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Research Article

A Finite Element Implementation of A Phenomenological Constitutive Model for Rubber-like Materials

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Abstract

In this paper, a finite element implementation of a recently proposed phenomenological constitutive model for rubber-like materials is represented based on the fundamentals of continuum mechanics and rubber elasticity. The phenomenological model is first fitted to the hyperelastic behavior of an unfilled silicon rubber subjected to five different uniform deformations. Then, a subroutine is written to import the model into the finite element software and an unfilled silicon rubber sheet is numerically modeled in the commercial finite element software. As performed in the experiments by Meunier et al (Meunier, Chagnon, Favier, Orgéas, & Vacher, 2008)., the rubber sheet is deformed 57.2 mm along the vertical axes in the simulations. Good agreement between the numerical model and experimental data is obtained.

Keywords: Rubber-like materials, Constitutive modeling, Hyperelasticity, FEM implementation.

Bir Fenomenolojik Yapısal Modelin Kauçuk Tipi Malzemeler İçin Sonlu Elemanlar Yöntemi Uygulaması

Öz

Bu makalede, kauçuk tipi malzemelerin yapısal olarak modellenmesi için yakın zamanda önerilmiş olan bir fenomenolojik model sürekli ortamlar mekaniği ve kauçuk elastisitesi temellerine dayanarak sonlu elemanlar yöntemi içerisine adapte edilmiştir. Model, ilk önce saf silikon kauçuğun beş farklı yükleme altında gösterdiği hiperelastik davranışlara göre kalibre edilmiştir. Sonrasında modeli sonlu elemanlar yöntemi yazılımına adapte etmek için altprogram yazılmış ve üzerinde delikler olan saf silikon kauçuk levha yazılım içerisinde nümerik olarak modellenmiştir. Meunier ve diğerlerinin (Meunier, Chagnon, Favier, Orgéas, & Vacher, 2008) deneylerde yaptığı gibi kauçuk levha simülasyon içerisinde 57.2 mm dikey deplasmana maruz bırakılmıştır. Yapılan ölçümlerde nümerik modelin ve deneysel verilerin örtüştüğü görülmüştür.

Anahtar Kelimeler: Kauçuk tipi malzemeler, Yapısal modelleme, Hiperelastisite, SEY uygulamaları.

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1. Introduction

Rubber-like materials are a class of polymers having a wide range of industrial applications ranging from tires to electronics, textile to isolation systems. Due to their application capability, their physical and mechanical characterization is crucial. However, rubber-like materials, such as carbon-black rubber, soft tissues, hydrogels, exhibit nonlinear inelastic features under large deformation. As such materials can sustain strain up to 20 times of initial length (Sun, et al., 2012), their mechanical characterization is complicated. Although, in literature, there have been many constitutive models in different scales proposed to describe the non-linearity and inelasticity of rubber-like materials (Boyce & Arruda, 2000), (Diani, Fayolle, & Gilormini, 2009), (Steinmann, Hossain, & Possart, 2012), the background of their complicated mechanical behavior is still indefinable. Therefore, since the beginning of the 1900s, there have been many studies carried out to identify structural properties of such materials during the course of deformation.

Constitutive modeling based on the fundamentals of continuum mechanics and rubber elasticity is a powerful tool to characterize behaviors of the materials, which can sustain large deformation. In general, constitutive modeling of rubber-like materials can be split into two parts: micro-mechanical and phenomenological. Despite higher computational costs and some idealization approaches, micro-mechanical models get more attention since they have interpreted the macroscopic material behavior with the change of the physical properties during deformation. This property is an important benefit compared to phenomenological models. Some of the micro-mechanical constitutive models for rubber-like materials are the 3-chain, 4-chain, 8-chain, and unit sphere models (Steinmann, Hossain, & Possart, 2012). However, in commercial applications, phenomenological models are broadly chosen due to computational cost. The commercial software, i.e., ANSYS®, ABAQUS®, also contains some of the very important phenomenological constitutive models ready to use.

