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PARAMETER ESTIMATION FOR A *K*-UNIT SERIES SYSTEM BASED ON THE PROGRESSIVELY CENSORED ERLANG-TRUNCATED EXPONENTIAL DATA WITH BINOMIAL REMOVALS

Çağatay Çetinkaya[1]

ABSTRACT

This study deals with point and interval estimations for the scale and shape parameters of the component lifetime distribution of a k-component series system when the component lifetimes are assumed to be independently and identically Erlang-truncated exponential distributions. It is assumed that the components are exposed to progressive Type-II censoring scheme. Each failure in this censoring plan is assumed to be random and subject to the binomial distribution. Parameter estimations are obtained by using the maximum likelihood method and their approximate confidence intervals are obtained by using the bootstrap method. The simulations are performed to evaluate the performances of the theoretical outcomes.

Keywords: Bootstrap, Erlang-Truncated Exponential Distribution, Maximum Likelihood, Progressive Censoring, Series system

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BİNOM KALDIRMALAR İLE AŞAMALI SANSÜRLENMİŞ ERLANG-KESİLMİŞ ÜSTEL VERİLERE DAYALI BİR K-BİRİMLİ SERİ SİSTEM İÇİN PARAMETRE TAHMİNİ

ÖZET

Bu çalışma, bileşen ömürlerinin bağımsız ve özdeş Erlang kesilmiş üstel dağılımına sahip olduğu varsayıldığında, bir k-bileşenli seri sistemin bileşen ömrü dağılımının ölçek ve şekil parametrelerinin parametre tahminlerini ele almaktadır. Bileşenlerin aşamalı Tip-II sansürleme şemasına maruz kaldığı varsayılmaktadır. Bu sansürleme planındaki her bir başarısızlığın rastgele olduğu ve binom dağılımına sahip olduğu varsayılır. Parametre tahminleri, maksimum olabilirlik yöntemi kullanılarak, yaklaşık güven aralıkları ise bootstrap yöntemi kullanılarak elde edilmiştir. Teorik sonuçların performanslarını değerlendirmek için simülasyon çalışmaları uygulanmıştır.

Anahtar Kelimeler: Bootstrap, Erlang Kesilmiş Üstel Dağılım, En Çok Olabilirlik, Aşamalı Sansürleme, Seri Sistem

1. INTRODUCTION

The Erlang-truncated exponential (ETE) distribution was introduced by El-Alosey (2007) as an extension of the classical exponential distribution. It is a mixture of the Erlang and the left truncated exponential distributions. Its probability density (pdf) and cumulative distribution function (cdf) are given by

$$f(x|\lambda,\beta) = \beta\left(1 - e^{-\lambda}\right)e^{-\beta\left(1-e^{-\lambda}\right)x}, \quad x > 0,\ \lambda,\beta > 0 \tag{1}$$

$$F(x|\lambda,\beta) = 1 - e^{-\beta\left(1-e^{-\lambda}\right)x} \tag{2}$$

where λ and β are the scale and shape parameters, respectively. The ETE distribution reduces to the one-parameter exponential distribution in the case of $\lambda \to \infty$. Recently, the ETE distribution was studied by various authors. Mohsin (2009) handled recurrence relations for single and product moments of record values. Khan et al. (2010) considered its moments of generalized order statistics and characterization. Rao (2013) handled one-sided cumulative sum control charts. Kulshrestha et al. (2013) studied the moment generating functions of generalized order statistics from ETE distribution. Kumar (2014a, 2014b) studied quotient moments based on records and relations of generalized order statistics. Gadde (2017) obtained reliability estimation in a multicomponent stress-strength model. Malik and Kumar (2017) considered relations for moments of progressively type-II right censored order statistics. Sarana et al. (2018) studied relationships for moments of generalized order statistics and related inference. Further, some generalizations and modifications of the ETE distribution was studied by various authors such as Nasiru et al. (2016), Okorie et al (2016, 2017a, 2017b), Jimoh et al. (2019) and Elbatal and Elgarhy (2020).

