COMMUNICATIONS

DE LA FACULTÉ DES SCIENCES DE L'UNIVERSITÉ D'ANKARA

Série A_1 : Mathématiques,

TOME : 33 ANNÉE : 1984

Solutions of Boundary Value Problems For a Class of Hyperbolic Equations With Singular Coefficients

by

İ. ETHEM ANAR

21

Faculté des Sciences de l'Université d'Ankara
Ankara, Turquie

Communications de la Faculté des Sciences
de l'Universite d'Ankara

La Revue "Communications de la Faculté des Sciences de l'Université d'Ankara" est un organe de publication englobant toutes les diciplines scientifique représentées à la Faculté des Sciences de l'Université d'Ankara.

La Revue, jusqu'à 1975 àl'exception des tomes I, II, III etait composé de trois séries

Série A: Mathématiques, Physique et Astronomie,

Série B: Chimie,

Série C: Sciences Naturelles.

A partir de 1975 la Revue comprend sept séries:

Série A_1: Mathématiques,

Série A_2: Physique,

Série A_3: Astronomie,

Série B: Chimie,

Série C_1: Géologie,

Série C_2: Botanique,

Série C_3: Zoologie.

A partir de 1983 les séries de C_2 Botanique et C_3 Zoologie on été réunies sous la seule série Biologie C et les numéros de Tome commencerons par le numéro 1.

En principle, la Revue est réservée aux mémories originaux des membres de la Faculté des Sciences de l'Université d'Ankara. Elle accepte cependant, dans la mesure de la place disponible les communications des auteurs étrangers. Les langues Allemande, Anglaise et Française serint acceptées indifféremment. Tout article doit être accompagnés d'un resume.

Les article soumis pour publications doivent être remis en trois exemplaires dactylographiés et ne pas dépasser 25 pages des Communications, les dessins et figuers portes sur les feulles séparées devant pouvoir être reproduits sans modifications.

Les auteurs reçoivent 25 extrais sans couverture.

l'Adresse : Dergi Yayın Sekreteri,
Ankara Üniversitesi,
Fen Fakültesi,
Beşevler—Ankara
TURQUIE

Solutions of Boundary Value Problems For a Class of Hyperbolic Equations With Singular Coefficients

İ. ETHEM ANAR

Department of Mathematics, Faculty of Arts and Sciences University of Gazi, Ankara, Turkey

(Received June 6, 1984 and accepted December 28, 1984)

ABSTRACT

In this article, a class of equations of the hyperbolic type with singular coefficients is studied. Using the multiple Fourier transforms in the space $(x_1,...,x_{n-1}, y) \in \mathcal{R}^n$, for the domain $|x| < y$ and in the space $(x_1,...,x_{n-1}, y, z) \in \mathcal{R}^{n+1}$ for the domain $(|x|^2 + z^2)^{\frac{1}{2}} < y$ boundary value problems containing the singular boundary are solved.

1. INTRODUCTION

The equations of hyperbolic type with singular coefficients have been studied by many authors. In this article, using the multiple Fourier transform we will obtain the solution of boundary value problems.

Let us denote a point in m - dimensional Euclidean space by $(x_1, \ldots, x_n, x_{n+1}, \ldots, x_m) \in \mathcal{R}^m$. Thus $(x, z) \in \mathcal{R}^{m+1}$. Now consider the following operators of the ultra-hyperbolic type with singular coefficients:

$$L \equiv \sum_{i=1}^{n-1} \frac{\partial^2}{\partial x_i^2} - \left(\sum_{j=n}^{m} \frac{\partial^2}{\partial x_j^2} + \frac{2k_j}{x_j} \frac{\partial}{\partial x_j} + k^2 \right), \qquad (I.I)$$

$$\Delta_\Sigma \equiv \sum_{i=1}^{n-1} \frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial z^2} - \left(\sum_{j=n}^{m} \frac{\partial^2}{\partial x_j^2} + \frac{2k_j}{x_j} \frac{\partial}{\partial x_j} \right) \qquad (1.2)$$

where $k_j, k \in \mathcal{R}^+$. By means of the transformation

$$y = \left(\sum_{j=n}^{m} x_j^2 \right)^{1/2}$$

the operators of (I.I) and (1.2) become,

$$L \equiv \sum_{i=1}^{n-1} \frac{\partial^2}{\partial x_i^2} - \left(\frac{\partial^2}{\partial y^2} + \frac{2\alpha}{y} \frac{\partial}{\partial y} + k^2 \right)$$

$$\Delta_\Sigma \equiv \sum_{i=1}^{n-1} \frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial z^2} - \left(\frac{\partial^2}{\partial y^2} + \frac{2\alpha}{y} \frac{\partial}{\partial y} \right),$$

where

$$2\alpha = m - n + 2 \sum_{j=n}^{m} k_j$$

$y = o$ is a singular hyperplane for the operators L and Δ_Σ.

