On Fuzzy 2-absorbing $\Gamma$-ideals in $\Gamma$-rings

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Received: 31-03-2021 • Accepted: 15-04-2022

Abstract. The goal of this paper is to give a definition of a generalization of fuzzy prime $\Gamma$-ideals in $\Gamma$-rings by introducing fuzzy 2-absorbing $\Gamma$-ideals and fuzzy weakly completely 2-absorbing $\Gamma$-ideals of commutative $\Gamma$-rings and to give their properties. Furthermore, we give a diagram which transition between definitions of fuzzy 2-absorbing $\Gamma$-ideals of a $\Gamma$-ring. Finally, we introduce fuzzy quotient $\Gamma$-ring of $R$ induced by the fuzzy weakly completely 2-absorbing $\Gamma$-ideal is a 2-absorbing $\Gamma$-ring.

2010 AMS Classification: 03E72, 08A72

Keywords: 2-absorbing, fuzzy 2-absorbing ideal, fuzzy 2-absorbing $\Gamma$-ideal, fuzzy weakly completely 2-absorbing $\Gamma$-ideal, fuzzy K-2-absorbing $\Gamma$-ideal.

1. Introduction


The concept of a 2-absorbing ideal, which is a generalization of prime ideal, was proposed by Badawi in [2] and also presented in [1, 3]. At present, study on the 2-absorbing ideal theory is progressing rapidly. It has been studied extensively by many authors (e.g. [4, 7, 14]). Darani [6] demonstrated the notion of $L$-fuzzy 2-absorbing ideals and has acquired interesting results on these concepts. Then, Darani and Hashempoor constructed the concept of $L$-fuzzy 2-absorbing ideals in semiring [8]. Elkettani and Kasem [11] clarified the notion of 2-absorbing $\delta$-primary $\Gamma$-ideal of $\Gamma$-ring and gave interesting results concerning these notions. Sönmez [26] described 2-absorbing primary fuzzy ideals of commutative rings and established relations between 2-absorbing primary fuzzy ideals and 2-absorbing primary ideals.

This paper provides a new algebraic structure of fuzzy prime $\Gamma$-ideal of commutative $\Gamma$-ring by 2-absorbing and weakly completely prime 2-absorbing ideal theory. We examine the notion of fuzzy 2-absorbing $\Gamma$-ideal of $\Gamma$-ring and fuzzy weakly completely 2-absorbing $\Gamma$-ideal of $\Gamma$-ring and explain some of its characterization of algebraic properties. Furthermore, we give definition of fuzzy strongly 2-absorbing $\Gamma$-ideal of $\Gamma$-ring and fuzzy $K$-2-absorbing $\Gamma$-ideal of $\Gamma$-ring. We investigate image and inverse image of fuzzy 2-absorbing $\Gamma$-ideal of $\Gamma$-ring and fuzzy weakly...
completely 2-absorbing $\Gamma$–ideal of $\Gamma$–ring. Then, we construct a diagram which transition between definitions of fuzzy 2-absorbing $\Gamma$–ideals of a $\Gamma$–ring as well as the relationship of these concepts with the notion of 2-absorbing $\Gamma$–ideal. Finally, we introduce fuzzy quotient $\Gamma$–ring of $R$ induced by the fuzzy weakly completely 2-absorbing $\Gamma$–ideal is a 2-absorbing $\Gamma$–ring.

2. Preliminaries

In this section, for the sake of completeness, we first recall some useful definitions and results. Throughout this paper, $\Gamma$–ring $R$ is a commutative with $1 \neq 0$ and $L = [0, 1]$ stands for a complete lattice.

Definition 2.1 (27). A fuzzy subset $\mu$ in a set $X$ is a function $\mu : X \to [0, 1]$.

Proposition 2.2 (22). Let $\mu$ and $\nu$ be fuzzy subset of $X$. We say that, $\mu$ is a subset of $\nu$ and write $\mu \subseteq \nu$, if $\mu(x) \leq \nu(x)$ for all $x \in X$.

Definition 2.3 (22). Let $\mu$ be any fuzzy subset of $X$ and $t \in L$. Then, the set $$
\mu_t = \{x \in X \mid \mu(x) \geq t\}
$$
is called the $t$–level subset of $X$ with respect to $\mu$.

Definition 2.4 (22). A fuzzy subset $\mu$ of $X$ is called a fuzzy point if $x \in X$ and $r \in L \setminus \{0\}$, is a fuzzy subset of $X$ and defined by

$$
x_r = \begin{cases} r, & \text{if } y = x; \\ 0, & \text{otherwise}. \end{cases}
$$

If $x_r$ is a fuzzy point of $X$ and $x_r \subseteq \mu$, then we write $x_r \in \mu$.

Definition 2.5 (10). Let $R$ and $\Gamma$ be two abelian additive groups. $R$ is called a $\Gamma$–ring, if there exist a mapping

$$
R \times \Gamma \times R \to R, 
(x, \alpha, y) \mapsto x\alpha y,
$$
satisfying the following conditions;

1. $(x + y)\alpha z = x\alpha z + y\alpha z$,
2. $x\alpha(y + z) = x\alpha y + x\alpha z$,
3. $x(\alpha + \beta)y = x\alpha y + x\beta y$,
4. $x\alpha(y\beta z) = (x\alpha y)\beta z$,

for all $x, y, z \in R$ and all $\alpha, \beta \in \Gamma$. A $\Gamma$–ring $R$ is called commutative, if $x\alpha y = \alpha x\beta$ for any $x, y \in R$ and $\alpha \in \Gamma$.

Definition 2.6 (10). A left (resp. right) ideal of a $\Gamma$–ring $R$ is a subset $A$ of $R$ which is an additive subgroup of $R$ and $R\alpha A \subseteq A$ (resp. $A\beta R \subseteq A$) where,

$$
R\alpha A = \{x\alpha y \mid x \in R, \alpha \in \Gamma, y \in A\}.
$$

If $A$ is both a left and a right ideal, then $A$ is called a $\Gamma$–ideal of $R$.

Definition 2.7 (9). A fuzzy set $\mu$ in a $\Gamma$–ring $R$ is called a fuzzy ideal of $R$, if the following requirements are satisfied:

1. $\mu(x - y) \geq \min \{\mu(x), \mu(y)\}$,
2. $\mu(x\alpha y) \geq \max \{\mu(x), \mu(y)\}$,

for all $x, y \in R$ and $\alpha \in \Gamma$.

