

Optimization of multi effect evaporation systems using a metaheuristic hybrid algorithm

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ABSTRACT

A computer program, capable of carrying out the mathematical modelling and optimization of multi effect evaporation (MEE) systems is developed in the scope of this study. C# programming language is used in the development of the computer program. A Particle Swarm Optimization (PSO) based algorithm is developed and hybridized with a Levenberg-Marquardt (LM) based algorithm. A computer program interface is developed in .NET platform for the user to give inputs such as feed and product streams flowrate and concentration. The optimization results is represented through this interface. Concentrating the sodium hydroxide content in the wastewater of the mercerization process is selected as the sample case.

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Nomenclature

Symbols

L_i	Outlet liquid flowrate of effect 'i', kg/h
V_i	Outlet vapor flowrate of effect 'i', kg/h
x_i	Solute concentration in the outlet liquid stream of effect 'i', kg solute/kg solution
h_i	Outlet liquid stream enthalpy of effect 'i', kJ/kg
H_i	Outlet vapor stream enthalpy of effect 'i', kJ/kg
$H_{vap,i}$	Outlet vapor stream vaporization enthalpy of effect 'i', kJ/kg
T_i	Operation temperature of effect 'i', K
P_i	Operation pressure of effect 'i', kPa
A_i	Heat transfer area of effect 'i', m ²
U_i	Overall heat transfer coefficient of effect 'i', W/m ² K
F	Feed stream flowrate, kg/h
V_0	Live steam flowrate, kg/h

Dimensionless parameters

$V_{j,d}$	PSO velocity of variable 'd' of particle 'j'
$X_{j,d}$	Value of variable 'd' of particle 'j'
$c_{j,d}$	Best known cluster position of variable 'd' of particle 'j'
$p_{j,d}$	Best known position of variable 'd' of particle 'j'
g_d	Global best known position of variable 'd'
ω	PSO constant of velocity
Φ_c	PSO constant of cluster gap
Φ_p	PSO constant of particle gap
Φ_g	PSO constant of global gap
r_c	PSO randomized value of cluster gap
r_p	PSO randomized value of particle gap
r_g	PSO randomized value of global gap

1 Introduction

Evaporation is the process of concentrating an aqueous solution by vaporizing the water content. In a MEE system, the solution is evaporated using saturated steam in the first effect and the vapour which is the outlet of each effect (V_i) is used as heating medium in the following effects. This design provides a huge steam economy.

There are several ways of feed sequences for MEE systems. The most widely used ones are forward feed and backward feed sequences. The live steam (V_0) is fed from the first effect in both of these options. Weak concentration -the feed stream- is fed from the first effect to flow parallel with the heating medium in the forward feed option while it is fed from the last effect to flow counter to the heating medium in the backward feed option.

The evaporation capacity of each effect depends on the evaporation capacity of the previous effect in forward feed sequence while it is dependent on the evaporation capacities of both the previous and next effects in backward feed sequence. An equation oriented approach is a better choice for simulation and design of MEE systems due to these

dependencies. Backward feed sequence and preheating has a positive effect on evaporation economy through an equation-oriented simulation model [1]. A MEE system with higher number of effects will have a better steam economy [2]. Falling film evaporator is a better choice due to the lower pressure loss [2]. But the falling film evaporators must be forced circulated, because it would be difficult to uniformly distribute the solution to all tubes.

By investigating both the operating cost and capital cost, optimum number of effects is found as three for an evaporation amount of 6750 kg/h [3]. A genetic based algorithm should be followed instead of Newton's method especially for higher number of effects [4]. Because very complex Jacobian matrixes can be encountered while dealing with higher number of effects. In the mathematical model, correlations can be used to calculate temperature dependent properties such as enthalpy and heat capacity for simplification [5]. These simplifications do not effect the selection of optimal feed sequence. The main condition of the optimization of an evaporation system is the product quality [6]. The product quality, in other words the concentration of the product must be determined before starting the modelling and all of the parameters are calculated as values that can catch the desired product concentration.