2. BOUNDARY VALUE PROBLEMS

We want to obtain the solution of the following boundary value problem for the operator L:

$$L\,[\,u(x, y)\,] = 0, \quad |x| < y \tag{2.1}$$

$$u(x, |x|) = g(x), \tag{2.2}$$

$$u(x, y) \to 0 \text{ as } |x|^2 + y^2 \to \infty, \quad |x| < y. \tag{2.3}$$

In this problem $n < 2\alpha + 1$ and $2\alpha > 1$, $n \geq 2$.

On the other hand boundary value problem for the operator Δ_Σ is as follows:

$$\Delta_\Sigma\,[\,W(x, y, z)\,] = 0, \quad (|x|^2 + z^2)^{1/2} < y \tag{2.4}$$

$$W\,[x, (|x|^2 + z^2)^{1/2}, z] = f(x, z), \tag{2.5}$$

$$W(x, y, z) \to 0 \text{ as } |x|^2 + y^2 + z^2 \to \infty, \quad (|x|^2 + z^2)^{1/2} < y. \tag{2.6}$$

Where $x = (x_1, \ldots, x_{n-1})$, (x,y) and (x, y, z) are the vectors in $\mathcal{R}^{n-1}$, $\mathcal{R}^n$ and $\mathcal{R}^{n+1}$ respectively. For both of the problems, $y = 0$ is a singular hyperplane. We will study the problem in the hypercone $(|x|^2+z^2)^{1/2} < y$. The given continuous functions g and f are the values of the functions u and W over the caracteristic cone $y = (|x|^2 + z^2)^{1/2}$ respectively. As it is well known the (n–1) and n dimensional Fourier transforms are given in the following forms [3],

$$\hat{\Phi}(\xi) \equiv \mathcal{F}_{(n-1)}\ [\Phi(x);\xi] = (2\pi)^{-\frac{1}{2}(n-1)} \int_{\mathcal{R}^{n-1}} \Phi(x)\, e^{i(\xi.x)}\, dx,$$

$$\hat{\Phi}(\xi,\eta) \equiv \mathcal{F}_{(n)}\ [\Phi(x,z);(\xi,\eta)] = (2\pi)^{-\frac{n}{2}} \int_{\mathcal{R}^{n}} \Phi(x,z)\, e^{i(\xi.x+\eta z)}\, dxdz$$

where $\xi = (\xi_1, ..., \xi_{n-1}) \in \mathcal{R}^{n-1}$ and $(\xi.x) = \xi_1 x_1 + ... + \xi_{n-1} x_{n-1}$. Let us denote the inverses of the operators $\mathcal{F}_{(n-1)}$ and $\mathcal{F}_{(n)}$ by $\mathcal{F}^*_{(n-1)}$ and $\mathcal{F}_{(n)}{}^*$.

3. SOLUTION OF BOUNDARY VALUE PROBLEM FOR THE OPERATOR L

Now let us apply the operator $\mathcal{F}_{(n-1)}$ to the both sides of the problem defined by (2.1), 2.2) and (2.3).

$$\mathcal{F}_{(n-1)}\ [L[u(x,y)];\ \xi] = \left(\frac{\partial^2}{\partial y^2} + \frac{2\alpha}{y}\frac{\partial}{\partial y} + k^2 + |\xi|^2\right)\hat{u}(\xi,y)$$

where

$$|\xi|^2 = \xi_1{}^2 + ... + \xi^2{}_{n-1}$$

and

$$\hat{u}(\xi,y) = \mathcal{F}_{(n-1)}\ [u(x,y);\ \xi].$$

In this manner we arrive at the following equivalent problem:

$$\left(\frac{\partial^2}{\partial y^2} + \frac{2\alpha}{y}\frac{\partial}{\partial y} + k^2 + |\xi|^2\right)\hat{u}(\xi,y) = 0 \tag{3.1}$$

