Öneri.C.9.S.34.
Temmuz 2010.195-201.

AGGREGATE PRODUCTION PLANNING MODEL BASED ON MIXED INTEGER LINEAR PROGRAMMING

Veli ULUÇAM

Dr.

AGGREGATE PRODUCTION PLANNING MODEL BASED ON MIXED INTEGER LINEAR PROGRAMMING

***Abstract:** This work presents a mixed integer linear programming method developed by using 0-1 variables for solving aggregate production planning problem with the following performance criteria: (1) maximize profit, (2) minimize costs. The production planning activities are called as aggregate production planning when both it is produced more than one goods and the demand changes period by period. The purpose of aggregate production planning is not to prepare detailed plans for each goods, it is to do plans for whole goods produced in the firm together to take in hand. Aggregate production planning is probably one of the most important, yet least understood, jobs that a manager performs. However, all parts of the organization, operations, marketing, finance, and so on, must work together in the planning process to ensure that they are moving in harmony with one another. Aggregate production planning is such a method that can move all parts of the organizations in same harmony.*

***Keywords:** Aggregate Production Planning, 0–1 Variables, Mixed Integer Linear Programming.*

KARMA TAMSAYILI DOĞRUSAL PROGRAMLAMA YÖNTEMİ İLE BÜTÜNLEŞİK ÜRETİM PLANLAMASI

***Özet:** Bu çalışmada; kazançların maksimize edilirken maliyetlerin minimize edilebileceği bir bütünleşik üretim planlama probleminin, 0–1 değişkenleri kullanılarak geliştirilmiş karma tamsayılı doğrusal programlama tekniği ile çözümü anlatılmaktadır. Birden fazla çeşitte ürünün bir arada üretildiği ve talebin dönemlere göre değişiklik gösterdiği durumlarda üretim planlama faaliyetleri bütünleşik üretim planlama olarak tanımlanır. Planlama çalışmalarının bütünleşik olma niteliği, bu yöntemin tek tek ürünlerin detaylı planlarının hazırlanması amacıyla değil, işletme tarafından üretilen tüm ürünlerin bir arada ele alınarak planlama çalışmalarının yapılmasıdır. Son yıllarda daha iyi anlaşıldığı üzere, bütünleşik üretim planlaması bir yöneticinin performansını etkileyen en önemli kriterdir. Bununla beraber bir işletmenin tüm bölümleri, üretim, pazarlama, finans ve diğerleri aynı ahenk içersinde ve uyumlu olarak çalışmak zorundadır. Bütünleşik üretim planlaması bu ahengi sağlayacak bir yöntemdir.*

***Anahtar Kelimeler:** Bütünleşik Üretim Planlaması, 0–1 Değişkenleri, Karma Tamsayılı Doğrusal Programlama.*

I. INTRODUCTION

The aggregate production planning, which might also be called macro production planning, addresses the problem of deciding how many employees the firm should retain and, for a manufacturing firm, the quantity and the mix of products to be produced [1]. Aggregate production planning methodology is designed to translate demand forecasts in to a blueprint for planning staffing and production levels for the firm over a predetermined planning horizon. The aggregate production-planning problem has been studied extensively, since it was first formulated in the 1950s. Interest in the problem stems from the ability such models provide to control production and inventory costs [2]. The costs associated with inventory management and production planning are a substantial portion of the total expenditure of the manufacturing companies. Aggregate production planning involves matching capacity to fulfill the demand of forecasted, fluctuating customer orders in the medium term from 3 to 18 months ahead [3].

II. THE PURPOSE OF AGGREGATE PRODUCTION PLANNING

The aggregate production planning and production plan disaggregating have two purposes [4]. Aggregate planning attempts to maximize total profit or minimize total cost while considering decision variables such as production levels, workforce levels, inventory carrying costs and hiring and layoff cost. Aggregate planning is necessary in POM because it provides for [5]:

- ✓ Fully loaded facilities and minimizes overloading and under loading, thus reducing production costs,
- ✓ Adequate production capacity to meet expected aggregate demand,
- ✓ A plan for the orderly and systematic change of production capacity to meet the peaks and valleys of expected customer demand,

✓ Getting the most output for the amount of resources available, which is important in times of scare production resources.

