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Araştırma Makalesi / Research Article

Matematiksel Modelleme Etkinliklerine Dayalı Öğrenme Ortamının İncelenmesi

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Özet – Bu araştırma, matematiksel modelleme etkinliklerine dayalı öğrenme ortamında öğrencilerin matematiksel bilgilerindeki değişime ilişkin sonuçlar sunmaktadır. Yedinci sınıfta öğrenim gören 6 öğrenciyle (13 yaş) yürütülen araştırmada, alan ölçme konusunun kazanımlarına yönelik olarak hazırlanan 8 adet matematiksel modelleme etkinliği uygulamaları, video ve ses kayıtları, öğrenci çözüm raporları ve araştırmacı notları aracılığıyla incelenmiştir. Uygulama sürecindeki verileri desteklemek amacıyla, öncesinde ve sonrasında öğrencilerle bireysel görüşmeler gerçekleştirilmiştir. Araştırmada elde edilen sonuçlar, matematiksel modelleme yöntemiyle yapılan öğretimin öğrencilerin alan ölçme bilgi ve becerilerini önemli ölçüde desteklediği yönündedir. Söz konusu gelişimin, matematiksel modelleme sürecinde ortaya çıkan öğrenme fırsatları yoluyla desteklendiği sonucuna ulaşılmıştır. Öğrencilerin alan ölçme bilgi ve becerilerinin gelişiminde birim kare kavramının oluşması ve alan ölçme bağıntısının birim kareyle ilişkilendirilerek açıklanması, bir lokomotif etkisi oluşturmuştur. Sonuçlar, matematiksel modelleme uygulamalarının öğretim programında yer alması gerektiğini gösterir niteliktedir.

Anahtar kelimeler: alan ölçme, matematiksel modelleme, öğrenme ortamı

Sorumlu yazar: Zeynep Çavuş Erdem, zcavuserdem@hotmail.com. Bu araştırma birinci yazarın “Matematiksel modelleme etkinliklerine dayalı öğrenim sürecinin alan ölçme konusu bağlamında incelenmesi” isimli doktora tezinden üretilmiştir. Aynı zamanda ‘4. Uluslararası Türk Bilgisayar ve Matematik Eğitimi Sempozyumu’nda sözlü bildiri olarak sunulmuş ve tam metin formatında yayınlanmıştır.

Geniş Özet

Giriş

Matematik öğretimin amacı, sadece okul ortamı değil, okul ortamının dışında da matematiği kullanabilen, uygulayabilen ve sorgulayabilen öğrencilerin yetiştirilmesini sağlamaktır (MEB, 2018). Bunu sağlamanın yollarından biri ise matematiksel modellemedir. Matematiksel modelleme, gerçek yaşamda karşılaşılan bir problem durumunun matematiksel yollarla çözüme ulaştırılıp elde edilen çözümün yorumlandığı ve değerlendirildiği bir süreçtir (Lesh ve Doerr, 2003). Matematiksel modelleme yoluyla öğrenciler, gerçek yaşamdaki matematiği keşfederek, matematiğin yaşamdan ayrı bir disiplin olmadığını, yaşamla iç içe olduğunu görme fırsatı yakalar. Matematiğe gerçek hayatta nasıl ihtiyaç duyulduğunu fark eder (Borromeo Ferri, 2018). Bu nedenle son yıllarda matematiksel modelleme uygulamalarının öğretim programlarında yer almasının önemi sıkça vurgulanmaktadır. Ortaokul matematik öğretim programında matematiksel modellemenin yer almadığı, modellemenin somutlaştırma ve görselleştirme olarak ifade edilen "matematiği modelleme" olarak yorumlandığı görülmektedir (MEB, 2018; Çavuş Erdem, Doğan, Gürbüz ve Şahin, 2017). Matematiksel modellemenin ülkemiz öğretim programlarında yer alması ve doğru bir şekilde ele alınması için matematiksel modellemenin öğrencilerin matematiksel gelişimlerine etkisini araştıran çalışmaların yapılması önem kazanmaktadır. Bu nedenle matematiksel modelleme etkinlikleri dayalı öğrenme ortamının incelendiği bu çalışmanın, bu anlamda literatüre katkı sağlayacağı düşünülmektedir.

Yöntem

Durum çalışması yöntemiyle yürütülen araştırmanın çalışma grubunu, araştırmacı tarafında kolay ulaşılabilir olması yönüyle belirlenen bir okulda öğrenim gören ve amaçlı örneklem yöntemiyle seçilen 6 tane yedinci sınıf öğrencisi oluşturmaktadır. Tüm veri toplama araçları öğretim programda yer alan, alan ölçme konusunun kazanımları dikkate alınarak geliştirilmiştir. Çalışmada, öğrencilerin bilgilerinde değişim olup olmayacağını net bir şekilde gözlemlemek için öğrencilerle uygulama öncesi ve sonrası bireysel görüşmeler gerçekleştirilmiştir. Uygulama süreci 8 adet matematiksel modelleme etkinliği ile yürütülmüştür. Verilerin analizinde gömülü teori kodlama (grounded coding) yöntemi kullanılmış ve rubrikle değerlendirme yapılmıştır. Araştırmada tüm veriler toplandıktan sonra, verilerin tamamı transkript edilmiştir. Transkriptlerin tamamlanmasının ardından kodlama sürecine geçilmiştir. Öncelikle toplanan veriler araştırmacılar tarafından ayrı ayrı kodlanmış, sonrasında kodlar karşılaştırılarak kod birliğine varılmış ve kodlama sürecine devam edilmiştir.

Bulgular

Araştırmada uygulama öncesi bazı öğrencilerde alan kavramının iki uzunluğun (en ve boy) çarpımı sonucunda elde edilen sayısal bir değer olarak oluştuğu, alan ölçmenin temel kavramlarından biri olarak birim kare kavramının oluşmadığı, bunun sonucu olarak öğrencilerin alan ölçme birimlerini tanımlamada ve birimler arası dönüşümde eksiklikleri bulunduğu belirlenmiştir. Üçgen (dik açılı üçgen hariç) ve paralelkenarın alanını hesaplarken, yanlış stratejiler kullanan ve hatalı işlemler yapan öğrencilerin, aynı zamanda, kenar uzunluğu ile alan arasındaki ilişkiyi fark edemedikleri belirlenmiştir. Uygulama sonrası öğrencilerin alan kavramını doğru bir şekilde açıkladığı ve alanı kaplama olarak ifade ettikleri belirlenmiştir. Tüm öğrencilerin çokgenlerin alanı doğru bir şekilde hesaplayabildiği, hatta kare, dikdörtgen ve üçgenin alan bağıntısını birim kare yardımıyla açıkladığı, bazı öğrencilerin ise bunlara ek olarak paralelkenar ve yamuğun alan bağıntısını da açıklayabildiği belirlenmiştir. Öğrencilerde yaşanan gelişmenin, matematiksel etkinlikleriyle meşgul olurken gerçekleştiği ve bu gelişmenin bilgilerin hatırlanması (çağrılması), modelleme sürecinde ortaya çıkan öğrenme fırsatları yoluyla değerlendirilmesi ve bunun sonucu olarak bilginin değiştirilmesi veya pekiştirilmesi şeklinde gerçekleştiği bulgularına ulaşılmıştır.

