Turk. J. Math. Comput. Sci.
14(1)(2022) 32–43
© MatDer
DOI : 10.47000/tjmcs.907088

On Fuzzy 2-absorbing Γ-ideals in Γ-rings

Serkan Onar

Department of Mathematical Engineering, Yildiz Technical University, 34200, Istanbul, Turkey.

Received: 31-03-2021 • Accepted: 15-04-2022

Abstract. The goal of this paper is to give a definition of a generalization of fuzzy prime Γ-ideals in Γ-rings by introducing fuzzy 2-absorbing Γ-ideals and fuzzy weakly completely 2-absorbing Γ-ideals of commutative Γ-rings and to give their properties. Furthermore, we give a diagram which transition between definitions of fuzzy 2-absorbing Γ-ideals of a Γ-ring. Finally, we introduce fuzzy quotient Γ-ring of R induced by the fuzzy weakly completely 2-absorbing Γ-ideal is a 2-absorbing Γ-ring.

2010 AMS Classification: 03E72, 08A72

Keywords: 2-absorbing, fuzzy 2-absorbing ideal, fuzzy 2-absorbing Γ-ideal, fuzzy weakly completely 2-absorbing Γ-ideal, fuzzy K-2-absorbing Γ-ideal.

1. Introduction

Zadeh in [27] introduced the notion of fuzzy subset and Rosenfeld [25] examined to apply fuzzy theory on algebraic structures. Then, many researchers have investigated about it. Liu [19] studied the concept of fuzzy ideal of a ring. N.Nobusawa [24] defined the notion of a $\Gamma-$ring, as more general than a ring. W.E. Barnes [5] weakened slightly the conditions in the definition of the $\Gamma-$rings in the sense of Nobusawa. W.E.Barnes [5], S.Kyuno [16, 17] and J.Luh [21] developed the structure of $\Gamma-$rings and acquired various generalizations analogous to corresponding parts in ring theory. In fuzzy commutative algebra, prime ideals are the most significant structures. Dutta and Chanda [9] described fuzzy prime ideals in $\Gamma-$rings. Ersoy [12] discussed fuzzy semiprime ideals in $\Gamma-$rings.

The concept of a 2-absorbing ideal, which is a generalization of prime ideal, was proposed by Badawi in [2] and also presented in [1, 3]. At present, study on the 2-absorbing ideal theory is progressing rapidly. It has been studied extensively by many authors (e.g. [4, 7, 14]). Darani [6] demonstrated the notion of $L-$fuzzy 2-absorbing ideals and has acquired interesting results on these concepts. Then, Darani and Hashempoor constructed the concept of $L-$fuzzy 2-absorbing ideals in semiring [8]. Elkettani and Kasem [11] clarified the notion of 2-absorbing $\delta-$primary $\Gamma-$ideal of $\Gamma-$ring and gave interesting results concerning these notions. Sönmez [26] described 2-absorbing primary fuzzy ideals of commutative rings and established relations between 2-absorbing primary fuzzy ideals and 2-absorbing primary ideals.

This paper provides a new algebraic structure of fuzzy prime $\Gamma-$ideal of commutative $\Gamma-$ring by 2-absorbing and weakly completely prime 2-absorbing ideal theory. We examine the notion of fuzzy 2-absorbing $\Gamma-$ideal of $\Gamma-$ring and fuzzy weakly completely 2-absorbing $\Gamma-$ideal of $\Gamma-$ring and explain some of its characterization of algebraic properties. Furthermore, we give definition of fuzzy strongly 2-absorbing $\Gamma-$ideal of $\Gamma-$ring and fuzzy $K-$2-absorbing $\Gamma-$ideal of $\Gamma-$ring. We investigate image and inverse image of fuzzy 2-absorbing $\Gamma-$ideal of $\Gamma-$ring and fuzzy weakly

Email address: serkan10ar@gmail.com (S. Onar)

completely 2-absorbing $\Gamma-$ideal of $\Gamma-$ring. Then, we construct a diagram which transition between definitions of fuzzy 2-absorbing $\Gamma-$ideals of a $\Gamma-$ring as well as the relationship of these concepts with the notion of 2-absorbing $\Gamma-$ideal. Finally, we introduce fuzzy quotient $\Gamma-$ring of R induced by the fuzzy weakly completely 2-absorbing $\Gamma-$ideal is a 2-absorbing $\Gamma-$ring.

2. Preliminaries

In this section, for the sake of completeness, we first recall some useful definitions and results. Throughout this paper, $\Gamma-$ring R is a commutative with $1 \neq 0$ and $L = [0, 1]$ stands for a complete lattice.

Definition 2.1 ([27]). A fuzzy subset μ in a set X is a function $\mu : X \to [0, 1]$.

Proposition 2.2 ([22]). *Let μ and ν be fuzzy subset of X. We say that, μ is a subset of ν and write $\mu \subseteq \nu$, if $\mu(x) \leq \nu(x)$ for all $x \in X$.*

Definition 2.3 ([22]). Let μ be any fuzzy subset of X and $t \in L$. Then, the set

$$\mu_t = \{x \in X \mid \mu(x) \geq t\}$$

is called the $t - level$ subset of X with respect to μ.

Definition 2.4 ([22]). A fuzzy subset μ of X is called a fuzzy point if $x \in X$ and $r \in L \setminus \{0\}$, is a fuzzy subset of X and defined by

$$x_r(y) = \begin{cases} r, & \text{if } y = x; \\ 0, & \text{otherwise.} \end{cases}$$

If x_r is a fuzzy point of X and $x_r \subseteq \mu$, then we write $x_r \in \mu$.

Definition 2.5 ([10]). Let R and Γ be two abelian additive groups. R is called a $\Gamma-$ring, if there exist a mapping

$$\begin{aligned} R \times \Gamma \times R &\to R \\ (x, \alpha, y) &\mapsto x\alpha y, \end{aligned}$$

satisfying the following conditions;

(1) $(x + y)\alpha z = x\alpha z + y\alpha z$,
(2) $x\alpha(y + z) = x\alpha y + x\alpha z$,
(3) $x(\alpha + \beta)y = x\alpha y + x\beta y$,
(4) $x\alpha(y\beta z) = (x\alpha y)\beta z$,

for all $x, y, z \in R$ and all $\alpha, \beta \in \Gamma$. A $\Gamma-$ring R is called commutative, if $x\alpha y = y\alpha x$ for any $x, y \in R$ and $\alpha \in \Gamma$.

Definition 2.6 ([10]). A left (resp. right) ideal of a $\Gamma-$ring R is a subset A of R which is an additive subgroup of R and $R\Gamma A \subseteq A$ (resp,$A\Gamma R \subseteq A$) where,

$$R\Gamma A = \{x\alpha y \mid x \in R, \alpha \in \Gamma, y \in A\}.$$

If A is both a left and a right ideal, then A is called a $\Gamma-$ideal of R.

Definition 2.7 ([9]). A fuzzy set μ in a $\Gamma-$ring R is called a fuzzy ideal of R, if the following requirements are satisfied:

(1) $\mu(x - y) \geq \min\{\mu(x), \mu(y)\}$,
(2) $\mu(x\alpha y) \geq \max\{\mu(x), \mu(y)\}$,

for all $x, y \in R$ and $\alpha \in \Gamma$.

Definition 2.8 ([10]). Let R and S be two $\Gamma-$rings, and f be a mapping of R into S. Then, f is called a $\Gamma-$homomorphism, if

$$f(a + b) = f(a) + f(b)$$

and

$$f(a\alpha b) = f(a)\alpha f(b),$$

for all $a, b \in R$ and $\alpha \in \Gamma$.