Developing models and simultaneous solution algorithm which comprises the complete complex structure of the multi effect evaporation systems is challenging and further research in this field will aim the energy saving weighted optimization [7].

2. Mathematical modelling

An evaporation system that is optimally designed, aims to concentrate the aqueous solution with an energy efficient way. Steam and electricity consumptions will be minimized by optimizing the operating parameters along with integer values such as the feed sequence and the number of effects. So, the mathematical model that is needed to optimize the system is a mixed integer nonlinear problem.

Figure 1 represents a single effect evaporation which is part of a MEE system. Equations 1 and 2 are derived from the total mass balance and solute component mass balance around each effect respectively. Eq. 3 is used as the energy balance equation of each effect.

$$V_i = L_{i\pm 1} - L_i \tag{1}$$

$$x_i = F * x_f / L_i \tag{2}$$

$$L_{i\pm 1} h_{i\pm 1} + V_{i-1} H_{vap,i-1} - V_i H_i - L_i h_i = 0 \tag{3}$$

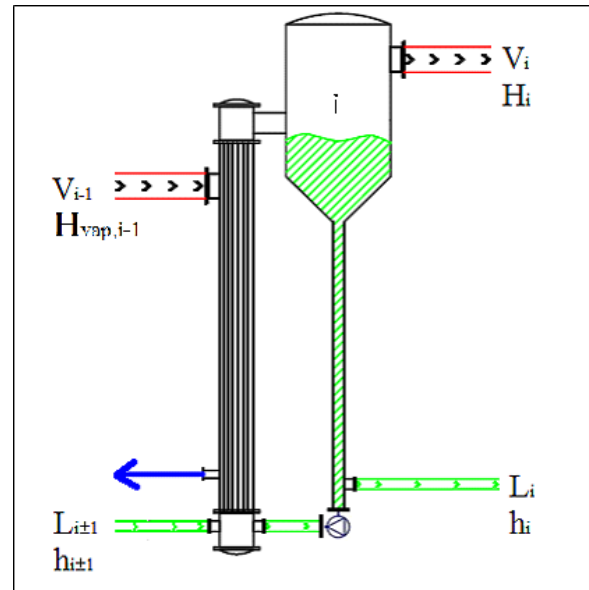


Figure 1. Single effect evaporation

Outlet vapor stream enthalpy (H_i) and outlet liquid stream enthalpy (h_i) can be calculated explicitly using operating temperature of each effect (T_i) which is dependent on two parameters. The first one is the operating pressure (P_i). The second one is the solute concentration (x_i) in the outlet liquid stream. The boiling point elevation (BPE) term represents the effect of the solute concentration on the boiling point of the solution and is calculated through a correlation of concentration particular to the solute type.

$$T_i = T_{sat,i} + BPE_i \tag{4}$$

Equations 5 and 6 are correlations derived from steam table data and are used to calculate saturation temperature and vaporization enthalpy [8]. The validity interval and the R-squared values of these equations are 10-1300 kPa and 0.995.

$$T_{sat,i} = 258.5 * P_i^{0.0803} \tag{5}$$

$$H_{vap,i} = 2 * 10^{-9} * P_i^4 - 4 * 10^{-6} * P_i^3 + 0.0033 * P_i^2 - 1.4124 * P_i + 2376.7 \tag{6}$$

Feed flowrate, feed concentration and product concentration are design variables because they are the design specifications. The remaining variables V_0 , L_i and P_i are live steam consumption (LSC), outlet liquid flowrate and operating pressure of effect 'i' respectively. P_i is determined by the optimization algorithm while L_i remains as an unknown variable except for the last effect which makes one unknown variable for each effect. Since there are one energy balance equation for each effect –Eq. 3– the degrees of freedom is zero.