The phenomenological models describe the macro-mechanical behavior based on the fundamentals of continuum mechanics and can be classified into two approaches: principal stretch- and invariant-based models. Ogden (Ogden, 1972) proposed a principal stretch-based model. The model is very flexible and can describe the mechanical behavior of various materials. Early strain invariant-based models are Mooney-Rivlin type (Mooney, 1940). The simplest expression is known as the neo-Hookean model that describes the material behavior for moderate deformations. Among the strain invariant-based models, Yeoh (Yeoh O. H., 1990), Gent (Gent, 1996), Yeoh and Fleming (Yeoh & Fleming, 1997), Carroll (Carroll, 2011) models are widely used in the literature. Recently, Blaise et al. (Blaise, Bien-aimé, Betchewe, Marckman, & Beda, 2020), Mansouri & Darijani (Mansouri & Darijani, 2014), and Darijani & Naghdabadi (Darijani & Naghdabadi, 2010) proposed successful phenomenological constitutive models.

In the present paper, the predictive capability of the model proposed by Külcü (Külcü, 2020) for complicated finite element simulations is examined. Firstly, the experimental data of five different loading modes are reproduced by the model having a constant value of material parameters for each deformation type. Then, the model is implemented into finite element simulation via subroutines. Lastly, the results of the numerical model have competed against the experimental data.

2. Modeling

2.1. The Fundamentals of Continuum Mechanics

The continuum mechanics approach is an effective tool, which does not consider discontinuities of microscopic systems. It is frequently used to develop mathematical models of material behavior to estimate the material deformation and motion. In the following, basic terms of the continuum mechanics used in this study are shown.

Let $\mathbf{F}$ be the deformation gradients that maps a material point $\boldsymbol{X}$ in the reference configuration to a point $\boldsymbol{x}$ in current configuration and shown as

$$\boldsymbol{x} = \mathbf{F}\boldsymbol{X} \tag{1}$$

Also, let $\Psi = \Psi(\boldsymbol{C}, \xi)$ be the strain energy function per unit volume of a rubber-like material, with ξ being some internal variables. Now, considering that the rubber-like material is nearly incompressible, the strain energy function can be additively decomposed into volumetric and isochoric parts (Flory, 1961) such as

$$\Psi(\boldsymbol{C}, \xi) = \mathrm{U}(J) + \Psi_{iso}(\boldsymbol{C}', \xi) \tag{2}$$

where the first term on the right-hand side is taken into account for the volumetric response of the material, whereas the second term is for the distortional deformation. In Eq. (2), $\boldsymbol{C}$ is the right Cauchy Green tensor, J is the volume change and $\boldsymbol{C}'$ is the unimodular tensor describing the distortional deformation.

$\boldsymbol{C}$ and $\boldsymbol{C}'$ can be written as follows

$$\mathbf{C} = \mathbf{F}^{\boldsymbol{T}}\mathbf{F}, \tag{3}$$

$$\boldsymbol{C}' = J^{-2/3}\boldsymbol{C}. \tag{4}$$

2.2. The Constitutive Model

In practice, to appraise the accuracy and applicability of a particular constitutive model, a simple experiment is performed and the results are compared with the data reproduced by the model. To the best of our knowledge, an ideal phenomenological constitutive model should contain the following requirements:

1. Include few material parameters as possible,
2. Describe different relatively complicated deformation behavior,
3. Reproduce the behavior of different materials.

However, the macro-mechanical models found in literature often suffer one or two of the above-mentioned requirements. Some of them contain more material parameters, some of them are not able to characterize the material behavior under different loading modes (see (Steinmann, Hossain, & Possart, 2012)). Such drawbacks result in difficulties in the structural analysis, which is carried out in the finite element simulations.

Recently, Külcü (Külcü, 2020) proposed a phenomenological constitutive model that represents material behavior with a constant material parameter for the classical Treloar experimental data (Treloar, 1944). Also, experimental data of rubber by Sasso et al. (Sasso, Palmieri, Chiappini, & Amodio, 2008) is successfully reproduced by the model. Lastly, experimental data of different materials, such as collagen and fibrin (Storm, Pastore, MacKintosh, Lubensky, & Janmey, 2005), is utilized to check the model accuracy.