Definition 2.9 ([5]). Let R be a $\Gamma-$ring. A proper ideal P of R is called a prime $\Gamma-$ideal, if for all pairs of ideals S and T of R,

$$S\Gamma T \subseteq P \text{ implies that } S \subseteq P \text{ or } T \subseteq P.$$

Proposition 2.10 ([18]). *If P is an ideal of a $\Gamma-$ring R, then the following conditions are equivalent:*

(1) *P is a prime $\Gamma-$ideal of R.*
(2) *If $x, y \in R$ and $x\Gamma R\Gamma y \subseteq P$, then $x \in P$ or $y \in P$.*

Definition 2.11 ([13]). A non-constant fuzzy ideal μ of a $\Gamma-$ring R is called a fuzzy prime $\Gamma-$ideal of R if for any two fuzzy ideals σ and θ of R,

$$\sigma\Gamma\theta \subseteq \mu \text{ implies that either } \sigma \subseteq \mu \text{ or } \theta \subseteq \mu.$$

Definition 2.12 ([22]). Let μ be a fuzzy subset of R. Then, the fuzzy ideal of R generated by μ is defined to be the intersection of all fuzzy ideals of R containing μ and denoted by $\langle\mu\rangle$.

Clearly, $\langle\mu\rangle$ is a fuzzy ideal of R. In fact, $\langle\mu\rangle$ is the smallest fuzzy ideal of R containing μ.

Lemma 2.13 ([9]). *Let R be a commutative $\Gamma-$ring with identity and let x_r and y_s be two fuzzy points of R. Then,*

(1) $x_r\alpha y_s = (x\alpha y)_{r\wedge s}$,
(2) $\langle x_r\rangle \alpha \langle y_s\rangle = \langle x_r\alpha y_s\rangle$.

Theorem 2.14 ([9]). *Let R be a commutative $\Gamma-$ring and μ be a fuzzy $\Gamma-$ideal of R. Then, the followings are equivalent:*

(1) *$x_r\Gamma y_t \subseteq \mu \Rightarrow x_r \subseteq \mu$ or $y_t \subseteq \mu$ where x_r and y_t are two fuzzy points of R.*
(2) *μ is a fuzzy prime ideal of R.*

Definition 2.15 ([2]). A proper ideal I of commutative ring M is called a 2-absorbing ideal of M if whenever $x, y, z \in M$ and $xyz \in I$, then $xy \in I$ or $xz \in I$ or $yz \in I$.

Definition 2.16 ([23]). A fuzzy ideal μ of R is said to be a fuzzy weakly completely prime ideal if μ is non-constant function and for all $x, y \in R$,

$$\mu(x, y) = \max\{\mu(x), \mu(y)\}.$$

Definition 2.17 ([15]). Let μ be a non-constant fuzzy ideal of R. μ is said to be a fuzzy $K-$prime ideal if for any $x, y \in R$,

$$\mu(xy) = \mu(0) \text{ implies either } \mu(x) = \mu(0)\, or \mu(y) = \mu(0).$$

Definition 2.18 ([11]). A proper $\Gamma-$ideal I of a $\Gamma-$ring R is called a 2-absorbing $\Gamma-$ideal of R if whenever $x, y, z \in R$, $\alpha, \beta \in \Gamma$ and $x\alpha y\beta z \in I$, then $x\alpha y \in I$ or $x\beta z \in I$ or $y\beta z$.

Proposition 2.19 ([11]). *Every prime Γ-ideal of Γ-ring R is a 2-absorbing Γ-ideal of R.*

3. Fuzzy 2-absorbing $\Gamma-$ideals of a $\Gamma-$ring

In this section, we investigate fuzzy 2-absorbing Γ-ideals of a Γ- ring. Throughout this paper, we assume that R is a commutative Γ-ring.

Definition 3.1. Let R be a commutative $\Gamma-$ring and μ be fuzzy $\Gamma-$ideal of $\Gamma-$ring R. μ is called fuzzy 2-absorbing $\Gamma-$ ideals of R if μ is non-constant and for any fuzzy points x_r, y_s, z_t of R and $\alpha, \beta \in \Gamma$ with

$$x_r\alpha y_s\beta z_t \in \mu \text{ implies that either } x_r\alpha y_s \in \mu \text{ or } x_r\beta z_t \in \mu \text{ or } y_s\beta z_t \in \mu.$$

Proposition 3.2. *Every fuzzy prime $\Gamma-$ideal of R is a fuzzy 2-absorbing $\Gamma-$ ideal of R.*

Proof. The proof is straightforward. □

Example 3.3. Let $R = \mathbb{Z}_4$ and $\Gamma = \mathbb{Z}$, define $\overline{x}\alpha\overline{y} = \overline{x\alpha y}$ for all $\overline{x}, \overline{y} \in \mathbb{Z}_4$ and $\alpha \in \mathbb{Z}$. So, $\mathbb{Z}_4$ is a $\Gamma-$ring. A fuzzy subset μ in $\mathbb{Z}_4$ is defined

$$\mu(x) = \begin{cases} 1, & x \in \{\overline{0}, \overline{2}\} \\ 0, & \text{otherwise.} \end{cases}$$

Then, μ is a fuzzy prime $\Gamma-$ideal and fuzzy 2-absorbing $\Gamma-$ ideal of $\mathbb{Z}_4$.

The following example shows that the converse of Proposition 3.2 is not necessarily true.

Example 3.4. Let $R = \mathbb{Z}_4$ and $\Gamma = \mathbb{Z}$, define $\overline{x}\alpha\overline{y} = \overline{x\alpha y}$ for all $\overline{x}, \overline{y} \in \mathbb{Z}_4$ and $\alpha \in \mathbb{Z}$. So, $\mathbb{Z}_4$ is a $\Gamma-$ring. A fuzzy subset μ in $\mathbb{Z}_4$ is defined

$$\mu(x) = \begin{cases} 1/2, & x \in \{\overline{0}, \overline{2}\} \\ 0, & \text{otherwise.} \end{cases}$$

Then, for fuzzy points $\overline{2}_{3/4}, \overline{3}_{1/2}$ of $\mathbb{Z}_4$ and $\alpha \in \mathbb{Z}$

$$\begin{aligned} \overline{2}_{3/4}\alpha\overline{3}_{1/2} &= \left(\overline{2}\alpha\overline{3}\right)_{3/4\wedge 1/2} = \left(\overline{2\alpha 3}\right)_{1/2}\left(\overline{2\alpha 3}\right) = 1/2 \\ &\leq \mu\left(\overline{2\alpha 3}\right) = 1/2. \end{aligned}$$

So, $\overline{2}_{3/4}\alpha\overline{3}_{1/2} \in \mu$. But since

$$\begin{aligned} \overline{2}_{3/4}\left(\overline{2}\right) &= 3/4 > 1/2 = \mu\left(\overline{2}\right), \text{ we get } \overline{2}_{3/4} \notin \mu \text{ and} \\ \overline{3}_{1/2}\left(\overline{3}\right) &= 1/2 > 0 = \mu\left(\overline{3}\right), \text{ we get } \overline{3}_{1/2} \notin \mu. \end{aligned}$$

Therefore, μ is not a fuzzy prime $\Gamma-$ideal of $\mathbb{Z}_4$. On the other hand, μ is a fuzzy 2-absorbing $\Gamma-$ideal of $\mathbb{Z}_4$.