After the equation set is solved and all of the flowrates are determined, the evaporators are designed. Design of

evaporator is similar to a vertical shell and tube heat exchanger design. Heating medium is passed through the shell side while the concentration is passed through the tube side. The tube side fluid consists of the inlet liquid stream and the circulated portion of the outlet liquid stream ($L_{c,i}$). So, the flowrate of the tube side is the sum of the flowrates of these streams. $L_{c,i}$ is found by Eq. 7 where circulation rate (cr_i) of each effect is an optimization parameter. Higher cr values mean lower purchase cost of evaporators but higher pumping cost. Due to the high rate of circulation, the temperature of the tube side is assumed as constant at the outlet liquid temperature.

$$L_{c,i} = cr_i \times L_i \quad (7)$$

Along with optimization parameters tube length and tube diameter, tube side flowrate is used to calculate tube side heat transfer coefficient, which is used in iterative overall heat transfer coefficient calculation. The film coefficient of the condensing steam will be too high to limit the overall heat transfer coefficient. So, shell side heat transfer coefficient can be assumed to be 8000 W/m² - K at all cases. Thermal conductivity of stainless steel is taken as 1.6 W/m-K.

There are two main reasons for the operating pressure difference among effects. One of them is the pressure loss due to the friction. The larger heat loss is the conversion of kinetic energy during the flash evaporation at the entering of the separator tanks.

The pump power (P) is calculated by Eq. 8 as kg.m²/s³ in other words as watts. The pump efficiency (η) is taken as 0.6. The mass flowrate (\dot{V}) of the pump must be in kg/s and the pressure difference (H_p) in mWH where 1 mWH is equal to 9.807 kPa.

$$P = \dot{V} g H_p / \eta \quad (8)$$

While calculating the power of the circulation pump (CP) of effect 'i', $L_{c,i}$ is used as \dot{V} and only the pressure loss due to the friction is used as H_p . While calculating the power of the feed pump (FP) or backward feed pump (BFP) of effect 'i', L_i is used as \dot{V} . As H_p the pressure difference between the outlet and inlet streams of the pump is converted to mWH and used in this wise.

Sum of the total purchase cost (TPC) and annual operating cost (AOC) is the value to be minimized as the objective of this study. TPC consists of mainly the cost of evaporators (EvC) and the cost of pumps (PuC). Costs of condenser, vacuum pump, piping and instrumentation are neglected because these costs are assumed to be close for each option for the same design specifications and have no effect on finding the optimum variables. The cost of the separator tanks are neglected because their prices would be negligibly small compared to the cost of evaporators.

$$TPC = EvC_T + PuC_T \quad (9)$$

The EvC is calculated using Hall method which is used for estimating the capital cost of stainless steel shell and tube heat exchangers [9].

$$EvC = 10000 + 324 \times A^{0.91} \quad (10)$$

Pump costs (PuC) is calculated through Eq. 12 where the purchase cost (PuC^0) of a carbon steel pump is calculated by Eq. 11 and the power of the pump (P) is in kW (Turton et al., 2012). The material factor, F_M is taken as 3.25 and the bare module factor, F_P is taken as 3 for centrifugal stainless steel pumps [10].

$$\log_{10} PuC^0 = 3.3892 + 0.0536 \log_{10}(P) + 0.1538 [\log_{10}(P)]^2 \quad (11)$$

$$PuC = PuC^0 (1.89 + 1.35 F_M F_P) \quad (12)$$

AOC consists of the price of steam and electricity used. Steam cost (StC) is calculated as \$/h through live steam consumption and unit price of electricity (UP_{st}). The plant is assumed to work 24 hours a day and 300 days in a year. The required conversions from [\$/h] to [\$/year] are needs to be performed while solving Eq. 13.