$$\hat{u}(\xi, |\xi|) = \hat{g}(\xi) \tag{3.2}$$

$$\hat{u}(\xi,y) \to o \qquad \text{as } y \to \infty \tag{3.3}$$

In equation (3.2),

$$\hat{g}(\xi) = \mathcal{F}_{(n-1)}\ [g(x);\ \xi].$$

The general solution of the equation (3.1) can be obtained as,

$$\hat{u}(\xi,y) = y^{\frac{1}{2}-\alpha}\{A(\xi)\, J_{\alpha-\frac{1}{2}}\ [y(k^2+|\xi|)^{\frac{1}{2}}] + B(\xi)\, N_{\alpha-\frac{1}{2}}\ [y(k^2+|\xi|^2)^{\frac{1}{2}}]\}, \tag{3.4}$$

by making the replacement

$$\hat{u}\ (\xi,\ y) = y^{\frac{1}{2}-\alpha}\ V\ (\xi,\ y)$$

with A (ξ) and B (ξ) beeing arbitrary functions. In equation (3.4) $J_{\alpha-\frac{1}{2}}$ and $N_{\alpha-\frac{1}{2}}$ are the Bessel functions of the first and second kind respectively. We know that the Bessel functions will be given for small values of their argument as follows [4]:

$$J_p\ (x) \sim \frac{1}{\Gamma(p+1)} \left(\frac{x}{2}\right)^p\ ;\ p \neq -1, -2, ...$$

$$N_p\ (x) \sim \begin{cases} \frac{2}{\pi}\ \text{In}\ x,\ p = o \\ \frac{-1}{\pi}\ \Gamma\ (p) \left(\frac{2}{x}\right)^p,\ p \neq 0 \end{cases} \qquad p \neq -1,\ -2,...$$

and for large values

$$J_p\ (x) \sim \left(\frac{2}{\pi x}\right)^{1/2} \cos\left[x - \left(p + \frac{1}{2}\right)\frac{\pi}{2}\right]$$

$$N_p\ (x) \sim \left(\frac{2}{\pi\, x}\right)^{1/2} \sin\left[x - \left(p + \frac{1}{2}\right)\frac{\pi}{2}\right].$$

In order to the solution (3.4) of (3.1) be finite as $y \to o$ we must have B (ξ) $=$ O. Besides in order to the solution

$$\hat{u}\ (\xi, y) = A\ (\xi)\ y^{\frac{1}{2}-\alpha}\ \ J_{\alpha-\frac{1}{2}}\ [\, y\ (k^2 + |\xi|^2)^{1/2}]$$

satisfy the conditions (3.2) and (3.3,)we should have

$$A\ (\xi) = \frac{\hat{g}\ (\xi)}{|\xi|^{\frac{1}{2}-\alpha}\ J_{\alpha-\frac{1}{2}}\ [\,|\xi|\ (k^2 + |\xi|^2)^{1/2}]}$$

Thus as a solution of the problem (3.1), (3.2), (3.3) we obtain

$$\hat{u}\ (\xi, y) = \hat{g}_1\ (\xi, y)\ P\ (\xi, y), \tag{3.5}$$

where the functions $\hat{g}_1(\xi)$ and $P(\xi, y)$ are given by

$$\hat{g}_1(\xi) = \frac{\hat{g}(\xi)}{[|\xi|(k^2+|\xi|^2)^{1/2}]^{\frac{1}{2}-\alpha} J_{\alpha-\frac{1}{2}}[|\xi|(k^2+|\xi|^2)^{1/2}]} \tag{3.6}$$

$$P(\xi, y) = [y(k^2+|\xi|^2)^{1/2}]^{\frac{1}{2}-\alpha} J_{\alpha-\frac{1}{2}}[y(k^2+|\xi|^2)^{1/2}]. \tag{3.7}$$

In this manner it can be seen easily that

$$\lim_{y=|\xi|\to 0} \hat{u}(\xi, y) = \hat{g}(0).$$

Applying the operator $\mathcal{F}_{(n-1)}{}^*$ to the both sides of the solution (3.5) and making use of the convolution theorem we obtain the solution of the problem (2.1), (2.2), (2.3). as [2],

$$u(x, y) = (2\pi)^{-\frac{1}{2}(n-1)} \int_{\mathcal{R}^{n-1}} g_1(s) K(x-s, y) ds. \tag{3.8}$$