Theorem 3.5. *Let μ and η be two distinct fuzzy prime $\Gamma-$ideals of R. Then, $\mu \cap \eta$ is a fuzzy 2-absorbing $\Gamma-$ideal of R.*

Proof. Assume that $x_r\alpha y_s\beta z_t \in \mu \cap \eta$ for some fuzzy points x_r, y_s, z_t of R, but $x_r\alpha y_s \notin \mu \cap \eta$ and $x_r\beta z_t \notin \mu \cap \eta$. Then, we have the following cases:

Case 1. If $x_r\alpha y_s \notin \mu$ and $x_r\beta z_t \notin \mu$, then since μ is a fuzzy prime $\Gamma-$ideal of R, we get $z_t \in \mu$ and so $x_r\beta z_t \in \mu$ which is a contradiction.

Case 2. In similar way, we get a contradiction if $x_r\alpha y_s \notin \eta$ and $x_r\beta z_t \notin \eta$. Hence, either $x_r\alpha y_s \notin \mu$ and $x_r\beta z_t \notin \eta$ or $x_r\alpha y_s \notin \eta$ and $x_r\beta z_t \notin \mu$.

Case 3. If the former case holds, then from $x_r\alpha y_s\beta z_t \in \mu \cap \eta$, we get $z_t \in \mu$ and $y_s \in \eta$. Therefore, $y_s\beta z_t \in \mu \cap \eta$.

Case 4. Similarly, we easily show that $y_s\beta z_t \in \mu \cap \eta$ if the latter case hold.

Finally $\mu \cap \eta$ is a fuzzy 2-absorbing $\Gamma-$ideal of R. □

Corollary 3.6. *The intersection of every pair of distinct fuzzy prime $\Gamma-$ideals of R is a fuzzy 2-absorbing $\Gamma-$ideal of R.*

Theorem 3.7. *Let μ be a fuzzy 2-absorbing $\Gamma-$ideal of R. Then, μ_a is a 2-absorbing $\Gamma-$ideal of R for every $a \in [0, \mu(0)]$ with $\mu_a \neq R$.*

Proof. Suppose that $x, y, z \in R$ and $\alpha, \beta \in \Gamma$ are such that $x\alpha y\beta z \in \mu_a$. Then, $\mu(x\alpha y\beta z) \geq a$ and we get $x_a\alpha y_a\beta z_a = (x\alpha y\beta z)_a \in \mu$. Since μ is a fuzzy 2-absorbing $\Gamma-$ideal of R, we get $(x\alpha y)_a = x_a\alpha y_a \in \mu$ or $(x\beta z)_a = x_a\beta z_a \in \mu$ or $(y\beta z)_a = y_a\beta z_a \in \mu$. If $k_a \in \mu$ for some $k \in R$, then $\mu(k) \geq a$ and so $k \in \mu_a$. Hence, $x\alpha y \in \mu_a$ or $x\beta z \in \mu_a$ or $y\beta z \in \mu_a$. Therefore, μ_a is a 2-absorbing $\Gamma-$ideal of R. □

The following example shows that the converse of Theorem 3.7 is not generally true.

Example 3.8. Let $R = \mathbb{Z}$ and $\Gamma = 3\mathbb{Z}$, then R is a $\Gamma-$ring. Define the fuzzy $\Gamma-$ideal of $\mathbb{Z}$ by

$$\mu(x) = \begin{cases} 1, & x = 0 \\ 1/4, & x \in 4\mathbb{Z} - \{0\} \\ 0, & x \in \mathbb{Z} - 4\mathbb{Z}. \end{cases}$$

Then,

$$\begin{aligned} t &\geq 0,\ \mu(x) \geq 0 \text{ and } x \in Z, \text{ we get } \mu_t = \mathbb{Z}, \\ t &\geq 1/4,\ \text{we get } \mu_t = 4\mathbb{Z}, \\ t &= 1,\ \text{we get } \mu_t = 0. \end{aligned}$$

Hence, μ_t is a 2-absorbing $\Gamma-$ideal of R for all $t \in Im\mu$. However, for $\alpha, \beta \in \mathbb{Z}$ we have

$$\begin{aligned} 2_{1/2}\alpha 2_{1/2}\beta 1_{1/4} &= (2\alpha 2\beta 1)_{(1/2\wedge 1/2\wedge 1/4)} = (2\alpha 2\beta 1)_{1/4}(2\alpha 2\beta 1) = 1/4 \\ &\leq \mu(2\alpha 2\beta 1) = 1/4. \end{aligned}$$

So, $2_{1/2}\alpha 2_{1/2}\beta 1_{1/4} \in \mu$.

$$\begin{aligned} 2_{1/2}\alpha 2_{1/2} &= (2\alpha 2)_{1/2\wedge 1/2} = (2\alpha 2)_{1/2}(2\alpha 2) = 1/2 \\ &> \mu(2\alpha 2) = 1/4. \end{aligned}$$

Thus, $2_{1/2}\alpha 2_{1/2} \notin \mu$ and

$$\begin{aligned} 2_{1/2}\beta 1_{1/4} &= (2\beta 1)_{1/2\wedge 1/4} = (2\beta 1)_{1/4}(2\beta 1) = 1/4 \\ &> \mu(2\beta 1) = 0. \end{aligned}$$

Hence, $2_{1/2}\beta 1_{1/4} \notin \mu$. We conclude that μ is not a fuzzy 2-absorbing $\Gamma-$ideal of $\mathbb{Z}$.

Corollary 3.9. *If μ is a fuzzy 2-absorbing $\Gamma-$ideal of R, then*

$$\mu_* = \{x \in R \mid \mu(x) = \mu(0)\}$$

is a 2-absorbing $\Gamma-$ideal of R.

Proof. Since μ is a non-constant fuzzy $\Gamma-$ideal of R, $\mu_* \neq R$. Now, the result follows from the above theorem. □

Definition 3.10. Let $1 \neq \sigma \in [0, \mu(0))$. Then, σ is called a 2-absorbing element if $r \wedge s \wedge t \leq \sigma$ implies that $r \wedge s \leq \sigma$ or $r \wedge t \leq \sigma$ or $s \wedge t \leq \sigma$ for all $r, s, t \in L$.

Proposition 3.11. *If μ is a fuzzy 2-absorbing $\Gamma-$ideal of R, then $\sigma = \mu(1)$ is a 2-absorbing element.*

Proof. Assume that $r \wedge s \wedge t \leq \sigma$ for some $r, s, t \in L$. Let $1_r, 1_s, 1_t$ are three fuzzy points of $\Gamma-$ring R and $\alpha, \beta \in \Gamma$ with $1_{(r\wedge s\wedge t)} = 1_r\alpha 1_s\beta 1_t \in \mu$. Since μ is a fuzzy 2-absorbing $\Gamma-$ideal of R, we get $1_{r\wedge s} = 1_r\alpha 1_s \in \mu$ or $1_{r\wedge t} = 1_r\beta 1_t \in \mu$ or $1_{s\wedge t} = 1_s\beta 1_t \subseteq \mu$. So, $r \wedge s \leq \mu(1) = \sigma$ or $r \wedge t \leq \mu(1) = \sigma$ or $s \wedge t \leq \mu(1) = \sigma$ and the result follows. □

Theorem 3.12. *Let I be a 2-absorbing $\Gamma-$ideal of R and σ be a 2-absorbing element. Then, the fuzzy subset of R defined by*

$$\mu(x) = \begin{cases} 1, & \text{if } x \in I \\ \sigma, & \text{otherwise} \end{cases}$$

is a fuzzy 2-absorbing $\Gamma-$ideal of R.