Definition 2.8 (10). Let $R$ and $S$ be two $\Gamma$–rings, and $f$ be a mapping of $R$ into $S$. Then, $f$ is called a $\Gamma$–homomorphism, if

$$
f(a + b) = f(a) + f(b)
$$

and

$$
f(ab) = f(a)f(b),
$$

for all $a, b \in R$ and $\alpha \in \Gamma$. 
Definition 2.9 ([5]). Let $R$ be a $\Gamma$–ring. A proper ideal $P$ of $R$ is called a prime $\Gamma$–ideal, if for all pairs of ideals $S$ and $T$ of $R$,

$$S \Gamma T \subseteq P$$

implies that $S \subseteq P$ or $T \subseteq P$.

Proposition 2.10 ([18]). If $P$ is an ideal of a $\Gamma$–ring $R$, then the following conditions are equivalent:

1. $P$ is a prime $\Gamma$–ideal of $R$.
2. If $x, y \in R$ and $x \Gamma R \Gamma y \subseteq P$, then $x \in P$ or $y \in P$.

Definition 2.11 ([13]). A non-constant fuzzy ideal $\mu$ of a $\Gamma$–ring $R$ is called a fuzzy prime $\Gamma$–ideal of $R$ if for any two fuzzy ideals $\sigma$ and $\theta$ of $R$,

$$\sigma \Gamma \theta \subseteq \mu$$

implies that either $\sigma \subseteq \mu$ or $\theta \subseteq \mu$.

Definition 2.12 ([22]). Let $\mu$ be a fuzzy subset of $R$. Then, the fuzzy ideal of $R$ generated by $\mu$ is defined to be the intersection of all fuzzy ideals of $R$ containing $\mu$ and denoted by $\langle \mu \rangle$.

Clearly, $\langle \mu \rangle$ is a fuzzy ideal of $R$. In fact, $\langle \mu \rangle$ is the smallest fuzzy ideal of $R$ containing $\mu$.

Lemma 2.13 ([9]). Let $R$ be a commutative $\Gamma$–ring with identity and let $x_r$ and $y_r$ be two fuzzy points of $R$. Then,

1. $x_r \alpha y_r = (x \alpha y)_r$,
2. $\langle x_r \rangle \cap \langle y_r \rangle = \langle x \alpha y_r \rangle$.

Theorem 2.14 ([9]). Let $R$ be a commutative $\Gamma$–ring and $\mu$ be a fuzzy $\Gamma$–ideal of $R$. Then, the followings are equivalent:

1. $x_r \Gamma y_r \subseteq \mu \Rightarrow x_r \subseteq \mu$ or $y_r \subseteq \mu$ where $x_r$ and $y_r$ are two fuzzy points of $R$.
2. $\mu$ is a fuzzy prime ideal of $R$.

Definition 2.15 ([2]). A proper ideal $I$ of commutative ring $M$ is called a 2-absorbing ideal of $M$ if whenever $x, y, z \in M$ and $xyz \in I$, then $xy \in I$ or $xz \in I$ or $yz \in I$.

Definition 2.16 ([23]). A fuzzy ideal $\mu$ of $R$ is said to be a fuzzy weakly completely prime ideal if $\mu$ is non-constant function and for all $x, y \in R$,

$$\mu(x, y) = \max\{\mu(x), \mu(y)\}.$$ 

Definition 2.17 ([15]). Let $\mu$ be a non-constant fuzzy ideal of $R$. $\mu$ is said to be a fuzzy $K$–prime ideal if for any $x, y \in R$,

$$\mu(xy) = \mu(0)$$

implies either $\mu(x) = \mu(0) \alpha \mu(y) = \mu(0)$.

Definition 2.18 ([11]). A proper $\Gamma$–ideal $I$ of a $\Gamma$–ring $R$ is called a 2-absorbing $\Gamma$–ideal of $R$ if whenever $x, y, z \in R$, $\alpha, \beta \in \Gamma$ and $xy \beta z \in I$, then $xy \in I$ or $x \beta z \in I$ or $y \beta z$.

Proposition 2.19 ([11]). Every prime $\Gamma$–ideal of $\Gamma$–ring $R$ is a 2-absorbing $\Gamma$–ideal of $R$.

### 3. Fuzzy 2-absorbing $\Gamma$–ideals of a $\Gamma$–ring

In this section, we investigate fuzzy 2-absorbing $\Gamma$-ideals of a $\Gamma$-ring. Throughout this paper, we assume that $R$ is a commutative $\Gamma$-ring.

Definition 3.1. Let $R$ be a commutative $\Gamma$–ring and $\mu$ be fuzzy $\Gamma$–ideal of $\Gamma$–ring $R$. $\mu$ is called fuzzy 2-absorbing $\Gamma$–ideals of $R$ if $\mu$ is non-constant and for any fuzzy points $x_r, y_r, z_r$ of $R$ and $\alpha, \beta \in \Gamma$ with

$$x_r \alpha y_r \beta z_r \in \mu$$

implies that either $x_r \alpha y_r \in \mu$ or $x_r \beta z_r \in \mu$ or $y_r \beta z_r \in \mu$.

Proposition 3.2. Every fuzzy prime $\Gamma$–ideal of $R$ is a fuzzy 2-absorbing $\Gamma$–ideal of $R$.

Proof. The proof is straightforward.
Example 3.3. Let $R = \mathbb{Z}_4$ and $\Gamma = \mathbb{Z}$, define $\overline{\alpha \alpha y} = \overline{\alpha y}$ for all $\overline{x}, \overline{y} \in \mathbb{Z}_4$ and $\alpha \in \mathbb{Z}$. So, $\mathbb{Z}_4$ is a $\Gamma$–ring. A fuzzy subset $\mu$ in $\mathbb{Z}_4$ is defined

$$\mu(x) = \begin{cases} 1, & x \in \overline{0, 2} \\ 0, & \text{otherwise.} \end{cases}$$

Then, $\mu$ is a fuzzy prime $\Gamma$–ideal and fuzzy 2-absorbing $\Gamma$–ideal of $\mathbb{Z}_4$.

The following example shows that the converse of Proposition 3.2 is not necessarily true.

Example 3.4. Let $R = \mathbb{Z}_4$ and $\Gamma = \mathbb{Z}$, define $\overline{\alpha \alpha y} = \overline{\alpha y}$ for all $\overline{x}, \overline{y} \in \mathbb{Z}_4$ and $\alpha \in \mathbb{Z}$. So, $\mathbb{Z}_4$ is a $\Gamma$–ring. A fuzzy subset $\mu$ in $\mathbb{Z}_4$ is defined

$$\mu(x) = \begin{cases} 1/2, & x \in \overline{0, 2} \\ 0, & \text{otherwise.} \end{cases}$$

Then, for fuzzy points $\overline{3}/4, \overline{3}/12$ of $\mathbb{Z}_4$ and $\alpha \in \mathbb{Z}$

$$\overline{3}/4 \overline{3}/12 = \overline{(2\alpha)} \overline{(3)/4(3)} = \overline{(2\alpha)} \overline{(3/2)} = 1/2 \leq \mu(2\alpha) = 1/2.$$

So, $\overline{3}/4 \overline{3}/12 \in \mu$. But since

$$\overline{3}/4(2) = 3/4 > 1/2 = \mu(2), \quad \text{we get } \overline{3}/4 \notin \mu \quad \text{and} \quad \overline{3}/12(3) = 1/2 > 0 = \mu(3), \quad \text{we get } \overline{3}/12 \notin \mu.$$

Therefore, $\mu$ is not a fuzzy prime $\Gamma$–ideal of $\mathbb{Z}_4$. On the other hand, $\mu$ is a fuzzy 2-absorbing $\Gamma$–ideal of $\mathbb{Z}_4$.