Tartışma, Sonuç ve Öneriler

Araştırmada matematiksel modellemeye dayalı öğrenme ortamında, öğrencilerin öğrenme fırsatları yoluyla gelişme sağladığı sonucuna ulaşılmıştır. Öğrencilerin bilgilerini değerlendirmesine ortam sağlayan öğrenme fırsatları bireysel keşifler, akran iş birliği veya rehberliği ya da öğretmen rehberliği şeklindedir. Öğrencinin dışardan hiç biri müdahale olmadan sadece model oluşturma sürecinde keşfettiği matematiksel anlayışlar araştırmada bireysel keşif olarak ifade edilmiştir. Lesh ve Doerr (2003), öğrencilerin güçlü, paylaşılabilir modeller oluştururken kavramsal yapılarını farkında olamadan gözden geçirdiğini ifade etmektedir. Öğrencilerin bilgileri sınamalarındaki bir diğer etken akran faktörüdür. Matematiksel modelleme etkinliklerinde, grup içinde öğrencilerin bilgileri sunması ve karşısındaki bireyin bilgilerine ulaşması, model oluşumu için bilginin grup üyelerinin onayından geçerek kullanılması doğal bir karşılaştırma ve değerlendirme sürecini barındırır. Akran değerlendirmesi olarak ifade edebileceğimiz bu durum öğrenciye rehberlik etmesi konusunda oldukça önemlidir (Lesh ve Harel, 2003). Bu çalışmada da grup üyelerinin, model oluşturma sürecindeki açıklamaları ve modeli değerlendirme basamağında diğer grup üyelerinin açıklamalarının öğrencilerin öğrenmelerinde etkileyici olduğu tespit edilmiştir. Çalışma bulguları ayrıca öğrencilerin gelişimlerini ve öğrenmelerini destekleyen bir diğer faktörün öğretmen rehberliği olduğunu göstermiştir. Modelleme etkinliklerinde öğretmen, bilgiyi aktaran bir kaynak olmanın ötesinde, bir rehber görevi üstlenmektedir (Dunne ve Galbraith, 2003; Ärlebäck, Doerr ve O'Neil, 2013). Modelleme etkinlikleri, gerek geleneksel olmayan çözümler gerektirdiğinden, gerek çözüm esnasında ortaya çıkan öğrenci diyaloglarından dolayı, öğrencilerin hem matematiksel bilgileri hem de matematik anlayışları hakkında derinlemesine bilgi çıkarmaya destektir (Brown ve Edwards, 2011). Burada öğretmenin doğru zamanda doğru sorularla öğrenci açıklamalarına ulaşması, derin bir değerlendirmenin yanı sıra öğrencilerin bilgilerini gözden geçirmelerine ve hatalarını fark etmelerine olanak tanımaktadır. Çalışmada öğrencilerin durumu gözlemleyen ve gerektiğinde kritik sorularla öğrencilerin muhakemelerini harekete geçiren araştırmacı, benzer bir öğretmen rolü üstlenmiştir. Araştırmacının öğrencilere yönlendirdiği kritik sorular, hem öğrencilerin düşüncelerini daha derin bir şekilde açığa çıkarmaya ve öğrenmenin ne düzeyde oluştuğunu belirmeye yardımcı olmuş, hem de etkinlikte fark edilmeyen noktaları ve matematiksel ilişkileri görmede tetikleyici bir rol üstlenmiştir.

Çalışmadan elde edilen sonuçlar matematiksel modelleme etkinliklerine dayalı öğrenme ortamının öğrencilerin öğrenmeleri destekleyici fırsatlar sunduğunu göstermiştir. Matematiksel modellemenin sağladığı katkıdan faydalanabilmek ve uygulamaları öğrenme ortamına taşıyabilmek için, öğretim programında matematiksel modelleme uygulamalarına yer verilmesi önemlidir. Bu anlamda matematiksel modellemenin öğrenmeye etkisini araştıran çalışmaların, bununla birlikte modelleme sürecinde öğrenmenin nasıl gerçekleştiği, matematiksel kavramların oluşumunu nasıl etkilediğiyle ilgili araştırmaların artırılması önem kazanmaktadır. Bu nedenle araştırma sonuçları dikkate alınarak, matematiksel modellemenin matematik öğrenimindeki etkisini araştıran çalışmaların artırılması ve öğretim programında modelleme uygulamalarına yer verilmesi gerektiği önerilebilir.

Investigation of Learning Environment Based on Mathematical Modelling Activities

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Abstract – This study aimed to examine change in students' mathematical knowledge in a learning environment based on mathematical modeling activities. The study conducted with six students (13-year-old) and examined the seventh graders' eight mathematical modeling activities prepared for the acquisition of the area measurement subject through applications, video and sound recordings, student solution reports, and researcher notes. To support the data in the implementation process, individual interviews were held with the students. The findings of the study revealed that teaching through mathematical modeling method supports students' area measurement knowledge and skills significantly. This development is supported through learning opportunities that arise in the mathematical modeling process. The formation of the unit square concept in the development of students' area measurement knowledge and skills and the explanation of the area measurement relation by associating it with the unit square created a locomotive effect. The findings of the study showed that mathematical modeling applications should be included in the curriculum.

Key words: area measurement, learning environment, mathematical modeling.

Corresponding author: Zeynep Çavuş Erdem, zcavuserdem@hotmail.com. This paper, has been produced from the first author's doctoral dissertation titled "Investigation of the learning process based on mathematical modeling activities in the context of area measurement".The paper was previously presented orally in the "4th International Symposium of Turkısh computer and Mathematics Education" and and it has been published in the proceedings.

Introduction

The purpose of teaching mathematics is to train students who can use, apply, and question mathematics in and out of school environment (Ministry of National Education [MEB]), 2018). One of the way to achieve this aim is mathematical modeling. In its most general form, mathematical modeling can be defined as the process of solving real-life problems using mathematics (Niss, Blum & Galbraith,2007). In this process, which involves creating a solution using mathematical structures and operations and evaluating the functionality of the solution, an individual can see the opportunity to comprehend that mathematics is not a discipline separate from life but is intertwined with life itself, and realizes how mathematics is needed in real life (Borromeo Ferri, 2018). Therefore, the importance of mathematical modeling applications has frequently been emphasized in the curriculum, international reports recently and research articles (National Council of Teachers of Mathematics [NCTM], 2000; Common Core State Standards for Mathematics [CCSM], 2011, Kaiser, 2020). It is also seen that mathematical modeling is handled as concrete material usage in some curricula and textbooks (Cavus Erdem, Dogan, Gurbuz, Sahin, 2017). However, mathematical modeling consists of a much more comprehensive structure. Mathematical modeling needs to be handled correctly in the curriculum. Because the goals in mathematics education programs can be achieved through mathematical modeling, it is expressed in modeling studies (Cavus Erdem, 2018). Besides, it is emphasized in studies that mathematical modeling has effect on the creation and deepening of mathematical concepts and information (Blum & Borromeo Ferri, 2009; Dogan, Gurbuz, Cavus Erdem & Sahin, 2019; Park, Park, Park, Cho & Lee, 2013). Thus, it is important to conduct studies investigating the effect of mathematical modeling on students' mathematical development to include and address mathematical modeling in the curriculum correctly.

There are few studies examining the effect of mathematical modeling on individuals' mathematical knowledge and skills (Stohlmann , DeVaul, Allen, Adkins, Ito, Lockett & Wong, 2016). The literature review shows that the majority of studies conclude that mathematical modeling affects students' learning positively (Harel & Lesh, 2003; Park et al.2013). Dunne and Gabraith (2003) examined the effect of a curriculum created with mathematical modeling activities on the development of students' mathematical concepts and skills. The researchers argued that mathematical modeling provides students with important opportunities to know how it is beyond knowing something and is a very effective tool to test the accuracy of students' knowledge gained through traditional teaching methods. Harel and Lesh (2003), in their study with three students with low academic success, examined conceptual changes of students during the application process of modeling activity in mathematics.

Their findings revealed that there were conceptual change and development in students and mathematical modeling effectively served this purpose. Park et al. (2013) argued that mathematical modeling activities provided discovery and conceptual developments related to the subject in their research and claimed that mathematical modeling has a facilitating role in learning. Ärlebäck, Doerr & O'Neil (2013) claimed that students provided improvement on the subject but had difficulty interpreting some situations. The researchers, addressing the difficulties experienced conceptually and contextually, stated that mathematical modeling activities contribute to the mathematical development of the students, and therefore suggested that further explanatory research should be conducted, especially on the subject. Freeman (2014) argued that mathematical modeling activities do not make a big difference compared to traditional teaching and do not affect students' attainment significantly. According to the findings of these studies, it would be safe to say that the algebraic issues are prioritized. More studies are required to examine the effect of mathematical modeling on mathematics teaching (Stohlmann et al., 2016). This study, in this sense, aimed to examine the learning environment based on mathematical modeling activities, including one of the areas of geometry sub-learning. Therefore, the study aimed to answer the following research question: "Are mathematical modeling activities an effective method in teaching the subject of field measurement?". This study investigated students' knowledge of area measurement at pre-implication and post-implication and their explanations during the implementation of activities.