Proof. Since I is a 2-absorbing $\Gamma-$ideal of R we get $I \neq R$ and so μ is non-constant. Suppose that $x_r\alpha y_s\beta z_t \in \mu$ but $x_r\alpha y_s \notin \mu$ and $x_r\beta z_t \notin \mu$ and $y_s\beta z_t \notin \mu$, where x_r, y_s, z_t are fuzzy points of R and $\alpha, \beta \in \Gamma$. Then, $\mu(x\alpha y) = \sigma$ and so $x\alpha y \notin I$. Similarly, $x\beta z \notin I$ and $y\beta z \notin I$. But I is assumed to be a 2-absorbing $\Gamma-$ideal of R. Thus, $x\alpha y\beta z \notin I$ and so $\mu(x\alpha y\beta z) = \sigma$ for $x, y, z \in R$ and $\alpha, \beta \in \Gamma$. Also, from $(x\alpha y\beta z)_{(r\wedge s\wedge t)} = x_r\alpha y_s\beta z_t \in \mu$ we get $r \wedge s \wedge t \leq \mu(x\alpha y\beta z) = \sigma$. Thus, $r \wedge s \leq \sigma$ or $r \wedge t \leq \sigma$ or $s \wedge t \leq \sigma$, since σ is a 2-absorbing element, which is a contradiction. Thus $x_r\alpha y \in \mu$ or $x_r\beta z_t \in \mu$ or $y_s\beta z_t \in \mu$. □

Example 3.13. We know that, every fuzzy prime $\Gamma-$ideal of R is a fuzzy 2-absorbing $\Gamma-$ideal of R as mentioned before. In this example, we show that the converse is not generally true. For example, consider $R = 2\mathbb{Z}$ and $\Gamma = 3\mathbb{Z}$.

$$\begin{aligned} R \times \Gamma \times R &\rightarrow R \\ (a, \alpha, b) &\mapsto a\alpha b, \end{aligned}$$

for all $a, b \in R$ and $\alpha \in \Gamma$. Then, R is a $\Gamma-$ring. Now, define $\mu : 2\mathbb{Z} \rightarrow [0, 1]$ by

$$\mu(x) = \begin{cases} 1, & \text{if } x \in 6\mathbb{Z} \\ 0, & \text{otherwise.} \end{cases}$$

Then, μ is a fuzzy 2-absorbing $\Gamma-$ideal of $2\mathbb{Z}$. Furthermore, $\mu_0 = I$ is a 2-absorbing $\Gamma-$ideal of $2\mathbb{Z}$ that is not a prime $\Gamma-$ideal. Therefore, μ is not a fuzzy prime $\Gamma-$ideal of $2\mathbb{Z}$.

Theorem 3.14. *Let $\{\mu_i \mid i \in I\}$ be a collection of fuzzy 2-absorbing $\Gamma-$ideals of R. Then, the fuzzy ideal $\mu = \underset{i\in I}{\cup}\mu_i$ is a fuzzy 2-absorbing $\Gamma-$ideal of R.*

Proof. Assume that $x_r\alpha y_s\beta z_t \in \mu$ and $x_r\alpha y_s \notin \mu$ for some x_r, y_s, z_t are fuzzy points of R and $\alpha,\beta \in \Gamma$. Then, there exist $j \in I$ such that $x_r\alpha y_s\beta z_t \in \mu_j$ and $x_r\alpha y_s \notin \mu_j$ for all $j \in I$. Since μ_j is a fuzzy 2-absorbing $\Gamma-$ideal of R then $y_s\beta z_t \in \mu_j$ or $x_r\beta z_t \in \mu_j$. Hence, $y_s\beta z_t \in \mu_j \subseteq \underset{i\in I}{\cup}\mu_i = \mu$ or $x_r\beta z_t \in \mu_j \subseteq \underset{i\in I}{\cup}\mu_i = \mu$. Therefore, $\mu = \underset{i\in I}{\cup}\mu_i$ is a fuzzy 2-absorbing $\Gamma-$ideal of R. □

Theorem 3.15. *Let $f : R \to S$ be a surjective $\Gamma-$ring homomorphism. If μ is a fuzzy 2-absorbing $\Gamma-$ideal of R which is constant on Kerf, then $f(\mu)$ is a fuzzy 2-absorbing $\Gamma-$ideal of S.*

Proof. Assume that $x_r\alpha y_s\beta z_t \in f(\mu)$, where x_r, y_s, z_t are fuzzy points of S and $\alpha,\beta \in \Gamma$. Since f is a surjective $\Gamma-$ring homomorphism then there exist $a, b, c \in R$ such that $f(a) = x$, $f(b) = y$, $f(c) = z$. Thus,

$$\begin{aligned} x_r\alpha y_s\beta z_t(x\alpha y\beta z) &= r \wedge s \wedge t \\ &\leq f(\mu)(x\alpha y\beta z) \\ &= f(\mu)(f(a)\alpha f(b)\beta f(c)) \\ &= f(\mu)(f(a\alpha b\beta c)) \\ &= \mu(a\alpha b\beta c). \end{aligned}$$

Because μ is constant on $Kerf$. Then, we get $a_r\alpha b_s\beta c_t \in \mu$. Since μ is a fuzzy 2-absorbing $\Gamma-$ ideal of R then,

$$a_r\alpha b_s \in \mu \text{ or } a_r\beta c_t \in \mu \text{ or } b_s\beta c_t \in \mu.$$

Thus,

$$\begin{aligned} r \wedge s &\leq \mu(a\alpha b) = f(\mu)(f(a\alpha b)) \\ &= f(\mu)(f(a)\alpha f(b)) \\ &= f(\mu)(x\alpha y), \end{aligned}$$

and so $x_r\alpha y_s \in f(\mu)$ or

$$\begin{aligned} r \wedge t &\leq \mu(a\beta c) = f(\mu)(f(a\beta c)) \\ &= f(\mu)(f(a)\beta f(c)) \\ &= f(\mu)(x\beta z). \end{aligned}$$

So, $x_r\beta z_t \in f(\mu)$ or

$$\begin{aligned} s \wedge t &\leq \mu(b\beta c) = f(\mu)(f(b\beta c)) \\ &= f(\mu)(f(b)\beta f(c)) \\ &= f(\mu)(y\beta z). \end{aligned}$$

So, $y_s\beta z_t \in f(\mu)$. Hence, $f(\mu)$ is a fuzzy 2-absorbing $\Gamma-$ideal of S. □

Theorem 3.16. *Let $f : R \to S$ be a $\Gamma-$ring homomorphism. If v is a fuzzy 2-absorbing $\Gamma-$ideal of S, then $f^{-1}(v)$ is a fuzzy 2-absorbing $\Gamma-$ideal of R.*

Proof. Suppose that $x_r\alpha y_s\beta z_t \in f^{-1}(v)$, where x_r, y_s, z_t any fuzzy points of R and $\alpha,\beta \in \Gamma$. Then,

$$\begin{aligned} r \wedge s \wedge t &\leq f^{-1}(v)((x\alpha y\beta z)) \\ &= v(f(x\alpha y\beta z)) \\ &= v(f(x)\alpha f(y)\beta f(z)). \end{aligned}$$

Let $f(x) = a$, $f(y) = b$, $f(z) = c \in S$. Hence, we have that $r \wedge s \wedge t \leq v(a\alpha b\beta c)$ and $a_r\alpha b_s\beta c_t \in v$. Since v is a fuzzy 2-absorbing $\Gamma-$ideal of R then $a_r\alpha b_s \in v$ or $a_r\beta c_t \in v$ or $b_s\beta c_t \in v$. If $a_r\alpha b_s \in v$, then

$$\begin{aligned} r \wedge s &\leq v(a\alpha b) = v(f(x)\alpha f(y)) \\ &= v(f(x\alpha y)) \\ &= f^{-1}(v(x\alpha y)). \end{aligned}$$

Thus, we conclude that $x_r\alpha y_s \in f^{-1}(v)$. In similar way, it can be see that $x_r\beta z_t \in f^{-1}(v)$ or $y_s\beta z_t \in f^{-1}(v)$. □

Definition 3.17. Let μ be a fuzzy $\Gamma-$ideal of R. μ is called a fuzzy strongly 2-absorbing $\Gamma-$ideal of R if it is non-constant and whenever λ, η, v are fuzzy $\Gamma-$ideal of R with $\lambda\Gamma\eta\Gamma v \subseteq \mu$, then $\lambda\Gamma\eta \subseteq \mu$ or $\lambda\Gamma v \subseteq \mu$ or $\eta\Gamma v \subseteq \mu$.