In reliability theory, many researchers are focused on the systems. Since the ETE distribution proposed a lifetime distribution, we focus on the lifetime distribution of a k-unit series system with ETE distributed lifetimes of the components.

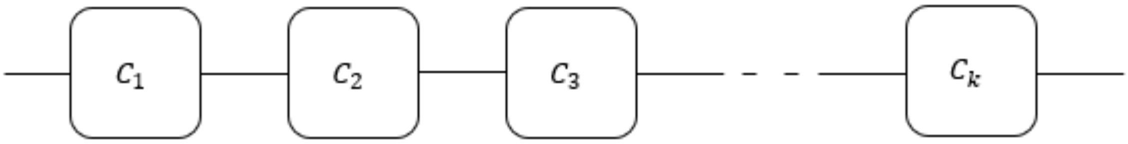

Figure 1. A configuration of the components $C_1, C_2, \cdots, C_k$ linked in series.

A series system functions when all the components are functioning. Thus, the structure function of a series system is given as (Smith, 2017)

$$\emptyset(X_1, X_2, \cdots, X_k) = \prod_{i=1}^{k} X_i = min(X_1, X_2, \cdots, X_k)$$

In this study, we consider a k-unit series system with independent and identically distributed components. Let X_i be the lifetime of the ith component with $X_i \sim ETE(\lambda, \beta)$. Let assume X denotes the lifetime of a k-unit series system. Then, the system lifetime is equal to $X = \min(X_1, X_2, \cdots, X_k)$. The distribution and probability functions of the system lifetime can be obtained with the theory of the order statistics. That is, the lifetime of the system denotes the minimum order statistics of the components $X_1, X_2, \cdots, X_k$. Thus, the distribution function of X is

$$G(x|\lambda, \beta) = 1 - e^{-k\beta(1-e^{-\lambda})x} \tag{3}$$

and its pdf is

$$g(x|\lambda,\beta) = k\beta\left(1-e^{-\lambda}\right)e^{-k\beta\left(1-e^{-\lambda}\right)x} \quad (4)$$

where $k = 1,2,\cdots\infty$.

On the other hand, it is known that lifetimes of the components cannot be observed exactly in many cases. In the case of many reliability problem, system components can be lost or removed from the experiments before they failed and censored datasets are observed in these cases. Many different censoring schemes are defined in the literature such as Type-I censoring, Type-II censoring, hybrid censoring which is a mixture of Type-I and Type II and introduced by Epstein (1954), progressive censoring schemes which let the experimenters to remove live units on failure times. Among them, we considered the progressive Type-II censoring in this scheme. In such a censoring scheme, the experiment starts with n independent and identical units which have $ETE(\lambda,\beta)$. Then, the test is terminated with the prefixed mth failure. When the first failure occurs R_1 live units are randomly removed from the experiment. At the second failure, R_2 live units are randomly removed from the experiment. This test terminates with mth failure and the remaining surviving units $R_m = n-m-\sum_{i=1}^{m-1} R_i$ are all removed from the experiment (see Figure 2). Here $R = (R_1, R_2, \cdots, R_m)$ and $\sum_{i=1}^{m} R_i = n-m$.