Theorem 3.5. Let $\mu$ and $\eta$ be two distinct fuzzy prime $\Gamma$–ideals of $R$. Then, $\mu \cap \eta$ is a fuzzy 2-absorbing $\Gamma$–ideal of $R$.

Proof. Assume that $x, xy, \beta z \in \mu \cap \eta$ for some fuzzy points $x, y, z \in R$, but $x, xy \notin \mu \cap \eta$ and $x, \beta z \notin \mu \cap \eta$. Then, we have the following cases:

Case 1. If $x, xy \notin \mu$ and $x, \beta z \notin \mu$, then since $\mu$ is a fuzzy prime $\Gamma$–ideal of $R$, we get $z \in \mu$ and so $x, \beta z \in \mu$ which is a contradiction.

Case 2. In similar way, we get a contradiction if $x, xy \notin \eta$ and $x, \beta z \notin \eta$. Hence, either $x, xy \notin \mu$ and $x, \beta z \notin \eta$ or $x, xy \notin \eta$ and $x, \beta z \notin \mu$.

Case 3. If the former case holds, then from $x, xy, \beta z \in \mu \cap \eta$, we get $z \in \mu$ and $y \in \eta$. Therefore, $y, \beta z \in \mu \cap \eta$.

Case 4. Similarly, we easily show that $y, \beta z \in \mu \cap \eta$ if the latter case hold.

Finally $\mu \cap \eta$ is a fuzzy 2-absorbing $\Gamma$–ideal of $R$.

Corollary 3.6. The intersection of every pair of distinct fuzzy prime $\Gamma$–ideals of $R$ is a fuzzy 2-absorbing $\Gamma$–ideal of $R$.

Theorem 3.7. Let $\mu$ be a fuzzy 2-absorbing $\Gamma$–ideal of $R$. Then, $\mu_a$ is a 2-absorbing $\Gamma$–ideal of $R$ for every $a \in [0, \mu(0)]$ with $\mu_a \neq R$.

Proof. Suppose that $x, y, z \in R$ and $\alpha, \beta \in \Gamma$ are such that $x y, y z, z \in \mu_a$. Then, $\mu(x y, y z) \geq a$ and we get $x a y z, y z \in \mu$. Since $\mu$ is a fuzzy 2-absorbing $\Gamma$–ideal of $R$, we get $(x y) z = x a y z \in \mu$ or $(x y) z = x a y z \in \mu$ or $(x y) z = x a y z \in \mu$. Therefore, $x y \in \mu_a$ or $y z \in \mu_a$ or $x z \in \mu_a$. Therefore, $\mu_a$ is a 2-absorbing $\Gamma$–ideal of $R$.

The following example shows that the converse of Theorem 3.7 is not generally true.

Example 3.8. Let $R = \mathbb{Z}$ and $\Gamma = 3\mathbb{Z}$, then $R$ is a $\Gamma$–ring. Define the fuzzy $\Gamma$–ideal of $\mathbb{Z}$ by

$$\mu(x) = \begin{cases} 1, & x = 0 \\ 1/4, & x \in 2\mathbb{Z} - \{0\} \\ 0, & x \in \mathbb{Z} - 4\mathbb{Z}. \end{cases}$$
Then,

\[
\begin{align*}
t & \geq 0, \mu(x) \geq 0 \text{ and } x \in \mathbb{Z}, \text{ we get } \mu_t = \mathbb{Z}, \\
t & \geq 1/4, \text{ we get } \mu_t = 4\mathbb{Z}, \\
t &= 1, \text{ we get } \mu_t = 0.
\end{align*}
\]

Hence, \( \mu_t \) is a 2-absorbing \( \Gamma \)-ideal of \( R \) for all \( t \in \text{Im} \mu \). However, for \( \alpha, \beta \in \mathbb{Z} \) we have

\[
2_{1/2} \alpha 2_{1/2} \beta 1_{1/4} = (2\alpha 2\beta 1_{1/2})_{1/2} = (2\alpha 2\beta 1_{1/4}) = (2\alpha 2\beta 1_{1/4}) = 1/4 \\
\leq 1/4.
\]

So, \( 2_{1/2} \alpha 2_{1/2} \beta 1_{1/4} \in \mu \).

\[
2_{1/2} \alpha 2_{1/2} = (2\alpha 2_{1/2})_{1/2} = (2\alpha 2_{1/2}) = 1/2 \\
> 1/4.
\]

Thus, \( 2_{1/2} \alpha 2_{1/2} \beta 1_{1/4} \in \mu \) and

\[
2_{1/2} \beta 1_{1/4} = (2\beta 1_{1/2})_{1/2} = (2\beta 1_{1/4}) = (2\beta 1_{1/4}) = 1/4 \\
> 1/4.
\]

Hence, \( 2_{1/2} \beta 1_{1/4} \notin \mu \). We conclude that \( \mu \) is not a fuzzy 2-absorbing \( \Gamma \)-ideal of \( \mathbb{Z} \).

**Corollary 3.9.** If \( \mu \) is a fuzzy 2-absorbing \( \Gamma \)-ideal of \( R \), then

\[
\mu_\ast = \{ x \in R \mid \mu(x) = \mu(0) \}
\]

is a 2-absorbing \( \Gamma \)-ideal of \( R \).

**Proof.** Since \( \mu \) is a non-constant fuzzy \( \Gamma \)-ideal of \( R \), \( \mu_\ast \neq R \). Now, the result follows from the above theorem. \( \square \)

**Definition 3.10.** Let \( 1 \neq \sigma \in [0, \mu(0)) \). Then, \( \sigma \) is called a 2-absorbing element if \( r \wedge s \wedge t \leq \sigma \) implies that \( r \wedge s \leq \sigma \) or \( s \wedge t \leq \sigma \) for all \( r, s, t \in L \).

**Proposition 3.11.** If \( \mu \) is a fuzzy 2-absorbing \( \Gamma \)-ideal of \( R \), then \( \sigma = \mu(1) \) is a 2-absorbing element.