Theorem 3.18. *Every fuzzy prime $\Gamma-$ideal of R is a fuzzy strongly 2-absorbing $\Gamma-$ideal of R.*

Proof. The proof is straightforward. □

Theorem 3.19. *Every fuzzy strongly 2-absorbing $\Gamma-$ideal of R is a fuzzy 2-absorbing $\Gamma-$ideal of R.*

Proof. Assume that μ is a fuzzy strongly 2-absorbing $\Gamma-$ideal of R. Suppose that $x_r, y_s, z_t \in \mu$ for some fuzzy points x_r, y_s, z_t of R. We get $\langle x_r\rangle \Gamma \langle y_s\rangle \Gamma \langle z_t\rangle = \langle x_r\Gamma y_s\Gamma z_t\rangle \subseteq \mu$. Since μ is a fuzzy strongly 2-absorbing $\Gamma-$ideal of R, we get $\langle x_r\Gamma y_s\rangle = \langle x_r\rangle \Gamma \langle y_s\rangle \subseteq \mu$ or $\langle x_r\Gamma z_t\rangle = \langle x_r\rangle \Gamma \langle z_t\rangle \subseteq \mu$ or $\langle y_s\Gamma z_t\rangle = \langle y_s\rangle \Gamma \langle z_t\rangle \subseteq \mu$. Hence, $x_r\Gamma y_s \subseteq \mu$ or $x_r\Gamma z_t \subseteq \mu$ or $y_s\Gamma z_t \subseteq \mu$ and then for $\alpha, \beta \in \Gamma$, $x_r\alpha y_s \in \mu$ or $x_r\beta z_t \in \mu$ or $y_s\beta z_t \in \mu$ which implies that μ is a fuzzy 2-absorbing $\Gamma-$ideal of R. □

4. Fuzzy Weakly Completely 2-absorbing Γ−ideals of a Γ− ring

Definition 4.1. Let μ be a fuzzy $\Gamma-$ideal of R and μ is called a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of R if

$$\mu(x\alpha y\beta z) = \mu(x\alpha y) \text{ or } \mu(x\alpha y\beta z) = \mu(x\beta z) \text{ or } \mu(x\alpha y\beta z) = \mu(y\beta z),$$

for all $x, y, z \in R$ and $\alpha, \beta \in \Gamma$.

Proposition 4.2. *Let μ be a non-constant fuzzy $\Gamma-$ideal of R. μ is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of R if and only if*

$$\mu(x\alpha y\beta z) = \max\{\mu(x\alpha y), \mu(x\beta z), \mu(y\beta z)\},$$

for every $x, y, z \in R$ and $\alpha, \beta \in \Gamma$.

Definition 4.3. A fuzzy $\Gamma-$ideal μ of R is called a fuzzy weakly completely prime $\Gamma-$ideal of R if μ is non-constant function and for all $x, y \in R$ and $\alpha \in \Gamma$,

$$\mu(x\alpha y) = \max\{\mu(x), \mu(y)\}.$$

Theorem 4.4. *Every fuzzy weakly completely prime $\Gamma-$ideal of R is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of R.*

Proof. Let μ be a fuzzy weakly completely prime $\Gamma-$ideal of R. Then, for every $x, y, z \in R$ and $\alpha, \beta \in \Gamma$,

$$\mu(x\alpha y\beta z) = \mu(x) \text{ or } \mu(x\alpha y\beta z) = \mu(y) \text{ or } \mu(x\alpha y\beta z) = \mu(z).$$

Suppose that $\mu(x\alpha y\beta z) = \mu(x)$. Then from $\mu(x\alpha y\beta z) \geq \mu(x\alpha y) \geq \mu(x)$ we get $\mu(x\alpha y\beta z) = \mu(x\alpha y)$. In similar way, we can easily show that if $\mu(x\alpha y\beta z) = \mu(y)$ or $\mu(x\alpha y\beta z) = \mu(z)$, then $\mu(x\alpha y\beta z) = \mu(y\beta z)$ or $\mu(x\alpha y\beta z) = \mu(x\beta z)$. Thus, μ is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of R. □

Theorem 4.5. *Let μ a fuzzy $\Gamma-$ideal of R. The following statements are equivalent:*

(1) *μ is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of R.*
(2) *For every $a \in [0, \mu(0)]$, the $a-$level subset μ_a of μ is a 2-absorbing $\Gamma-$ideal of R.*

Proof. (1) $\Rightarrow$ (2) : Suppose that μ is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of R and let $x, y, z \in R$, $\alpha, \beta \in \Gamma$ and $x\alpha y\beta z \in \mu_a$ for some $a \in [0, \mu(0)]$. Then,

$$\max\{\mu(x\alpha y), \mu(x\beta z), \mu(y\beta z)\} = \mu(x\alpha y\beta z) \geq a.$$

Hence, $\mu(x\alpha y) \geq a$ or $\mu(x\beta z) \geq a$ or $\mu(y\beta z) \geq a$, which implies that $x\alpha y \in \mu_a$ or $x\beta z \in \mu_a$ or $y\beta z \in \mu_a$. Hence, μ_a is a 2-absorbing $\Gamma-$ideal of R.

(2) $\Rightarrow$ (1) : Admit that μ_a is a 2-absorbing $\Gamma-$ideal of R for every $a \in [0, 1]$. For $x, y, z \in R$ and $\alpha, \beta \in \Gamma$, let $\mu(x\alpha y\beta z) = a$. Then $x\alpha y\beta z \in \mu_a$ and μ_a is 2-absorbing $\Gamma-$ideal it gives $x\alpha y \in \mu_a$ or $x\beta z \in \mu_a$ or $y\beta z \in \mu_a$. Hence, $\mu(x\alpha y) \geq a$ or $\mu(x\beta z) \geq a$ or $\mu(y\beta z) \geq a$, that is $\max\{\mu(x\alpha y), \mu(x\beta z), \mu(y\beta z)\} \geq a = \mu(x\alpha y\beta z)$. Also, since μ is a fuzzy $\Gamma-$ ideal of R, we get

$$\mu(x\alpha y\beta z) \geq \max\{\mu(x\alpha y), \mu(x\beta z), \mu(y\beta z)\}.$$

Thus, $\mu(x\alpha y\beta z) = \max\{\mu(x\alpha y), \mu(x\beta z), \mu(y\beta z)\}$, that is μ is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of R. □

Theorem 4.6. *Let $f : R \to S$ be a surjective $\Gamma-$ring homomorphism. If μ is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of R which is constant on $Kerf$, then $f(\mu)$ is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of S.*