$$AOC = StC + EIC \quad (13)$$

$$StC = V_0 \times UP_{st} \quad (14)$$

$$EIC = \left(P_{FP} + \sum_{i=1}^n P_{CP,i} \right) \times UP_{el} \quad (15)$$

$$EIC = \left(P_{FP} + \sum_{i=1}^n P_{BFP,i} + \sum_{i=1}^n P_{CP,i} \right) \times UP_{el} \quad (16)$$

Equations 15 and 16 represent the method of calculating the electricity cost (EIC) as \$/h in the cases of forward feed and backward feed respectively. Unit price of electricity (UP_{el}) is the cost of 1 kWh electricity. In the case of backward feed, the outlet liquid stream of each effect is fed to the previous effect as the inlet liquid stream, which has a higher operating pressure. So there is need of a backward feed pump at each effect except the first effect. There is no such need in the case of forward feed.

3. Optimization algorithm

Particle swarm optimization (PSO) is a computational optimization method that aims to find the optimum point of a function. PSO algorithm is based on simulating the social behavior of a bird swarm that tries to find the area which contains better food [11]. The bird which finds the best food sings and calls the other members of the swarm to his surroundings. But each bird keeps the memory of the location where he found the best food resource until he finds a better food. Each bird in the swarm is named as ‘particle’.

Coordinates and velocity vectors of each particle is randomly initialized. The coordinate’s vectors of the particles are moved by the velocity vectors at each iteration. Each particle has a memory that keeps the best-known point of that particle. The global best-known point of all particles is also kept in the parent memory. The velocity is recalculated in each iteration. There are three values that effect the velocity vector. The first of them is the difference between the global best-known position and the particle’s current position. The second one is the difference between the particle’s best-known position and the particle’s current position. The third one is the particle’s previous velocity.

This algorithm allows the particles gather around the position that has a higher possibility to have optimum results. Also, local optimum points can be found by this algorithm. But that doesn’t prevent the algorithm to find the global optimum. When a particle is trapped in a position and no longer moves, that point is marked as local optimum, the particle is taken out of the swarm and the algorithm continues with the remaining swarm. So, the particle which is trapped in the local optimum can no longer attract the swarm to the local optimum surroundings.

The main mathematical operations of the PSO are the velocity vector calculation and the movement of each particle by the velocity vector.

3.1. Hybridization of PSO with LM

A hybrid algorithm using PSO and a modified Levenberg-Marquardt (LM) method has been developed for this study. The modified LM method is a combination of Newton-Raphson and Gradient Descent methods reinforced by a homothopy parameter which keeps the results in physically meaningful area [12]. The variables that is determined by the PSO algorithm are sent to LM algorithm to solve the nonlinear equation set that is constructed using the energy balance equations. The LSC and flowrates between the effects are calculated as a result of the modified LM algorithm. By these values the evaporators are designed and the objective function result (Y) is calculated. Objective function is the combination of AOC and TPC by a coefficient called capital recovery factor (CRF).

$$Y = AOC + CRF * TPC \quad (17)$$

The main algorithm is represented in Figure 2. After the design specifications are taken from the user inputs and the initial swarm is created by randomization, the PSO algorithm is triggered. At each iteration of the PSO, alias at the beginning of each LM operation, all of the optimization parameters are checked if they satisfy the constraints. If they don’t satisfy the constraints, the algorithm returns the value of -1 which means this particle must be destroyed. The procedure that is going to be followed when there is a particle that must be destroyed, is described in section 3.2.

3.2. Modification of PSO

To apply PSO method to constrained mixed integer programming, some modifications are needed. Most variables in our mathematical model are integer and have minimum and maximum boundaries. In fact, some of the boundaries are dependent on other variables. For example, the operating pressure of each effect must be high enough for the outlet vapor stream to heat the next effect. The necessary value is dependent on the operating pressure and outlet liquid stream concentration of the next effect. Another conflict is the dependency of most of the variables on the number of effects (NoE). When NoE of a particle increase during the optimization, the number of variables of that particle increases too. That makes the recently added variables unable to make their way, because neither previous velocity value for that particle nor the best-known position value exist for those variables.