Theoretical Framework

Mathematical Modeling as a Tool for Teaching Mathematics

Studies on mathematical modeling showed that modeling is defined differently and used for different purposes (Galbraith, 2012; Kaiser & Sriraman, 2006). The use of mathematical modeling to teach and reinforce mathematical concepts is the approach in which modeling is considered a tool (Galbraith, 2012). As the mathematical structures and concepts targeted in the problems used in this approach create a requirement for students, it is aimed to provide more meaningful learning. Examples of perspectives adopting this approach in the literature are educational modeling and contextual modeling (model and modeling perspective [MMP]) perspectives defined by Kaiser and Sriraman (2006). Although there are fundamental differences in the meaning attributed to modeling activities between these two perspectives, both include the idea that mathematical concepts can be taught through mathematical modeling activities (Abassian, Safi, Bush & Bostic, 2020).

In this research, the approach in which mathematical modeling is adopted as a tool and some issues highlighted in the aforementioned approaches were taken into consideration in research design. Mathematical modeling activities are considered realistic, complex, and open-ended problems with a set of principles (Dogan, 2020; Gurbuz, et al., 2019; Lesh & Zawojewski, 2007; Sahin, 2019; Sahin, Dogan, Cavus Erdem, Gurbuz & Temurtas, 2019), however, it might be simpler and closed-ended when it is used as a tool for teaching mathematics (Bukova Güzel, Dede, Hıdıroğlu, Kula Ünver & Özaltun Çelik, 2016). This issue adopted by the educational modeling approach was considered while designing activities in the research, and mathematical modeling activities were designed in a more structured way (Table 2). Another approach proposed by Lesh and Doerr (2003), the MMP, takes mathematical modeling as a tool. According to this approach, students develop important mathematical and conceptual structures while engaging in modeling activities. The features emphasized by the researchers dealing with the topics mentioned in their research in a much deeper way and the features taken into consideration in this research design can be listed as follows:

According to MMP, an individual in mathematical modeling activities explains the problem's situation with his own experiences and creates his/her model with his conceptual structures. If a person first creates the model and then examines it, his/her thinking can be changed in basic ways, and this change in the individual can be observed through external representations and models used in the modeling process. This study, in this sense, examined mathematical structures and models in the modeling processes to determine how the students achieved the desired goals with modeling activities.

MMP proposes applying mathematical modeling activities with groups of three or four students, explaining this on cognitive and social bases. In any case, it is not always possible for an individual to handle and interpret a subject with all its components. Researchers argued that it would be appropriate to carry out applications by forming groups because of the same concept being perceived and interpreted differently by different people and the individual not being aware of the details while looking at the big picture or not being able to see the big picture while focusing on the details. Based on this idea, mathematical modeling activities in this study were applied to groups of three students.

Concept of and Measurement of Area

In this study, mathematical modeling activities are designed to include the gains of the measurement subject. The area measurement, which is considered one of the important topics of the geometry-learning field, determines how many of a limited plane will be covered by the same type and appropriate measurement unit (Reynolds & Wheatley, 1996). To understand area measurement, it is essential to comprehend the area and measurement correctly, as these two concepts are different from each other in the area measurement, and two concepts can be mixed and the students with no sufficient knowledge about the area interpret the area concept as the area measurement (Huang & Witz, 2013). The area is a certain amount that covers the surface of a limited area, and the purpose of area measurement is to determine this amount (Baturo &

Nason, 1996). Simon and Blume (1994) stated that the area measurement should be handled in two different stages as evaluating the area as a limited region and determining the amount of this area. The first stage involves understanding that the area is a planar region and conceptually interpreting the area. The second stage includes being able to determine the amount of the region. Baturo and Nason (1996) emphasized that it is very important to correctly understand the measurement tool, the unit, in determining this amount. Likewise, Outhred and Mitchelmore (2000) stated that systematic counting comes to the fore in covering the region with the same type of unit and determining the unit amount. As a result of covering a rectangle with a suitable and identical unit, it is necessary to turn it into a systematic count to calculate how many units are on each line and how many lines are available, because it is very important to see the column-row coordination and correlate this with the product of the edge lengths in the transition of the area measurement to the product (Huang & Witz, 2013). Thus, students can understand the multiplication process in the area formula conceptually (Outhred & Mitchelmore, 2000).

Another important concept in learning area measurement at a conceptual level is conservation. Conservation includes information on how changing the shape and dimensions of a surface will change the surface area. The concept of field conservation is generally neglected in teaching (Stephan & Clements, 2003). However, ensuring conservation in measurement is very important for conceptual development (Lesh & Carmona, 2003). Studies reported that students have difficulty in accepting that the area remains the same when they cut a certain area and rearrange it to create another shape (Lehrer, 2003). Therefore, the concept of the area, the concept of the unit, and the conservation of the area should be gained basically for the teaching of the subject measurement. Then, teaching the area relations by repeating the units should be targeted. In this research, while the mathematical modeling activities were being designed and the order of implementation of the activities, the aforementioned issues were being taken into consideration, an application aimed at creating the perception for the students that the area is a region, covering this region with standard and non-standard units, and then the transition to the area relation.

Method

Research Method

Due to the in-depth analysis of the learning environment, a case study method is adopted in this study. In the most general sense, the purpose of a case study is to examine and describe a case in its real context (Yin, 2009). In this study, a case study is considered an instrumental case. The instrumental case study aims to provide an idea about a subject and make a generalization, and a limited situation is selected from the population to achieve this goal. The situation under consideration serves as a tool to reach general information in this type of research. This study aimed to obtain general findings of the learning environment based on mathematical modeling; therefore, the instrumental case study was adopted.

Participants

The study group consists of six seventh-grade students (13-year- old) who study at a school that can be easily accessible by the researchers. The participants of the study were selected through a purposeful sampling method. In the selection process, students' knowledge levels regarding the subject of area measurement were taken into consideration. The subject of area measurement is included in the mathematics curriculum from the third grade of primary school (MEB, 2018), and therefore, it is considered that the application group has preliminary information on the subject. Based on this assumption, in the first stage, a form titled "Area Knowledge Evaluation Form" was developed by the researchers and applied to determine the students' prior knowledge on the subject. The form was applied to 160 people and six students (three girls, three boys) who were randomly identified among the student groups that were determined to have incomplete knowledge due to the application, and the study group was formed. Students were coded under the following names: Serhat, Mehmet, Ali, Meral, Esma, and Pelin.

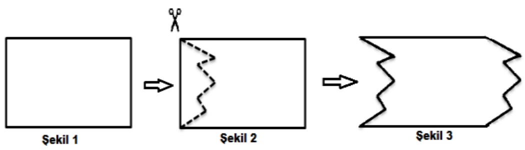
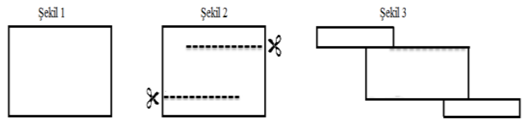
Data Collection Tools

Different data collection tools were used in this study. The main tools are mathematical modeling activities and interview forms made before and after the implementation.

Interview Forms

Semi-structured individual interviews were conducted with students before and after the implementation. To create the interview questions, a literature review has been conducted (Hart, 1981; Orhan, 2013), and some basic concepts and achievements regarding area measurement have been taken into account (MEB, 2018). The questions are arranged to reveal how students define the area, the ability to measure the area of polygons, knowledge of area conservation, unit square information, and area-perimeter-edge length relation. Two sample questions related to forms are presented in Table 1.

Table 1. Sample Questions from Pre-Interview and Last Interview Form

Pre- interview form	Last interview form
Q.3. Consider a rectangle. How does the rectangle's area change when its length of all sides is doubled? Please explain.	Q.4. Think of a square. When two units increase a square's side length and decrease the other side's length, does it change the area?
	(the table continues)
Q.5. Yasemin cut a rectangular paper in Figure 1, as shown in Figure 2. She then slid the cut piece to the right of the rectangle to form Figure 3. In your opinion, how does the area of the 3rd Shape change? Please explain. 	Q.3. A rectangular paper given in Figure 1 below is cut slightly on both sides, as shown in Figure 2, folded in the opposite direction to the side where it was cut, and Figure 3 is formed. In your opinion how does the area of the 3rd Shape change? 

