COMMUNICATIONS

DE LA FACULTÉ DES SCIENCES
DE L'UNIVERSITÉ D'ANKARA

Série A_1 : Mathématiques

TOME : 33 ANNÉE : 1984

Some Common Fixed Point Theorems In Uniform Spaces

by

M.S. KHAN and M. IMDAD

14

Faculté des Sciences de l'Université d'Ankara
Ankara, Turquie

Communications de la Faculté des Sciences de l'Université d'Ankara

La Revue "Communications de la Faculté des Sciences de l'Université d'Ankara" est un organe de publication englobant toutes les diciplines scientifique représentées à la Faculté des Sciences de l'Université d'Ankara.

LaĞeuve, jusqu'à 1975 àl'exception des tomes I, II, III etait composé de trois séries

Série A: Mathématiques, Physique et Astronomie,
Série B: Chimie,
Série C: Sciences Naturelles.

A partir de 1975 la Revue comprend sept séries:

Série A_1: Mathématiques,
Série A_2: Physique,
Série A_3: Astronomie,
Série B: Chimie,
Série C_1: Géologie,
Série C_2: Botanique,
Série C_3: Zoologie.

A partir de 1983 les séries de C_2 Botanique et C_3 Zoologie on été réunies sous la seule série Biologie C et les numéros de Tome commencerons par le numéro 1.

En principle, la Revue est réservée aux mémories originaux des membres de la Faculté des Sciences de l'Université d'Ankara. Elle accepte cependant, dans la mesure de la place disponible les communications des auteurs étrangers. Les langues Allemande, Anglaise et Française serint acceptées indifféremment. Tout article doit être accompagnés d'un resume.

Les article soumis pour publications doivent être remis en trois exemplaires dactylographiés et ne pas dépasser 25 pages des Communications, les dessins et figuers portes sur les feulles séparées devant pouvoir être reproduits sans modifications.

Les auteurs reçoivent 25 extrais sans couverture.

l'Adresse : Dergi Yayın Sekreteri,
Ankara Üniversitesi,
Fen Fakültesi,
Beşevler—Ankara
TURQUIE

Some Common Fixed Point Theorems In Uniform Spaces

M.S. KHAN and M. IMDAD

(Received September 12, 1983; accepted January 30, 1984)

Dept. of Mathematics, Aligarh Muslim Univ. Aligarh 202001, INDIA

ABSTRACT

Some results on common fixed points for a class of mappings defined on a sequentially complete Hausdorff uniform space have been obtained. Our work extends fixed point theorems due to Hardy-Rogers, Jungck, and Acharya. Convergence theorems are also established.

1. INTRODUCTION

Let (X, d) be a metric space. A mapping $S : X \longrightarrow X$ is called a a contraction mapping if

$$d(Sx, Sy) \leq k\, d(x,y),$$

where $x,y, \in X$ and $k \in (0,1)$. The well-known Banach Contraction Principle says that every contraction mapping on a complete metric space has a unique fixed point. A number of extensions and generalizations of this celebrated theorem have appeared in recent years.

It may be observed that a fixed point of a mapping $S : X \longrightarrow X$ is clearly a common fixed point of S and the identity mapping I_X on X. Motivated by this fact, Jungck [5] obtained the following extension of Banach Contraction Principle replacing I_X by a continuous mapping $T : X \longrightarrow X$.

Theorem A (Jungck [5]). Let T be a continuous mapping of a complete metric space (X, d) into itself. Then T has a fixed point in X if and only if there exists $k \in (0, 1)$ and a mapping $S : X \longrightarrow X$ which commutes with T and satisfies $S(x) \subset T(X)$ and (J) —— $d(Sx, Sy) \leq k\, d(Tx, Ty)$, for all $x,y \in X$. Indeed, S and T have a unique common fixed point Evidently, if $T = I_X$,Jungck's theorem reduces to that of Banach. Here it is also worth noting that the continuty of the mapping S is a consequence of the condition (J) and was used in the proof of Theorem A.

Further extensions, generalizations and applications of Jungck's theorem have been obtained by Kasahara [7], Meade and Singh [11], Park [12], [13], [14]; Park and Park [15].