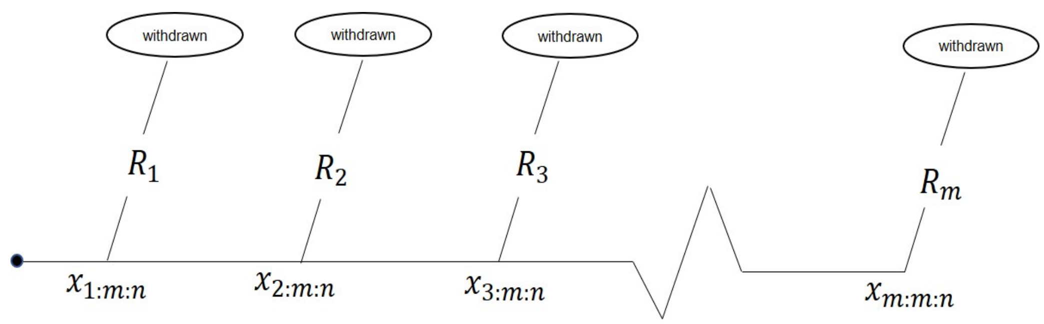

Figure 2. Structure of a progressive type-II censoring scheme

In progressive censoring schemes, the removals $R_1, R_2, \cdots, R_m$ are mostly pre-determined before the experiment. However, in the reliability problems the amounts of units removed from the test may not always be determined exactly and these amounts can be observed as random variables. In this study, we consider the failures of the ETE components with binomial removals in a k-unit series system. For this purpose, it is assumed that the R_i quantities which are removed randomly from the test follow the binomial distributions with probability

p_i. That means each component has the same probability of being removed from the test as p. Then, the probability of units removed after the ith failure time can be obtained as

$$P(R_i = r_i) = \binom{n-m}{r_i} p_i^{r_i}(1-p_i)^{n-m-r_i}$$

and

$$P(R_i = r_i | R_{i-1} = r_{i-1}, \cdots, R_1 = r_1) = \binom{n-m-\sum_{j=1}^{i-1} r_j}{r_i} p_i^{r_i}(1-p_i)^{n-m-\sum_{j=1}^{i-1} r_j}$$

where $0 \le r_i \le n-m-\sum_{j=1}^{i-1} r_j$ and $i = 1,2,3,\cdots,m-1$. With considering $R = R_1, R_2, \cdots, R_m$ and $r = r_1, r_2, \cdots, r_m$ we obtain

$$\begin{aligned} P(R = r) &= P(R_m = r_m | R_{m-1} = r_{m-1}, \cdots, R_1 = r_1) \cdots P(R_1 = r_1) \\ &= \frac{(n-m)!}{\prod_{i=1}^{m-1}\left(n-m-\sum_{j=1}^{i-1} r_j\right)!} p^{\sum_{i=1}^{m-1} r_i}(1-p)^{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i} \end{aligned}$$

In literature, there are many different studies based on progressive censoring schemes with binomial removals. For example, Weibull distributed lifetimes are considered in this plan by Tse et al. (2000). Wu and Chang (2002) handled exponential, Yan et al. (2011) generalized exponential, Mubarak (2012) Frèchet distributions in the same context.

In this study, we aimed to obtain the parameter estimations of a k-unit series system based on the ETE components under progressive type-II censoring scheme with binomial removals. In this purpose, we obtained the maximum likelihood estimations (MLE) of the shape and scale parameters in the Section 2. As an approximate confidence interval, we used bootstrap method in Section 3. Finally, the whole theoretical outcomes are illustrated with simulation schemes in Section 4.

2. MAXIMUM LIKELIHOOD ESTIMATION

We suppose that n identical k-unit linked series units are put on life test. Let $x_{(i)}$ be the order-statistics from a progressively Type-II censored sample of size n with removals $R_1, R_2, \cdots, R_m$ being the progressive censoring scheme. Under the assumptions of binomial removals with probability p, the likelihood function of the observed sample can be obtained as

$$L(\lambda,\beta) = L_x(\lambda,\beta)\, P(R=r)$$
$$= C_1 C_2 \prod_{i=1}^{m} g\big(x_{(i)};\lambda,\beta\big)\big[1-G\big(x_{(i)};\lambda,\beta\big)\big]^{R_i} p^{\sum_{i=1}^{m-1} r_i}(1-p)^{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i}$$