After the interview forms were developed, two mathematics education experts were shown, and their ideas were taken, and revised accordingly. After the arrangements, the implementation was carried out with four students determined for the pilot study in the ''Area Knowledge Evaluation Form'' application group. As a result of the implementation, one of the questions was revised according to the expert opinion, and the interview forms were finalized.

Mathematical Modeling Activities

One of the eight activities, another data collection tool used in the application process, and seven mathematical modeling activities were created based on the literature review (Doruk, 2010). The following points were taken into account during the designing the activities: a) The problematic situation given in the activity is in a way that the context of real-life can make sense of the students with their past experiences, and the real-life situation coincides with the reality of the student (Lesh et al.,2000; Maaß, 2006), b) As aiming to teach mathematical concepts is a primary goal, the problem is more structured rather than completely open-ended in events (Bukova Güzel, et al., 2016).

The developed activities are designed to support both conceptual and operational information on area measurement. The ‘‘Recycling Adventure Event,’’ ‘‘Patchwork Pillow,’’ and ‘‘Swimming Pool’’ problems are designed both to improve the unit square concept and to support operational information related to area measurement, to make the area feel like a covering. “Kamil's Sheep” and “Almond Claim” events emphasize the protection of the area and the relationship between the area-perimeter-edge length and support these achievements. “School Party,’’ “Halva with Cheese,’’ and “Inheritance Sharing” activities are designed to highlight the ability to measure the area. While determining the application order of the activities, a sequence was followed considering the conservation, unit square, expressing a region with different unit squares, the conversion of units, and measuring the area of polygons. Although the activities are designed to support different concepts related to the area, it is considered that they would function as a whole in gaining the area concept and area measurement skills and will support the development of other achievements in the area apart from the targeted activities. The activities were revised according to expert opinion and finalized as a result of the pilot implementation. One exemplary activity is presented in Figure1.

Recycling Adventure Problem

Mrs. Ayşe is planning to contribute to her household income by covering the old used materials leftover from household chores with fabric and selling them as ornaments. To increase the contribution, she wants to use the fabrics she will cover with the minimum number of materials. However, she has no idea how many pieces of fabric that should be used to cover the materials. She asks for help from you, as a mathematician, in this matter. Your task is to develop a measuring tool (unit) that will identify the piece of fabric covering the tin can that is given to you. Indicate in detail what aspects you considered when developing the measuring tool.

Figure 1. Mathematical Modeling Activity

Applications were recorded through a video camera and voice recorder. In the research, students’ activity solution papers and researchers’ notes consist of the other data collection tools.

Data Analysis

The data analysis of this research was performed in the following two stages: an embedded theory coding method (grounded coding) and a rubric evaluation method. In the first stage, all data were analyzed through the embedded theory coding method that requires creating categories and continuous comparison to analyze the data (Jones & Alony, 2011). In this approach, the following three main coding stages were employed to analyze the data through coding: open, axial, and selective coding. The data analysis process of the study was as follows: After all the data were collected, they were transcribed first, and then empowerment method was applied to the data (Ellis et al.,2016). All of the video and sound recordings of each application were watched again, taking into account the transcript text of that application, and were enriched with verbal expressions, gestures, attitudes, drawings, and images. Some transcripts were individually coded by the researchers. After the coding, the researchers compared and evaluated the codes together. For the compatibility between the codes, Miles and Huberman's (1994) intercoder reliability formula (Compatible codes / [Compatible codes + Incompatible codes] x100) was applied, and it was determined that they were 84%, 80%, and 82% for the first interview, the event, and the last interview, respectively. Incompatible codes were evaluated, associated with codes with the same meaning according to the situation, and new codes were created according to the situation. In axial coding, related codes are collected under the same title. As a result of selective coding, two important titles emerged, and data analysis was performed separately under these two titles.

The first topic is related to the students' area measurement knowledge and skills. ''Area Concept and Area Measurement Knowledge Evaluation Rubric'' has been developed with the codes obtained. In this way, the data collected in the preliminary interview and the final interview were evaluated according to the developed rubric, and it was aimed to examine the change in students' knowledge by supporting the rubric assessment with student explanations. The rubric consists of five main titles and 11 subtitles. Each subtitle is rated within itself (Appendix A). The students' explanations in the pre-interview and the last interview were graded according to rubrics by taking into account their development in the process. The second title emerging in the analysis process is related to the learning environment based on mathematical modeling activities. The codes and sample explanations are discussed in the results section of the study.

Results

Results Related to Students' Area Measurement Knowledge and Skills

In this research, the changes in students' area measurement knowledge and skills were presented through the findings obtained from the pre-interview and the final interview and the solution reports they presented during the modeling process. The students' scores from the evaluation rubric based on their pre-interview and final interviews' responses are presented in Table 2.

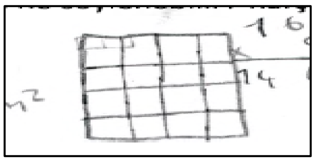
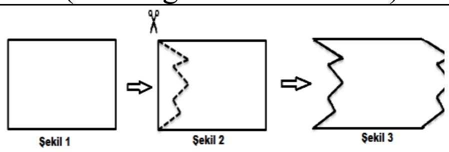
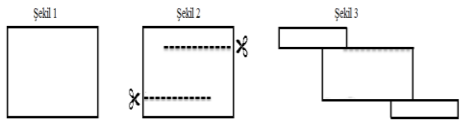
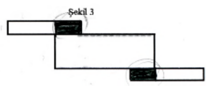
Table2. Students' Scores from the Evaluation Rubric in the Pre-Interview and Final Interview Form

Concept of area and area measurement skill	**HS***	**Serhat**		**Ali**		**Pelin**		**Meral**		**Mehmet**		**Esma**	
		P*	**L***	**P**	**L**	**P**	**L**	**P**	**L**	**P**	**L**	**P**	**L**
Perception of area concept	3	3	3	3	3	1	3	1	3	3	3	3	3
Area computing perception	4	1	4	1	4	2	3	0	3	1	3	0	3
Unit square	5	1	5	0	4	0	4	0	4	0	4	1	4
Area measurement units and conversion of units	5	1	5	2	5	2	4	1	4	1	4	0	4
Area conservation	1	1	1	1	1	0	1	0	1	0	1	1	1
Ability to calculate area of square and rectangle	2	1	2	1	2	1	2	1	2	1	2	1	2
Ability to calculate the area of a triangle	4	1	4	1	3	1	3	1	3	1	3	1	3
Ability to calculate the area of the parallelogram	3	0	2	0	2	0	2	0	2	0	1	0	2
Side length - area relation	2	0	2	0	2	0	2	0	2	0	2	0	2
Circumference- area relationship	2	1	2	1	2	0	1	0	1	0	1	0	1
Area- circumference relationship	2	1	1	1	1	0	1	0	1	0	1	0	1
TOTAL (0-6) unsatisfactory (7-13) fairly unsatisfactory (14-20) partially satisfactory (21-27) fairly satisfactory (28-33) satisfactory	33	11	31	11	29	7	26	4	26	7	26	7	26

* HS: Highest score possible , P: Preliminary interview, L: Last interview,

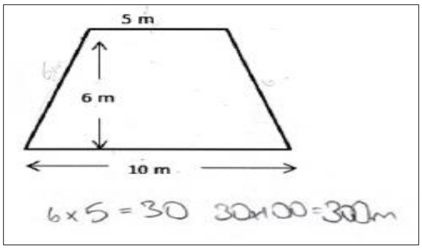
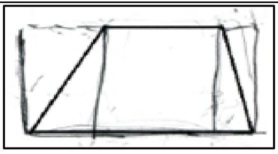
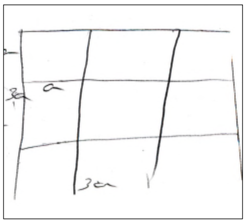
Table 2 shows that the students' knowledge about the subject has reached a sufficient category as a result of the application process. Although an individual difference has been observed in the students' development, it can be said that the learning environment based on mathematical modeling activities has a positive effect on all students' scores. To examine the development of the students more clearly, sample explanations are presented based on their answers to the questions in the preliminary and final meetings (Table 3).