Proof. Suppose that $f(\mu)(x\alpha y\beta z) \neq f(\mu)(x\alpha y)$ for any $x, y, z \in S$ and $\alpha, \beta \in \Gamma$. Since f is a surjective $\Gamma-$ring homomorphism then,

$$f(a) = x,\ f(b) = y,\ f(c) = z \text{ for some } a, b, c \in R.$$

Hence,

$$\begin{aligned} f(\mu)(x\alpha y\beta z) &= f(\mu)(f(a)\alpha f(b)\beta f(c)) = f(\mu)(f(a\alpha b\beta c)) \\ &\neq f(\mu)(x\alpha y) = f(\mu)(f(a)\alpha f(b)) = f(\mu)(f(a\alpha b)). \end{aligned}$$

Since μ is constant on $Kerf$,

$$\begin{aligned} f(\mu)(f(a\alpha b\beta c)) &= \mu(a\alpha b\beta c) \text{ and} \\ f(\mu)(f(a\alpha b)) &= \mu(a\alpha b). \end{aligned}$$

It means that,

$$f(\mu)(f(a\alpha b\beta c)) = \mu(a\alpha b\beta c) \neq \mu(a\alpha b) = f(\mu)(f(a\alpha b)).$$

Since μ is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of R, then

$$\begin{aligned} \mu(a\alpha b\beta c) &= f(\mu)(f(a)\alpha f(b)\beta f(c)) = f(\mu)(x\alpha y\beta z) \\ &= \mu(a\beta c) = f(\mu)(f(a\beta c)) = f(\mu)(f(a)\beta f(c)) = f(\mu)(x\beta z). \end{aligned}$$

So, we get $f(\mu)(x\alpha y\beta z) = f(\mu)(x\beta z)$ or

$$\begin{aligned} \mu(a\alpha b\beta c) &= f(\mu)(f(a)\alpha f(b)\beta f(c)) = f(\mu)(x\alpha y\beta z) \\ &= \mu(b\beta c) = f(\mu)(f(b\beta c)) = f(\mu)(f(b)\beta f(c)) = f(\mu)(y\beta z). \end{aligned}$$

We have $f(\mu)(x\alpha y\beta z) = f(\mu)(y\beta z)$. Therefore, $f(\mu)$ is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of S. □

Theorem 4.7. *Let $f : R \to S$ be a $\Gamma-$ring homomorphism. If ν is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of S, then $f^{-1}(\nu)$ is a fuzzy weakly completely 2-absorbing $\Gamma-$ ideal of R.*

Proof. Suppose that $f^{-1}(\nu)(x\alpha y\beta z) \neq f^{-1}(\nu)(x\alpha y)$ for any $x, y, z \in R$ and $\alpha, \beta \in \Gamma$. Then,

$$\begin{aligned} f^{-1}(\nu)(x\alpha y\beta z) &= \nu(f(x\alpha y\beta z)) = \nu(f(x)\alpha f(y)\beta f(z)) \\ &\neq f^{-1}(\nu)(x\alpha y) = \nu(f(x\alpha y)) = \nu(f(x)\alpha f(y)). \end{aligned}$$

Since ν is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of S we have that

$$\begin{aligned} \nu(f(x)\alpha f(y)\beta f(z)) &= f^{-1}(\nu)(x\alpha y\beta z) \\ &= \nu(f(x)\beta f(z)) = \nu(f(x\beta z)) \\ &= f^{-1}(\nu)(x\beta z) \end{aligned}$$

or

$$\begin{aligned} v(f(x)\alpha f(y)\beta f(z)) &= f^{-1}(v)(x\alpha y\beta z) \\ &= v(f(y)\beta f(z)) = v(f(y\beta z)) \\ &= f^{-1}(v)(y\beta z). \end{aligned}$$

Hence, $f^{-1}(v)$ is a fuzzy weakly completely 2-absorbing Γ−ideal of R. □

Corollary 4.8. *Let f be a Γ−ring homomorphism from R onto S. f induces a one to one inclusion preserving correspondence between fuzzy weakly completely 2-absorbing Γ−ideal of S in such a way that if μ is a fuzzy weakly completely 2-absorbing Γ−ideal of R constant on Kerf, then $f(\mu)$ is the corresponding fuzzy weakly completely 2-absorbing Γ−ideal of S, and if v is a fuzzy weakly completely 2-absorbing Γ−ideal of S, then $f^{-1}(v)$ is the corresponding fuzzy weakly completely 2-absorbing Γ−ideal of R.*

5. Fuzzy $K-$2-absorbing Γ−ideals of a Γ−ring

Definition 5.1. Let μ be a fuzzy Γ−ideal of R. μ is called a fuzzy $K-$ 2-absorbing Γ−ideal of R if for all $x, y, z \in R$ and $\alpha, \beta \in \Gamma$,

$$\mu(x\alpha y\beta z) = \mu(0) \text{ implies that } \mu(x\alpha y) = \mu(0) \text{ or } \mu(x\beta z) = \mu(0) \text{ or } \mu(y\beta z) = \mu(0).$$

Theorem 5.2. *Every fuzzy weakly completely 2-absorbing Γ−ideal of R is a fuzzy $K-$2-absorbing Γ−ideal of R.*

Proof. Assume that μ is a fuzzy weakly completely 2-absorbing Γ-ideal of R. If $\mu(x\alpha y\beta z) = \mu(0)$ for any $x, y, z \in R$ and $\alpha, \beta \in \Gamma$, then we get

$$\begin{aligned} \mu(0) &= \mu(x\alpha y\beta z) \le \mu(x\alpha y) \le \mu(0) \quad \text{or} \\ \mu(0) &= \mu(x\alpha y\beta z) \le \mu(x\beta z) \le \mu(0) \quad \text{or} \\ \mu(0) &= \mu(x\alpha y\beta z) \le \mu(y\beta z) \le \mu(0). \end{aligned}$$

Because μ is a fuzzy weakly completely 2-absorbing Γ-ideal of R. From this, the following result is obtained:

$$\mu(x\alpha y) = \mu(0) \text{ or } \mu(x\beta z) = \mu(0) \text{ or } \mu(y\beta z) = \mu(0).$$

It means that, μ is a fuzzy $K-$ 2-absorbing Γ-ideal of R. □

The following example shows that the converse of the above theorem is need not to be true.

Example 5.3. Let $R = \mathbb{Z}$ and $\Gamma = 2\mathbb{Z}$, so R is a Γ−ring. Define the fuzzy Γ−ideal μ of R by

$$\mu(x) = \begin{cases} 1, & \text{if } x = 0 \\ 1/3, & \text{if } x \in 27\mathbb{Z} - \{0\} \\ 1/4, & \text{if } x \in \mathbb{Z} - 27\mathbb{Z}. \end{cases}$$

Then, μ is a fuzzy $K-$2-absorbing Γ−ideal of R. However, for $\alpha, \beta \in 2\mathbb{Z}$ we have

$$\mu(3\alpha 3\beta 15) = 1/3 > 1/4 = \max\{\mu(3\alpha 3), \mu(3\beta 15), \mu(3\beta 15)\}.$$

Hence, μ is not a fuzzy weakly completely 2-absorbing Γ−ideal of R.

Definition 5.4. Let μ be a fuzzy Γ-ideal of R and μ is called a fuzzy $K-$prime Γ− ideal of R if

$$\mu(x\alpha y) = \mu(0) \text{ implies that } \mu(x) = \mu(0) \text{ or } \mu(y) = \mu(0),$$

for $x, y \in R$ and $\alpha, \beta \in \Gamma$.

