



## A Mathematical modelling of success in education

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*Mathematics,*  
*Mathematical model,*  
*Education,*  
*Success*

**Abstract.** In this study, to determine the value of success within any time interval or specified time intervals, a mathematical model has been developed by taking into account “Discipline Factors”, “Teacher Factors”, “Negative Factors” and “Student Factors” as factors affecting the success of the student, class or school, positively or negatively. These factors, determining the value of success, which are the parameters of the mathematical model, was defined as real-valued linear functions, considering internal and external influences and any interventions and with the help of these functions, situations in which success is increasing, decreasing, or stable in a time interval examined by this model.

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### 1. Introduction

Education and training are directly related to all segments of society because they are processes in which some outcomes, such as learning, knowing, getting habits, gaining behaviour, social, cultural and scientific development, getting a professional and growing up in a special field. Therefore, everyone involved in the training process is willing to see positive effects for the educated ones. Obviously, at the end of each training process, obtaining the predicted achievements provided that some criteria are provided, means to be successful. However, being successful or achieving the desired success is not an easy task. There are multiple factors that directly or indirectly affect the process of being successful and the education process. Particularly, Ministry, for the Elementary and Secondary School affiliated with Ministry of National Education, Directorate of National Education, school administration, environment, curriculum, teachers, disciplinary rules, school regulations and practices, student’s family, economic reasons and many factors as an institution, person and situation that are uncontrolled in-school or out-of-school interventions concerning the school and the student are included in the education process. These or similar factors are also pointed for Universities and students of the Council of Higher Education. Because schools belonging to both institutions have similar directions in social and administrative terms and have similar academic, social, and cultural ideals. The goal of every school and every student is the success ultimately, that is, to be educated in both academic and social and cultural fields.

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Making specific classifications, factors affecting the success, will often and often find their place within these classifications. To ensure this, in this study, by determining the “set of disciplinary factors”, “set of teacher performance factors”, “set of negative factors”, the state of functions helping the model will be proposed according to the real provisions of the values in these sets and their averages. Therefore, the multiplicity of elements of these clusters in terms of values means considering the vast majority of education factors that are essential and which have the possibility of intervention.

In this study, “disciplinary factors”, “teacher performance factors”, and “negative factors” that may cause the success graph to increase, remain stable or decrease by concerning the school and the student primarily are considered. Functions are created for the values considered, and it is stated how to calculate the minimum, maximum and average real values for a certain time or period with the help of these functions. After calculating these real values, an exponential function has been considered as a criterion for evaluating success because it is thought that more explicit results of the success value graph can be seen with the help of an exponential function.

Mathematics has its alphabet and therefore a language, which is also an internationally used language of science. Mathematics takes its place in the school programs of all countries at a significant concentration and is taught and taught under various names.

Some of the reasons why mathematics is indispensable in human life can be listed as follows:

- 1) The fact that everyone can accept the rules of mathematics and that the claims, in theory, can be proved,
- 2) There is always mathematics in the person himself, his speech and his movements,
- 3) With the help of the univariate or multivariate functions and the solution methods of initial value problems, which are the main topics of mathematics, there is the ability to build and use mathematical models in many different fields such as water sciences, climate and atmosphere, energy sciences, food and agricultural sciences, biology, health and medical sciences, human social relations, radioactive decay [1].
- 4) With the help of mathematical topics in which differential equations, statistics, probability, analysis, numeric analysis are used predominantly, there is not only the ability to analyse movements but also the establishment of mathematical models in both natural sciences and earth sciences and technological fields such as medicine, forensic medicine, earthquake analysis, economics, gene movement, signals, all engineering, architecture, physics, chemistry, astronomy and robotic technology.
- 5) The password, formula, or correlation of almost any mystery found in the universe can be translated into mathematics.

It is possible to determine the success of a student, a class, or a school in a process by using the fundamental equation of change based on the analysis and formulation of mathematics. Therefore, models can be formed to determine the real value of success at a time point or the course of success in a time interval by taking into consideration the factors that affect students’ attitudes, behaviours, work, motivation and learning, as well as the teacher’s performance factors and discipline rules and rules.

A general definition of mathematical modelling is expressed in the most general sense as the process of trying to express mathematical or non-mathematical events, phenomena, relationships between events and to reveal mathematical patterns within these events and phenomena [2].

In the literature, there are mathematical models developed by Malthus [3] and Verhulst [4] on population growth, as an example of mathematical models in this sense. As an example of Malthus and Verhulst’s models, the study of Pingle [5] on Malthus’s population principle using dynamic models, Brillhante, Gomes and Pestana [6] on the extension of the classical Verhulst’s logistics model, Şekerci and

Petrovskii [7] on oxygen and plankton dynamics, Bender [8] on the details of mathematical modelling, and Junior [9] on the population growth of Sergipe can be given as source works.