This model is represented as

$$\Psi = \mu\left[f(I_1, \alpha) + f\left(I_2, -\frac{\alpha}{16}\right) + \ln\left(\frac{1}{\alpha} f\left(I_{1,}1\right) + 1\right)\right], \tag{5}$$

where I_1 and I_2 are the first and second strain invariants of right Cauchy-Green tensor, respectively, μ is the shear modulus, α is the scalar material parameter and

$$\mathrm{f}(x, y) = \frac{\alpha}{y}\left[e^{y[x-3]} - 1\right]. \tag{6}$$

The strain invariants can be calculated as

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2, \tag{7}$$

$$I_2 = \lambda_1^2\lambda_2^2 + \lambda_1^2\lambda_3^2 + \lambda_2^2\lambda_3^2, \tag{8}$$

where λ_i $(i = 1,2,3)$ is the principal stretch. The principal stretches can be written for different uniform deformation modes as shown in Table 1.

Table 1. Principle stretches

	$\boldsymbol{\lambda_1}$	$\boldsymbol{\lambda_2}$	$\boldsymbol{\lambda_3}$
Uniaxial	λ	$\lambda^{0.5}$	$\lambda^{0.5}$
Equibiaxial	λ	λ	$\lambda^{0.5}$
Pure shear	λ	λ^{-1}	1

Finally, by considering the incompressibility condition for rubber-like materials, the first Piola-Kirchhoff stress is written as

$$P_i = \frac{\partial\Psi}{\partial\lambda_i} - \frac{1}{\lambda_i}\mathrm{p} \quad (i = 1,2,3), \tag{9}$$

where p is the Lagrange multiplier. The first Piola-Kirchoff stress for the uniaxial, equibiaxial, and pure shear deformations are represented as

$$P_{uniaxial} = 2\mu\left(\lambda - \frac{1}{\lambda^2}\right)\left(\alpha\, e^{(\alpha[I_1-3])} + \alpha\frac{1}{\lambda}e^{\left(\frac{-\alpha[I_2-3]}{16}\right)} + 1\right), \tag{10}$$

$$P_{equibiaxial} = 2\mu\left(\lambda - \frac{1}{\lambda^5}\right)\left(\alpha\, e^{(\alpha[I_1-3])} + \alpha\lambda^2 e^{\left(\frac{-\alpha[I_2-3]}{16}\right)} + 1\right), \tag{11}$$

$$P_{pureshear} = 2\mu\left(\lambda - \frac{1}{\lambda^3}\right)\left(\alpha\, e^{(\alpha[I_1-3])} + \alpha\, e^{\left(\frac{-\alpha[I_2-3]}{16}\right)} + 1\right). \tag{12}$$

To implement the model into finite element application, Abaqus® is utilized. The model is coded for user subroutine UHYPER.

3. Results and Discussion

The aim to constitute a material model is to predict material behavior under complicated deformations. Implementing a material model into the finite element simulations is significant to perform such analysis. In this contribution, the Külcü model is implemented by writing a code and importing the code into a commercial finite element software. Experimental data of unfilled silicon rubber by Meunier et al. (Meunier, Chagnon, Favier, Orgéas, & Vacher, 2008) is utilized to check the accuracy of the model in finite element simulations. Therefore, the model is first fitted to the five different uniform deformations. In Figure 1, the fitting of the model against uniaxial tension/compression, equibiaxial tension, and tensile/compressive pure shear tests are represented. The model is able to reproduce all of the deformations with constant values of material parameters.