Where $s = (s_1, \ldots, s_{n-1}) \in \mathcal{R}^{n-1}$ and

$$g_1(x) = \mathcal{F}_{(n-1)}{}^* [\hat{g}_1(\xi); x)]$$

$$= (2\pi)^{-\frac{1}{2}(n-1)} \int_{\mathcal{R}^{n-1}} \hat{g}(\xi) \frac{[|\xi|(k^2+|\xi|^2)^{1/2}]^{\alpha-\frac{1}{2}}}{J_{\alpha-\frac{1}{2}}[|\xi|(k^2+|\xi|^2)^{1/2}]} e^{-i(\xi.x)} d\xi \tag{3.9}$$

and

$$K(x, y) = \mathcal{F}_{(n-1)}{}^* [P(\xi, y); x].$$

The Hankel transform of order ν of the function $f(r)$ is

$$\hat{f}_\nu(\xi) \equiv \mathcal{H}_\nu [f(r); \xi] = \int_0^\infty r f(r) J_\nu(\xi r) dr.$$

Its inverse transform is,

$$f(r) \equiv \mathcal{H}_\nu [\hat{f}(\xi); r)].$$

We know the following relationship [3] between Fourier and Hankel transforms

$$f(x_1,\dots,x_n) = (2\pi)^{-\frac{1}{2}n} \int_{\mathcal{R}^n} F(\rho_1)\, e^{-i(\xi.x)}\, d\xi \qquad (3.10)$$

$$r_1^{\frac{1}{2}n-1} f(r_1) = \int_0^\infty \rho_1 [\rho_1^{\frac{1}{2}n-1} F(\rho_1)]\, J_{\frac{1}{2}n-1}(\rho_1 r_1)\, d\rho_1 , \qquad (3.11)$$

where

$r_1 = |x| = (x_1^2 + \dots + x_n^2)^{1/2}$, $\rho_1 = |\xi| = (\xi_1^2 + \dots + \xi_n^2)^{1/2}$

So, if we set

$r = |x| = (x_1^2 + \dots + x_{n-1}^2)^{1/2}$, $\rho = |\xi| = (\xi_1^2 + \dots + \xi_{n-1}^2)^{1/2}$

we can write,

$$K(x,y) = (2\pi)^{-\frac{1}{2}(n-1)} \int_{\mathcal{R}^{n-1}} P(\xi,y)\, e^{-i(\xi.x)}\, d\xi$$

$$= r^{\frac{1}{2}(3-n)}\, y^{\frac{1}{2}-\alpha}$$

$$x \int_0^\infty \rho^{\frac{1}{2}(n-1)} (k^2+\rho^2)^{\frac{1}{2}(\frac{1}{2}-\alpha)} J_{\alpha-\frac{1}{2}} [y(k^2+\rho^2)^{1/2}]\, J_{\frac{1}{2}(n-3)}(\rho r)\, d\rho.$$

From [I] we know the following integral

$$\int_0^\infty J_\mu(bt)\, J_\nu [a(t^2+z^2)^{1/2}]\, (t^2+z^2)^{-\frac{1}{2}\nu}\, t^{\mu+1}\, dt$$

$$= \begin{cases} 0 , \ a < b,\ \mathrm{Re}\,\nu > \mathrm{Re}\,\mu > -1 \\ b^{\mu} a^{-\nu} z^{1+\mu-\nu} (a^2-b^2)^{\frac{1}{2}(\nu-\mu-1)} J_{\nu-\mu-1} [z(a^2-b^2)^{1/2}],\ a > b, \mathrm{Re}\nu > \mathrm{Re}\mu > -1 \end{cases} \qquad (3.12)$$

Using (3.12) as a kernel funsction we obtain,

$$K(x,y) = k^{\frac{n}{2}-\alpha}\, y^{1-2\alpha}\, (y^2-r^2)^{\frac{2\alpha-n}{4}}\, J_{\alpha-\frac{n}{2}} [k(y^2-r^2)^{1/2}]. \qquad (3.13)$$