On the other hand, many studies on the factors that affect student success [10] and some studies on the discipline factor and environmental conditions in education [11,12] have been conducted.

In this study, “discipline factors”, “teacher factors”, and “negative factors” were taken into consideration as the factors affecting the success of the student, class or school, and it was also stated that the personal contribution of the student could be taken into consideration. In this context, a mathematical model of success was tried to be proposed. First, an exponential success relation was obtained with the help of an initial value problem, and the coefficient of the time variable in the correlation was kept dependent on the mentioned factors. Since these factors may vary with time, they are functions according to time-independent variables. Therefore, since the graph of these functions will be formed according to the real values taken by these factors in any time interval, the graphs will be constant, increasing or decreasing according to the time variable so that it is a curve as convex or concave or a line in a plane.

For example, it is found a point for each factor in the set of disciplinary factors for any time interval, where the arithmetic means of the real discipline values predicted and obtained accordingly. Likewise, a point is obtained at any time. Depending on the disciplinary values obtained from the first point to the second point, it may be reached along a line or curve. If the process between  $t_1$  and  $t_2$  can be observed, and it is determined that the arrival from the first point to the second point is along a curve, and if the function of this curve can be determined, the disciplinary values between these two points can be studied with the help of the determined function. However, if the disciplinary values are not observed in the process between  $t_1$  and  $t_2$  or if they are determined that the arrival from the first point to the second point is along a line or almost a line, the disciplinary values between these two points are studied with the help of linear function.

In this research, it has been accepted that obtaining the values of the factors affecting the success both in points and between these points is with the help of linear functions and non-linearity case is explained in the methods and applications sections.

## 2. Method

In establishing the criterion, the basic principle of change, differential equations, the solution of the initial value problem, mathematical models provided by [3,4] were considered.

### 2.1. Establishment of Student Success Model in Education

#### Preliminary Definitions

$t$ : Independent time variable

$S(t)$ : Success value obtained at any time  $t$

$\Delta t$  : Time interval in variation

$D$  : Discipline factors

The cases such as positive or negative behaviours and behaviours in personal attitudes, behaviours and habits of the student, compliance or not compliance with the rules and rules of environment, everything's being free or too extreme rules for the student, there being the necessary sanctions or not, the student's wellbeing managed or not wellbeing managed [11].

$T$  : Teacher performance factors

The teacher is the class administrator, trainer, instructor and leader, both socially and academically. The teacher is the person who knows and applies the rules and principles of teaching, duties and responsibilities. The ability of the teacher to carry and maintain the teaching mission is called teacher performance [10].

$N$  : Negative factors

The negative intervention of the family to education, economic conditions or of future anxiety that negatively affect the student's work, causing the student not to be motivated or worried about his goals, external conditions such as whether the environment is suitable for education or not, whether the environment is clean and decent or not, passing a course without learning, and not failing because of discontinuity [10].

In this study, functions defined as  $D = f(t)$ ,  $T = g(t)$  and  $N = h(t)$  for disciplinary factors, teacher performance factors and negative factors, respectively and given their detail in the following sections are real-valued functions, and in terms of the integrity study, all three are considered as linear functions. However, any one or all of these functions may not be linear. An explanation of this situation is given in "Applications (B)".

Taking these definitions into consideration as Malthusian population modelling, let  $k$  be the variation of the success,  $\Delta t$  be the small-time increment of the independent time variable  $t$ , the change of success function  $S(t)$  is

$$S(t + \Delta t) - S(t) = kS(t)\Delta t \quad (1)$$

Then,

$$\frac{S(t + \Delta t) - S(t)}{\Delta t} = kS(t)$$

And if we take limit for  $\Delta t \rightarrow 0$ , we obtain the differential equation as follows:

$$\frac{dS(t)}{dt} = kS(t)$$

At the first instance, that is  $t = t_0$ , let the available success value be  $S_0$ , we can write the following initial value problem with this equation:

$$\frac{dS(t)}{dt} = kS(t), S(t_0) = S_0$$

When solved this problem, we obtain the below formula (relation)

$$S(t) = e^{k(t-t_0)}S_0 \quad (2)$$

If we take the first time when success value is determined as  $t_0 = 0$ , the formula (2) will be

$$S(t) = e^{kt}S_0 \quad (3)$$

Here, the value of  $k$ , at time  $t = t_j$ , will be determined by  $D = f(t_j)$ ,  $T = g(t_j)$  and  $N = h(t_j)$

it will be  $k = k_j$  for the time value in question.