where $C_1 = n(n-1-R_1)(n-1-R_1-R_2)\cdots(n-m+1-\sum_{i=1}^{m-1} R_i)$ and

$$C_2 = (n-m)!\Big/\prod_{i=1}^{m-1}\left(n-m-\sum_{j=1}^{i-1} r_j\right)!$$

Thus, the likelihood function for the k-unit linked series ETE data is obtained as

$$L(\lambda,\beta) \propto k^m \beta^m \big(1-e^{-\lambda}\big)^m e^{-k\beta\left(1-e^{-\lambda}\right)^m \sum_{i=1}^{m} x_{(i)}(1+R_i)}$$
$$\times\, p^{\sum_{i=1}^{m-1} r_i}(1-p)^{(m-1)(n-m)-\sum_{i=1}^{m-1}(m-i)r_i}$$

and the log-likelihood function is obtained as

$$l(\lambda,\beta) \propto m\big(\log k + \log\beta + \log\big(1-e^{-\lambda}\big)\big) - k\beta\big(1-e^{-\lambda}\big)\sum_{i=1}^{m} x_{(i)}(1+R_i)$$

To obtain the MLEs of the parameters, denoted by $\hat{\lambda}$ and $\hat{\beta}$, we should equate the partial derivates of $l(\lambda,\beta)$ to zero with respect to λ and β respectively as given in the following

$$\frac{\partial l}{\partial\lambda} = \frac{m}{e^{\lambda}-1} - k\beta e^{-\lambda}\sum_{i=1}^{m} x_{(i)}(1+R_i) = 0$$
$$\frac{\partial l}{\partial\beta} = \frac{m}{\beta} - k\big(1-e^{-\lambda}\big)\sum_{i=1}^{m} x_{(i)}(1+R_i) = 0$$

These non-linear equations cannot be solved analytically and iterative methods such as Newton-Raphson method are needed. Thus, approximate solutions of the system of these non-linear equations be the MLEs of the parameters.

3. APPROXIMATE CONFIDENCE INTERVALS

In this section, we firstly handled the asymptotic normality property of the maximum likelihood estimators to obtain approximate confidence intervals for the parameters. However, by performing the simulations we observed that regularity conditions for the maximum likelihood estimators cannot be satisfied in this problem. Particularly, the Fisher information matrix cannot be always obtained as a positive definite matrix in a neighborhood of the parameter space. Therefore, we considered bootstrap confidence intervals for approximate confidence intervals of $\hat{\lambda}$ and $\hat{\beta}$. The bootstrap percentile method (boot-p) is used for constructing bootstrap confidence intervals (see Efron (1994) for details).

The following steps can be used to construct a $100(1-\gamma)\%$ parametric percentile bootstrap confidence interval for one replicate.

Step 1: Draw a random sample $(x_1, x_2, \cdots, x_m)$ from $ETE(\lambda, \beta)$.

Step 2: Compute the maximum likelihood estimates of all parameters and denote them as $\hat{\lambda}$ and $\hat{\beta}$.

Step 3: Generate independent bootstrap sample $(x_1^*, x_2^*, \cdots, x_m^*)$ from $ETE(\hat{\lambda}, \hat{\beta})$ by using the inverse transformation method.

Step 4: Compute the MLEs of all parameters based on the bootstrap sample and denote them as $\hat{\lambda}^*$ and $\hat{\beta}^*$.

Step 5: Repeat step 3 B times to get other independent bootstrap samples from $ETE(\hat{\lambda}, \hat{\beta})$ and a set of bootstrap estimates of λ and β and denote as $\{\hat{\lambda}_i^*, \hat{\beta}_i^*; i = 1,2,\cdots, B\}$.