It was shown in Kalisch ([6], p. 937) that the topology of every Hausdorff Uniform space X can be described completely in terms of convergence of nets in X with respect to a certain Kalisch metric. This led Reich [16] to observe that the condition

$$d(Tx, Ty) \leq ad(x, Tx) + bd(y, Ty) + cd(x, y),$$

where $x,y \in X$, a,b,c are non-negative, and $a+b+c < 1$, ensures the existence of a unique fixed point of $T : X \longrightarrow X$ where X is a sequentially complete Hausdorff Uniform space and d is the Kalisch metric associated with it.

Reich [16] also suggested the problem of formulating and proving a similar theorem using only the members of the uniformity, without appealing to a generalized distance (or to pseudo-metrics).

In this paper, we shall extend Jungck's Theorem to uniform spaces, when the involved mappings satisfy certain conditions in terms of members of the uniformity only. The contractive condition we use is actually patterned after the notion of generalized contractions due to Ćirić [2].

2. PRELIMINARIES.

Throughout this paper, (X,U) stands for a sequentially complete Hausdorff uniform space. Let P be a fixed family of pseudo-metrics on X which generates the uniformity U. Following Kelley ([8], Chapter 6), we let

(i) $V(\rho,r) = \{(x,y) : x,y \in X, \rho(x,y) < r, r > 0\}$

(ii) $G = \{V : V = \bigcup_{i=1}^{n} V(\rho_i,r_i), \rho_i \in P, r_i > 0, i = 1,2 \text{ —— } n\}$,

(iii) For $\alpha > 0$,

$$\alpha V = \{\bigcup_{i=1}^{n} V(\rho_i, \alpha r_i) : \rho_i \in P, r_i > 0, i = 1,2 \text{ —— } n\}.$$

The following results are taken from Acharya [1].

Lemma 2.1. If $V \in G$ and $\alpha, \rho > 0$, then

(a) $\alpha(\beta v) = (\alpha\beta) v$,

(b) $\alpha V o \beta V \subset (\alpha + \beta) V$,

(c) $\alpha V \subset \beta V$ for $\alpha < \beta$.

Lemma 2.2. Let ρ be any pseudo metric on X and $\alpha, \beta > 0$. If $(x,y) \in \alpha V(\rho, r_1) o \beta V \rho(, r_2)$ then $\rho(x_1 y) < \alpha r_1 + \beta r_2$.

Lemma 2.3. If $x,y \in X$, then for every V in G there is positive number λ such that $(x,y) \in \lambda V$.

Lemma 2.4. For an arbitrary $V \in G$ there is pseudo metric ρ on X such that $V = V(\rho, 1)$.

The pseudo-metric ρ of Lemma 2.4 is called the Minkowski pseudo-metric corresponding to V.

3. RESULTS ON COMMON FIXED POINTS.

The following lemma is the key in proving our main result. Its proof is similar to that of Jungck ([5]).

Lemma 3.1. Let $\{y_n\}$ be a sequence in a complete pseudo-metric space (X, ρ). If there exists a $k \in (0, 1)$ such that $\rho(y_{n+1}, y_n) \leq k\rho(y_n, y_{n-1})$ for all n, then $\{y_n\}$ converges to a point in X.

Theorem 3.2. Let A, S and T be self-mappings of X such that the following hold:

(i) $A(x) \subset S(x) \cap T(x)$;

(ii) $SA = AS$, $AT = TA$,;

(iii) S and T are continuous;

(iv) Let $V_i \in G$ $(i = 1,2,3,4,5)$ and $x,y \in X$. Further suppose

that $(Sx, Ax) \in V_1$, $(Ty, Ay) \in V_2$, $(Sx, Ay) \in V_3$, $(Ty, Ax) \in V_4$ and $(Sx, Ty) \in V_5$ implies that

$$(*) \ldots\ldots \quad (Ax, Ay) \in \alpha_1 V_1 o \alpha_2 V_2 o \alpha_3 V_3 o \alpha_4 V_4 o \alpha_5 V_5,$$

where $\alpha_i \geq 0$ $(i = 1,2,3,4,5)$, $\sum_{i=1}^{5} \alpha_i < 1$, $\alpha_3 = \alpha_4$.