Table 4 Sample Explanations of the Interviews

Category	Example explanation from the preliminary interview	Example explanation from the last interview
Perception of area concept	Researcher: What comes to your mind when you think of the area, Pelin? Pelin: The product of the width and length of something. Researcher: Can you show me the area here, where is the area here? (on the square) Pelin: This part, like that (she shows by drawing the width and length of the shape. Area is equal to the product of two sides.	Researcher: What comes to your mind when you think of the area, Pelin? Pelin: This is where something takes up, its interior. Researcher: Well, will there be areas of irregular shapes? Pelin: Yes, there is a place it occupies, but we cannot find it.
Unit square	Ali: As it is a square, 5 times 5 is 25 cm^2. Researcher: Well, can you show me this 25 cm^2 here? Ali: 25 cm^2 is inside the area. Researcher: Can you explain it a little more? Can you show what quantity is 25, that is 25 cm^2? Ali: No. Researcher: Why? Ali: (nodding ‘‘I don't know’’).	 Researcher: Can you draw me a square with an area of 16 br^2? Ali: Yes, I can draw. It would be 4 laterally and 4 vertically. So, that’s what you see here.
Area conservation	 Meral: I think it doesn’t change. Researcher: Why is that? Meral: Because it moves the amount it cut here. Researcher: What if I paste it above? Meral: When we paste it above, it may change. Because its height is expanding. We cannot calculate it because this is (cut length) a curve. I don’t know, perhaps it might change that way.	  Meral: It adds the same amount to itself there. This part comes on top of the bottom. Researcher: So, does the area change? Meral: I think the area gets smaller. It overlaps, and the bottom overlaps as well. I think the area is decreasing. Researcher: How much does it decrease then? Meral: As much as the shape of this (filling the area).

(the table is continues)

Table 3. *Continuation of….*

Area measurement skill	 Esma: If here is 5 m and here is 10 m, then here is 6 m. Now (calculating), we multiply 6 by 5. We also multiply 30 by 10. Researcher: Why? Esma: The thing here is 5 m, these two are not equal. As it is not equal, we cannot multiply only one. We have to multiply all three.	Esma: To find the trapezoid area, we should complete the rectangle and then subtract these completed places.  Researcher: What lengths should be known? Esma: For this place, the base of the shape must be known. We also need to know the height of the shape. Researcher: Can you show where you call the base? Esma: The base needs to be known, but these two parts need to be known. (triangle bases on the sides). And this middle piece. In total, there are three lengths for the base.
Area- side length relationship	Researcher: Does the area of a rectangle change, if its side lengths double? Serhat: When it is doubled, we use 4 cm for 2 cm and 8 cm for 4 cm. One of the areas is 8 and the other 32, enlarges 4 times. Researcher: So, why is it 4 times larger? Why is that? Serhat: Why could it be? Teacher, 4, hmm, I don't know. Researcher: Okay. Then what happens if the side lengths triple? Serhat: The area becomes 3 times larger.	Researcher: When the side length of a square is increased by 3 times, how many times does the area increase?  Serhat: We can find it by using value. Researcher: I want you to do it without using a value. Serhat: If a becomes 3a. I t will be 3a. The other side will be 3a (drawing the shape). Serhat: We divide this place by 3. (divides for 7s). Then we can see that 9 of these can fit inside. Becomes 9 times bigger.

Although the pre-application levels of the students differed, the examples were presented over the same student to observe the development of the students more clearly. Considering all the examples, it would be safe to say that the concept of space was formed as a numerical value obtained by multiplying two lengths (width and length) in some students before the application (n = 2), and the concept of the unit square did not occur as one of the basic concepts of area measurement (n = 4). The findings of the analysis revealed that students have deficiencies in defining area measurement units, and conversion of units (n = 6), and some students think that the area changes depending on the shape without changing the quantity (n = 3). When calculating the area of the triangle (except the right-angled triangle) and the parallelogram, it can be said that students using wrong strategies and perform erroneous operations (n = 6) also did not notice the relationship between side length and area (n = 6). In the explanations after the application, it has been observed that the students correctly explain the concept of space and express the area as a covering one (n = 6). It has been observed that all students can correctly calculate the area of the polygons, and even explain the area relation of the square, rectangle, and triangle with the help of unit square (n = 6).Also, some students can explain the area relation of the parallelogram and trapezoid (n = 2). Table 4 shows that all students have conservation. The change in students' knowledge can be seen from the models and solution reports that emerged during the modeling process. The solution report of the students in the first activity is presented in Figure 2.

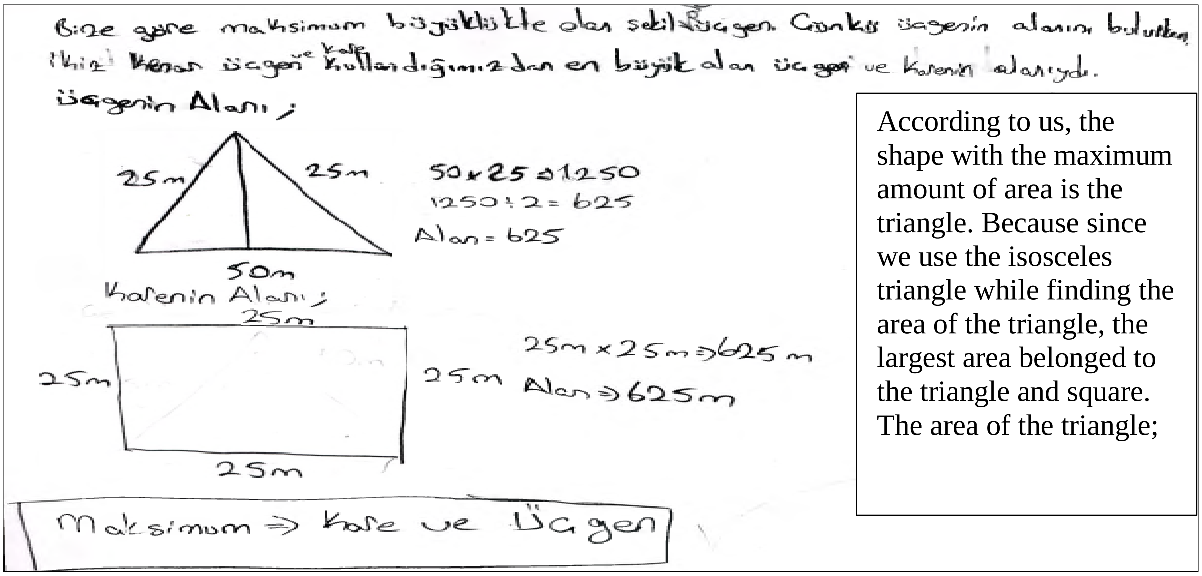


Figure 2. First Activity Solution Report

In the first activity, students were asked to find the maximum area they can create a 100-m fence. Students' solution report shows that there is an error in their knowledge. When calculating the triangle area, the students miscalculated the area taking into account one length of the sides instead of the height. At the same time, it is seen that the students expressed the measurement result with the wrong unit. This activity aimed for students to realize the relationship between the sides and circumference length. The next activity has the same purpose. The students reported that the circumference of the two rectangles with the same area at the stage they presented their solutions might be different. This is a mathematical development for students who think that the perimeter of the rectangles whose area is equal before the application will also be equal (n = 4), and this can be regarded as one of the first improvements during the modeling activities application process. In the next activity, students were asked to determine the amount of fabric that would cover a can. In the activity in which students used a spotted (polka dot) piece of fabric, they expressed the amount of fabric in unit squares. In this activity, an important progress by the students has been observed and it was seen that students expressed the area using the unit square. In this activity, students expressing the area with only unit squares started to express the same area with different units in the next activity. In this activity, students showed the area of the same area in both cm^2 and m^2, and they also showed the measurement result in m^2 correctly. Although it was not aimed to include in the activity outcomes, as a result of the question directed by the researcher while students discuss their models, they discovered the relationship between m^2 and cm^2 and wrote the result they found in m^2. Considering the incorrect student explanations regarding the relationship between the standard measurement units, this can be considered an important development. In the last activities aimed at the development of the students' area measurement skills, it is seen that the students started to calculate the area of the polygons correctly. The solution report regarding the seventh activity is presented as a sample calculation in Figure 3.