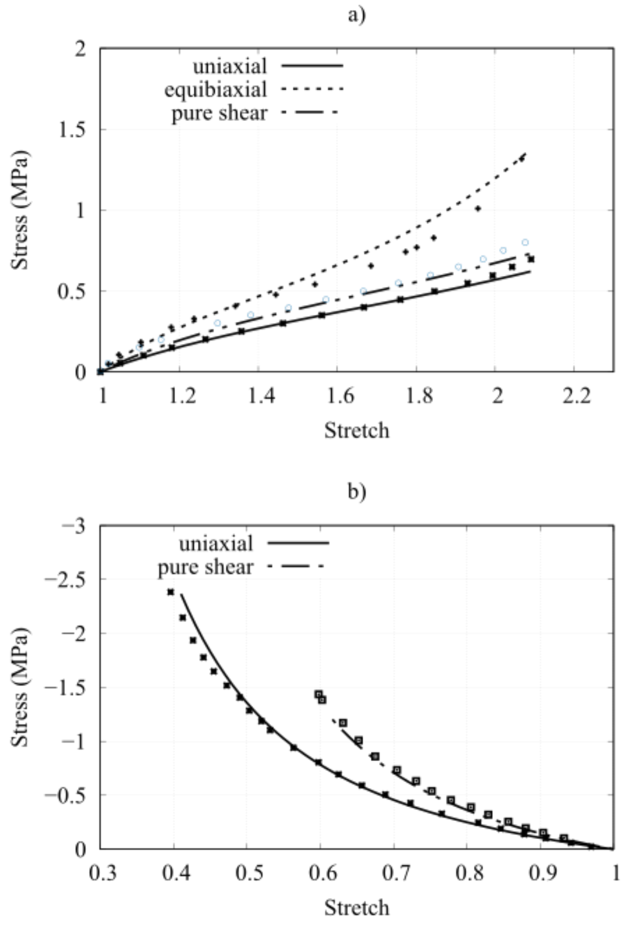

Figure 1. Comparison of the Külcü model against the experimental data of unfilled silicon rubbers by Meunier et al. (Meunier, Chagnon, Favier, Orgéas, & Vacher, 2008) $(\mu = 0.1, \alpha = 0.28)$ *a. tensile tests, b. compressive test*

The error margin of the model for this particular comparison is calculated as

$$\mathrm{Error}^2 = \frac{1}{N}\sum_{i=1}^{N}\left[P_{model}(\lambda_i) - P_{exp}(\lambda_i)\right]^2, \tag{13}$$

where N is the number of experimental data. In Table 2, the mean error margin of the model in comparison to the experimental data shown in Figure 1 is represented. A relatively small error margin is achieved for the model comparison.

Table 2. The mean error margin of the model compared to experimental data by Meunier et al. (Meunier, Chagnon, Favier, Orgéas, & Vacher, 2008)

Deformation	Error (MPa)
Uniaxial	0.027
Equibiaxial	0.051
Pure shear	0.040
Compressive uniaxial	0.097
Compressive pure shear	0.022

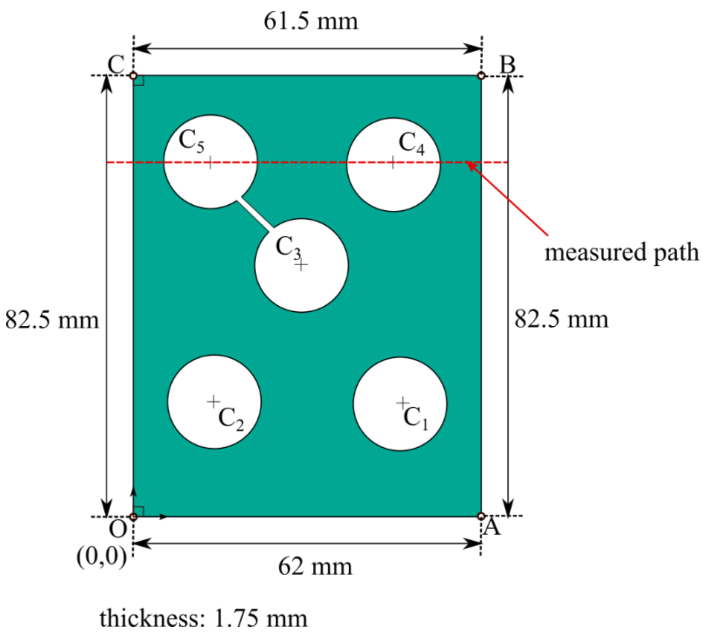

Figure 2. A rubber specimen simulated using the finite element method

Further, an unfilled silicon rubber specimen is accounted for in the finite element simulations. Figure 2 represents the dimensions of the rubber sheet tested in the experiments, whereas Table 3 shows the locations of the holes placed on the rubber sheet (Khiêm & Itskov, 2016). There is a cut between C_3 and C_5.