These problems are solved by some modifications on the original PSO algorithm. The main modification is the cluster term. The particles which have the same NoE and feed sequence are the members of a cluster. The NoE of the particles are initialized to numbers between 2 and 6. As there are two feed sequences, there are 12 numbers of clusters at the beginning of the algorithm. If the NoE of any particle moves to a larger value than 6, a new cluster is created.

$$v_{j,d} = \omega v_{j,d} + \Phi_c r_c (c_{j,d} - x_{j,d}) + \Phi_p r_p (p_{j,d} - x_{j,d}) + \Phi_g r_g (g_d - x_{j,d}) \quad (18)$$

The best-known position of the clusters is also kept like the global and particle best known positions and is included to the velocity calculation formula as seen in Eq. 18. If any particle is the best-known particle of its cluster, NoE and the feed sequence of that particle is not moved, in other words that particle is not allowed to move out of its cluster.

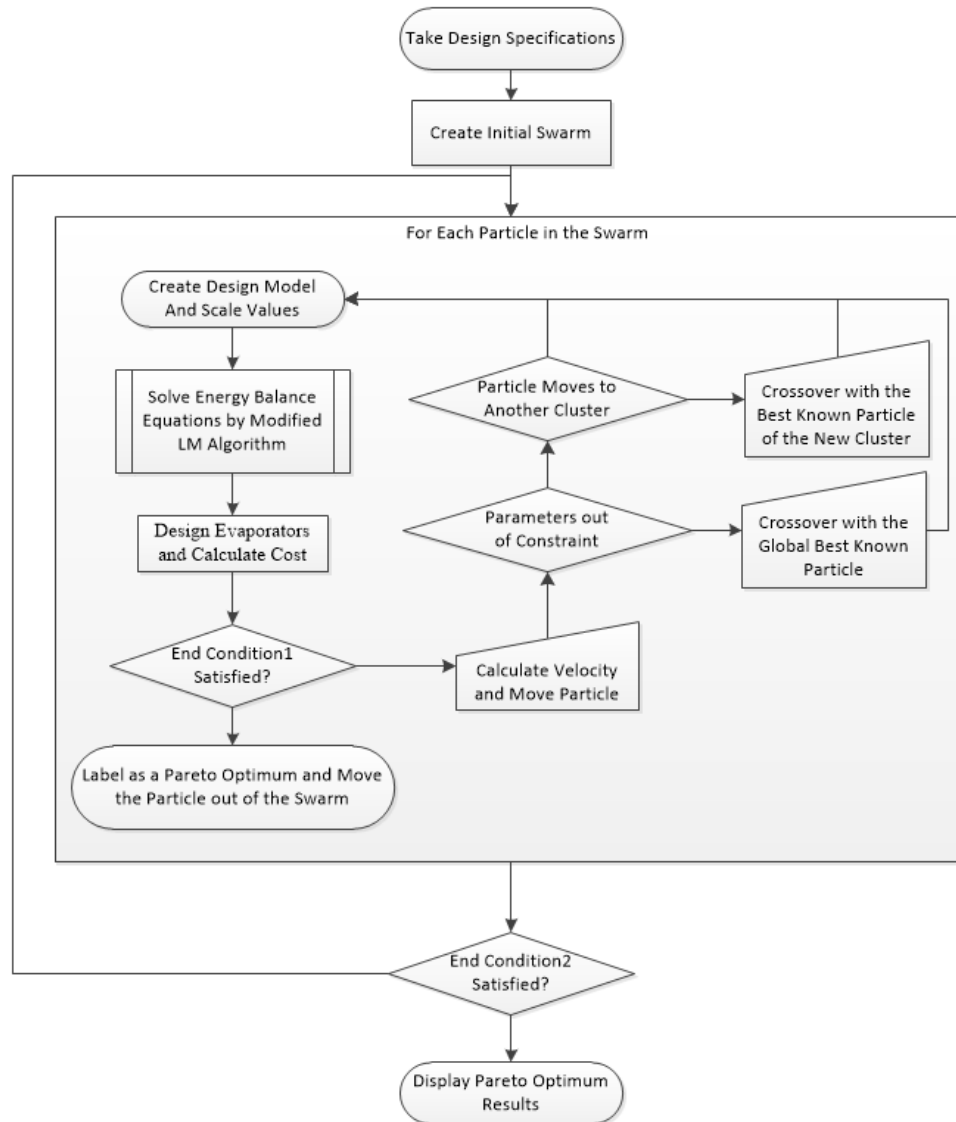


Figure 2. Main Algorithm

In some cases, the particle moves to an uncommon position. In those cases, uncommon operations are needed to be performed.

3.2.1. Out of Constraint Parameters

Whenever a particle moved out of constraints, that particle is destroyed. This operation is similar to the natural selection event. Out of constraint particles are destroyed because in the debug mode it is observed that these particles are tend to stay out of constraints. If natural selection occurs, new particles are needed to replace the destroyed ones. To create the new particle, the destroyed particle