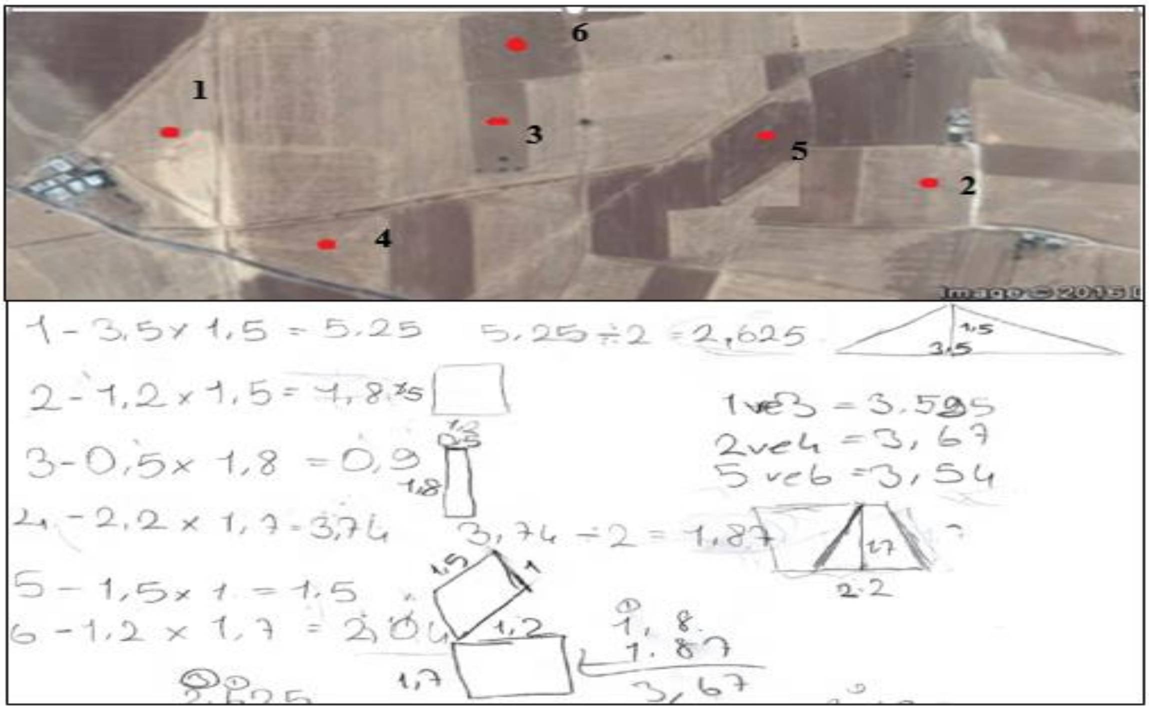


Figure 3. Solution Report Regarding the Seventh Activity

In the seventh activity, the students were asked to divide the six fields among the three siblings in the fairest way. As shown in Figure 3, the students correctly calculate the area of all polygons, including the trapezoid, parallelogram, and triangle. Students used the parallelogram to find the trapezoidal field area and measured the area by converting the trapezoid to a parallelogram. It is considered that it is an important progress that students correctly calculate the trapezoid area they have not encountered before. At the same time, students who can only calculate the area of the right triangle before the application (n = 6), who measure the area of the parallelogram incorrectly (n = 6), show that the area measurement skills of the students have improved because they made correct calculations during the application process.

Results Related to the Learning Environment Based on Mathematical Modeling Activities

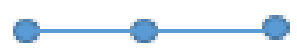
Results from the research show that mathematical modeling activities significantly affected students' learning. Student explanations regarding the learning of the unit square concept will be presented to show how the situation in question has been handled. The third mathematical modeling activity of the research applied for the unit square concept is "Recycling Adventure," as shown in Table 2. During the application process, students were given an activity and asked to evaluate individually for five minutes and then create a model with their groups. The dialogue between students at this stage is as follows (Figure 4).

Meral: What do we do now, any ideas?
Mehmet: Let's find out how much fabric we have. Look, there are gaps here. Let's say 1 cm for each gap.
Pelin: No, it will probably be more than 1 cm, it cannot be 1 cm. Let's think. Okay, you will take it as a unit.
Mehmet: OK, anyway, I'll make my calculation (counting the width and length of the fabric). You tell me, 1 m^2 equals to 1000 cm^2, right?
Pelin: Yeah. It was 1000 cm^2. But why do you multiply to find the amount of fabric?
Meral: No, I think it was 100 cm^2. 1 m^2 equals to 100 cm^2.
Mehmet: No. It was 1000 cm^2. Sides came out to be 22 cm and 19 cm. It makes 4180 when I multiply. It is 4 m^2.

Figure 4. Student Descriptions Regarding the Third Activity

The explanations that the students try to transform the problem into the language of mathematics also show that they try to remember their existing knowledge. The dialogues show that the students made both true and false statements. It is the correct approach that Pelin sees the distance between the two spots as a unit, and Mehmet multiplies the sides to determine the amount of fabric. Also, it is seen that students misunderstood the relationship between square meters and square centimeters. The most important point that attracts our attention here is that the students shared their information and presented it to the peer's evaluation. At this stage, it has been determined that the emergence of situations enabling students to evaluate their knowledge is very important in the realization of learning. The findings of the study show that students have the opportunity to learn by evaluating their knowledge a) individually, b) through peer guidance, and c) through teacher guidance. Some samples for these three situations will be presented below.

Peer guidance can be expressed as the development that the students have due to the explanations of his group friend. Dialogues between the students to find the amount of fabric they determined in the activity will be presented as an example of peer guidance. As mentioned above, Mehmet considered the distance between the two spots 1 cm, while Pelin stated that the distance between the two spots was more than 1 cm and accepted the range as 1 unit and showed a correct approach. This acceptance of Pelin played a supportive role in learning the concept of the unit square. A sample dialog for this situation is presented below.

Pelin: Yes, the unit square is a unit square because each gap is a unit.
Researcher: So, can you show me the unit square here?
Pelin: Look, teacher, this is a unit square (draws a side length).
Researcher: Can you show me how it is exactly?
Pelin: This is the unit (side length-). The product of these two is a unit square. (shows two unit lengths - ).
Meral: The product of these sides becomes a unit square (pointing the sides)
Pelin: Teacher, these two lengths and the product of these units will be a unit square.
Meral: I think just here (square) is a unit square.
Researcher: Well, Meral, can you show me the cm2?
Meral: Then it becomes a square with its sides being 1 cm.
Researcher: Pelin, what do you think?
Pelin: I think it makes more sense to consider it as a unit square, or rather it looks like that.

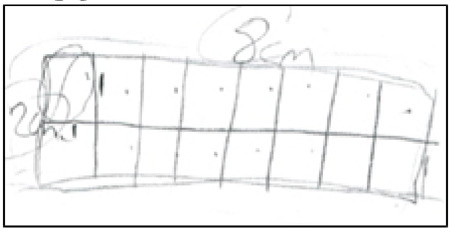
It appears that the knowledge of the unit square concept of Pelin, Mehmet, and Meral before the application is insufficient (Table 4). Pelin’s explanations to determine the amount of fabric that will cover the tin can show that she maintains her incorrect perception. Expressing the distance between the two spots on the fabric as one unit and the unit square as the length of the two units, Pelin questioned her knowledge after Meral’s explanation. It seems that Meral explained the concept of the unit square correctly. The fact that she defined a square centimeter correctly supports the idea that she learned the unit square correctly. The dialogue given above may not be sufficient to show that learning process has been completed, especially for Pelin, but in the continuation of the solution process, it has been determined that both students addressed the unit square correctly. Some examples will be presented below. Meral's influence on Pelin's concept of learning can be expressed as peer guidance. Likewise, Pelin's acceptance of the distance between two spots as a unit affected Meral's learning. Although the occurrence of Meral's development has not been fully reflected in the activation process, it is thought that it is effective to associate the unit length with the square region.

