also

$$\sum_{k=P+1}^{\infty} | a_{nk} | \, | x_k | \leq \sum_{k=P+1}^{\infty} | a_{nk} | \left(| x_k |^{P_k / H} \right)^{H/P_k}$$

$$< \sum_{k=P+1}^{\infty} | a_{nk} | \left(\frac{\varepsilon}{4LM^{1/H}} \right)^{H/P_k}$$

using (2.5)

$$< \frac{\varepsilon}{4L} \sum_{k=P+1}^{\infty} | a_{nk} | M^{-1/P_k}$$

$$< \varepsilon / 4 \text{ using } (2.3)$$

Similary $\sum_{k=P+1}^{\infty} | a_{n+ri,k} | \, | x_k | < \frac{\varepsilon}{4}$ so that $S_2 < \frac{\varepsilon}{2}$

Hence (2.7) gives $| y_n - y_{n+ri} | < \varepsilon$ so that $(y_n) \in Q$.

COROLLARY 2a (Theorem 1 [1]). $A \in (c_0, Q)$ if and only if

(i) each column of the matrix $A = (a_{nk})$ belongs to Q, and

(ii) $\Sigma \, | a_{nk} | \leq M$ independent of n.

PROOF: Take $p_k = 1$ for all. k.

COROLLARY 2 b (Theorem 4 [1]). $A \in (\Gamma, Q)$ if and only if

(i) each column of the matrix $A = (a_{nk})$ belongs to Q , and

(ii) $| a_{nk} |^{1/k} \leq D$ independent of n and k.

PROOF: Take $p_k = 1 / k$ so that $\Sigma M^{-k} < \infty$ for $M > 1$.

THEOREM 3. If for the set of all $p = (p_k)$, there exists a $N > 1$ such that $\Sigma N^{\frac{-1}{P_k}} < \infty$ then $A \in (l_\infty (p), Q)$ if and only if

(3.1) each column of the matrix $A = (a_{nk})$ belongs to Q, and

(3.2) $\operatorname{Sup}_n \Sigma \, | a_{nk} | M^{1/P_k} < \infty$ for every integer $M > 1$.

PROOF : Let $A \in (l_\infty (p), Q)$. Since $e_k \in l_\infty (p)$, trivially (3.1.) is necessary.

Since $Q \subset m$, the necessity of (3.2) follows from Lemma C.

Conversely, let (3.1) and (3.2) hold and $(x_k) \varepsilon l\infty$ (p). By (3.2) we have.

(3.3) $\Sigma \mid a_{nk} \mid M^{1/P_k} \leq L$ independent of n.

Since for the set of all $p = (p_k)$, there exists a $N > 1$ such that $\Sigma N^{-1/P_k} < \infty$, given an $\varepsilon > 0$, there exists a $P \geq 1$ such that

(3.4) $$\sum_{k=P+1}^{\infty} N^{-1/P_k} < \frac{\varepsilon}{4L}$$

When P is fixed, since $(x_k) \in l_\infty$ (p), we have

(3.5) $| x_k | \leq R^{1/P_k} \leq S$ where $S = \max (1, R^{1/p_k})$; $k=1, 2, \ldots, P$.

By (3.1), for $\varepsilon > 0$ and for n and r, there exists i_k, ; $k = 1, 2, \ldots P$.

such that $| a_{nk} - a_{n+ri_k,k} | < \frac{\varepsilon}{2PS}$. Then choosing i to be the least

common multiple of i_k; $k = 1,2, \ldots, P$, we have

(3.6) $$\sum_{k=1}^{\infty} | a_{nk} - a_{n+ri,K} | > \frac{\varepsilon}{2S}$$

As in Theorem 1, we have

(3.7) $| y_n - y_{n+ri} | \leq S_1 + S_2$

Now $S_1 < \varepsilon / 2$ using (3.5) and 3.6). Further

$$\sum_{k=P+1}^{\infty} | a_{nk} | \, | x_k | \leq \sum_{k=P+1}^{\infty} LM^{-1/p_k} R^{1/p_k} \leq L \sum_{k=P+1}^{\infty} \left(\frac{R}{M}\right)^{1/P_k}$$

Now choosing $M \geq NR$, we have

$$\sum_{k=P+1}^{\infty} | a_{nk} | \; | x_k | \leq L \sum_{k=P+1}^{\infty} N^{-1/P_k} \frac{\varepsilon}{4}$$

Similarly $\sum_{k=P+1}^{\infty} | a_{n+ri,k} | \; | x_k | < \frac{\varepsilon}{4}$ so that $S_2 < \frac{\varepsilon}{2}$.

Hence (3.7) gives $| y_n - y_{n+ri} | < \varepsilon$ so that $(y_n) \in Q$.

COROLLARY 3a. $A \in (\Gamma^*, \varrho)$ if and only if

(i) each column of the matrix $A = (a_{nk})$ belongs to Q, and

(ii) $\sup_n \Sigma \mid a_{nk} \mid M^k < \infty$ for every integer $M > 1$.

PROOF: Take $p_k = 1/k$.