is crossed over by the global best-known particle before destruction. This crossover operation is a similar operation in the genetic algorithm [9]. But only one individual is created instead of two by using two random values as the crossover constants (ct). The ct_1 value is

randomized between -1 and 1 as the first step. After this, ct_2 is randomized between -1 and $\min(1, |1.5 - ct_1|)$ if ct_1 is positive. If ct_1 is negative, then ct_2 is randomized between $\max(-1, |ct_1 - 1.5|)$ and 1. By this approach, ct_1 and ct_2 are kept between -1 and 1, and their sum is kept between -1.5 and 1.5.

$$x_{offspring} = \frac{1 + ct_1}{2} x_1 + \frac{1 + ct_2}{2} x_2 \quad (19)$$

3.2.2. Movement Out of the Cluster

If a particle moves out of the cluster which means NoE or feed sequence parameter of that particle is changed during the execution of the algorithm, that particle is destroyed similar to another natural selection event just as in the 'Out of Constraint

Parameters' case. Moving out of the cluster means getting into another cluster. Instead of moving just the same particle to its new cluster, a crossover operation is performed just as in the 'Out of Constraint Parameters' case. But this time the particle is crossed over with its new cluster's best-known position instead of the global best-known position.

Before executing the algorithm on evaporation problem, PSO algorithm is run with a 2 variable optimization problem to observe the movement of the particles. The related problem is described in Eq. 20 and the movement of the particles is represented in Figure 3.