III. TECHNIQUES FOR AGGREGATE PRODUCTION PLANNING

Numerous Aggregate Production Planning models with varying degrees of sophistication have been introduced in the last four decades [6].

Holt et al. Developed one of the first Aggregate Production Planning models, commonly known as LDR model. It was applied to a paint factory to generate a production plan using quadratic approximations to the actual operational costs of the firm. Silva et al. Extended the LDR model to develop a decision rule that considers a constant level of employment during the entire planning period.

Hanssman and Hess developed a model based on the linear programming approach using a linear cost structure of the decision variables. Haehling extended the Hanssman and Hess model for multiproduct, multistage production systems in which optimal disaggregation decisions can be made under capacity constraints.

III.1. The Mathematical Approach

Mathematical programming (or optimization theory) is that branch of mathematics dealing with techniques for maximizing or minimizing an objective function subject to linear, nonlinear, and integer constraints on the variables [7]. One of the difficulties faced with discrete mathematical programming for aggregate production planning is combinatorial complexity, which increases dramatically with the size of the problem [8].

III.2. Linear Programming

Linear programming is concerned with the maximization or minimization of a linear objective function in many variables subject to linear equality and inequality constraints [7]. Linear programming (LP) can be used in aggregate planning if the costs of various resources are assumed to be linear functions of the amount of those resources used by the aggregate plan [9]. LP can be used to plan production over some horizon for an actual product or for some pseudo product. LP is a method that can be used to analyze the effects of long range financial plans, aggregate plans, production plans, holding inventory, distribution problems, promotion decisions, back ordering, the use of overtime, and the use of subcontracting.

It's possible to enumerate some of the reasons of choosing linear programming in production management [10].

a. To plan the production planning to minimize the production and the storage costs so as to cover the demand for a product.

b. Determine the production families by using available machines and the workforce, to maximize the firm benefits.

c. Determine the raw materials combination to make the production for the minimal production costs.

d. To plan the establishment of the firm so as to make the costs minimal for production and distribution.

e. To assign the workforce for the machines.

III.3. Mixed Integer Linear Programming

A general integer programming problem is a linear programming model in which the decision variables can only accept whole or integer numbers [11]. For example, such variables may represent the number of employees to hire, the number of the trucks to build, or the number of tankers to schedule. When all the decision variables are required to be integer, the problem is called a **pure integer programming** problem. If only some of the variables are required to have integer values (so the divisibility assumption holds for the rest), this model is referred to as **mixed integer programming** (MIP) [12].

III.4. 0-1 Variables

There are three basic types of integer linear programming models: a total integer model, a 0-1 integer model, and a mixed integer model [13]. In a total integer model all of the decision variables are required to have integer solution values. In a 0-1 integer model all of the decision variables have integer values of zero or one. Finally, in a mixed integer model some of the decision variables (but not all) are required to have integer solutions.

When extra conditions are imposed on a linear programming model, 0–1 variables are usually introduced and "linked" to some of the continuous variables in the problem to indicate certain states [14]. For example, suppose that x represents the quantity of an ingredient to be included in a blend. We may well wish to use an indicator variable ∂ to distinguish between the state where $X = 0$ and the state where $X > 0$. By introducing the following constraint we can force ∂ to take the value 1 when $X > 0$;

$$X - M\partial \leq 0$$

M is a constant coefficient representing a known upper bound for X.

Logically we have achieved the condition;

$$X > 0 \rightarrow \partial = 1$$

where "$\rightarrow$" stands for "implies".