Since the increase in the value of  $D = f(t)$  and  $T = g(t)$  will affect the success in a positive sense and the increase in the value of  $N = h(t)$  will affect the success in a negative sense, the coefficient  $k_j$  at  $t = t_j$  time will be determined as

$$k_j = \frac{f(t_j)g(t_j)}{h(t_j)} \in \mathbb{R}$$

Therefore, the value of success at time  $t = t_j$ , will be obtained as

$$S(t_j) = e^{k_j(t_j-t_0)}S_0 \quad (4)$$

according to the equality (2) and as

$$S(t_j) = e^{k_j t_j} S_0 \quad (5)$$

## 2.2. Aim

This study aims to create a mathematical model to examine the success of the student, class or school over a certain period of  $[t_0, t_v]$  time. Informing this model, firstly, the factors that are certain to affect the success positively or negatively were determined. To determine the status of success, the course of success within the time interval considered will be examined with the help of this model.

First of all, the task is to determine the coefficients  $k_j$  in time values  $t_j \in [t_0, t_v]$  utilizing linear functions of “disciplinary factors”, “teacher factors”, and “negative factors”.

The values  $S(t_j)$  can then be determined using (4) or (5), and comments can be made on the success of the time interval.

Let us now give detailed descriptions, explanations and assumptions of the unknowns of  $D$ ,  $T$  and  $N$  by considering the formulas (2) and (3).

## 3. Results

### 3.1. The Case $t \in [0, t_v]$

#### 1) Disciplinary factors $D$ .

Disciplinary factors in an educational environment are determined by the help of the function  $D = f(t)$  defined as

$$f: [0, t_v] \rightarrow \mathbb{R}, \quad a_1, b_1 \in \mathbb{R}.$$

Here,

a) For  $t = 0$ , the first moment of disciplinary value determination,  $f(t) = b_1$ . If  $f(t) = 0$  in time  $t = 0$ , that is, if disciplinary values are not available, the disciplinary function is in the form of as

$$f(t) = a_1 t$$

b) If  $f(t) > 0$  in the time interval  $[0, t_v]$ , the arithmetic means of the values obtained during this time interval is as following

$$\overline{f(t)} = \frac{1}{n} \sum_{j=1}^n f(t_j) = \alpha$$

where the mean value of discipline function  $f(t)$  is  $\alpha \in \mathbb{R}$ .

c) If an increasing discipline function at a time  $t = t_1$  continues to increase after reaching the necessary and sufficient legal and humanitarian disciplinary values at a time  $t = t_1$ , discipline values are considered negative and

$$f(t) = -a_1t - b_1$$

is considered as a disciplinary function after this point. Where  $a, b > 0$ , since there will be excessive discipline after this point. The maximum point where the disciplinary value of transition from normal to extreme discipline will be expressed as  $t = t_j = t_{max}$  in  $t = t_j \in [0, t_v]$ . According to this,

$$\max(f(t_{max})) = a_1t_{max} + b_1 = c_1$$

where  $c_1 \in \mathbb{R}$ .

Here, excessive discipline is when the student is anxious, intimidated and subjected to violence beyond the normal rules of discipline. It is thought that in the case of excessive discipline, success is negatively affected. In this case, disciplinary function  $f(t)$  is

$$f(t) = \begin{cases} a_1t + b_1, & 0 \leq t \leq t_{max} \\ -a_1t - b_1, & t_{vmax} \end{cases}$$

where  $a_1, b_1 > 0$ .

d) In time interval  $[0, t_v]$ , if  $f(t) > 0$  and the value of disciplinary function  $f(t)$  is constant, then

$$f(t) = b_1$$

where  $b_1 > 0$ .

e) If the disciplinary values for the time interval  $[0, t_v]$  do not exist, then

$$f(t) = 0$$

shall be taken.

f) As long as the disciplinary function  $f(t)$  in the time interval  $[0, t_v]$  is under the  $t$ -axis until  $f(t) = 0$ , that is if the disciplinary situation in the environment is absolutely undesirable, then since the disciplinary function is in the following type

$$f(t) = -a_1t,$$

$$f(t) = -a_1t - b_1,$$

$$f(t) = -b_1$$

where  $a_1, b_1 > 0$ , in this case,  $f(t) < 0$  and we consider

$$f(t) = f\left(\frac{t_1}{2}\right)$$

2) The function  $T = g(t)$ , expressing the teacher's performance factors, is defined as follows.

$$g: [0, t_v] \rightarrow (0, m], g(t) = a_2t + b_2, a_2, b_2, m \in \mathbb{R}$$

a) The average contribution value of teacher performance over the time interval  $[0, t_v]$  will be the arithmetic means of the values obtained during this period, that is,

$$\overline{g(t)} = \frac{1}{n} \sum_{j=1}^n f(t_j) = \beta$$

where  $\beta \in (0, m]$ .

b) It is thought that teacher contribution will be positive in good faith as it should be.

c) Anyway, if  $g(t) \leq 0$ , success will decrease compared to the first situation by formula.

d) The reason why the right endpoint is not included in the interval  $(0, m]$  is that each teacher can have a maximum performance value. According to this, the maximum expression of teacher value is

$$\max(g(t_{\max})) = a_2 t_{\max} + b_2 = m$$

where the maximum point is  $t = t_j = t_{\max}$ , for  $t = t_j \in [0, t_v]$ .