**Proof.** Assume that \( r \wedge s \wedge t \leq \sigma \) for some \( r, s, t \in L \). Let \( 1, 1, 1, 1 \) are three fuzzy points of \( \Gamma \)-ring \( R \) and \( \alpha, \beta \in \Gamma \) with

\[
1_{(r \wedge s \wedge t)} = 1 \land \alpha \land \beta 1_t \in \mu.
\]

Since \( \mu \) is a fuzzy 2-absorbing \( \Gamma \)-ideal of \( R \), we get \( 1_{(r \wedge s \wedge t)} = 1 \land \alpha 1_t \in \mu \) or \( 1_{(r \wedge s \wedge t)} = 1 \land \beta 1_t \in \mu \). So, \( r \wedge s \wedge t \leq \mu(1) = \sigma \) or \( s \wedge t \leq \mu(1) = \sigma \). Now, the result follows. \( \square \)

**Theorem 3.12.** Let \( I \) be a 2-absorbing \( \Gamma \)-ideal of \( R \) and \( \sigma \) be a 2-absorbing element. Then, the fuzzy subset of \( R \)

\[
\mu(x) = \begin{cases} 
1, & \text{if } x \in I \\
\sigma, & \text{otherwise}
\end{cases}
\]

is a fuzzy 2-absorbing \( \Gamma \)-ideal of \( R \).

**Proof.** Since \( I \) is a 2-absorbing \( \Gamma \)-ideal of \( R \) we get \( I \neq R \) and so \( \mu \) is non-constant. Suppose that \( x_{\alpha \gamma \beta} \in \mu \) but \( x_{\alpha \gamma} \notin \mu \) and \( x_{\beta} \notin \mu \). Then, \( \mu(x_{\alpha \gamma}) = \sigma \) and so \( x_{\alpha \gamma} \notin \mu \). Similarly, \( \beta \notin \mu \) and \( \gamma \notin \mu \). But \( I \) is assumed to be a 2-absorbing \( \Gamma \)-ideal of \( R \). Thus, \( x_{\alpha \gamma \beta} \notin \mu \) and so \( \mu(x_{\alpha \gamma \beta}) = \sigma \) for \( x, \gamma, \beta \in R \) and \( \alpha, \beta \in \Gamma \). Also, from \( x_{\alpha \gamma \beta} \in \mu \) we get \( r \wedge s \leq \mu(1) = \sigma \), which is a contradiction. Thus \( x_{\alpha \gamma \beta} \in \mu \) or \( x_{\beta} \in \mu \) or \( x_{\gamma} \in \mu \).

**Example 3.13.** We know that, every fuzzy prime \( \Gamma \)-ideal of \( R \) is a fuzzy 2-absorbing \( \Gamma \)-ideal of \( R \) as mentioned before. In this example, we show that the converse is not generally true. For example, consider \( R = 2\mathbb{Z} \) and \( \Gamma = 3\mathbb{Z} \).

\[
R \times \Gamma \times R \to R \\
(a, \alpha, b) \mapsto aab
\]

for all \( a, b \in R \) and \( \alpha \in \Gamma \). Then, \( R \) is a \( \Gamma \)-ring. Now, define \( \mu : 2\mathbb{Z} \to [0, 1] \) by

\[
\mu(x) = \begin{cases} 
1, & \text{if } x \in 6\mathbb{Z} \\
0, & \text{otherwise}
\end{cases}
\]
Then, $\mu$ is a fuzzy 2-absorbing $\Gamma$–ideal of $2\mathbb{Z}$. Furthermore, $\mu_0 = I$ is a 2-absorbing $\Gamma$–ideal of $2\mathbb{Z}$ that is not a prime $\Gamma$–ideal. Therefore, $\mu$ is not a fuzzy prime $\Gamma$–ideal of $2\mathbb{Z}$.

**Theorem 3.14.** Let $\{\mu_i \mid i \in I\}$ be a collection of fuzzy 2-absorbing $\Gamma$–ideals of $R$. Then, the fuzzy ideal $\mu = \bigcup_{i \in I} \mu_i$ is a fuzzy 2-absorbing $\Gamma$–ideal of $R$.

**Proof.** Assume that $x, \alpha, \gamma, \beta, z \in \mu$ and $x, \alpha, \gamma, \beta, z \notin \mu$ for some $x, \alpha, y, z, \gamma \in R$. Then, there exist $j \in I$ such that $x, \alpha, \gamma, \beta, z \in \mu_j$ and $x, \alpha, \gamma, \beta, z \notin \mu_j$ for all $j \in I$. Since $\mu_j$ is a fuzzy 2-absorbing $\Gamma$–ideal of $R$, $y, \beta, z \in \mu_j$ or $x, \beta, z \in \mu_j$. Hence, $y, \beta, z \in \mu_j \subseteq \bigcup_{i \in I} \mu_i = \mu$ or $x, \beta, z \in \mu_j \subseteq \bigcup_{i \in I} \mu_i = \mu$. Therefore, $\mu = \bigcup_{i \in I} \mu_i$ is a fuzzy 2-absorbing $\Gamma$–ideal of $R$. \hfill $\square$

**Theorem 3.15.** Let $f : R \rightarrow S$ be a surjective $\Gamma$–ring homomorphism. If $\mu$ is a fuzzy 2-absorbing $\Gamma$–ideal of $R$ which is constant on $\text{Ker} f$, then $f(\mu)$ is a fuzzy 2-absorbing $\Gamma$–ideal of $S$.

**Proof.** Assume that $x, \alpha, \gamma, \beta, z \in f(\mu)$, where $x, \alpha, y, z, \beta, \gamma$ are fuzzy points of $S$ and $\alpha, \beta \in \Gamma$. Since $f$ is a surjective $\Gamma$–ring homomorphism then there exist $a, b, c \in R$ such that $f(a) = x$, $f(b) = y$, $f(c) = z$. Thus,

$$x, \alpha, \gamma, \beta, z (x\alpha y \beta z) = r \wedge s \wedge t$$

$$\leq f(\mu)(x\alpha y \beta z)$$

$$= f(\mu)(f(a)f(b)f(c))$$

$$= f(\mu)(f(ab\beta c))$$

$$= \mu(ab\beta c).$$

Because $\mu$ is constant on $\text{Ker} f$, then we get $a, \alpha, \beta, \gamma, \beta, \gamma \in \mu$. Since $\mu$ is a fuzzy 2-absorbing $\Gamma$–ideal of $R$ then,

$$a, \alpha, \beta, \gamma, \beta, \gamma \in \mu.$$  

Thus,

$$r \wedge s \leq \mu(ab) = f(\mu)(f(ab))$$

$$= f(\mu)(f(a)f(b))$$

$$= f(\mu)(x\alpha y),$$

and so $x, \alpha, y \in f(\mu)$ or

$$r \wedge t \leq \mu(ab) = f(\mu)(f(ab))$$

$$= f(\mu)(f(a)f(b))$$

$$= f(\mu)(f(\alpha)y)$$

$$= f(\mu)(x\alpha y).$$

So, $x, \beta, \gamma \in f(\mu)$ or

$$s \wedge t \leq \mu(b\beta c) = f(\mu)(f(b\beta c))$$

$$= f(\mu)(f(b)f(c))$$

$$= f(\mu)(y\beta z).$$

So, $y, \beta, \gamma \in f(\mu)$. Hence, $f(\mu)$ is a fuzzy 2-absorbing $\Gamma$–ideal of $S$. \hfill $\square$

**Theorem 3.16.** Let $f : R \rightarrow S$ be a $\Gamma$–ring homomorphism. If $\nu$ is a fuzzy 2-absorbing $\Gamma$–ideal of $S$, then $f^{-1}(\nu)$ is a fuzzy 2-absorbing $\Gamma$–ideal of $R$.