This can also be written in another form (Theorem 5 [1]) as $A \in (\Gamma^*, A)$ if and only if

(iii) each column of the matrix $A = (a_{nk})$ belongs to Q ,and

(iv) the sequence $\{f_n(z)\}$ of integral functions is uniformly bounded on every compact set (of the comlpex plane) where $f_n(z) = \Sigma a_{nk} z^k$; n =1,2,..
PROOF: The proof in the same as that of Corollary 2 of Theorem 4 (12]

REMARK. Theorem 3 is false in the general case even when we replace (3.1) by the stronger assumption that each column of the matrix $A = (a_{nk})$ is periodic. As for example, take $p_k = 1$ (all k), so that (3.2) reduces to

$$(R_1) \quad \sup_n \Sigma \mid a_{nk} \mid < \infty$$

Now define

$$a_{nk} = \begin{cases} 1 & (\text{n odd multiple of } 2^{k-1};\ k=1, 2,....) \\ 0 & (\text{otherwise}) \end{cases}$$

Thus each column is periodic of period 2^k. Also any positive integer n can be experessed uniquely in the form $\mu . 2^r$ (μ odd, r an integer); we just take 2^r as the highest power of 2 dividing n (Of course if n is odd, then $r = 0$). Thus, for even n, there is just one value of k for which n is an odd multiple of 2^{k-1}. Hence each row of the matrix A has one element equal to 1, all the other elements being 0. Thus (R_1) is satisfied.

Now define $x = (x_k)$ by

$$x_k = \begin{cases} 1 & (\text{k odd}) \\ 0 & (\text{k even}) \end{cases}$$

Then $x \in l_\infty(p)$, so that if we show that $y = (y_n) = Ax$ does not belong to Q, the falsity of the sufficiency part of the theorem will be established.

Now $y_n = a_{nk} x_k$ where $k = k(n)$ is the unique k for which $a_{nk} \neq 0$. Hence we deduce that

$$(R_2)\ y_n = \begin{cases} 1 & (\text{n an odd multiple of } 2^{2r}\ ;\ r = 0, 1, 2, \ldots) \\ 0 & (\text{n an odd multiple of } 2^{r+1};\ r = 0, 1, 2, \ldots) \end{cases}$$

Now any sequence of 0's and 1's which belong to Q must be periodic; for the only positive values of $| x_k - x_{k+ri} |$ are 0,1; so that if we take $\varepsilon < 1$, the inequality $| x_k - x_{k+ri} | < \varepsilon$ gives

$$x_k - x_{k+ri} = 0;$$

in otherwords (x_k) is of period r. So it is enough to verify that y is not periodic.

Suppose y were periodic. Its period must be a positive integer so that by what has already been said, its period can be expressed in the form $\mu . 2^\delta$ with μ odd, $\delta > 0$, δ an integer.

Now

$2^{\delta+1}$ is an odd multiple of $2^{\delta+1}$ (since 1 is odd)

$2^{\delta+1} + \mu . 2^\delta = (\mu + 2) . 2^\delta$ is an add multiple of 2^δ. (since μ is odd, so the $\mu + 2$ is odd).

Thus by (R_2), y_n takes differentvalue s for $n = 2^{\delta+1}$; $n = 2^{\delta+1} + \mu . 2^\delta$. This contraidcts the assumption that y_n has period $\mu . 2^\delta$.

ACKNOWLEDGEMENT

The authors wish to express their deep sense of gratutide to Prof. B. Kuttner for providing the example to prove that the Thorem 3 is false in the general case.

REFERENCES

1– **Chandrasekhara Rao. K** Matrix transformations of some sequence spaces- III. Bulletin U.M.I. (4) 5 (1972), 7–13.

2– **Ganapathy Iyer, V.** On the space of integral functions-I. J. Indian Math. Socs. (2) 12 (1948), 13–30.

3– **Lascarides,** C.G. A study of certain sequence spaces of Maddox and a generalization of a theorem of Iyer. Pac. J. Marh., 38 (1971). 487–500.

4– **Lascarides, C.G. and Maddox, I.J.** Matrix transformations betveen some classes of sequences. Proc. Camb. Phil. Soc., 68 (1970), 99–104.

5– **Maddox, I.J.** Spaces of strongly summbable sequences. Quarterly J. Math. Oxford. (2) 18 (1967), 345–355.

6– **Maddox, I.J.** Paranormed sequence spaces generated by infinite matrices. Proc. Camb. Phil. Soc., 64 (1968), 335–340.

7– **Maddox, I.J.** Continuous and Kothe-Teeplitz duals of certain sequence spaces. Proc. Camb. Phil. Sec., 65 (1969), 431–435.

8– **Maddox, I.J.** Some properties of paranormed sequence spaces. J. London. Math. Soc., (2) 1 (1969), 316-322.

9– **Maddox, I.J.** Operators on the generalized entire sequences. Proc. Camb. Phil. Soc., 71 (1972), 491–494.

10– **Simons, S.** The sequence spaces $l(P_v)$ and m (P_v). Proc. Lond. Math. Soc., (3) 15 (1965), 422–436.

11– **Sirajudeen, S.M.** Inclusion Theorems on matrix transformations of some generalized sequence spaces. Soochow J.M. 7 (1981), 165–174.

12– **Sirajudeen, S.M.** Matrix transformations of some generalized sequence spaces into semiperiodic sequence space. Communications de la Faculte des Sciences de l'Universite d'-Ankara. Series A. 30 (1981), 45–53.

13– **Somasundaram, D and Sirajudeen, S.M.** Matrix tansformations of l_p d into (Γ, Γ^*, Γ_c and q). Proc. Nat. Acad. Sci., India. 51 (A) 11 (1981), 272–276.

DEPARTMENT OF MATHEMATICS, JAMAL MOHAMED COLLECE, TIRUCHIRAPALLI-620 020. TAMIL NADU (INDIA).

DEPARTMENT OF MATHEMATICS, BHARATHIAR UNIVERSITY, COIMBATORE 641 046. TAMIL NADU (INDIA).