In many applications provides a sufficient link between X and ∂. There applications however, where we also wish to impose the condition.

$$X = 0 \rightarrow \partial = 0$$

is another way of saying;

$$\partial = 1 \rightarrow X > 0$$

and together it can be written as;

$$\partial = 1 \leftrightarrow X > 0$$

where "$\leftrightarrow$" stands for "if and only if".

$$\partial = 1 \rightarrow X > M$$

This condition can be imposed by the constraint;

$$X - m\partial \geq 0.$$

III.5. Software Options for Solving Such Models

All of the software packages featured in your OR Courseware (Excel, LINGO/LINDO, and MPL/CPLEX) include an algorithm for solving (pure or mixed) BIP models, as well as an algorithm for solving general (pure or mixed) IP models where variables need to be integer but not binary [12]. However, since binary variables are considerably easier to deal with than general integer variables, the former algorithm generally can solve substantially larger problems than the latter algorithm.

When using the Excel Solver, the procedure is basically the same as for linear programming. The one difference arises when you click on the "Add" button on the Solver dialogue box to add the constraints. In addition to the constraints that fit linear programming, you also need to add the integer constraints. In the case of integer variables that are not binary, this is accomplished in the Add Constraint dialogue box by choosing the range of integer-restricted variables on the left-hand side and then choosing "int" from the pop-up menu. In the case of binary variables, choose "bin" from the pop-up menu instead.

In a LINDO model, the binary or integer constraints are inserted after the END statement. A variable X is specified to be a general integer variable by entering GIN X. Alternatively, for any positive integer value of n, the statement GIN n specifies that the first n variables are general integer variables. Binary variables are handled in the same way except for substituting the word INTEGER for GIN.

IV. Problem Formulation

IV.1. Notation

Indices

i = number of raw material families.

j = number of product families.

t = number of periods (months) in the planning horizon.

Input Parameters

$a_{j,t}$ per unit sales revenue of product j in period t ($/tone).

$d_{j,t}$ per unit demands of product j in period t (tone).

$c_{i,j,t}$ percentage of products that can be produced from the raw material i in period t (%/tone)

$e_{j,t}$ per unit exported sales revenue of product j in period t (tone/month)

$o_{j,t}$ per unit imported sales revenue of product j in period t (tone/month).

$h_{i,t}$ amount of chemical components per raw material in period t (tone).

$p_{i,t}$ amount of material components per raw material in period t (tone).

$b_{i,t}$ purchasing costs per unit of raw materials in period t ($/tone).

$r_{1,t}$ cost of chemical components per raw material in period t ($/tone).

$r_{2,t}$ cost of material components per raw material in period t ($/tone).

$q_{i,t}$ cost to hold one unit of raw material i in stock in period t ($/tone).

$f_{j,t}$ cost to hold one unit of product i in stock in period t ($/tone).

Decision Variables

$(X_{i,t})$ amount of raw material i to be bought in period t (tone/month)

$(Y_{i,t})$ amount of raw material i to be used in period t (tone/month)

$(I_{i,t})$ inventory of raw material i at the end of period t (tone/month)

$(I_{i,t-1})$ inventory of raw material i at the beginning of period t (tone/month)

$(S_{j,t})$ inventory of product j at the end of period t (tone/month)

$(m_{i,t})$ amount of product j that can be exported in period t (tone/month).

$(n_{i,t})$ amount of product j that can be imported in period t (tone/month).

Constraints

For each period, the following constraints apply;

1) The amount of raw material bought, should not be greater than the available capacity.

u: the capacity of the main raw material tank.

$$\sum_{i=1}^{10} \sum_{t=1}^{3} x_{i,t} \le u$$

u = 1.110.531,69 (tone/month)

Upper limit of the production capacity.

$$\sum_{i=1}^{10} \sum_{t=1}^{3} c_{i,1,t} \le 37.658$$

$$\sum_{i=1}^{10} \sum_{t=1}^{3} c_{i,2,t} \le 87.085$$

$$\sum_{i=1}^{10} \sum_{t=1}^{3} c_{i,3,t} \le 135.272$$

$$\sum_{i=1}^{10} \sum_{t=1}^{3} c_{i,4,t} \le 152.563$$

$$\sum_{i=1}^{10} \sum_{t=1}^{3} c_{i,5,t} \le 250.215$$

$$\sum_{i=1}^{10} \sum_{t=1}^{3} c_{i,6,t} \le 46.998$$

$$\sum_{i=1}^{10} \sum_{t=1}^{3} c_{i,7,t} \le 253.197$$

$$\sum_{i=1}^{10} \sum_{t=1}^{3} c_{i,8,t} \le 143.543$$

2) Not to stop the production, the production rate must be 60% of the upper limit of the capacity.