Step 6: Compute $\left(\hat{\lambda}^{*(\gamma/2)}, \hat{\lambda}^{*(1-\gamma/2)}\right)$ and $\left(\hat{\beta}^{*(\gamma/2)}, \hat{\beta}^{*(1-\gamma/2)}\right)$ where $\hat{\lambda}^{*(\gamma)}$ and $\hat{\beta}^{*(\gamma)}$ are the γ −percentile of $\{\hat{\lambda}_i^*, \hat{\beta}_i^*; i = 1,2,\cdots, B\}$ that is a number such that $\frac{1}{B}\sum_{i=1}^{B} I\left(\hat{\lambda}_i^* \leq \hat{\lambda}^{*(\gamma)}\right) = \gamma$ and $\frac{1}{B}\sum_{i=1}^{B} I\left(\hat{\beta}_i^* \leq \hat{\beta}^{*(\gamma)}\right) = \gamma$ for $0 < \gamma < 1$.

where $I(\cdot)$ denotes the indicator function.

4. SIMULATIONS

In this section, we present some simulation studies to illustrate the theoretical findings. Firstly, the actual parameter values (λ, β) are randomly selected as $(0.5, 1.5)$ and $(1, 2)$. We considered sample sizes ana corresponding failure numbers (n, m) as $(20,12)$, $(20,16)$, $(30,20)$, $(30,25)$, $(50,36)$ and $(50,40)$. For removal probabilities we considered three values of the binomial parameter p as 0.3, 0.5 and 0.7, respectively. We handled two different system type by taking $k = 3$ and $k = 6$ components.

In order to obtain progressive censored samples from ETE distribution, we used the algorithm proposed by Balakrishnan and Sandhu (1995) as given in the following

Step 1: Generate m independent $W_1, W_2, \cdots, W_m$ observations from $U(0,1)$.
Step 2: Set $V_i = W_i^{1/(1+R_m+R_{m-1}+\cdots+R_{m-i+1})}$ for $i = 1,2,\cdots,m$
Step 3: Set $U_i = 1 - V_m V_{m-1} \cdots V_{m-i+1}$ for $i = 1,2,\cdots,m$ as the required progressive Type-II censored sample from the $U(0,1)$.
Step 4: Set $X_i = F^{-1}(U_i)$ for $i = 1,2,\cdots,m$ where $F^{-1}(U_i)$ is the inverse cdf of the distribution which is given in equation (3) and obtained as

$$F^{-1}(U_i) = -\log(1-U_i)/\left[k\beta\left(1-e^{-\lambda}\right)\right].$$

Then, $X_1, X_2, \cdots, X_m$ is the required progressive Type-II censored sample from the ETE distribution.

Thus, we run the simulations for 1000 replication, and we used 500 bootstrap sample for each replicate. Then, we reported the biases, mean squared errors (MSE) and average lengths (AL) of the bootstrap confidence intervals of the MLEs of the parameters. We report the results for $(\lambda = 0.5, \beta = 1.5)$ in the case of $k = 3$ in Table 1 and in Table 2 for $k = 6$. The results for $(\lambda = 1, \beta = 2)$ are given in Tables 3-4 for the cases of $k = 3$ and $k = 6$, respectively.

Table 1. The biases and MSEs of the parameters with the corresponding ALs of their bootstrap confidence intervals for $\lambda = 0.5$, $\beta = 1.5$ and $k = 3$.

			$\hat{\lambda}$			$\hat{\beta}$		
p	n	m	*Bias*	*MSE*	AL	*Bias*	*MSE*	AL
0.3	20	12	0.06097	0.06940	0.84447	0.06257	0.00563	0.48692
	20	16	0.04744	0.04856	0.70748	0.05520	0.00472	0.40638
	30	20	0.02843	0.03077	0.64449	0.05105	0.00454	0.33880
	30	25	0.01690	0.01964	0.57217	0.04394	0.00459	0.30308
	50	36	0.00944	0.01385	0.46583	0.04296	0.00423	0.28017
	50	40	0.00373	0.01082	0.44077	0.04177	0.00454	0.27271
0.5	20	12	0.05305	0.06887	0.83940	0.05718	0.00549	0.47574
	20	16	0.04390	0.04423	0.72729	0.05058	0.00434	0.38692
	30	20	0.02426	0.03107	0.63857	0.04596	0.00454	0.33622
	30	25	0.01257	0.01866	0.56463	0.04446	0.00476	0.30222
	50	36	0.00576	0.01221	0.46822	0.04449	0.00471	0.27490
	50	40	0.00117	0.01126	0.43360	0.04146	0.00446	0.27072
0.7	20	12	0.05152	0.06754	0.84920	0.06233	0.00761	0.47314
	20	16	0.02640	0.04042	0.70939	0.05245	0.00477	0.37323
	30	20	0.02755	0.02744	0.63170	0.04892	0.00509	0.34551
	30	25	0.02712	0.02271	0.58473	0.04496	0.00491	0.31101
	50	36	0.00894	0.01339	0.46850	0.04147	0.00476	0.28031
	50	40	0.00240	0.01060	0.43945	0.03877	0.00454	0.27118