Then A,S and T have a unique common fixed point.

Proof. Let $V \in G$ be arbitrary and ρ the Minkowski pseudometric of V. For $x,y \in X$, let us take $\rho(Sx, Ax) = r_1$, $\rho(Ty, Ay) = r_2$, $\rho(Sx, Ay) = r_3$, $\rho(Ty, Ax) = r_4$ and $\rho(Sx, Ty) = r_5$. Take $\varepsilon > 0$. Then $(Sx, Ax) \in (r_1 + \varepsilon) V$, $(Ty, Ay) \in (r_2, + \varepsilon) V$, $(Sx, Ay) \in (r_3 + \varepsilon) V$, $(Ty, Ax) \in (r_4 + \varepsilon) V$ and $(Sx, Ty) \in (r_5 + \varepsilon) V$. Therefore, by (*), we have.

$(Ax, Ay) \in \alpha_1(r_1 + \varepsilon) V_0\alpha_2 (r_2 + \varepsilon) V_0\alpha_3 (r_3 + \varepsilon) V_0\alpha_4 (r_4 + \varepsilon) Vo\alpha_5 (r_5 + \varepsilon) V$.

Using Lemma 2.1 (a), Lemma 2.2 and Lemma 2.3, we get

$$\begin{aligned}\rho(Ax, Ay) &< \alpha_1(r_1 + \varepsilon) + \alpha_2(r_2 + \varepsilon) + \alpha_3 (r_3 + \varepsilon) + \alpha_4(r_4 + \varepsilon) \\ &+ \alpha_5(r_5 + \varepsilon) = \alpha_1 \rho(Sx, Ax) + \alpha_2 \rho(Ty, Ay) + \alpha_3 \rho(Sx, \\ &\quad Ay) + \alpha_4 \rho(Ty, Ax) + \alpha_5 \rho(Sx, Ty) + \\ &+ \left(\sum_{i=1}^{5} \alpha_i\right) \varepsilon.\end{aligned}$$

Since ε is arbitrary, we have

$$(^{**})\ldots\ \rho(Ax, Ay) \leq \alpha_1 \rho(Sx, Ax) + \alpha_2 \rho(Ty, Ay) + \alpha_3 \rho(Sx, Ay) + \alpha_4 \rho(Ty, Ax) + \alpha_5 \rho(Sx, Ty).$$

Let x_0 be an arbitrary point of X. Since A(X) is contained in S (X), we can always pick up a point x_1 in X such that $Sx_1 = Ax_0$. Further, as A (X) is contained in T (X), we can select a point x_2 in X satisfying $T x_2 = Ax_1$. So, in general, $Sx_n = Ax_{n-1}$, when n is odd, and $Tx_n = Ax_{n-1}$, when n is even.

Now, using (**), it is easy to see that

$$\rho(A_{x2n+1}, Ax_{2n}) \leq \left(\frac{\alpha_2 + \alpha_4 + \alpha_5}{1 - \alpha_1 - \alpha_4}\right) \rho(Ax_{2n}, Ax_{2n-1}).$$

Therefore, by Lemma 3.1, $\{Ax_n\}$ converges to some $Z \in X$. Since sequences $\{Sx_{2n+1}\}$. and $\{Tx_{2n}\}$ are subsequences of $\{Ax_n\}$, they have the same limit z.

Then continuity of S and T imply that $SAx_n \to Sz$ and $TAx_n \to Tz$. Since S and T commute with A, we conclude that $ASx_n \to Sz$ and