3) The inventory level of the raw material can be written as given below;

$$I_{i,t-1} \quad X_{i,t} - Y_{i,t} = I_{i,t}$$

4) The inventory level of the raw material i at the beginning of period t (tone)

$I_{1,t-1} = 3.600$ tone

$I_{3,t-1} = 156.100$ tone

$I_{8,t-1} = 34.300$ tone

5) The inventory level of the product families can be written as given below;

$$S_{j,t-1} + c_{i,j,t} - d_{j,t} - m_{j,t} + n_{j,t} = S_{j,t}$$

6) Storage capacity of the sum of products at the end of the period t.

Inventory level of the products (400.000 tone/month)

$$\sum_{j=1}^{8}\sum_{t=1}^{3} S_{j,t} \leq 400.000 \quad t=1,2,3$$

7) The raw material (for i = 7) amount must be less than %25 percent of the total amount of the all raw materials.

$$X_{7,t} \leq 0.25 \sum_{i=1}^{10} X_{i,t}$$

8) Non-negativity constraints.

$$X_i \ , Y_i \ \ I_i \ \ S_J \ , n_j \ , m_j \geq 0$$

0–1 Variable

After the demand required, the rest of the products can be exported if needed.

$W_{j,t} = 0-1$ variable

$$W_{j,t} = \begin{cases} 1 & \text{(prod. level bigger than the capacity)} \\ 0 & \text{(prod. level smaller than the capacity)} \end{cases}$$

$$\sum_{i=1}^{10} c_{i,j,t} \geq k_{j,t} \ W_{j,t}$$

Q; the limit of the imports. It's given 100.000 (tone/month)

$$n_{j,t} \leq W_{j,t} \ Q$$

The amount of products should be more than 10.000 ton to be imported.

So, the constraint can be written as below.

$$n_{j,t} \geq 10.000 \ W_{j,t}$$

IV.2. The Mixed Integer Linear Programming Model for Profit Maximization Function

$$Max.Z = \underbrace{\sum_{j=1}^{8}\sum_{t=1}^{3} a_{j,t} d_{j,t}}_{\text{Sales Revenue}} + \underbrace{\sum_{j=1}^{8}\sum_{t=1}^{3} e_{j,t} m_{j,t}}_{\text{Export Revenue}} - \underbrace{\sum_{j=1}^{8}\sum_{t=1}^{3} o_{j,t} \ n_{j,t}}_{\text{Import Costs}}$$

$$- \underbrace{\sum_{i=1}^{10}\sum_{t=1}^{3} b_{i,t} X_{i,t}}_{\text{Buying Costs}} - \underbrace{\sum_{i=1}^{10}\sum_{t=1}^{3} h_{i,t} Y_{i,t} r_{1,t}}_{\text{Chemical Costs}} - \underbrace{\sum_{i=1}^{10}\sum_{t=1}^{3} p_{i,t} Y_{i,t} r_{2,t}}_{\text{Material Costs}}$$

$$- \underbrace{\sum_{i=1}^{10}\sum_{t=1}^{3} q_{i,t} I_{i,t}}_{\text{Raw Material Storage Costs}} - \underbrace{\sum_{j=1}^{8}\sum_{t=1}^{3} f_{j,t} S_{j,t}}_{\text{Storage Costs For Products}}$$

IV.3. An Illustrative Example

According to firm's aggregate production planning results, at the end of the planning periods, the firm's net benefit becomes 114.696.791$. For the first period the firm's profit is -57.720.310$, this means the firm losses for this period. But the second period profit becomes 62.575.745$. Finally for the last third period the firm's profit rises to 109.841.356$, and totally the profit for the planning periods is 114.696.791$. The firm's profit/loss table is given below at Table.1 in detail.