We observed that the biases, MSEs and average lengths of the approximate confidence intervals decrease in parallel to increasing on sample size. There is not any important difference on the estimates depend on the number of components (k). Further, the probability of removals does not have a particular effect on estimates. All the results in three cases of probabilities have similar simulation performances. Consequently, we obtained consistent results in all cases.

Table 2. The biases and MSEs of the parameters with the corresponding ALs of their bootstrap confidence intervals for $\lambda = 0.5,\ \beta = 1.5$ and $k = 6$.

			$\hat{\lambda}$			$\hat{\beta}$		
p	n	m	*Bias*	*MSE*	AL	*Bias*	*MSE*	AL
0.3	20	12	0.07030	0.08150	0.83815	0.06354	0.00606	0.50764
	20	16	0.04234	0.04586	0.72219	0.05469	0.00510	0.39249
	30	20	0.02869	0.02841	0.64501	0.05062	0.00468	0.34083
	30	25	0.01300	0.01976	0.56148	0.04337	0.00440	0.30330
	50	36	0.00516	0.01169	0.46611	0.04578	0.00465	0.27590
	50	40	0.00474	0.01253	0.43711	0.04182	0.00459	0.27523
0.5	20	12	0.06024	0.06522	0.85010	0.05954	0.00615	0.48695
	20	16	0.03810	0.04188	0.71511	0.05260	0.00442	0.39463
	30	20	0.02754	0.03160	0.63684	0.05198	0.00461	0.34509
	30	25	0.02109	0.02156	0.58106	0.04435	0.00448	0.30417
	50	36	0.00014	0.01243	0.46125	0.04449	0.00438	0.27514
	50	40	0.00746	0.01235	0.44253	0.04119	0.00409	0.27586
0.7	20	12	0.05720	0.07554	0.84534	0.05960	0.00632	0.47743
	20	16	0.04914	0.04955	0.72545	0.05357	0.00496	0.39973
	30	20	0.01967	0.02756	0.63881	0.05139	0.00447	0.33068
	30	25	0.01625	0.02352	0.56296	0.04805	0.00447	0.31099
	50	36	0.00385	0.01171	0.46686	0.04487	0.00449	0.27583
	50	40	0.00869	0.01078	0.44548	0.03858	0.00476	0.27243

Table 3. The biases and MSEs of the parameters with the corresponding ALs of their bootstrap confidence intervals for $\lambda = 1,\ \beta = 2$ and $k = 3$.