$ATx_n \to Tz$. Furthermore, we also find that $SSx_{2n+1} \to Sz$, $TSx_{2n+1} \to Tz$ and $STx_{2n} \to Sz$. We now claim that $ASx_{2n+1} \to Az$. To do this, we observe that

$$\begin{aligned}\rho(ASx_{2n+1}, Az) &\le \alpha_1 \rho(SSx_{2n+1}, ASx_{2n+1}) + \alpha_2 \rho(Tz, Az) + \alpha_3 \\ &\quad \rho(SSx_{2n+1}, Az) + \alpha_4 \rho(Tz, Asx_{2n+1}) + \alpha_5 \rho(SSx_{2n+1}, Tz) \\ &\le \alpha_1 \rho(SSx_{2n+1}, Sz) + \alpha_1 \rho(Sz, ASx_{2n+1}) \\ &\quad + \alpha_2 \rho(Tz, STx_{2n}) + \alpha_2 \rho(STx_{2n}, Sz) + \\ &\quad \alpha_2 \rho(Sz, ASz_{2n+1}) + \alpha_2 \rho(ASx_{2n+1}, Az) \\ &\quad + \alpha_3 \rho(SSx_{2n+1}, Sz) + \alpha_3 \rho(Sz, Azx_{2n+1}) \\ &\quad + \alpha_3 \rho(ASx_{2n+1}, Az) + \alpha_4 \rho(Tz, ST x_{2n}) \\ &\quad + \alpha_4 \rho(STx_{2n}, Sz) + \alpha_4 \rho(Sz, ASx_{2n+1}) \\ &\quad + \alpha_5 \rho(SSx_{2n+1}. Sz) + \alpha_5 \rho(Sz, STx_{2n}) + \alpha_5 \rho(STx_{2n}, Tz).\end{aligned}$$

Therefore,

$$\begin{aligned}(1-\alpha_2-\alpha_3)\rho(ASx_{2n+1}, Az) \le{}& (\alpha_1 + \alpha_3 + \alpha_5)\rho(SSx_{2n+1}, Sz) \\ &+ (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4)\rho(Sz, ASx_{2n+1}) \\ &+ (\alpha_2 + \alpha_4 + \alpha_5)\rho(Sz, STx_{2n}) \\ &+ (\alpha_2 + \alpha_4 + \alpha_5)\rho(Tz, STx_{2n}),\end{aligned}$$

which implies that $ASx_{2n+1} \to Az$ as $n \to \infty$. Hence $Sz = Az$. Similarly, one can prove that $Tz = Az$. Using the commutativity of S and T with A, it follows that $TTz = TSz = SSz = SAz = ASz = AAz$ we also have

$$\begin{aligned}\rho(Az, AAz) &\le \rho(Az, ASx_{2n+1}) + \rho(ASx_{2n+1}, AAz) \\ &\le \rho(Az, ASx_{2n+1}) + \alpha_1\rho(SSx_{2n+1}, ASx_{2n+1}) \\ &\quad + \alpha_2 \rho(TAz, AAz) + \alpha_3 \rho(SSx_{2n+1}, AAz) \\ &\quad + \alpha_4 \rho(TAz, ASx_{2n+1}) + \alpha_5 \rho(SSx_{2n+1}, TAz) \\ &\le \rho(Az, ASx_{2n+1}) + \alpha_1 \rho(SSx_{2n+1}, Sz) \\ &\quad + \alpha_1 \rho(Az, ASx_{2n+1}) + \alpha_3 \rho(SSx_{2n+1}, Sz) \\ &\quad + \alpha_3 \rho(Az, AAz) + \alpha_4 \rho(AAz, Az) \\ &\quad + \alpha_4 \rho(Az, ASx_{2n+1}) + \alpha_5 \rho(SSx_{2n+1}, Sz) \\ &\quad + \alpha_5 \rho(Az, AAz).\end{aligned}$$

Thus last inequality yields

$$(1-\alpha_3-\alpha_4-\alpha_5)\,\rho\,(Az, AAz) \leq (1+\alpha_1+\alpha_4)\,\rho\,(Az, ASx_{2n+1}) + (\alpha_1+\alpha_3+\alpha_5)\,\rho\,(SSx_{2n+1}, Sz).$$