Table.1. The Firms Profit / Loss

	1.Period	2.Period	3.Period	TOTAL
Total Sale Returns ($)	535.847.801	546.994.905	756.353.008	1.839.195.714
Total Exporting Returns ($)	265.181.878	178.186.676	165.034.696	608.403.250
Imports Expense ($)	121.219.308	120.945.200	94.550.557	336.715.065
Raw Material Purchasing Cost ($)	706.370.920	510.391.310	685.310.228	1.902.072.458
Chemical Costs ($)	1.421.164	1.340.309	1.367.156	4.128.629
Material Costs ($)	1.629.360	1.536.660	1.567.440	4.733.460
Raw Material Storage costs ($)	1.172.726	873.096	1.188.058	3.233.879
Storage costs for products ($)	994.888	1.577.638	1.621.286	4.193.812
TOTAL	—31.778.687	88.517.368	135.782.979	**192.521.660**
FIXED EXPENSE ($)	25.941.623	25.941.623	25.941.623	77.824.869
PROFIT / LOSS ($)	—57.720.310	62.575.745	109.841.356	**114.696.791**

According to alternative plan, at the end of planning periods the firm's net profit is put forward to be 875.912.076$. For the first period the firm's profit is 47.720.507$, but the second period profit becomes -28.634.011$, this means the firm losses for this period. Finally for the last third period the firm's profit rises to 857.825.580$, and totally the profit for the planning periods is 875.912.076$. The firm's and the alternative plan's profit/loss comparison table is given below at Table.2 in detail.

Table.2. The Firm's And The Alternative Plan's Profit / Loss

	FIRM	PLAN
Total Sale Returns	1.839.195.714	1.839.195.714
Total Exporting Returns	608.403.250	1.012.267.746
Imports Expense	336.715.065	298.833.443
Raw Material Purch.Costs	1.902.072.458	1.583.112.708
Chemical Costs	4.128.629	4.588.274
Material Costs	4.733.460	5.260.441
Raw Material Storage costs	3.233.879	2.164.550
Storage costs for products	4.193.812	3.767.099
Fixed Expense	77.824.869	77.824.869
PROFIT / LOSS	**114.696.791**	**875.912.076**

According to firm's aggregate production planning results, the net profit for the planning periods is 114.696.791$ and totally the firm has 561.868.676$ valuable products to put up for sale and the raw materials to be given for the production. When all of these worth's are taken into accounts, the firm's profit revenue becomes 676.565.467$.

The benefit difference between the alternative and the firm's aggregate production planning is occurs as 225.284.805$.

V. CONCLUSION

This paper presents a multiple criteria mixed integer linear programming model to solve aggregate production planning problems. The model has been developed to optimize four criteria for a quantity of purchasing raw material, quantity of raw material to be used in production, quantity of products to be exported, and the quantity of products to be imported. In order to enhance its application in practice, a decision support system based on the model has also been included.

In this study Excel Solver has been used to solve the model.

According to alternative aggregate production planning study it has been found out that, the quantity of raw materials to be purchased, also has been reduced in totally, is clear enough to cover the anticipated demand for the production. The amount of raw material has been found out to be 2.728.466 tone in the research. But the firm's planning result gives us the amount of 2.762.100 tone raw materials. It becomes out that 33.634 tone raw materials is decreased, and this comes into existence of increasing benefits. In this study, the reduced amount of raw materials increased the benefits, because the production level is also increased. The reason how production level is increased is related with the type of raw materials. In addition to this, the rise of the exporting amounts makes the benefits go up for another factor.