3.3. Optimization Check

$$y = e^{-(x_1^2+x_2^2)} + 2e^{-(x_1-1.7)^2+(x_2-1.7)^2} \tag{20}$$

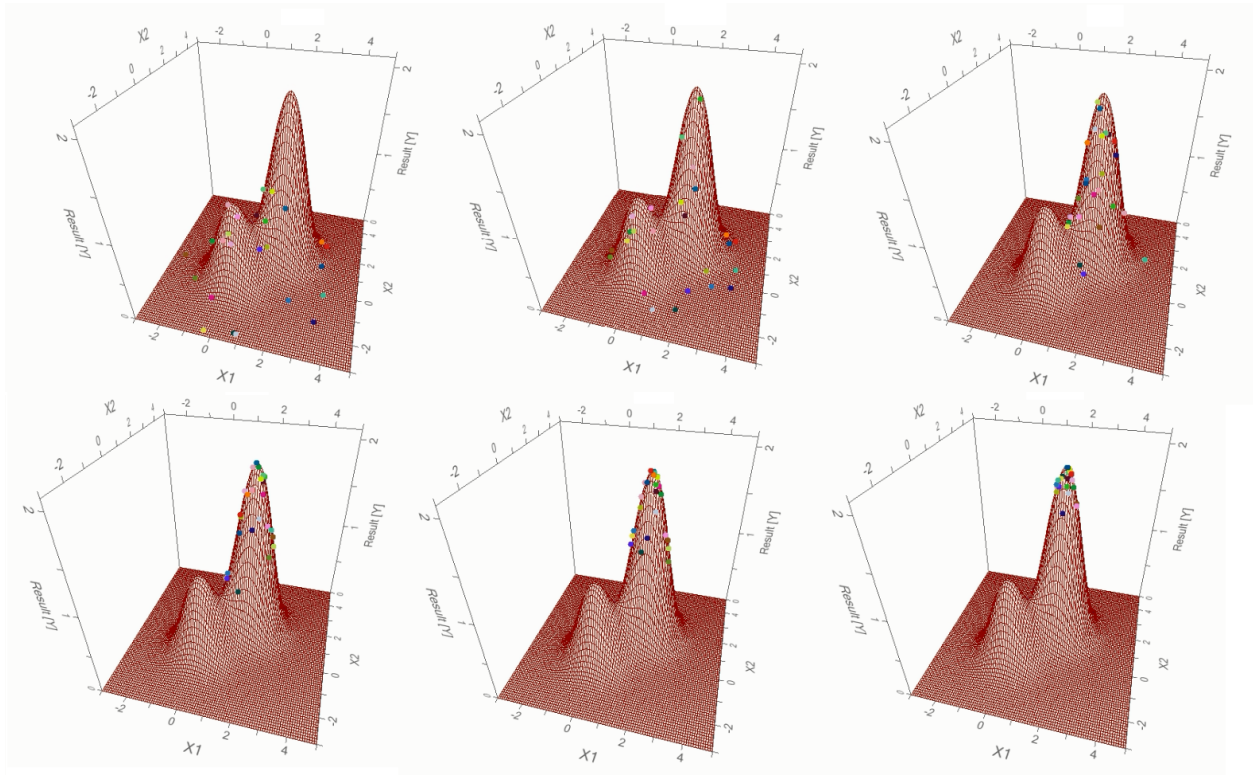


Figure 3. Movement of PSO Particles

As seen in Fig. 3, almost all of the particles are gathered around the optimum point at six iterations and the optimum point is found as $x_1=x_2=1.7$ and the optimum result is 2.

3.4. Development of the Computer Program and User Interfaces

A computer program including the mathematical models, nonlinear equation set solver, the hybrid optimization algorithm and other auxiliary methods was developed using C# programming language in .NET Framework 4.5 Environment using Microsoft Visual Studio. The pipe standards used in the calculation of overall heat transfer coefficient are kept at a related database.

The inputs form tabs are represented in Figures 4-6. As seen in Figure 5, user can select to enter some optimization parameters instead of the algorithm to optimize those parameters. The results form is represented in Figure 7.