Theorem 5.5. *Every fuzzy $K-$prime Γ−ideal of R is a fuzzy $K-$2-absorbing Γ−ideal of R.*

Proof. Let μ be a fuzzy $K-$prime Γ− ideal of R. Then, for every $x, y, z \in R$ and $\alpha, \beta \in \Gamma$,

$$\mu(x\alpha y\beta z) = \mu(0) \text{ implies that } \mu(x) = \mu(0) \text{ or } \mu(y) = \mu(0) \text{ or } \mu(z) = \mu(0).$$

Admit that $\mu(x) = \mu(0)$. Then, from

$$\mu(0) = \mu(x) \le \mu(x\alpha y) \le \mu(x\alpha y\beta z) = \mu(0),$$

we get $\mu(x\alpha y) = \mu(0)$ or similarly, we can easily show that $\mu(x\beta z) = \mu(0)$ or $\mu(y\beta z) = \mu(0)$. Therefore, μ is a fuzzy $K-$ 2-absorbing Γ− ideal of R. □

Theorem 5.6. *Let $f : R \to S$ be a surjective $\Gamma-$ring homomorphism. If μ is a fuzzy $K-2$-absorbing $\Gamma-$ideal of R which is constant on $Kerf$, then $f(\mu)$ is a fuzzy $K-2$-absorbing $\Gamma-$ideal of S.*

Proof. Assume that $f(\mu)(a\alpha b\beta c) = f(\mu)(0_S)$ for any $a, b, c \in S$ and $\alpha, \beta \in \Gamma$. Then, $f(x) = a$, $f(y) = b$, $f(z) = c$ for some $x, y, z \in R$ since f is a surjective $\Gamma-$ring homomorphism. Thus,

$$\begin{aligned} f(\mu)(a\alpha b\beta c) &= f(\mu)(f(x)\alpha f(y)\beta f(z)) \\ &= f(\mu)(f(x\alpha y\beta z)) \end{aligned}$$

and

$$f(\mu)(0_S) = \vee\{\mu(x) \mid f(x) = 0_S\}.$$

From here, we get $x \in Kerf$ and so μ is constant on $Kerf$, $\mu(x) = \mu(0)$

$$f(\mu)(0_S) = \vee\{\mu(x) \mid f(x) = 0_S\},$$

which implies that

$$f(\mu)(f(x\alpha y\beta z)) = \mu(x\alpha y\beta z) = \mu(0).$$

Due to fact that μ is a fuzzy $K-2$-absorbing Γ-ideal of R,

$$\mu(x\alpha y\beta z) = \mu(0) \text{ implies that } \mu(x\alpha y) = \mu(0) \text{ or } \mu(x\beta z) = \mu(0) \text{ or } \mu(y\beta z) = \mu(0).$$

By the previous theorem, the rest of proof can easily show and we see that $f(\mu)$ is a fuzzy $K-2$-absorbing Γ-ideal of S. □

Theorem 5.7. *Let $f : R \to S$ be a $\Gamma-$ring homomorphism. If v is a fuzzy $K-2$-absorbing $\Gamma-$ideal of S, then $f^{-1}(v)$ is a fuzzy $K-2$-absorbing $\Gamma-$ideal of R.*

Proof. Suppose that $f^{-1}(v)(x\alpha y\beta z) = f^{-1}(v)(0)$ for any $x, y, z \in R$ and $\alpha, \beta \in \Gamma$. Then, from

$$\begin{aligned} f^{-1}(v)(x\alpha y\beta z) &= v(f(x\alpha y\beta z)) = v(f(x)\alpha f(y)\beta f(z)) \\ &= f^{-1}(v)(0) = v(f(0)) = v(0) \end{aligned}$$

we get $v(f(x)\alpha f(y)\beta f(z)) = v(0)$ since f is a surjective $\Gamma-$ring homomorphism. Then, we have

$$\begin{aligned} v(f(x)\alpha f(y)\beta f(z)) &= v(0) \text{ implies that} \\ v(f(x)\alpha f(y)) &= v(0) \text{ or } v(f(x)\beta f(z)) = v(0) \text{ or } v(f(y)\beta f(z)) = v(0), \end{aligned}$$

since v is a fuzzy $K-2$-absorbing Γ-ideal of S. From this, we have

$$\begin{aligned} v(f(x)\alpha f(y)) &= v(f(x\alpha y)) = f^{-1}(v)(x\alpha y) \\ &= v(0) = v(f(0)) = f^{-1}(v)(0) \\ f^{-1}(v)(x\alpha y) &= f^{-1}(v)(0) \text{ or} \end{aligned}$$

similarly, we can show that $f^{-1}(v)(x\beta z) = f^{-1}(v)(0)$ or $f^{-1}(v)(y\beta z) = f^{-1}(v)(0)$. Finally, $f^{-1}(v)$ is a fuzzy $K-2$-absorbing Γ-ideal of R. □

Corollary 5.8. *Let f be a $\Gamma-$ring homomorphism from R onto S. f induces a one to one inclusion preserving correspondence between fuzzy $K-2$-absorbing $\Gamma-$ideal of S in such a way that if μ is a fuzzy $K-2$-absorbing $\Gamma-$ideal of R constant on Kerf, then $f(\mu)$ is the corresponding fuzzy $K-2$-absorbing $\Gamma-$ ideal of S, and if v is a fuzzy $K-2$-absorbing $\Gamma-$ideal of S, then $f^{-1}(v)$ is the corresponding fuzzy $K-2$-absorbing $\Gamma-$ideal of R.*

Remark 5.9. The following table summarizes findings of fuzzy 2-absorbing $\Gamma-$ideals of a $\Gamma-$ring.

$$\begin{array}{ccc} & & f.\,strongly\ 2-abs.\,\Gamma-ideal \\ & \nearrow & \downarrow \\ f.prime\,\Gamma-ideal & \rightarrow & f.\ 2-abs.\ \Gamma-ideal \\ \downarrow & & \downarrow \\ f.w.c.p.\Gamma-ideal & \rightarrow & f.w.c.2-abs.\ \Gamma-ideal \\ & & \downarrow \\ & & f.K-2-abs.\ \Gamma-ideal \end{array}$$

6. Fuzzy Quotient $\Gamma-$ring of R Induced by Fuzzy 2-absorbing $\Gamma-$ideal

Now, we remind the notion of fuzzy quotient $\Gamma-$ring induced by fuzzy $\Gamma-$ideal of R. Let μ be a fuzzy $\Gamma-$ideal of a $\Gamma-$ring R. For any $x, y \in R$, define a binary relation $\sim$ on R which is a congruence relation of R by $x \sim y$ if and only if

$$\mu(x-y) = \mu(0),$$

where 0 is the zero element of R. Let $\mu[x] = \{y \in R \mid y \sim x\}$ be the equivalence class containing x and $R/\mu = \{\mu[x] \mid x \in R\}$ the set of all equivalence classes of R. Define two operations by

$$\begin{aligned} \mu[x] + \mu[y] &= \mu[x+y] \quad \text{and} \\ \mu[x]\alpha\mu[y] &= \mu[x\alpha y], \end{aligned}$$

for $x, y \in R, \alpha \in \Gamma$. Then, R/μ is a fuzzy $\Gamma-$ring with two operations and call it fuzzy quotient $\Gamma-$ring of R induced by the fuzzy $\Gamma-$ideal μ [20].