**Proof.** Suppose that $x, \alpha, \gamma, \beta, z \in f^{-1}(\nu)$, where $x, \alpha, y, z, \gamma, \beta$ any fuzzy points of $R$ and $\alpha, \beta \in \Gamma$. Then,

$$r \wedge s \wedge t \leq f^{-1}(\nu)((x\alpha y \beta z))$$

$$= v(f(x\alpha y \beta z))$$

$$= v(f(x)\alpha f(y)\beta f(z)).$$
Let \( f(x) = a, f(y) = b, f(z) = c \in S \). Hence, we have that \( r \land s \land t \leq v(ab\beta c) \) and \( a, ab, \beta c, v \in v \). Since \( v \) is a fuzzy 2-absorbing \( \Gamma \)-ideal of \( R \) then \( a, ab, \beta c, v \in v \) or \( b, \beta c, v \in v \). If \( a, ab, \beta c, v \in v \), then
\[
\begin{align*}
r \land s & \leq v(ab) = v(f(x)af(y)) \\
& = v(f(xay)) \\
& = f^{-1}(v(xay)).
\end{align*}
\]
Thus, we conclude that \( x, ay \in f^{-1}(v) \). In similar way, it can be see that \( x, \beta c, v \in f^{-1}(v) \).

**Definition 3.17.** Let \( \mu \) be a fuzzy \( \Gamma \)-ideal of \( R \). \( \mu \) is called a fuzzy strongly 2-absorbing \( \Gamma \)-ideal of \( R \) if it is non-constant and whenever \( \lambda, \eta, \nu \) are fuzzy \( \Gamma \)-ideal of \( R \) with \( \lambda \Gamma \eta \Gamma \nu \subseteq \mu \), then \( \lambda \Gamma \eta \subseteq \mu \) or \( \lambda \Gamma \nu \subseteq \mu \) or \( \eta \Gamma \nu \subseteq \mu \).

**Theorem 3.18.** Every fuzzy prime \( \Gamma \)-ideal of \( R \) is a fuzzy strongly 2-absorbing \( \Gamma \)-ideal of \( R \).

**Proof.** The proof is straightforward.

**Theorem 3.19.** Every fuzzy strongly 2-absorbing \( \Gamma \)-ideal of \( R \) is a fuzzy 2-absorbing \( \Gamma \)-ideal of \( R \).

**Proof.** Assume that \( \mu \) is a fuzzy strongly 2-absorbing \( \Gamma \)-ideal of \( R \). Suppose that \( x, y, z \in \mu \) for some fuzzy points \( x, y, z \subseteq \mu \). We get \( \langle x, \Gamma \rangle \Gamma \langle y, \Gamma \rangle \Gamma \langle z, \Gamma \rangle \subseteq \mu \). Since \( \mu \) is a fuzzy strongly 2-absorbing \( \Gamma \)-ideal of \( R \), we get \( \langle x, \Gamma \rangle \subseteq \mu \) or \( \langle y, \Gamma \rangle \subseteq \mu \) or \( \langle z, \Gamma \rangle \subseteq \mu \). Hence, \( x, \Gamma \subseteq \mu \) or \( y, \Gamma \subseteq \mu \) or \( z, \Gamma \subseteq \mu \).

4. **Fuzzy Weakly Completely 2-absorbing \( \Gamma \)-ideals of a \( \Gamma \)-ring**

**Definition 4.1.** Let \( \mu \) be a fuzzy \( \Gamma \)-ideal of \( R \) and \( \mu \) is called a fuzzy weakly completely 2-absorbing \( \Gamma \)-ideal of \( R \) if
\[
\mu \langle xayb \rangle = \mu \langle xay \rangle \quad \text{or} \quad \mu \langle xayb \rangle = \mu \langle x\beta z \rangle \quad \text{or} \quad \mu \langle xayb \rangle = \mu \langle y\beta z \rangle,
\]
for all \( x, y, z \in R \) and \( x, \beta \in \Gamma \).

**Proposition 4.2.** Let \( \mu \) be a non-constant fuzzy \( \Gamma \)-ideal of \( R \). \( \mu \) is a fuzzy weakly completely 2-absorbing \( \Gamma \)-ideal of \( R \) if and only if
\[
\mu \langle xayb \rangle = \max \{ \mu \langle xay \rangle, \mu \langle x\beta z \rangle, \mu \langle y\beta z \rangle \},
\]
for every \( x, y, z \in R \) and \( x, \beta \in \Gamma \).

**Definition 4.3.** A fuzzy \( \Gamma \)-ideal \( \mu \) of \( R \) is called a fuzzy weakly completely prime \( \Gamma \)-ideal of \( R \) if \( \mu \) is non-constant function and for all \( x, y \in R \) and \( x, \beta \in \Gamma \),
\[
\mu \langle xay \rangle = \max \{ \mu \langle x \rangle, \mu \langle y \rangle \}.
\]

**Theorem 4.4.** Every fuzzy weakly completely prime \( \Gamma \)-ideal of \( R \) is a fuzzy weakly completely 2-absorbing \( \Gamma \)-ideal of \( R \).

**Proof.** Let \( \mu \) be a fuzzy weakly completely prime \( \Gamma \)-ideal of \( R \). Then, for every \( x, y, z \in R \) and \( x, \beta \in \Gamma \),
\[
\mu \langle xayb \rangle = \mu \langle x \rangle \quad \text{or} \quad \mu \langle xayb \rangle = \mu \langle y \rangle \quad \text{or} \quad \mu \langle xayb \rangle = \mu \langle \beta z \rangle.
\]
Suppose that \( \mu \langle xayb \rangle = \mu \langle x \rangle \). Then from \( \mu \langle xayb \rangle \geq \mu \langle xay \rangle \geq \mu \langle x \rangle \) we get \( \mu \langle xayb \rangle = \mu \langle xay \rangle \). In similar way, we can easily show that if \( \mu \langle xayb \rangle = \mu \langle y \rangle \) or \( \mu \langle xayb \rangle = \mu \langle \beta z \rangle \), then \( \mu \langle xayb \rangle = \mu \langle y \beta z \rangle \) or \( \mu \langle xayb \rangle = \mu \langle x \beta z \rangle \). Thus, \( \mu \) is a fuzzy weakly completely 2-absorbing \( \Gamma \)-ideal of \( R \).