Table 3. Location of the holes on the rubber sheet

	X(mm)	Y(mm)
C1	47.5	21.2
C2	14.0	23.0
C3	31.5	40.5
C4	47.5	59.0
C5	14.5	58.0

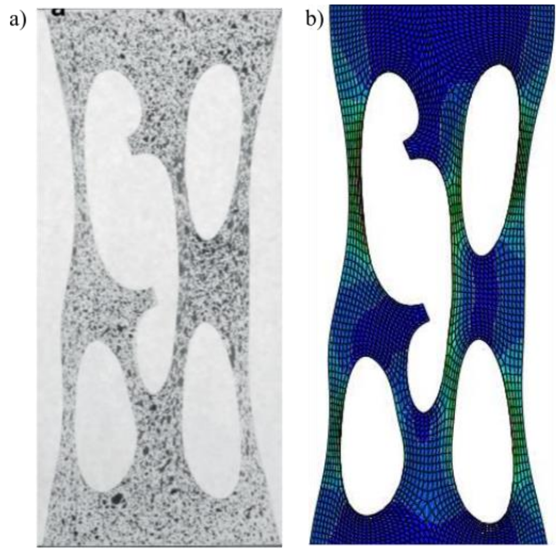

Figure 3. A rubber specimen simulated using the finite element method: a) experiment, b) simulation

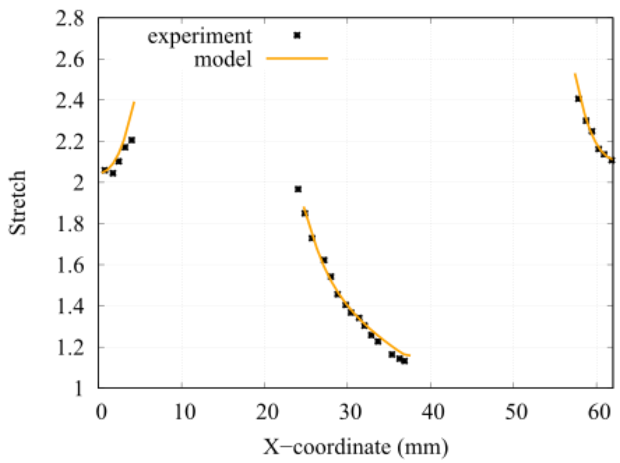

Figure 4. Comparison of the numerical model with the experimental data taken from the measured path

The rubber specimen discretized by 6566 linear hexahedral elements of type C3D8H is fixed at the bottom and subjected to the vertical deformation of 57.2 mm as performed in the experiment (Meunier, Chagnon, Favier, Orgéas, & Vacher, 2008). Figure 3 shows the deformed configuration of the rubber specimen in the experiment and simulation. In the simulation, a deformed configuration that is similar to one obtained in the experiment is captured. This fact indicates that the model qualitatively describes the experimental data.

In Figure 4, a comparison of the numerical model and the experimental data taken from the measured path shown in Figure 2 is illustrated. In the numerical model, the values of the material parameters are considered to be $\mu = 0.1$, $\alpha = 0.28$ as in Figure 1. The deformation along the measured path in the numerical model and the experimental data shows a good match. Therefore, the results of the finite element simulation show quantitatively good agreement with the experimental data as well.

4. Conclusions and Recommendations

Rubber-like materials are commercially widely-used, and the characterization of their properties is therefore significant. In this contribution, the numerical implementation of a phenomenological model proposed by Külcü (Külcü, 2020) is studied. The model is first fitted to the experimental data of five different uniform deformations by Meunier et al. (Meunier, Chagnon, Favier, Orgéas, & Vacher, 2008). Then, the model is utilized in the finite element simulation by writing a user-defined subroutine. A silicon rubber specimen having five holes, in which two of them have a cut in between, is accounted for in the numerical analysis. The model is discretized by the approximately 6500 elements and loads are applied as done in the experiments. Qualitatively and quantitatively good agreement between the numerical model and experimental data is obtained. Further, the predictive capability of the model may be checked on the fiber-reinforced rubber-like materials.

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