The firm's product stocks (for all planning periods) to be turned over for the next period is becomes totally 350.800 tone, but response to this, for the alternative plan the amount reduces to 30.998 tone. From this point of view, the quantity of demand is being covered, at the same time the unrequired stocks of product have been reduced. When the firm's selling amounts and the costs is examined, at the end of planning periods, it is clear that positive differences occurs in the profit/loss tables. If these benefits and losses are taken into account, an increase of 225.284.805$ worth becomes in the firm's profit.

According to the sensitive analysis for the first period of planning horizon; utilizing the variables for the reduced costs, the raw materials (for all variables) amount bought, is the optimum quantities and another purchasing doesn't make reduce in the profit maximization function.

According to the sensitive analysis for the second period of planning horizon; utilizing the variables for the reduced costs, if a purchase is done for i=6 variable: 239,85$ reduction occurs for one unit.

According to the sensitive analysis for the third period of planning horizon; utilizing the variables for the reduced costs, if a purchase is done for **i=2, i=4, i=5** and **i=6** variable: 1,27$, 1,19$, 3,35$ and 221,15$ reduction occurs for one unit in order.

The model that has been developed in this study is so elastic that it provides a facility in working against this kind of studies to find the optimal solutions. This model can be applied to different manufacturing firms by doing some modifications in variables and parameters.

REFERENCES

[1] Nahmias, S. (2005). *Production and Operation Analysis*. 5th Ed. New York: Irwin Mc Graw Hill.

[2] Jain, A. & Palekar, U.S. (2005). Aggregate Production Planning for a Continuous Reconfigurable Manufacturing Process. *Computers & Operations Research*, 32(5), 1213-1236.

[3] Wang, R.C. & Liang, T.F. (2005). Aggregate Production Planning with Multiple Fuzzy Goals. *International Journal of Adv. Manufacture Technologies*, 25(7), 589-587.

[4] Yalçın, A. & Boucher, T.O. (2004). A Continuous-time for Disaggregation of Hierarchical Production Plans. *IIE Transactions*, 36(10), 1023-1035.

[5] Gaither, N. (1996). *Productions and Operations Management*. 7th Ed. California: Duxbury Press.

[6] Da Silva, C.G.; Figueria, J.; Lisboa, J. & Barman, S. (2006). An Interactive Decision Support System for an Aggregate Production Model Based on Multiple Criteria Mixed Integer Linear Programming. *The International Journal of Management Science*, 34(16), 166-167.

[7] Dantzig, G.B. & Thapa, M.N. (1997). *Linear Programming*, California: Springer.

[8] Singhvi, A.; Madhavan, K.P. & Shenoy, U.V. (2004). Pinch Analysis for Aggregate Production Planning in Supply Chains. *Computers and Chemical Engineering*, 28(10), 993-998.

[9] Dilworth, J.B. (1993). *Production and Operations Management: Manufacturing and Services*. 5th Ed. New Jersey: Mc Graw Hill.

[10] Üreten, S. (2002). *Üretim İşlemler Yönetimi Stratejik Kararlar ve Karar Modelleri*. 3. Baskı. Ankara: Gazi Kitabevi,.

[11] Pegels, C.C. & Lofti, V. (1996). *Decision Support Systems for Operations Management and Management Science*. 3rd Ed. New Jersey: Irwin.

[12] Hillier, F.S. & Lieberman, G.J. (2005). *Introduction to Operations Research*. 8th Ed. New York: Tata Mc Graw Hill.

[13] Taylor III, B.W. (1999). *Introduction to Management Science*. 6th Ed. Upper Saddle River, New Jersey: Prentice Hall.

[14] Williams, H.P. (2003). *Model Building in Mathematical Programming*. 4th Ed. New York: John Wiley & Sons Ltd.

Veli ULUÇAM

veliu2001@yahoo.com

He is graduated from the Istanbul University, Faculty of Forestry. He earned his master degree in Production Management and Marketing from the Marmara University, 2001. Dr. degree in Production Management and Marketing from the Marmara University, 2008. His major research interests focus on the production management, supply chain and E-Business.