			$\hat{\lambda}$			$\hat{\beta}$		
p	n	m	*Bias*	*MSE*	AL	*Bias*	*MSE*	AL
0.3	20	12	0.01727	0.14345	1.00151	0.20391	0.06430	1.55047
	20	16	0.00760	0.12160	0.90089	0.17249	0.04474	1.26196
	30	20	0.01283	0.09564	0.83383	0.15410	0.02569	1.12320
	30	25	0.01331	0.08726	0.77273	0.14315	0.02101	0.95903
	50	36	0.01619	0.07471	0.70575	0.11857	0.01614	0.78106
	50	40	0.03218	0.06752	0.68455	0.11654	0.01190	0.73962
0.5	20	12	0.03521	0.14584	1.00377	0.21151	0.07450	1.59286
	20	16	0.00185	0.12563	0.89244	0.18014	0.04656	1.25016
	30	20	0.01092	0.10407	0.81456	0.15482	0.03225	1.12985
	30	25	0.00382	0.09400	0.75545	0.14648	0.02628	0.98921
	50	36	0.00479	0.07316	0.68935	0.11427	0.01536	0.82130
	50	40	0.02208	0.06503	0.68145	0.10944	0.01297	0.75505
0.7	20	12	0.03761	0.15484	0.98146	0.21400	0.07623	1.62090
	20	16	0.02614	0.12077	0.89880	0.17864	0.04285	1.29916
	30	20	0.00227	0.10221	0.81540	0.15043	0.02725	1.12054
	30	25	0.00164	0.08828	0.76343	0.13621	0.02279	0.98271
	50	36	0.01549	0.07078	0.69996	0.12125	0.01628	0.79451
	50	40	0.00308	0.06747	0.67549	0.10821	0.01355	0.78754

Table 4. The biases and MSEs of the parameters with the corresponding ALs of their bootstrap confidence intervals for $\lambda = 1,\ \beta = 2$ and $k = 6$.

			$\hat{\lambda}$			$\hat{\beta}$		
p	n	m	*Bias*	*MSE*	AL	*Bias*	*MSE*	AL
0.3	20	12	0.04035	0.15425	0.99655	0.21166	0.08066	1.58733
	20	16	0.02112	0.12347	0.88820	0.17598	0.04283	1.29655
	30	20	0.01423	0.10455	0.82374	0.15706	0.03097	1.12677
	30	25	0.00953	0.09017	0.76600	0.13271	0.02073	0.97943
	50	36	0.00581	0.07264	0.69369	0.11653	0.01341	0.81110
	50	40	0.01761	0.06868	0.68364	0.11309	0.01386	0.75519
0.5	20	12	0.01738	0.11963	0.89555	0.17869	0.04469	1.28519
	20	16	0.00730	0.11728	0.90559	0.17456	0.04480	1.26237
	30	20	0.00948	0.10377	0.82158	0.15060	0.02636	1.09565
	30	25	0.00093	0.09112	0.76493	0.14171	0.02346	0.97332
	50	36	0.00558	0.07241	0.69744	0.11780	0.01462	0.80725
	50	40	0.01564	0.06758	0.66947	0.11532	0.01337	0.77856
0.7	20	12	0.02181	0.14438	0.99082	0.20551	0.06194	1.56433
	20	16	0.00548	0.12176	0.88975	0.17694	0.04447	1.28427
	30	20	0.01437	0.10081	0.80788	0.15438	0.02974	1.15162
	30	25	0.00477	0.09139	0.75898	0.13893	0.02136	0.97482
	50	36	0.02201	0.06699	0.71090	0.11792	0.01401	0.78432
	50	40	0.00777	0.06801	0.67141	0.11158	0.01406	0.78040

5. CONCLUSION

In this study, a k-unit series system with Erlang-truncated exponential components is handled under a progressive censoring scheme. We considered removals in progressive censoring under the binomial distribution. It is seen that the maximum likelihood estimations and approximate confidence intervals perform well. Further, we considered asymptotic confidence intervals for the maximum likelihood estimators but regularity conditions were not satisfied always. For this reason, we discarded the asymptotic method and preferred the bootstrap method. As a result, the ETE distribution can be used for such a system under different progressive censoring schemes and parameter estimation can be obtained by using these theoretical findings.

ETHICAL DECLARATION

In the writing process of the study titled "Parameter Estimation for k-unit series system basedon the progressively censored erlang-truncated exponential data with binomial removals", there were followed the scientific, ethical and the citation rules; was not made any falsification on the collected data and this study was not sent to any other academic media for evaluation.

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