Note: If  $g(t) < 0$  and  $f(t) < 0$ , then it does not mean that  $\frac{f(t)g(t)}{h(t)} > 0$ .

3) The function  $N = h(t)$ , which expresses the state of negative factors, is defined as

$$h: [0, t_v] \rightarrow [1, n], h(t) = a_3 t + b_3, a_3, b_3, n \in \mathbb{R}$$

a) The average contribution value of negative factors to student achievement in the time interval  $[0, t_v]$  that is, the arithmetic means of the values obtained during this time interval will be

$$\overline{h(t)} = \frac{1}{n} \sum_{j=1}^n f(t_j) = \delta$$

where  $\delta \in [1, n)$ .

b) The reason why the lower bound of the interval  $[1, n)$  is  $\delta = 1$  that in the event that the negative value is only the minimum value  $\delta = 1$ , the success will not be adversely affected. Moreover, the case that  $\delta > 1$  is the state of  $A$  negatively affecting success.

c) In the case of  $\delta < 1$ , it will be revealed as if negative situations would positively contribute to success.

d) The reason why the upper bound of the interval  $[1, n)$  is not included is due to the fact that there may be an upper limit of negative conditions but not the maximum value.

### 3.2. The Case $t \in [t_0, t_v]$ ( $t_0 < t_v$ and $t_0 \neq 0$ ):

1) For the discipline value  $D$ , linear discipline function  $D = f(t)$  is as following:

$$f: [t_0, t_v] \rightarrow \mathbb{R}, f(t) = a_1 t + b_1, a_1, b_1 \in \mathbb{R}.$$

Here,

a) If  $f(t) = 0$  in  $t = t_0$  time, that is, there are no disciplinary values, then the disciplinary function is as follows:

$$f(t) = a_1 t + b_1 = 0$$

b) The average mean of the disciplinary function  $D = f(t)$  in the time interval  $[t_0, t_v]$  is again the arithmetic mean of the values obtained during this period, that is,

$$\overline{f(t)} = \frac{1}{n} \sum_{j=1}^n f(t_j) = \alpha$$

where  $\alpha \in \mathbb{R}$ .

c) If an increasing  $f(t) > 0$ , the disciplinary function is still increasing after the moment  $t = t_2$  in time interval  $[t_0, t_v]$ , that is, if there is excessive discipline as stated in item 1.c regarding the case

$t \in [0, t_v]$ , we consider the

$$f(t) = -a_1 t - b_1$$

as a disciplinary function where  $a_1, b_1 > 0$ . Here too, the maximum point  $t = t_j$ , which is a transition from normal disciplinary values to extreme disciplinary values, will be expressed as  $t = t_j = t_{max}$ . According to this,

$$\max(f(t_{max})) = a_1 t_{max} + b_1 = c_1$$

where  $c_1 \in \mathbb{R}$ .

In this case, the disciplinary function is

$$f(t) = \begin{cases} a_1 t + b_1, & t_0 \leq t \leq t_{max} \\ -a_1 t - b_1, & t_{1max} \end{cases}$$

where  $a_1, b_1 > 0$ .

d) In time interval  $[t_0, t_v]$ , if  $f(t) > 0$  and the value of the disciplinary function  $f(t)$  is constant, then

$$f(t) = b_1$$

where  $b > 0$ .

e) If there is no disciplinary provision in the time interval  $[t_0, t_v]$ , then again

$$f(t) = 0$$

is considered.

f) As long as the disciplinary function  $f(t)$  is below the  $t$ -axis until  $f(t) = 0$ , in the time interval  $[t_0, t_v]$  that is, if the disciplinary situation in the environment is absolutely undesirable, then since the disciplinary function will be as

$$f(t) = -a_1 t,$$

$$f(t) = -a_1 t - b_1,$$

$$f(t) = -b_1$$

where  $a_1, b_1 > 0$ , then  $f(t) < 0$  and the value of discipline is obtained as

$$f(t) = f\left(\frac{t_0 + t_1}{2}\right)$$

2) The function  $T = g(t)$  expressing the value of teacher's performance in the time interval  $[t_0, t_v]$  is defined as follows:

$$g: [t_0, t_v] \rightarrow (0, m], g(t) = a_2 t + b_2, a_2, b_2, m \in \mathbb{R},$$

where  $a$  and  $b$  are positive real numbers.