Letting $n \to \infty$, we get $\rho\,(Az, AAz) = 0$. This means that $(Az, AAz) \in V$. As X is Hausforrf and V is arbitrary, it follows that $AAz = Az$. Exactly in the same way, one can prove that $SSz = Sz$ and $TTz = Tz$. Now $Sz = Tz = Az$ implies that S. T and A have a common fixed point. To prove the unicity of common fixed points of A and S, suppose that z_1 and z_2 are two distinct common fixed points of A and S. Choose any $V \in G$. Then $(Az_1, Sz_1) = (z_1. z_1) \in V$ and $(Az_2, Sz_2) = (z_2, z_2) \in V$. By Lemma 2.1 (b) and Lemma 2.1 (c), this implies that $(z_1. z_2) \in (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5) V \subset V$. As V is arbitrary, we get $z_1 = z_2$. Exactly. with same repeated arguments one can prove that S and T have a unique common fixed point. Now combining the unicity of A and S together with S and T, we at once conclude that A, S and T have a unique common fixed point. This completes the proof.

Remarks (i) The requirement $\alpha_3 = \alpha_4$ in the statement of Theorem 3.2 is also a part of the definition of generalized contraction studied by Ciric [2].

(ii) We can replace constants α_i by functions α_i (x, y) with Sup

$$\left(\sum_{\substack{i=1 \\ x,y,\in X}}^{5} \alpha_i\right) < 1.$$

(iii) Theorem 3.2 refines the result of Khan and Fisher [10].

As consequences of Theorem 3.2, we derive the following results.

Corollary 3.3 Let A and T be two commuting self-mappings of X such that T is continuous, $A(X) \subset T(X)$ and

$V_i \in G$ $(i = 1,2,3,4,5)$; $x,y \in X$, $(Tx, Ax) \in V_1$, $(Ty, Ay) \in V_2$ $(Tx, Ay) \in V_3$, $(Ty, Ax) \in V_4$ and $(Tx, Ty) \in V_5$ implies

$(Ax, Ay) \in \alpha_1 V_1 o \alpha_2 V_2 o \alpha_3 V_3 o \alpha_4 V_4 o \alpha_5 V_5$,

where $\alpha_i \geq 0$ $(i = 1,2,3,4,5)$ $\sum_{i=1}^{5} \alpha_i < 1$, $\alpha_3 = \alpha_4$.

Then A and T have a unique common fixed point.

Remark: Corollary 3.3 is proved by Khan [9].

Corollary 3.4. Let A be a self-mapping on X such that for $V_i \in G$ (i = 1,2,3,4,5) x,y $\in$ X, (x, Ax) $\in V_1$, (y, Ay) $\in V_2$, (x, Ay) $\in V_3$, (y, Ax) $\in V_4$ and (x, y) $\in V_5$, we have

$$(Ax, Ay) \in \alpha_1 V_1 o \alpha_2 V_2 o \alpha_3 V_3 o \alpha_4 V_4 o \alpha_5 V_5,$$

where $\alpha_i \geq 0$, $\sum_{i=1}^{5} \alpha_i < 1$, $\alpha_3 = \alpha_4$. Then S has a unique fixed pcint.

Remark. Corollary 3.4 is the uniform space version of a fixed point theorem due to Hardy and Rogers [3].

Corollary 3.5. Let T be a continuous mapping of X into itself Then T has a fixed point in X if there exists a real number k $\in$ (0,1) and a mapping A : X $\rightarrow$ X which commutes with T satisfying

(a) A (X) $\subset$ T(X)

(b) (Ax, Ay) $\in$ kV if (Tx, Ty) $\in$ V for all x,y in X and V $\in$ G. Indeed,

S and T have a unique common fixed point.

Remark. Corollary 3.5 may be regarded as an extension of Jungck's Theorem to uniform spaces.

Corollary 3.6. (Acharya [1], Theorem 3.1). Let A be a self-mapping on X such that for any V $\in$ G and x,y in X

$$(Ax, Ay) \in kV \text{ if } (x,y) \in V,$$

where $0 < k < 1$, Then A has a unique fixed point in X.

Next result extends a result of Iseki, [4] to uniform spaces.