Let us apply the convolution theorem for the equation (3.9),

$$g_1(x) = (2\pi)^{-\frac{1}{2}(n-1)} \int_{\mathcal{R}^{n-1}} g(\tau)\, \Phi(x-\tau)\, d\tau,$$

where $\tau = (\tau_1,\dots,\tau_{n-1}) \in \mathcal{R}^{n-1}$ and

$$\Phi\ (x-\tau) = \mathcal{F}_{(n-1)}{}^{*} \left\{ \frac{[\,|\xi|\,(k^2+|\xi|^2)^{1/2}]^{\alpha-\frac{1}{2}}}{J_{\alpha-\frac{1}{2}}\ [\,|\xi|\,(k^2+|\xi|^2)^{1/2}]};\ x-\tau \right\}$$

$$= (2\pi)^{-\frac{1}{2}(n-1)} \int_{\mathcal{R}^{n-1}} \frac{[\,|\xi|\,(k^2+|\xi|^2)^{1/2}]^{\alpha-\frac{1}{2}}}{J_{\alpha-\frac{1}{2}}\ [\,|\xi|\,(k^2+|\xi|^2)^{1/2}]}\ e^{-i\,[\xi.(x-\tau)]}\ d\xi$$

Considering (3.10) and (3.11), we get

$$\Phi\,(x-\tau) = |x-\tau|^{\frac{3-n}{2}} \int_0^\infty \rho^{\frac{1}{2}(n-1)} \frac{[\rho\,(k^2+\rho^2)^{1/2}]^{\alpha-\frac{1}{2}}}{J_{\alpha-\frac{1}{2}}\ [\rho\,(k^2+\rho^2)^{1/2}]} J_{\frac{n-3}{2}}[\rho\,|x-\tau|]\,d\rho$$

where

$$\rho = |\xi|,\ |x-\tau| = [(x_1-\tau_1)^2 + \dots + (x_{n-1}-\tau_{n-1})^2]^{1/2}$$

Thus as the value of the function g_1,

$$g_1(s) = (2\pi)^{-\frac{1}{2}(n-1)} \int_{\mathcal{R}^{n-1}} g(\tau) \left\{ |s-\tau|^{\frac{3-n}{2}} \int_0^\infty \frac{[\rho\,(k^2+\rho^2)^{1/2}]^{\alpha-\frac{1}{2}}}{J_{\alpha-\frac{1}{2}}\ [\rho\,(k^2+\rho^2)^{1/2}]} \right.$$

$$\left. \times\ J_{\frac{n-3}{2}}\ [\rho\ |s-\tau|]\ d\rho \right\} d\tau \tag{3.14}$$

is obtained. If we replace (3.14) into (3.8) the solution of the problem (2.1), (2.2), (2.3) is

$$u\,(x,y) = (2\pi)^{1-n} \int_{\mathcal{R}^{2(n-1)}} \left\{ \int_0^\infty D\,(\rho,\,|s-\tau|)\ K\,(x-s,\,y)\ g(\tau)\ d\rho \right\} d\tau ds$$

where

$$D\,(\rho,\ |s-\tau|) = \frac{[\,\rho\ (k^2+\rho^2)^{1/2}]^{\alpha-\frac{1}{2}}}{J_{\alpha-\frac{1}{2}}\ [\rho\,(k^2+\rho^2)^{1/2}]}\ |s-\tau|^{\frac{3-n}{2}}\ J_{\frac{n-3}{2}}\ [\rho\ |s-\tau|].$$

4. SOLUTION OF BOUNDARY VALUE PROBLEM FOR THE OPERATOR Δ_Σ

Following a similar idea and applying the operator $\mathcal{F}_{(n)}$ to the both sides of boundary value problem (2.4), (2.5), (2.6) we obtain

$$\left(\frac{\partial^2}{\partial y^2} + \frac{2\alpha}{y}\frac{\partial}{\partial y} + |\xi|^2 + \eta^2\right)\hat{W}(\xi, y, \eta) = o, \tag{4.1}$$

$$\hat{W}\left[\xi, (|\xi|^2 + \eta^2)^{1/2}, \eta\right] = \hat{f}(\xi, \eta). \tag{4.2}$$

$$\hat{W}(\xi, y, \eta) \to 0, \text{ as } y \to \infty. \tag{4.3}$$

Where

$$\hat{W}(\xi, y, \eta) = \mathcal{F}_{(n)}\left[W(x, y, z); (\xi, \eta)\right],$$

$$\hat{f}(\xi, \eta) = \mathcal{F}_{(n)}\left[f(x, z); (\xi, \eta)\right].$$

By use of the transformation

$$\hat{W}(\xi, y, \eta) = y^{\frac{1}{2}-\alpha} W_1(\xi, y, \eta)$$

the general solution of the equation (4.1) is obtained as

$$\hat{W}(\xi, y, \eta) = y^{\frac{1}{2}-\alpha}\left\{ C(\xi, \eta) J_{\alpha-\frac{1}{2}}\left[y(|\xi|^2 + \eta^2)^{1/2}\right] + D(\xi, \eta) N_{\alpha-\frac{1}{2}}\left[y(|\xi|^2 + \eta^2)^{1/2}\right]\right\} \tag{4.4}$$