**Theorem 4.5.** Let \( \mu \) a fuzzy \( \Gamma \)-ideal of \( R \). The following statements are equivalent:

1. \( \mu \) is a fuzzy weakly completely 2-absorbing \( \Gamma \)-ideal of \( R \).
2. For every \( a \in [0, \mu(0)] \), the \( a \)-level subset \( \mu \) is a 2-absorbing \( \Gamma \)-ideal of \( R \).
Proof. (1) $\Rightarrow$ (2): Suppose that $\mu$ is a fuzzy weakly completely 2-absorbing $\Gamma$–ideal of $R$ and let $x, y, z \in R$, $\alpha, \beta \in \Gamma$ and $x\alpha y\beta z \in \mu_a$ for some $a \in [0, \mu(0)]$. Then,

$$\mu(\alpha x) \cdot \mu(\beta y) \cdot \mu(\gamma z) = \mu(x\alpha y\beta z) \geq a.$$ 

Hence, $\mu(x\alpha y) \geq a$ or $\mu(x\beta y) \geq a$ or $\mu(y\alpha z) \geq a$, which implies that $x\alpha y \in \mu_a$ or $x\beta y \in \mu_a$ or $y\alpha z \in \mu_a$. Hence, $\mu_a$ is a 2-absorbing $\Gamma$–ideal of $R$.

(2) $\Rightarrow$ (1): Admit that $\mu_a$ is a 2-absorbing $\Gamma$–ideal of $R$ for every $a \in [0, 1]$. For $x, y, z \in R$ and $\alpha, \beta \in \Gamma$, let $\mu(x\alpha y\beta z) = a$. Then $x\alpha y\beta z \in \mu_a$ and $\mu_a$ is 2-absorbing $\Gamma$–ideal it gives $x\alpha y \in \mu_a$ or $x\beta y \in \mu_a$ or $y\alpha z \in \mu_a$. Hence, $\mu(x\alpha y) \geq a$ or $\mu(x\beta y) \geq a$ or $\mu(y\alpha z) \geq a$, that is $\mu(x\alpha y) \cdot \mu(x\beta y) \cdot \mu(y\alpha z) \geq a = \mu(x\alpha y\beta z)$. Also, since $\mu$ is a fuzzy $\Gamma$–ideal of $R$, we get

$$\mu(\alpha x) \cdot \mu(\beta y) \cdot \mu(\gamma z) = \mu(x\alpha y\beta z) \geq \mu(\alpha y) \cdot \mu(\beta y) \cdot \mu(\gamma z).$$ 

Thus, $\mu(x\alpha y\beta z) \geq \max \{ \mu(x\alpha y), \mu(x\beta y), \mu(y\alpha z) \}$, that is $\mu$ is a fuzzy weakly completely 2-absorbing $\Gamma$–ideal of $R$. □

**Theorem 4.6.** Let $f : R \rightarrow S$ be a surjective $\Gamma$–ring homomorphism. If $\mu$ is a fuzzy weakly completely 2-absorbing $\Gamma$–ideal of $R$ which is constant on $Ker f$, then $\mu(f)\mu$ is a fuzzy weakly completely 2-absorbing $\Gamma$–ideal of $S$.

Proof. Suppose that $f(\mu)(x\alpha y\beta z) \neq f(\mu)(x\alpha y)$ for any $x, y, z \in S$ and $\alpha, \beta \in \Gamma$. Since $f$ is a surjective $\Gamma$–ring homomorphism then,

$$f(a) = x, \ f(b) = y, \ f(c) = z \text{ for some } a, b, c \in R.$$ 

Hence,

$$f(\mu)(x\alpha y\beta z) = f(\mu)(f(a)\alpha f(b)\beta f(c)) = f(\mu)(f(a\alpha b\beta c)) \neq f(\mu)(x\alpha y) = f(\mu)(f(a)\alpha f(b)) = f(\mu)(f(a\alpha b)).$$ 

Since $\mu$ is constant on $Ker f$,

$$f(\mu)(f(a\alpha b\beta c)) = \mu(a\alpha b\beta c) \text{ and } f(\mu)(f(a\alpha b)) = \mu(a\alpha b).$$ 

It means that,

$$f(\mu)(f(a\alpha b\beta c)) = \mu(a\alpha b\beta c) = f(\mu)(f(a\alpha b)).$$ 

Since $\mu$ is a fuzzy weakly completely 2-absorbing $\Gamma$–ideal of $R$, then

$$\mu(a\alpha b\beta c) = f(\mu)(f(a)\alpha f(b)\beta f(c)) = f(\mu)(x\alpha y\beta z) = f(\mu)(f(a)\alpha f(b)c) = f(\mu)(f(a)\beta f(c)) = f(\mu)(x\alpha y)\beta f(c).$$ 

So, we get $f(\mu)(x\alpha y\beta z) = f(\mu)(x\beta z)$ or

$$\mu(a\alpha b\beta c) = f(\mu)(f(a)\alpha f(b)\beta f(c)) = f(\mu)(x\alpha y\beta z) = f(\mu)(f(a\alpha b\beta c)) = f(\mu)(f(b)\beta f(c)) = f(\mu)(y\alpha z).$$ 

We have $f(\mu)(x\alpha y\beta z) = f(\mu)(y\alpha z)$, Therefore, $f(\mu)$ is a fuzzy weakly completely 2-absorbing $\Gamma$–ideal of $S$. □

**Theorem 4.7.** Let $f : R \rightarrow S$ be a $\Gamma$–ring homomorphism. If $v$ is a fuzzy weakly completely 2-absorbing $\Gamma$–ideal of $S$, then $f^{-1}(v)$ is a fuzzy weakly completely 2-absorbing $\Gamma$–ideal of $R$.