a) The average contribution value of teacher performance is again the arithmetic mean of the values obtained during this time interval  $[t_0, t_v]$ , that is,

$$\overline{g(t)} = \frac{1}{n} \sum_{j=1}^n f(t_j) = \beta$$

where  $\beta \in (0, m]$ .

b) Maximum expression of teacher value is

$$\max(g(t_{max})) = a_2 t_{max} + b_2 = m$$

where  $m \in \mathbb{R}$  and the maximum point is  $t = t_j = t_{max}$  for  $t = t_j \in [t_0, t_v]$ .

The function  $N = h(t)$  expressing the state of negative factors in the time interval  $[t_0, t_v]$  is defined as



$$h: [t_0, t_v] \rightarrow [1, n), h(t) = a_3 t + b_3, a_3, b_3, n \in \mathbb{R},$$

where  $a_3$  and  $b_3$  are positive real numbers.

The average contribution value of negative factors to the success of the student is the arithmetic mean of the values obtained during this period, and it is as follows:

$$\overline{h(t)} = \frac{1}{n} \sum_{j=1}^n f(t_j) = \delta$$

where  $\delta \in [1, n)$ . Other features of the function  $h(t)$  are expressed in item 3 of case  $t \in [0, t_v]$ .

## 4. Conclusion and Suggestions

### 4.1. Some Evaluations of Success Formula

1) If  $f(t_1) = \alpha_1 = 0$ , since  $S(t) = S_0$ , it means no success has been achieved regardless  $t_1 \in [t_0, t_v]$ ,  $\delta_1 \in [1, n)$  and of teacher value  $\beta_1 \in (0, m]$  is.

2) If  $f(t) < 0$  for  $t_1 \in [t_0, t_v]$ , negative case  $\delta_1 \in [1, n)$  and the value of teacher  $\beta_1 \in (0, m]$ , for example, let  $f(t_1) = \alpha_1 = -2$ . In this case, since

$$S(t_1) = e^{-\frac{2\beta_1}{\delta_1}(t_1-t_0)} S_0 = \frac{S_0}{e^{\frac{2\beta_1}{\delta_1}(t_1-t_0)}}$$

our success is even behind the point we started in time. Again, let  $t_1 > t_{max}$  where the other conditions are the same. In this case, since  $f(t_1) = at_1 + b = \alpha_2 < 0$ ,

$$S(t_1) = e^{\frac{\alpha_2\beta_1}{\delta_1}(t_1-t_0)} S_0 = \frac{S_0}{e^{\frac{\alpha_2\beta_1}{\delta_1}(t_1-t_0)}}$$

is obtained, and again a negative result is obtained.

3) In the case of  $f(t) > 0$ , decreasing the contrary intervention value  $\delta_1 \in [1, n)$ , increasing the teacher value  $\beta_1 \in (0, m]$ , then the success of the student will increase.

4) We're where we started on behalf of success, since  $S(t) = S_0$ , where  $f(t) > 0$  and  $n \rightarrow \infty$ .

5) According to formula (2), to obtain an average achievement in education, it must be reached

$$\overline{S(t)} = e^{\frac{(\text{ort}f(t))(\text{ort}g(t))}{\text{ort}(h(t))}(t-t_0)} S_0 = e^{\frac{\alpha\beta}{\delta}(t-t_0)} S_0$$

6) According to formula (2), for maximum success in education,

$$\max(S(t)) = e^{\frac{(\max f(t))(\max g(t))}{\min h(t)}(t-t_0)} S_0 = e^{\frac{c_1 m}{1}(t-t_0)} S_0 = e^{c_1 m(t-t_0)} S_0$$

must be reached.

7) Let  $t_1, t_2 \in [t_0, t_v]$ , ( $t_2 < t_3$ ), disciplinary value  $f(t_j) = \alpha_j$ , teacher value  $g(t_j) = \beta_j$ , contrary intervention value  $h(t_j) = \delta_j$  for ( $j = 1, 2$ ).