Theorem 3.7. Let $\{T_n\}$ be a sequence of self-mappings of X satisfying: for any $V_i \in G$ (i = 1,2,3,4,5) and x,y $\in$ X, (x,y) $\in V_1$, (x, T_ix) $\in V_2$, (x, T_jy) $\in V_3$, (y, T_ix) $\in V_4$, and (y T_jy) $\in V_5$ implies (T_ix T_jy) $\in \alpha_1 V_1$ $o \alpha_2 V_2 o \alpha_3 V_3 o \alpha_4 V_4 o \alpha_5 V_5$, where $\alpha_i > 0$, (i = 1,2,3,4,5), $\sum_{i=1}^{5} \alpha_i < 1$, $\alpha_3 = \alpha_4$.

Then $\{T_n\}$ have a unique common fixed point.

Proof. Following the argument of Theorem 3.2, one can prove

$$\rho(T_i x\ T_j y) \leq \alpha_1 \rho(x,y) + \alpha_2 \rho(x, T_i x) + \alpha_3 \rho(x, T_j y) + \alpha_4 \rho(y, T_i x) + \alpha_5 \rho(y, T_j y).$$

Let $x_0 \in X$ be an arbitrary point. Now define a sequence $\{x_n\}$ by putting $x_n = T_n x_{n-1}$, $(n = 1,2 \ldots)$.

By a routine calculation, we easily conclude that sequence $\{x_n\}$ converges to some point z in X. For the point z

$$\begin{aligned}\rho(z, T_n z) &\leq \rho(z, x_{m+1}) + \rho(T_{m+1} x_m, T_n z)\\ &\leq \rho(z, x_{m+1}) + \alpha_1 \rho(x_m, z) + \alpha_2 \rho(x_m, T_{m+1} x_m)\\ &\quad + \alpha_3 \rho(x_m, T_n z) + \alpha_4 \rho(z, T_{m+1} x_m) + \alpha_5 \rho(z, T_n z).\end{aligned}$$

Letting $m \to \infty$, we get

$\rho(z, T_n z) \leq (\alpha_3 + \alpha_5) \rho(z, T_n z)$ a contradiction and hence $\rho(z, T_n z) = 0$. This means that $(z, T_n z) \in V$. As X is Hausdorff and V is arbitrary, it follows that $T_n z = z$ is a common fixed point of all T_n. Uniqueness of common fixed point follows easily.

4. STABILITY OF FIXED POINTS

Now we wish to discuss the convergence of sequences of mappings and their fixed points in uniform spaces.

Theorem 4.1. Let $\{A_n\}$, $\{S_n\}$ and $\{T_n\}$ be sequences of self-mappings on X converging uniformly to self-mappings A, S and T on X, respectively. Suppose that for each $n \geq 1$, x_n is a common fixed point of A_n and S_n, and y_n is a common fixed point of A_n and T_n. Further, let A, S and T satisfy condition (IV) of Theorem 3.2. If x_0 is the common fixed point of A.S and T, then $x_n \to x_0$ and $y_n \to x_0$.

Proof. Let $V \in G$ be arbitrary with corresponding Minkowski pseudo metric ρ. Since $A_n \to A$ uniformly, there exists a positive integer N_1 such that for all $n \geq N_1$, $(A_n x_n, Ax_n) \in V$ for all x_n. Also $S_n \to S$ uniformly, therefore as earlier $(S_n x_n, Sx_n) \in V$, for all $n \geq N_2$.

Now

$$\begin{aligned}\rho(x_n, x_0) &\leq \rho(x_n, Ax_n) + \rho(Ax_n, Ax_0)\\ &\leq \rho(A_n X_n, Ax_n) + \alpha_1 \rho(Sx_n, Ax_n) = \alpha_2 \rho(Tx_0, Ax_0)\\ &\quad + \alpha_3 \rho(Sx_n, Ax_0) + \alpha_4 \rho(Tx_0, Ax_n) + \alpha_5 \rho(Sx_n, Tx_0)\end{aligned}$$

$$\leq \rho(A_n x_n, Ax_n) + \alpha_1 \rho(Sx_n, S_n x_n) + \alpha_1 \rho(A_n x_n, Ax_n) + \alpha_3 \rho(Sx_n, S_n x_n) + \alpha_3 \rho(x_n, x_0) + \alpha_4 \rho(x_0, x_n) + \alpha_4 \rho(A_n x_n, Ax_n) + \alpha_5 \rho(Sx_n, S_n x_n) + \alpha_5 \rho(x_n, x_0).$$