The situations in which the students made progress in the activity solution process without any external intervention can be considered as individual discovery (interaction with the activity). An example of an explanation for this situation is given below:

Researcher: What did you guys do?
Serhat: We calculated the area of the piece of fabric. These gaps can only be 1 cm anyway.
Esma: We took a centimeter as an estimate.
Ali: It would be 1,3 cm. More than 1 cm.
Researcher: Well, if you accept that it is not 1 cm, then how will you express it?
(They think for a while)
Ali: We would follow the unit calculation.
Serhat: Yes, it makes sense, we'll do it that way. Eleven units, this here is four units. Friends, the area is 44 unit squares.
Ali: Yes, this was a nice progress.
Researcher: How did you decide that it is a unit square?
Ali: Now, because we don't know the size of this square, we called it a unit, when we calculate it, it looks like a square shape anyway. When we multiply this side with that side, it becomes the square of a unit. For this reason, it is a unit square. Yeah, of course, that's very good indeed.

All in all, Ali uses correct expressions regarding the concept of the unit square. The explanations of Ali, who could not explain the concept of the unit square before the application, showed that he discovered the concept of the unit square upon accepting it as a unit between two spots. Ali's interaction with the activity allowed him to establish a mathematical relationship, and the situations that occurred in this way in the research were named individual discovery. In the modeling process of the students, the questions asked by the researchers were another factor supporting the development of the students, except for the conversations they had with their group friends and when they noticed individually. A sample dialogue that arises with the questions asked by the researchers to the students who determined the area of the fabric is presented below (Figure 5).

Researcher: Friends, you said that by multiplying 11 by 4, you found the area of this fabric as 44 unit squares. Can you show me here without multiplying to get 44?
Mehmet: For example, here (first row) there are 11 squares, in this order, there are 11, 44 in total.
Researcher: So, why are we multiplying the sides?
Pelin: Because, teacher, there are four 11s. For example, as there are four lines longitudinally and 11 lines transversely, we multiply 4 with 11, hence 44.



Researcher: Can you draw a rectangle with a long side of 8 cm and a short side of 2 cm to show its area in the unit square?
Meral: Isn't it like this? If we divide it by 2 transversely, we divide it into 8 parts longitudinally. It becomes 16 cm^2.
Ali: I think she's doing it correctly.
Researcher: Then what do those squares represent?
Pelin: It represents a square of 1 cm.

Figure 5. An Example of an Explanation for Teacher Guidance

The dialogue shows that the students explain the area relation of the rectangle by associating it with the unit square. The questions posed by the researcher played a supportive role in establishing this relationship. To fully reveal and reinforce the knowledge of the students, the correct answer to the second question asked by the researcher supports the idea that they have learned the unit square, which is one of the basic concepts of the area measurement. Conversion information between the students who learned the concept of the unit square was carried out in the next mathematical modeling activity. In the "Patchwork Pillow" modeling activity, the students were asked to determine the amount of each color fabric for the pillow cover. The students found the area of the different color fabrics used on the pillow cover in both unit square and square centimeters. The researcher then asked the students to express the results in m^2 to observe the change in their unit transformation knowledge. Mehmet, tried to explain the conversion of units algebraically (Figure 6).

Researcher: If I ask you to write this in m^2, how can you write it?
Esma: Ladder calculation.
Serhat: Then, we will convert cm to m. We were going up two steps when converting it to meters. If we add two zeros.
Meral: No, if we delete two zeros.
Mehmet: There is also something like 1 m^2, 1,000 cm^2.
Meral: Becomes 100. Because 1 m is 100 cm.
Mehmet: Okay, but it has a square. 1 m^2 is 1,000 cm^2. For example, do you think m and m^2 are the same?
Serhat: No, it wouldn't be the same.
Mehmet: Look, this is a m. This is m x m. See if this is 100, 100 times 100 makes 10,000. Ooooh, I was absurd.

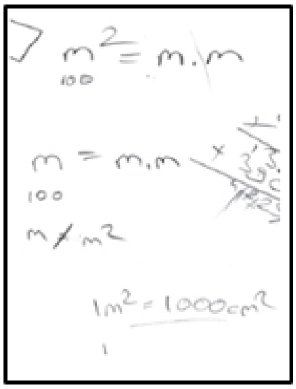
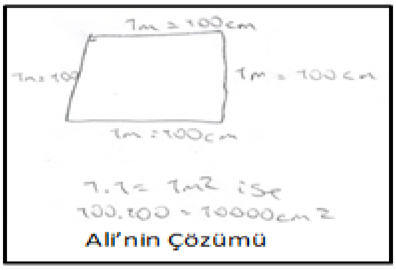


Figure 6. Student Descriptions Regarding the Unit Transformation

The ladder calculation, which teachers frequently apply in teaching the conversion of measurement units and coverts the relationship of the measurement units with their lower and upper counterparts into a visual form, was the first method students have applied. While the students explained and evaluated their knowledge, Mehmet questioned whether two units were different and tried to explain the difference between the two algebraically. Based on Mehmet's approach and the correct calculation he made, Serhat and Ali tried to find the conversion by doing some mathematical calculations (Figure 6).

Serhat: Let's draw a shape. When it continues from the calculation that Mehmet said, it turns out to be 10,000. Let's show it on the figure.
Ali: I did it. Can I say my opinion? Because I found 10,000.
Pelin: I think it's 10,000 as well
Ali: If 1 m equals 100 cm, I calculated it as its square. As it is a square of 1 m, it becomes 100 x 100, which equals to 10,000.



Esma: He found the correct result, and I think it is very reasonable.
Serhat: We thought so too. Here is 1 m, if we multiply 1 by 1, it becomes 1 m square. For example, if we convert 1 m to 100 cm, this contains 100 cm in 1 m. If we divide this part into 100 pieces like this (he draws a line parallel to the bottom of the square). One hundred pieces come on top of each other. This is 1 m in the same way, as a square (drawing another line parallel to the other side). So, there is a total of 10,000 of these squares here. (The little square indicated by the red circle)

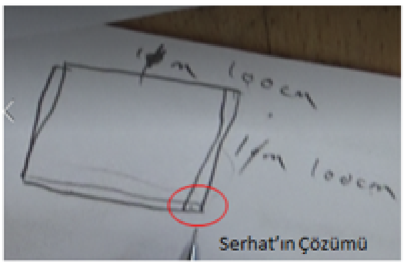


Figure 7. An Example of an Explanation for Teacher Guidance

The explanations showed that students explained the conversion of units in both algebraic and geometrical ways. Students did the unit conversion not by heart, but using the unit square and explaining the measurement units algebraically. This development, which was achieved by the students on the subject of area measurement, was supported by the question directed by the researcher and Mehmet's guiding explanation. Teacher and peer guidance, one of the learning opportunities that emerge in mathematical modeling activities, is observed in the dialogues given above.

Discussion, Conclusions and Suggestions

The results of the study show that mathematical modeling is effective in teaching the basic concepts of area measurement subjects such as unit square, area conservation, area measurement, and gaining skills. It would be appropriate to examine the results consistent with the literature (Dunne & Gabrailth, 2003; Park et al., 2013), both from the viewpoint of the applied method, that is mathematical modeling, and the subject taught. Considering the role of mathematical modeling in the said development, it can be said that activities offer important opportunities for students to question and evaluate their knowledge. The students' activation of their knowledge to form a model and its evaluation by interacting with the activity, peers, and teacher (researcher) involves a natural learning process. The first step in the exchange of information carried out through learning opportunities is remembering the information. In modeling activities, students engage in intensive cognitive activities and, thus, take an active role (Blum & Niss, 1991). For this reason, while trying to mathematicise the real-life situation and solve the model in the activities, the students run (remember) their existing knowledge and share it with their group friends. Information sharing within the group also includes a natural evaluation process. In this way, the students who reached the information of their group friends and presented their information to the approval of their peers had the opportunity to notice their incorrect information. Group support (Harel & Lesh, 2003), which is very important in guiding the student, was expressed as peer guidance in the research. Ng (2011) claimed that students are significantly influenced by group members in understanding modeling activity, interpreting context, using mathematical knowledge, and reasoning. In addition to Ng's (2011) findings, the results of this study also showed that students outside the group also have an impact and guide students. During the evaluation phase, students had the opportunity to evaluate and improve their knowledge, where they presented and discussed their models in the classroom. Hitt and González-Martín (2015) argued that other students undertook the duty of a locomotive in the modeling process and prepared the ground for learning. The study results showed that students both in and out of the group guide students to access new information.