where $C(\xi, \eta)$ and $D(\xi, \eta)$ are arbitrary functions. In order to (4.4) be finite for $y \to o$, we get $D(\xi, \eta) = o$. Using the conditions (4.2) and (4.3) we obtain the solution of the problem (4.1), (4.2), (4.3)

$$\hat{W}(\xi, y, \eta) = \hat{f}_1(\xi, \eta)\, Q(\xi, y, \eta) \tag{4.5}$$

where

$$\hat{f}_1(\xi, \eta) = \frac{\hat{f}(\xi, \eta)}{(|\xi|^2 + \eta^2)^{\frac{1}{2}-\alpha} J_{\alpha-\frac{1}{2}}(|\xi|^2 + \eta^2)} \tag{4.6}$$

$$Q(\xi, y, \eta) = \left[y(|\xi|^2 + \eta^2)^{1/2}\right]^{\frac{1}{2}-\alpha} J_{\alpha-\frac{1}{2}}\left[y(|\xi|^2 + \eta^2)^{1/2}\right] \tag{4.7}$$

The solution (4.5) satisfies the boundary conditions and takes the following value in the limit:

$$\lim_{y=(|\xi|^2+\eta^2)^{1/2}\to 0} \hat{W}(\xi,y,\eta) = \hat{f}(0,0).$$

When we apply the operator $\mathcal{F}_{(n)}^*$ of (4.5) we can obtain the solution of boundary value problem (2,4), (2.5), (2.6) as

$$W(x,y,z) = (2\pi)^{-\frac{1}{2}n} \int_{\mathcal{R}^n} f_1(s,t)\, M(x-s, y, z-t)\, ds dt. \tag{4.8}$$

Where the kernel function M is given as follows

$$M(x,y,z) = \mathcal{F}^*_{(n)} [Q(\xi,y,\eta); (x,z)]$$

$$= \frac{y^{\frac{1}{2}-\alpha}}{(2\pi)^{\frac{n}{2}}} \int_{\mathcal{R}^n} (|\xi|^2+\eta^2)^{\frac{1}{2}(\frac{1}{2}-\alpha)} J_{\alpha-\frac{1}{2}}[y(|\xi|^2+\eta^2)^{1/2}]\, e^{-i(\xi.x+\eta z)}\, d\xi d\eta \tag{4.9}$$

and

$$f_1(x,z) = \mathcal{F}_{(n)}^* [\hat{f}_1(\xi,\eta); (x,z)].$$

Let

$$\tilde{\rho} = (|\xi|^2+\eta^2)^{1/2}, \quad \tilde{r} = (|x|^2+z^2)^{1/2}.$$

From (3.10), (3.11) we get

$$M(x,y,z) = y^{\frac{1}{2}-\alpha}\, \tilde{r}^{1-\frac{n}{2}} \int_0^\infty \tilde{\rho}^{\frac{1}{2}(n-2\alpha+1)} J_{\alpha-\frac{1}{2}}(y\tilde{\rho})\, J_{\frac{n}{2}-1}(\tilde{\rho}\,\tilde{r})\, d\tilde{\rho}.$$

We know from [1] that,

$$\int_0^\infty J_\mu(at)J_\nu(bt)t^{-\gamma}\, dt = \frac{a^\mu\, \Gamma[\frac{1}{2}(1+\nu+\mu-\gamma)]}{2^\gamma b^{\mu-\gamma+1}\, \Gamma(\mu+1)\, \Gamma[\frac{1}{2}(1+\nu+\gamma-\mu)]}$$

$$x_2F_1\left[\tfrac{1}{2}(1+\nu+\mu-\gamma),\ \tfrac{1}{2}(1+\mu-\nu-\gamma);\ \mu+1;\ \frac{a^2}{b^2}\right], \tag{4.10}$$

$$\text{Re}\,(\nu+\mu-\gamma+1) > 0,\ \text{Re}\,\gamma > -1,\ 0 < a < b.$$

By use of (4.10) the kernel function becomes

$$M(x,y,z) = \frac{2^{\frac{1}{2}(n+1-2\alpha)}}{\Gamma\left[\frac{1}{2}(2\alpha-n+1)\right]} y^{-n} {}_2F_1\left[\frac{n}{2}, \frac{1}{2}(n-2\alpha+1); \frac{n}{2}; \frac{\tilde{r}^2}{y^2}\right].$$