a) To continue the success of  $B(t)$ , it must be as following:

$$\left. \frac{\alpha_j \beta_j}{\delta_j} \right|_{t=t_{j+1}} > \left. \frac{\alpha_j \beta_j}{\delta_j} \right|_{t=t_j}$$

b) In the case that  $\frac{\alpha_j \beta_j}{\delta_j} = 1$  for  $t_j \in [t_0, t_v]$ , then  $S(t_j) = e^{t_j - t_0} S_0$ . In this case, since the values of  $D, T$  and  $N$  do not affect success and  $S_0$  has only a time-dependent coefficient, success will be considered as

$$S(t) = e^{t - t_0} S_0 = \text{constant}$$

c) In the case that  $\left. \frac{\alpha_j \beta_j}{\delta_j} \right|_{t=t_j} > 1$  for  $t_j \in [t_0, t_v]$ , since  $\alpha_j \beta_j > \delta_j$  must be, if the value  $\delta_j$  increase, the value of  $\beta_j$  will have to grow steadily to achieve inequality because the value of  $\alpha_j$  can increase to a maximum of  $c$ . In fact, to increase the success, while  $\alpha_j \beta_j > 1$ ,  $\delta_j \rightarrow 1$  is the desired situation.

Now, here is a comparative example. Let  $[0, t_v]$  be any time interval. In  $t_j = 2 \in [0, t_v]$  time, assume that the disciplinary value is the number  $f(2) = \alpha_1 = 3$  as regards disciplinary function, where  $\alpha_1 \in \mathbb{R}$ , the negative case value is the number  $h(2) = \delta_1 = 2$  as regards the negative factors function and teacher value is the number  $g(2) = \beta_1 = 10$  as regards the teacher factor function, where  $\beta_1 \in (0, m]$ . In this case,

$$S(t) = e^{\frac{3 \cdot 10}{2} \cdot 2} S_0 = e^{30} S_0$$

is obtained. Now, let us just take  $h(2) = 5$  and not change the others. In this case,

$$S(t) = e^{\frac{3 \cdot 10}{5} \cdot 2} S_0 = e^{12} S_0$$

is obtained. As seen, the increasing value of  $h(t)$  decreases the value of success. Under the same conditions, if  $\alpha_1$  is to be fixed and  $\delta_1 > 30$ ,  $\beta_1 > 10$  should be to ensure inequality since  $\alpha_1 \beta_1 = 30$ , which means that the teacher should exceed his current performance.

#### 4.2. The Case of Consideration of Student Contribution

Considering that the student personally can contribute to the student achievement formula (2) and (3), a function that expresses the student's contribution value in terms of personal and school interest and interest can be considered. Since social issues related to students' attitude, behaviour and habit movements are included in the discipline values, students' contribution to academic achievements such as studying, doing homework, questioning, research, interest, and attention to the course can only be considered.

Here if the academic contribution function to the success of a student's class or school is shown as  $R = \varphi(t)$ , then it is defined as follows:

$$\varphi: [t_0, t_v] \rightarrow (0, r], \varphi(t) = a_4 t + b_4, \quad a_4, b_4, r \in \mathbb{R}.$$

Since the student's academic contribution is considered to be positive, the value set of the function is taken as a positive interval. Here,

- a) The fact that zero is not included in the value set is that the student is at least in the classroom is considered a positive value.
- b) The inclusion of the right endpoint in the value set is that the student can make a maximum contribution to success.
- c) Since  $k = \frac{DT}{N}$ , when considering the student's academic contribution, as this coefficient is  $k = \frac{RDT}{N}$ , the success formula in the time interval  $[0, t_v]$  is as following:

$$S(t) = e^{\frac{RDT}{N} t} S_0$$

Moreover, in the time interval  $[0, t_v]$ , the mean value of the function  $R(t)$  is the arithmetic mean of the values obtained during this time interval and shall be

$$\overline{\varphi(t)} = \frac{1}{n} \sum_{j=1}^n f(t_j) = \gamma$$

If the maximum point of the function  $R(t)$  is  $t = t_j = t_{max}$ , then

$$\max(\varphi(t_{max})) = a_4 t_{max} + b_4 = r$$

d) The success formula in the time interval  $[t_0, t_v]$  will be

$$S(t) = e^{\frac{RDT}{N}(t-t_0)} S_0.$$

On the other hand, in the time interval  $[t_0, t_v]$ , the mean value of the  $R(t)$  function will again be the arithmetic mean of the values obtained during this time interval, i.e.,

$$\overline{\varphi(t)} = \frac{1}{n} \sum_{j=1}^n f(t_j) = \gamma$$

Therefore, the average value of success is as follows:

$$\overline{S(t)} = e^{\frac{\overline{\varphi(t)f(t)g(t)}}{h(t)}(t-t_0)} S_0 = e^{\frac{\gamma\alpha\beta}{\delta}(t-t_0)} S_0.$$

The maximum value will be

$$\max(\varphi(t_{max})) = a_4 t_{max} + b_4 = r$$

where the point  $t = t_j = t_{max}$  is the maximum contribution value of the student is achieved academically. In this case, the maximum value of the success is obtained as:

$$\begin{aligned} \max(S(t)) &= e^{\frac{(\max \varphi(t))(\max f(t))(\max g(t))}{\min h(t)}(t-t_0)} S_0 \\ &= e^{\frac{rc_1 m}{1}(t-t_0)} S_0 \\ &= e^{rc_1 m(t-t_0)} S_0 \end{aligned}$$

Here, for  $\overline{S(t)}$  and  $\max(S(t))$ ,  $t_0 = 0$  in time interval  $[0, t_v]$ .