Another learning opportunity that supports students' development in a mathematical modeling environment is determined as teacher guidance. In modeling activities, the teacher acts as a guide, rather than a source of knowledge (Ärlebäck et al.,2013; Dunne & Galbraith, 2003). The fact that the teacher reaches the student explanations with the right questions at the right time allows students to review their knowledge and realize their mistakes as well as an in-depth assessment. The researcher, who observed the students' situation in the research and mobilized the students' reasoning with critical questions when necessary, assumed the role of a similar teacher. Critical questions directed by the researcher helped reveal students' thoughts more deeply and determine the level of learning and played a triggering role in seeing the points and mathematical relationships that were not noticed in the activity. The explanation of the rectangle's area relation by linking the rectangle with the unit square is an example of this situation. Hitt and González-Martín (2015) explained this as the institutionalizing role of the teacher. The researchers claimed that the teacher's statements in the mathematical modeling activities to support the students' mathematical understanding emerging in the process and explaining the mathematical relations after the activity would increase the permanence of the learning. The results of the research are also in line with this idea.

One of the last learning opportunities is the changes that the student experiences due to interaction with the activity and is expressed as an individual discovery. Individual discovery is the mathematical insights that the students discover in the model building process only without any external intervention, such as peer or teacher guidance. In the process that students unconsciously review their conceptual structure (Lesh & Doerr, 2003), it is thought that they see the whole by combining the parts and, thus, construct mathematical understanding. It should be noted that the learning opportunities that arise during the research process are not independent of each other. It is thought that other learning opportunities have an impact on the realization of each. Learning is considered a long and complicated process, and the process has evolved through the synchronous operation of the three opportunities that emerged in the research.

When students' development is evaluated based on area measurement, it is possible to say that the third activity is a turning point. In the activity where some students got acquainted with the unit square concept and associated the unit square with the area relation of the rectangle, students mathematically comprehended why two lengths are multiplied in the area relation. The students' comprehension of the area relation by associating it with the unit square was also observed during the mathematical explanation of the area relation of the triangle in the same activity. As Harel and Lesh (2003) stated, student transferred the mathematical understanding that emerged in the model. These findings, which are also parallel with the explanations of Stephan and Clements (2003) and Zacharos (2006), showed that the concept of the unit square is very important for the area measurement subject, because covering the interior of a polygon with a unit square will naturally create the perception that the area is a region. At the same time, associating the number of unit squares covering the region with the area relation will enable the structuring of area measurement as reported by Stephan and Clements (2003).The results of the research showing that the perception of the students who perceive the area as ‘‘width x height’’ before the application has changed and turned into area conservation, also confirms the idea that the unit square is an effective tool in overcoming the learning difficulties in the width x height aspect (Kamii & Kysh, 2006). The mathematical structuring of the students’ area relation also contributed to the development of the area measurement skill naturally. It also made it easier for them to understand how to convert the standard area measurement units to cm^2 or m^2.

To summarize, the support of mathematical modeling activities is carried out through the opportunities it offers. Achieving all this development is possible with mathematical modeling activities by the concept and acquisition hierarchy of the subject planned to be taught. Besides, although there are opportunities to support learning in this research, it is not explained how learning occurs in the students’ minds during the mathematical modeling process. Also, research results are limited to the subject of area measurement. To show whether mathematical modeling is an effective tool for teaching mathematics, it is recommended to conduct further studies in different learning areas and conduct studies that detail the modeling process’ learning.

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Appendix A:

Concept of Area and Area Measurement Information Evaluation Rubric

		Levels	Description
Perceptions of the concept of area	Perception of area concept	Level 0 (0 points)	Not making a statement about the concept of the area or making a false statement.
		Level 1 (1 point)	Thinking the concept of area as a numerical value obtained by multiplying the width and height of the shape. (width x height perception)
		Level 2 (2 points)	Seeing the concept of area as the place occupied by shape and thinking it only a property of smooth shapes.
		Level 3 (3 points)	Thinking the concept of area as the place occupied by an object.
	Area computing perception	Level 0 (0 points)	Not making a statement about the area calculation or having a wrong perception.
		Level 1 (1 point)	Thinking the area of a shape as the product of two sides that intersect without looking for a 90-degree condition.
		Level 2 (2 points)	Thinking the area of a shape as the product of the shape's base and the height of the base.
		Level 3 (3 points)	A mathematical explanation of the area computing relations of the rectangle. (explaining relations by associating them with unit squares.)
		Level 4 (4 points)	A mathematical explanation of the area calculation relations of all polygons. (explaining relations by associating them with unit squares.)
Unit square	Unit square	Level 0 (0 points)	Not making a statement about the concept of unit square or making a false statement.
		Level 1 (1 point)	Measuring and expressing the area of polygons area intersect with 90 degrees on all sides, in square units.
		Level 2 (2 points)	Measuring and expressing the area of the rectangle with unit squares of different sizes.
		Level 3 (3 points)	Measuring and expressing the areas of all polygons in square units.
		Level 4 (4 points)	Measuring and expressing the area of a polygon with unit squares of different sizes.
		Level 5 (5 Points)	Measuring and expressing the area of a polygon in a nonstandard unit.

	Area measurement units and conversion of units	Level 0 (0 points)	Not making a statement about the area measurement units and using the area measurement unit incorrectly.
		Level 1 (1 point)	Using the area measurement unit correctly.
		Level 2 (2 points)	Defining area measurement units correctly and algebraically, but not geometrically.
		Level 3 (3 points)	Correctly defining the area measurement units geometrically.
		Level 4 (4 points)	Converting the area measurement units.
		Level 5 (5 Points)	Ability to explain the conversion of area measurement units mathematically.
Conservation	Area conservation (AC)	Level 0 (0 points)	Not having area conservation.
		Level 1 (1 point)	Having area conservation.
Area seasurement skill	Ability to calculate the area of square and rectangle	Level 0 (0 points)	Inability to calculate areas of square and rectangle or incorrect calculation.
		Level 1 (1 point)	Correct calculation of the areas of the square and rectangle.
		Level 2 (2 points)	Creating the area relation of the square and rectangle and explaining them mathematically.
	Ability to calculate the area of the triangle	Level 0 (0 points)	Inability to calculate the area of the triangle or incorrect calculation.
		Level 1 (1 point)	Correct calculation of the area of the right triangle.
		Level 2 (2 points)	Correct calculation of the area of all triangles.
		Level 3 (3 points)	Calculating the area of triangles by taking different sides as the base.
		Level 4 (4 points)	Creating the area relation of the triangle and explaining mathematically.
	Ability to calculate the area of the parallelogram	Level 0 (0 points)	Inability to calculate the area of the parallelogram or miscalculation.
		Level 1 (1 point)	Correct calculation of the parallelogram area.
		Level 2 (2 points)	Calculating the area by taking different sides of the parallelogram as the base.
		Level 3 (3 points)	Creating the area relation of the parallelogram and explaining mathematically.
Side length -circumference-area relation	Side length-area relation	Level 0 (0 points)	Failure or misrepresentation of the relationship between side length and area.
		Level 1 (1 point)	Explaining the relationship between side length and area in a limited way (by way of examples).
		Level 2 (2 points)	Generalizing the relationship between side length and area.

	Circumference- area relationship	Level 0 (0 points)	Not being able to create different polygons with the same circumference.
		Level 1 (1 point)	Ability to create different polygons with the same perimeter.
		Level 2 (2 points)	From polygons with the same circumference, seeing the area grow with the convergence of the side lengths.
	Area- circumference relationship	Level 0 (0 points)	Not being able to create different polygons with the same area.
		Level 1 (1 point)	Creating different polygons with the same area.
		Level 2 (2 points)	From the polygons with the same area, being able to see that the circumference increases with increasing distance of the sides from each other.