Now in order to calculate the function f_1, let us apply the operator $\mathcal{F}_{(n)}^*$ of the equation (4.6) and use the convolution theorem for the Fourier transform we get

$$f_1(x,z) = (2\pi)^{-\frac{n}{2}} \int_{\mathcal{R}^n} f(\sigma, \zeta)\, G(x-\sigma, z-\zeta)\, d\sigma\, d\zeta,$$

where

$$\sigma = (\sigma_1, \ldots, \sigma_{n-1}) \in \mathcal{R}^{n-1}, \ (\sigma,\zeta) \in \mathcal{R}^n$$

$$G(x-\sigma, z-\zeta) = (2\pi)^{-\frac{n}{2}} \int_{\mathcal{R}^n} \frac{(|\xi|^2+\eta^2)^{\alpha-\frac{1}{2}}}{J_{\alpha-\frac{1}{2}}(|\xi|^2+\eta^2)}\, e^{-i[\xi.(x-\sigma) + \eta(z-)\zeta]}\, d\xi d\eta$$

Again forom (3.10) and (3.11)

$$G(x-\sigma, z-\zeta) = [\,|x-\sigma|^2 + (z-\zeta)^2]^{\frac{2-n}{4}}$$

$$x \int_0^\infty \frac{\tilde{\rho}^{\frac{1}{2}(n+4\alpha-2)}}{J_{\alpha-\frac{1}{2}}(\tilde{\rho}^2)}\, J_{\frac{n}{2}-1}\{\tilde{\rho}\,[\,|x-\sigma|^2 + (z-\zeta)^2]^{1/2}\}\, d\tilde{\rho},$$

where

$$|x-\sigma|^2 + (z-\zeta)^2 = (x_1-\sigma_1)^2 + \ldots + (x_{n-1} - \sigma_{n-1})^2 + (z-\zeta)^2.$$

Hence the solution of boundary value problem (2.4), (2.5), (2.6) is,

$$W(x,y,z) = (2\pi)^{-n} \int_{\mathcal{R}^{2n}} \left\{ \int_0^\infty E\,[\tilde{\rho}, |s-\sigma|^2 + (t-\zeta)^2] \right.$$

$$\left. x\ M(x-s,y,z-t)\ f(\sigma, \zeta)\, d\tilde{\rho} \right\} ds\, d\sigma,$$

where

$$E\,[\tilde{\rho}, |s-\sigma|^2 + (t-\zeta)^2] = \frac{\tilde{\rho}^{\frac{1}{2}(n+4\alpha-2)}\, [\,|s-\sigma|^2 + (t-\zeta)^2]^{\frac{2-n}{4}}}{J_{\alpha-\frac{1}{2}}(\tilde{\rho}^2)}$$

$$x\ J_{\frac{n}{2}-1}\{\tilde{\rho}\ [\,|s-\sigma|^2 + (t-\zeta)^2]^{1/2}\}.$$

ÖZET

Bu makalede hiperbolik türden olan tekil katsayılı bir, denklem sınıfı incelenmiştir. $(x_1, \ldots, x_{n-1}, y) \in \mathcal{R}^n$ de $|x| < y$ bölgesinde ve $(x_1, \ldots, x_{n-1}, y, z) \in \mathcal{R}^{n+1}$ de $(|x|^2 + z^2)^{\frac{1}{2}} < y$ bölgesinde çok boyutlu Fourier dönüşümü kullanılarak tekil sınırı kapsayan sınır değer problemleri çözülmüştür.

REFERENCES

Erdelyı, A., Higher Transcendental Fuctions. Vol. II Mc Graw Hill, 1952.

2. **Sneddon I,N.,** A relation between the solution of the halfspace Dirichlet problems for Helmholtz'sequation in $\mathcal{R}^n$ and Laplace's equation in $\mathcal{R}^{n+1}$. J. of Engineering Math. Vol. 8, 1974, 177–180.

3. **Sneddon I.N.,** Fourier Transforms, McGraw Hill, 1951.

4. **Watson G.N.,** Theory of Bessel Functions, (Cambridge University Press), 1962.