Proof. Suppose that $f^{-1}(v)(x\alpha y\beta z) \neq f^{-1}(v)(x\alpha y)$ for any $x, y, z \in R$ and $\alpha, \beta \in \Gamma$. Then,

$$f^{-1}(v)(x\alpha y\beta z) = v(f(x\alpha y\beta z)) = v(f(x)\alpha f(y)\beta f(z)) \neq f^{-1}(v)(x\alpha y) = v(f(x)\alpha f(y)) = v(f(x)\alpha f(y)) = v(f(x)\beta f(z)) = f^{-1}(v)(x\beta z).$$ 

Since $v$ is a fuzzy weakly completely 2-absorbing $\Gamma$–ideal of $S$ we have that

$$v(f(x)\alpha f(y)\beta f(z)) = f^{-1}(v)(x\alpha y\beta z) = v(f(x)\beta f(z)) = v(f(x)\beta f(z)) = f^{-1}(v)(x\beta z).$$
we get
\[ \mu(x) = \begin{cases} 1, & \text{if } x = 0 \\ 1/3, & \text{if } x \in 2\mathbb{Z} - \{0\} \\ 1/4, & \text{if } x \in \mathbb{Z} - 2\mathbb{Z}. \end{cases} \]

Then, \( \mu \) is a fuzzy \( K - 2 \)-absorbing \( \Gamma \)-ideal of \( R \). However, for \( \alpha, \beta \in 2\mathbb{Z} \) we have
\[ \mu(3\alpha3\beta15) = 1/3 > 1/4 = \max \{\mu(3\alpha3), \mu(3\beta15), \mu(3\beta15)\}. \]

Hence, \( \mu \) is not a fuzzy weakly completely \( 2 \)-absorbing \( \Gamma \)-ideal of \( R \).

**Definition 5.4.** Let \( \mu \) be a fuzzy \( \Gamma \)-ideal of \( R \) and \( \mu \) is called a fuzzy \( K - 2 \)-absorbing \( \Gamma \)-ideal of \( R \) if for all \( x, y, z \in R \) and \( \alpha, \beta \in \Gamma \),
\[ \mu(xy\beta) = \mu(0) \text{ implies that } \mu(xy) = \mu(0) \text{ or } \mu(x\beta) = \mu(0) \text{ or } \mu(y\beta) = \mu(0). \]

**Theorem 5.5.** Every fuzzy \( K - 2 \)-absorbing \( \Gamma \)-ideal of \( R \) is a fuzzy \( K - 2 \)-absorbing \( \Gamma \)-ideal of \( R \).

**Proof.** Let \( \mu \) be a fuzzy \( K - 2 \)-prime \( \Gamma \)-ideal of \( R \). Then, for every \( x, y, z \in R \) and \( \alpha, \beta \in \Gamma \),
\[ \mu(xy\beta) = \mu(0) \text{ implies that } \mu(x) = \mu(0) \text{ or } \mu(y) = \mu(0) \text{ or } \mu(z) = \mu(0). \]
Admit that \( \mu(x) = \mu(0) \). Then, from
\[ \mu(0) = \mu(x) \leq \mu(xy) \leq \mu(xy\beta) = \mu(0), \]
we get \( \mu(xy) = \mu(0) \) or similarly, we can easily show that \( \mu(x\beta) = \mu(0) \) or \( \mu(y\beta) = \mu(0) \). Therefore, \( \mu \) is a fuzzy \( K - 2 \)-absorbing \( \Gamma \)-ideal of \( R \). \( \square \)
Theorem 5.6. Let \( f : R \to S \) be a surjective \( \Gamma \)-ring homomorphism. If \( \mu \) is a fuzzy \( K \)-2-absorbing \( \Gamma \)-ideal of \( R \) which is constant on \( \ker f \), then \( f (\mu) \) is a fuzzy \( K \)-2-absorbing \( \Gamma \)-ideal of \( S \).

Proof. Assume that \( f (\mu)(a b \beta c) = f (\mu)(0_S) \) for any \( a, b, c \in S \) and \( \alpha, \beta \in \Gamma \). Then, \( f (x) = a, f (y) = b, f (z) = c \) for some \( x, y, z \in R \) since \( f \) is a surjective \( \Gamma \)-ring homomorphism. Thus,

\[
f (\mu)(a b \beta c) = f (\mu)(f (x) \alpha f (y) \beta f (z)) = f (\mu)(f (x y \beta z))
\]

and

\[
f (\mu)(0_S) = \bigvee \{ \mu (x) \mid f (x) = 0_S \}.
\]

From here, we get \( x \in \ker f \) and so \( \mu \) is constant on \( \ker f \), \( \mu (x) = \mu (0) \)

\[
f (\mu)(0_S) = \bigvee \{ \mu (x) \mid f (x) = 0_S \},
\]

which implies that

\[
f (\mu)(f (x y \beta z)) = \mu (x y \beta z) = \mu (0) .
\]

Due to fact that \( \mu \) is a fuzzy \( K \)-2-absorbing \( \Gamma \)-ideal of \( R \),

\[
\mu (x y \beta z) = \mu (0) \quad \text{implies that} \quad \mu (x y) = \mu (0) \quad \text{or} \quad \mu (x \beta z) = \mu (0) \quad \text{or} \quad \mu (y \beta z) = \mu (0) .
\]

By the previous theorem, the rest of proof can easily show and we see that \( f (\mu) \) is a fuzzy \( K \)-2-absorbing \( \Gamma \)-ideal of \( S \). \( \square \)

Theorem 5.7. Let \( f : R \to S \) be a \( \Gamma \)-ring homomorphism. If \( \nu \) is a fuzzy \( K \)-2-absorbing \( \Gamma \)-ideal of \( S \), then \( f^{-1}(\nu) \) is a fuzzy \( K \)-2-absorbing \( \Gamma \)-ideal of \( R \).

Proof. Suppose that \( f^{-1}(\nu)(x y \beta z) = f^{-1}(\nu)(0) \) for any \( x, y, z \in R \) and \( \alpha, \beta \in \Gamma \). Then, from

\[
f^{-1}(\nu)(x y \beta z) = v(f(x y \beta z)) = v(f(x) \alpha f(y) \beta f(z)) = f^{-1}(\nu)(0) = v(f(0)) = v(0)
\]

we get \( v(f(x) \alpha f(y) \beta f(z)) = v(0) \) since \( f \) is a surjective \( \Gamma \)-ring homomorphism. Then, we have

\[
v(f(x) \alpha f(y) \beta f(z)) = v(0) \quad \text{implies that} \quad v(f(x) \alpha f(y)) = v(0) \quad \text{or} \quad v(f(x) \beta f(z)) = v(0) \quad \text{or} \quad v(f(y) \beta f(z)) = v(0) ,
\]

since \( \nu \) is a fuzzy \( K \)-2-absorbing \( \Gamma \)-ideal of \( S \). From this, we have

\[
v(f(x) \alpha f(y)) = v(f(x y)) = f^{-1}(\nu)(x y) = v(0) = v(f(0)) = f^{-1}(\nu)(0)
\]

\[
f^{-1}(\nu)(x y) = f^{-1}(\nu)(0) \quad \text{or}
\]

similarly, we can show that \( f^{-1}(\nu)(x \beta z) = f^{-1}(\nu)(0) \) or \( f^{-1}(\nu)(y \beta z) = f^{-1}(\nu)(0) \). Finally, \( f^{-1}(\nu) \) is a fuzzy \( K \)-2-absorbing \( \Gamma \)-ideal of \( R \). \( \square \)

Corollary 5.8. Let \( f \) be a \( \Gamma \)-ring homomorphism from \( R \) onto \( S \). \( f \) induces a one to one inclusion preserving correspondence between fuzzy \( K \)-2-absorbing \( \Gamma \)-ideal of \( R \) in such a way that if \( \mu \) is a fuzzy \( K \)-2-absorbing \( \Gamma \)-ideal of \( R \) constant on \( \ker f \), then \( f (\mu) \) is the corresponding fuzzy \( K \)-2-absorbing \( \Gamma \)-ideal of \( S \), and if \( \nu \) is a fuzzy \( K \)-2-absorbing \( \Gamma \)-ideal of \( S \), then \( f^{-1}(\nu) \) is the corresponding fuzzy \( K \)-2-absorbing \( \Gamma \)-ideal of \( R \).