Therefore,

$$\rho(x_n, x_0) \leq \left(\frac{1+\alpha_1+\alpha_4}{1-\alpha_3-\alpha_4-\alpha_5}\right) \rho(A_n x_n, Ax_n) + \left(\frac{\alpha_1+\alpha_3+\alpha_5}{1-\alpha_3-\alpha_4-\alpha_5}\right) \rho(S_n x_n, Sx_n)$$

$$\leq \left(\frac{1+2\alpha_1+\alpha_3+\alpha_4+\alpha_5}{1-\alpha_3-\alpha_4-\alpha_5}\right) \max\{\rho(A_n x_n, Ax_n), \rho(S_n x_n, Sx_n)\}$$

So that $(x_n, x_0) \in \left(\frac{1+2\alpha_1+\alpha_3+\alpha_4+\alpha_5}{1-\alpha_3-\alpha_4-\alpha_5}\right) V$ for all $n \geq N = \max\{N_1, N_2\}$.

Since V is arbitrary $x_n \to x_0$. Similarly, we can show that $y_n \to x_0$. This completes the proof.

From Theorem 4.1 we can derive the following result.

Theorem 4.2. Let $\{A_n\}$, $\{S_n\}$ and $\{T_n\}$ be the sequences of self-mappings whose uniform limits are A, S and T, respectively. If A, S and T satisfy condition (iv) of Theorem 3.2, then the sequence $\{x_n\}$ of unique common fixed points of A_n, S_n and T_n converges to the unique common fixed point x_0 of A, S and T.

ACKNOWLEDGEMENT.

Work of the second author was supported by a research grant from C.S.I.R., New Delhi, India.

REFERENCES

1. **S.P. Acharya,** Some results on fixed points in uniform spaces, Yokohama Math. J., 22 (1-2) (1974), 105-116.

2. **Lj. Ciric,** Generalized contractions and fixed point theorems, Publ. Inst. Math., 12 (66) (1971), 19-26.

3. **G.E. Hardy and T.D. Rogers,** A generalization of a fixed point theorem of Simeon Reich, Canad. Math. Bull., 16 (2) (1973), 201-206.

4. **K. Iseki,** On common fixed points of mappings, Bull. Austral. Math. Soc., 10 (3) (1974), 365-370.

5. **G. Jungck,** Commuting mappings and fixed points, Amer. Math. Monthly. 83 (1976), 261-263.

6. **G.K. Kalisch,** On uniform spaces and topological algebras, Bull. Amer. Math. Soc., 52 (1946), 936-939.

7. **S. Kasahara,** Iff fixed point criterions in L-spaces, Math. Seminar Notes 4 (1976), 205-210.

8. **J.L. Kelley,** General Topology (1955), D. Van Nostrand Co.

9. **L.S. Khan,** Commuting mappings and fixed points in uniform spaces, Bull. Acad. Polon. Sci., Ser Sci. Math. Astr. Phys. (29), 1981, 499-507.

10. **M.S. Khan,** and B. Fisher, Common fixed points in uniform spaces, Math. Seminar Notes, 10 (1982), 373-377.

11. **B.A. Meade and S.P. Singh,** An extension of a fixed point theorem, Maths. Seminar Notes, 5 (1977), 343-346.

12. **S. Park,** A generalization of a theorem of Janos and Edelstein, Proc. Amer. Math. Soc., 66 (2) (1977), 344-346.

13. ——— Fixed points of f-contractive maps, Rocky Mountain J. Math. 8(4) (1978), 743-750.

14. **S. Park,** Fixed point of contractive maps on compact metric spaces, Bull. Korean Math. Soc., 14 (1) (1977), 33-37.

15. **S. Park and K. Park,** Generalized f-contractions and fixed point theorems J. Korean Math. Soc., 14 (1) (1977), 127-133.

16. **S. Reich,** Fixed points of contractive functions, Boll. Un. Mat. Ital., 5 (1972), 26-42.