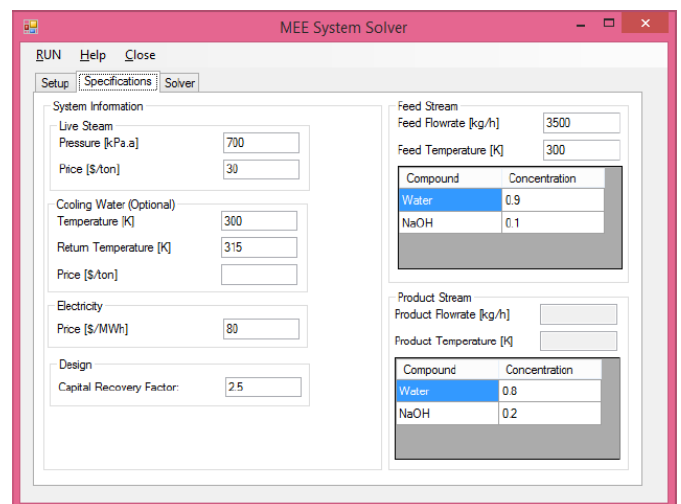


Figure 4. Specifications tab

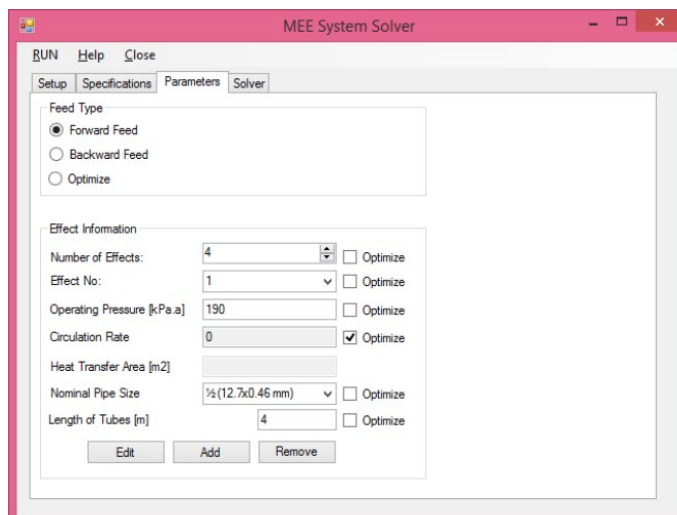


Figure 5. Parameters tab

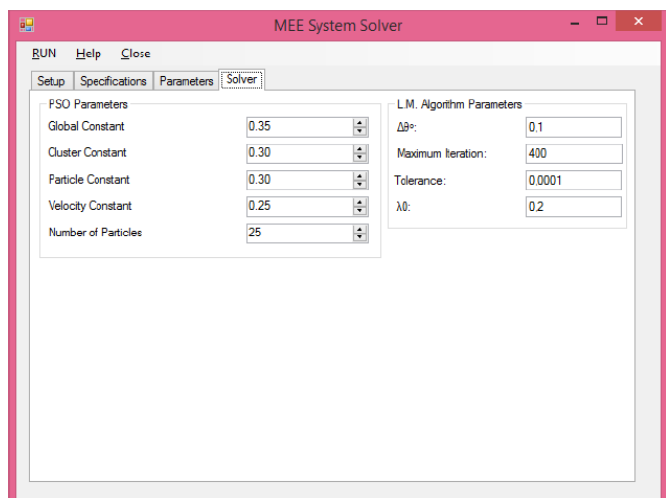


Figure 6. Solver tab



Figure 7. Results form

4. Case study and results

Concentrating a 8500 kg/h mercerization waste water having 10% NaOH to 30% is selected as the case study. Live steam pressure is selected as 700 kPa. Unit prices of steam and

electricity are taken as 20 \$/kg and 50 \$/MWh respectively. The initial swarm of the optimization algorithm contains 5*NoE particles of each cluster. The Φ_g , Φ_c , Φ_p , and ω are taken as 0.35, 0.3, 0.3 and 0.25 respectively. Optimization algorithm is run with a CRF of 1 and best three pareto optimum results (PO) are taken into consideration. Problem is solved in 14 iterations. The total vaporization amount is calculated as 5667 kg/h and the live steam temperature is calculates as 437 K. the results are represented in tables.

Table 1. Pareto optimum points

	PO-1	PO-2	PO-3
Y [\$]	410504	468547	480251
AOC [\$ /year]	350934	430289	430733
TPC [\$]	59510	38269	49518
Feed Sequence	Backward	Backward	Forward
NoE	4	3	4
V₀	2428.7	2982.1	2981.5

Table 2. Detailed results of PO-1

	1st Effect	2nd Effect	3rd Effect	4th Effect
P [kPa]	336	230	130	10
T [K]	427.4	405.3	385.6	314.2
L_c [kg/h]	29707	43963	42243	67397
L [kg/h]	2833	5028	6945	7843
c	0.3	0.17	0.12	0.11
V [kg/h]	2194	1918	898	657

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