### Explanation

1) The fact that  $S(t) = S_0$  and especially  $0 < S(t) < 1$  is definitely a failure situation. In such a situation, each value of success increases negatively, that is  $k \leq 0$ . Since the value of success  $S(t)$  would also represent a success above the status of failure can be graded in terms of success.

2) Since only the coefficient  $k$  determines the value of success  $S(t)$  if it is desired the multiplier  $(t - t_0)$  to be ineffective, the length of time intervals can be selected to be fixed; that is, we can take  $t_{j+1} - t_j$  is constant.

## 5. Applications

**A) The case where the functions  $D = f(t)$ ,  $T = g(t)$  and  $N = h(t)$  are linear:**

Let  $D$  be the set consisting of disciplinary values  $D_1, D_2, D_3, D_4, D_5$ , i.e.

$$D = \{D_1, D_2, D_3, D_4, D_5\}$$

and the corresponding the real numbers of the disciplinary value  $D_1, D_2, D_3, D_4, D_5$  is  $d_{j1}, d_{j2}, d_{j3}, d_{j4}, d_{j5}$  ( $j = 1, 2$ ), respectively in time interval  $t = t_j$ .

For  $t = t_1$ , we consider the number

$$d_{t_1} = \frac{d_{11} + d_{12} + d_{13} + d_{14} + d_{15}}{5}$$

and for  $t = t_2$ ,

$$d_{t_2} = \frac{d_{21} + d_{22} + d_{23} + d_{24} + d_{25}}{5}$$

as disciplinary value. According to this, since  $f(t_1) = a_1 t_1 + b_1$  and  $f(t_2) = a_1 t_2 + b_1$ , we obtain the following system:

$$\begin{cases} a_1 t_1 + b_1 = d_{t_1} \\ a_1 t_2 + b_1 = d_{t_2} \end{cases}$$

From this system, the coefficients  $a$  and  $b$  are found as follows:

$$a_1 = \frac{d_{t_2} - d_{t_1}}{t_2 - t_1} \text{ and } b_1 = \frac{t_2 d_{t_1} - t_1 d_{t_2}}{t_2 - t_1}$$

So, the equation of a line passing through points  $(t_1, d_{t_1})$  and  $(t_2, d_{t_2})$  will be

$$f(t) = a_1 t + b_1 = \frac{d_{t_2} - d_{t_1}}{t_2 - t_1} t + \frac{t_2 d_{t_1} - t_1 d_{t_2}}{t_2 - t_1}$$

A disciplinary value between points  $t = t_1$  and  $t = t_2$  can also be estimated using this line equation. Here, disciplinary values in the set  $D = \{D_1, D_2, D_3, D_4, D_5\}$  is reproducible or reducible.

Same way, after determining the  $T = \{T_1, T_2, T_3, T_4, T_5\}$  for the teacher value  $T$ , and

$N = \{N_1, N_2, N_3, N_4, N_5\}$  for the negative factors  $A$ , real values for these values can be given, and linear functions  $g(t)$  and  $h(t)$  can be written.

As a result, after obtaining the coefficient

$$k_j = \frac{f(t_j)g(t_j)}{h(t_j)}$$

and by determining the real value of  $f(t_j), g(t_j)$  and  $h(t_j)$ , the success value  $S(t_j)$  is found in any time  $t = t_j$ . If desired, considering the student's academic contribution value, the success value  $S(t_j)$  will be determined with the help of the coefficient

$$k_j = \frac{\varphi(t_j)f(t_j)g(t_j)}{h(t_j)}$$

### B) The case where the functions $D = f(t)$ , $T = g(t)$ and $N = h(t)$ are not linear:

Any or all of these functions are nonlinear. For example, assume that the disciplinary function is quadratic as following:

$$D = f(t) = at^2 + bt + c$$

Moreover, let disciplinary values be  $d_{t_1}, d_{t_2}$  and  $d_{t_3}$ , respectively for  $t = t_1, t = t_2$  and  $t = t_3$ , where  $t_1 < t_2 < t_3$ . According to this, we obtain the following system:

$$\begin{cases} at_1^2 + bt_1 + c = d_{t_1} \\ at_2^2 + bt_2 + c = d_{t_2} \\ at_3^2 + bt_3 + c = d_{t_3} \end{cases}$$