Remark 5.9. The following table summarizes findings of fuzzy 2-absorbing \( \Gamma \)-ideals of a \( \Gamma \)-ring.

\[
\begin{array}{ccc}
f \text{ strongly } 2-\text{abs. } \Gamma \text{-ideal} & \downarrow & \text{f. prime } \Gamma \text{-ideal } \rightarrow f. \text{ 2- } \text{abs. } \Gamma \text{-ideal} \\
\downarrow & \downarrow & \downarrow \\
f \text{ w. c. p. } \Gamma \text{-ideal } \rightarrow f. \text{ w. c. 2- } \text{abs. } \Gamma \text{-ideal} \\
\downarrow & \downarrow & \downarrow \\
f. \text{ K - 2- } \text{abs. } \Gamma \text{-ideal}
\end{array}
\]
6. Fuzzy Quotient $\Gamma$–ring of $R$ Induced by Fuzzy 2-absorbing $\Gamma$–ideal

Now, we remind the notion of fuzzy quotient $\Gamma$–ring induced by fuzzy $\Gamma$–ideal of $R$. Let $\mu$ be a fuzzy $\Gamma$–ideal of a $\Gamma$–ring $R$. For any $x, y \in R$, define a binary relation $\sim$ on $R$ which is a congruence relation of $R$ by $x \sim y$ if and only if

$$\mu(x - y) = \mu(0),$$

where 0 is the zero element of $R$. Let $\mu[x] = \{y \in R \mid y \sim x\}$ be the equivalence class containing $x$ and $R/\mu = \{\mu[x] \mid x \in R\}$ the set of all equivalence classes of $R$. Define two operations by

$$\mu[x] + \mu[y] = \mu[x + y] \quad \text{and} \quad \mu[x] \alpha \mu[y] = \mu[\alpha xy].$$

for $x, y \in R, \alpha \in \Gamma$. Then, $R/\mu$ is a fuzzy $\Gamma$–ring with two operations and call it fuzzy quotient $\Gamma$–ring of $R$ induced by the fuzzy $\Gamma$–ideal $\mu$ [20].

**Theorem 6.1.** Let $\mu$ be a non-constant fuzzy $\Gamma$–ideal of $R$. Then, $\mu$ is a fuzzy $K$–2-absorbing $\Gamma$–ideal of $R$ if and only if $R/\mu$ is a 2-absorbing $\Gamma$–ring.

**Proof.** Suppose that $\mu$ is a fuzzy $K$–2-absorbing $\Gamma$–ideal of $R$ and let $\mu[x], \mu[y], \mu[z] \in R/\mu$ be such that

$$\mu[x] \alpha \mu[y] \beta \mu[z] = \mu[0].$$

Since $\mu[x] \alpha \mu[y] \beta \mu[z] = \mu[xy\beta z]$, we get

$$\mu(xy\beta z) = \mu(xy\beta z - 0) = 1 = \mu(0).$$

As $\mu$ is considered to be fuzzy $K$–2-absorbing $\Gamma$–ideal of $R$,

$$\mu(xy) = \mu(0) = 1 \quad \text{or} \quad \mu(x\beta z) = \mu(0) = 1 \quad \text{or} \quad \mu(y\beta z) = \mu(0) = 1.$$

It means that,

$$\mu[xy] = \mu[x] \alpha \mu[y] = \mu[0] \quad \text{or} \quad \mu[x\beta z] = \mu[x\beta z] = \mu[0] \quad \text{or} \quad \mu[y\beta z] = \mu[y] \beta \mu[z] = \mu[0].$$

So, $R/\mu$ is a 2-absorbing $\Gamma$–ring. Conversely, suppose that $R/\mu$ is a 2-absorbing $\Gamma$–ring and let $\mu(xy\beta z) = \mu(0) = 1$ for $x, y, z \in R$ and $\alpha, \beta \in \Gamma$. Then, we get

$$\mu[x] \alpha \mu[y] \beta \mu[z] = \mu[xxy\beta z] = \mu[0].$$

Since $R/\mu$ is a 2-absorbing $\Gamma$–ring,

$$\mu[xy] = \mu[0] \quad \text{or} \quad \mu[x\beta z] = \mu[0] \quad \text{or} \quad \mu[y\beta z] = \mu[0],$$

which implies that $\mu$ is a fuzzy $K$–2-absorbing $\Gamma$–ideal of $R$. \hfill \Box

**Corollary 6.2.** If $\mu$ is a fuzzy weakly completely 2-absorbing $\Gamma$–ideal of $R$, then $R/\mu$ is a 2-absorbing $\Gamma$–ring.

7. Conclusion

In this paper, we have characterized fuzzy 2-absorbing $\Gamma$–ideals of a $\Gamma$–ring. Also, the notions of fuzzy weakly completely 2-absorbing $\Gamma$–ideals of a $\Gamma$–ring and fuzzy $K$–2-absorbing $\Gamma$–ideals of a $\Gamma$–ring and their properties are proposed. Moreover, we have given a diagram which transition between definitions of fuzzy $\Gamma$–ideals of $\Gamma$–ring. Finally, we have shown that if $\mu$ is a fuzzy weakly completely 2-absorbing $\Gamma$–ideal, then fuzzy quotient $\Gamma$–ring of $R$ induced by the fuzzy $\Gamma$–ideal is a 2-absorbing $\Gamma$–ring. To extend this study, one could study other algebraic structures and do some further study on the properties them. In our future work, we have planed to define an intuitionistic fuzzy 2-absorbing $\Gamma$–ideal of a $\Gamma$–ring and an intuitionistic fuzzy weakly completely 2-absorbing $\Gamma$–ideal of a $\Gamma$–ring and to discuss its related properties.

**Conflicts of Interest**

The author declares that there are no conflicts of interest regarding the publication of this article.
The author has read and agreed to the published version of the manuscript.

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