Theorem 6.1. *Let μ be a non-constant fuzzy $\Gamma-$ideal of R. Then, μ is a fuzzy $K-2$-absorbing $\Gamma-$ideal of R if and only if R/μ is a 2-absorbing $\Gamma-$ring.*

Proof. Suppose that μ is a fuzzy $K-2$-absorbing $\Gamma-$ideal of R and let $\mu[x], \mu[y], \mu[z] \in R/\mu$ be such that

$$\mu[x]\alpha\mu[y]\beta\mu[z] = \mu[0].$$

Since $\mu[x]\alpha\mu[y]\beta\mu[z] = \mu[x\alpha y\beta z]$, we get

$$\mu(x\alpha y\beta z) = \mu(x\alpha y\beta z - 0) = 1 = \mu(0).$$

As μ is considered to be fuzzy $K-2$-absorbing $\Gamma-$ideal of R,

$$\mu(x\alpha y) = \mu(0) = 1 \text{ or } \mu(x\beta z) = \mu(0) = 1 \text{ or } \mu(y\beta z) = \mu(0) = 1.$$

It means that,

$$\begin{aligned} \mu[x\alpha y] &= \mu[x]\alpha\mu[y] = \mu[0] \quad \text{or} \\ \mu[x\beta z] &= \mu[x]\beta\mu[z] = \mu[0] \quad \text{or} \\ \mu[y\beta z] &= \mu[y]\beta\mu[z] = \mu[0]. \end{aligned}$$

So, R/μ is a 2-absorbing $\Gamma-$ring. Conversely, suppose that R/μ is a 2-absorbing $\Gamma-$ring and let $\mu(x\alpha y\beta z) = \mu(0) = 1$ for $x, y, z \in R$ and $\alpha, \beta \in \Gamma$. Then, we get

$$\mu[x]\alpha\mu[y]\beta\mu[z] = \mu[x\alpha y\beta z] = \mu[0].$$

Since R/μ is a 2-absorbing $\Gamma-$ring,

$$\mu[x\alpha y] = \mu[0] \text{ or } \mu[x\beta z] = \mu[0] \text{ or } \mu[y\beta z] = \mu[0],$$

which implies that μ is a fuzzy $K-2$-absorbing $\Gamma-$ideal of R. □

Corollary 6.2. *If μ is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal of R, then R/μ is a 2-absorbing $\Gamma-$ring.*

7. Conclusion

In this paper, we have characterized fuzzy 2-absorbing $\Gamma-$ideals of a $\Gamma-$ring. Also, the notions of fuzzy weakly completely 2-absorbing $\Gamma-$ideals of a $\Gamma-$ring and fuzzy $K-$2-absorbing $\Gamma-$ideals of a $\Gamma-$ring and their properties are proposed. Moreover, we have given a diagram which transition between definitions of fuzzy $\Gamma-$ideals of $\Gamma-$ring. Finally, we have shown that if μ is a fuzzy weakly completely 2-absorbing $\Gamma-$ideal, then fuzzy quotient $\Gamma-$ring of R induced by the fuzzy $\Gamma-$ideal is a 2-absorbing $\Gamma-$ring. To extend this study, one could study other algebraic structures and do some further study on the properties them. In our future work, we have planed to define an intuitionistic fuzzy 2-absorbing $\Gamma-$ideal of a $\Gamma-$ring and an intuitionistic fuzzy weakly completely 2-absorbing $\Gamma-$ideal of a $\Gamma-$ring and to discuss its related properties.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this article.

Authors Contribution Statemen

The author has read and agreed to the published version of the manuscript.

References

[1] Anderson, D.F., Badawi, A., *On n-absorbing ideals of commutavie rings*, Commutative Algebra, **39**(2011), 1646–1672.
[2] Badawi, A., *On 2-absorbing ideals of commutative rings*, Bull. Austral Math. Soc., **75**(2007), 417–429.
[3] Badawi, A., Tekir, U., Yetkin, E., *On 2-absorbing primary ideals in commutative rings*, Bull. Austral. Math. Soc., **51**(4)(2014), 1163–1173.
[4] Badawi, A., Darani, A.Y., *On weakly 2-absorbing ideals of commutative rings*, Houston J. Math., **39**(2013), 441–452.
[5] Barnes, W.E., *On the* $\Gamma-$*rings of Nobusawa*, Pacific J. Math., **18**(1966), 411–422.
[6] Darani, A.Y., *On L–fuzzy 2-absorbing ideals*, Italian Journal of Pure and Appl. Math., **36**(2016), 147–154.
[7] Darani, A.Y., Puczylowski, E.R., *On 2-absorbing commutative semigroups and their applications to rings*, Semigroup Forum, **86**(2013), 83–91.
[8] Darani, A.Y., Hashempoor, A., *L–fuzzy 0-(1- or 2- or 3-) 2-absorbing ideals in semiring*, Annals of Fuzzy Math. and Inform., **7**(2)(2014), 303–311.
[9] Dutta, T.K., Chanda, T., *Fuzzy prime ideals in* $\Gamma-$*rings*, Bull. Malays. Math. Sci. Soc., **30**(2007), 65–73.
[10] Dutta, T.K., Chanda, T., *Structures of fuzzy ideals of* $\Gamma-$*ring*, Bull. Malays. Math. Sci. Soc., **28**(2005), 9–18.
[11] Elkettani, M.Y., Kasem, A., *On 2-absorbing* $\delta-$*primary gamma ideal of gamma ring*, Int. J. Pure and Appl. Math., **106**(2)(2016), 543–550.
[12] Ersoy, B.A., *Fuzzy semiprime ideals in* $\Gamma-$*rings*, Int. J. Physical Sciences, **5**(4)(2010), 308–312.
[13] Jun, Y.B., *On fuzzy prime ideals of* $\Gamma-$*rings*, Soochow J. Math., **21**(1)(1995), 41–48.
[14] Kumar, P., Dubey, M.K., Sarohe, P., *Some results on 2-absorbing ideals in commutative semirings*, J. Math. and Appl., **38**(2015), 77–84.
[15] Kuraoka, T., Kuroki, T., *On fuzzy quotient rings induced by fuzzy ideals*, Fuzzy Sets and Systems, **47**(1992), 381–386.
[16] Kyuno, S., *On prime gamma rings*, Pacific J. Math. **75**(1)(1978), 185–190.
[17] Kyuno, S., *A gamma ring with the right and left unities*, Math. Jopanica, **24**(2)(1979), 191–193.
[18] Kyuno, S., *Prime ideals in gamma rings*, Pacific J. Math, **98**(2)(1982), 375–379.
[19] Liu, W.J., *Operation on fuzzy ideals*, Fuzzy Sets and Systems, **11**(1983), 31–41.
[20] Liu, X., *Idealistic soft* $\Gamma-$*rings*, Journal of Hyperstructures, **2**(2)(2013), 136–150.
[21] Luh, J., *On the theory of simple* $\Gamma-$*rings*, Michigan Math. J., **16**(1969), 65–75.
[22] Malik, D.S., Mordeson, J.N., Fuzzy Commutative Algebra, World Scientific Publishing, 1998.
[23] Mukherjee, T.K., Sen, M.K., *Prime fuzzy ideals in rings*, Fuzzy Sets and Systems, **32**(1989), 345–350.
[24] Nobusawa, N., *On a generalization of the ring theory*, Osaka J. Math., **1**(1964), 81–89.
[25] Rosenfeld, A., *Fuzzy groups*, J. Math. Anal. Appl., **35**(1971), 512–517.
[26] Sönmez, D., Yeşilot, G., Onar, S., Ersoy, B.A., Davvaz, B., *On 2-absorbing primary fuzzy ideals of commutative rings*, Mathematical Problems in Engineering, (2017), Article ID 5485839.
[27] Zadeh, L.A., *Fuzzy sets*, Inform and Control; **8**(1965), 338–353.