The coefficients  $a$ ,  $b$  and  $c$  belonging to disciplinary functions are determined as following from the system:

$$\begin{aligned} a &= \frac{d_{t_1}t_2 + d_{t_3}t_1 + d_{t_2}t_3 - d_{t_3}t_2 - d_{t_1}t_3 - d_{t_2}t_1}{t_1^2t_2 + t_3^2t_1 + t_2^2t_3 - t_3^2t_2 - t_1^2t_3 - t_2^2t_1} \\ b &= \frac{d_{t_2}t_1^2 + d_{t_1}t_3^2 + d_{t_3}t_2^2 - d_{t_2}t_3^2 - d_{t_3}t_1^2 - d_{t_1}t_2^2}{t_1^2t_2 + t_3^2t_1 + t_2^2t_3 - t_3^2t_2 - t_1^2t_3 - t_2^2t_1} \\ c &= \frac{d_{t_3}t_1^2t_2 + d_{t_2}t_3^2t_1 + d_{t_1}t_2^2t_3 - d_{t_1}t_3^2t_2 - d_{t_2}t_1^2t_3 - d_{t_3}t_2^2t_1}{t_1^2t_2 + t_3^2t_1 + t_2^2t_3 - t_3^2t_2 - t_1^2t_3 - t_2^2t_1} \end{aligned}$$

The disciplinary value can now be determined at a time  $t = t_j$  using the disciplinary function whose coefficients are known. Namely, if the functions  $T = g(t)$  and  $N = h(t)$  are quadratic, their coefficients can be determined as above.

Two of these functions can be quadratic, and the third is linear, or one can be quadratic, and the other two are linear. To determine the coefficients of the quadratic function, we use the points  $(t_1, d_{t_1})$ ,  $(t_2, d_{t_2})$  and  $(t_3, d_{t_3})$  above. Since there will be time interval  $[t_1, t_3]$  for a linear function, the coefficients of this function are calculated separately for time intervals  $[t_1, t_2]$  and  $[t_2, t_3]$  two functions are determined concerning these time intervals. That is to say, the coefficients of the first function will be obtained with the help of the points  $(t_1, d_{t_1})$ ,  $(t_2, d_{t_2})$  concerning the linear function and the coefficients of the second function will be obtained with the help of the points  $(t_2, d_{t_2})$ ,  $(t_3, d_{t_3})$  in time interval  $[t_1, t_3]$ . So, this linear function will be as following:

$$\begin{cases} a_1t + b_1, t \in [t_1, t_2] \\ a_2t + b_2, t \in [t_2, t_3] \end{cases}$$

For any time  $t_j \in [t_1, t_3]$ , the nonlinear function will be used directly and for the linear function, the function of the time interval in which the time value  $t = t_j$  is to be considered.

## 6. Results and Discussion

In this study, some real results will be obtained with the help of the functions installed for the “discipline factors”, “teacher performance factors”, and “negative factors”. Therefore, in the case of the value of the success desired to be learnt is the negative according to the results, the function or the function or the functions which cause the failure can be determined. The function or functions which adversely affect the success of the function are improved. For example, if the failure is due to more disciplinary values, the real value of decreasing the mean by decreasing the average is small; it will make to correct that value or that values by examining the earlier determined discipline values. After determining the disciplinary values for two points of a time interval, the linear line equation of the function can be determined with the help of their averages. By considering the line of this equation, it is possible to determine the disciplinary values for a time value between these time points. On the other hand, the model can be implemented with the help of the model and the functions depending on the model to achieve positive results for success because it is not intended to determine success only. Besides, it is possible to make suggestions based on the results obtained in terms of the elements of the sets of values.

Of course, comments can be made in the name of values derived from functions established in the name of “discipline factors”, “teacher factors”, and “negative factors”. On the other hand, since there will be a linear function for the mentioned values, it will first determine the failing or featured value by reading the status of these functions by good reading. For this reason, the study aims to make an application in the name of success by meaning the clusters of the mentioned values, determining the status of the success for an instant or a time interval, or making a suggestion in cases or cases where the success is negative.

This study aims to establish a criterion for examining the success of the student, class, or school in a certain period. The way of success can be examined in the time taken into consideration with the help of this criterion. For this purpose, the first thing to do is to determine the coefficient in time values with the help of linear functions installed for the values, which will be defined as “discipline factors”, “teacher factors”, and “negative factors”. Then, the success values can be determined depending on the mentioned values by using the relation obtained as a solution of an initial value problem and comments on success can be made in the aforesaid time interval.

## Conflicts of Interest

The author declares no conflict of interest.

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