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Some Pair Difference Cordial Graphs

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Abstract — Let $G = (V,E)$ be a (p,q) graph. Define

$$\rho = \begin{cases} \frac{p}{2}, & \text{if } p \text{ is even} \\ \frac{p-1}{2}, & \text{if } p \text{ is odd} \end{cases}$$

and $L = \{\pm 1, \pm 2, \pm 3, \cdots, \pm\rho\}$ called the set of labels. Consider a mapping $f : V \longrightarrow L$ by assigning different labels in L to the different elements of V when p is even and different labels in L to p-1 elements of V and repeating a label for the remaining one vertex when p is odd.The labeling as defined above is said to be a pair difference cordial labeling if for each edge uv of G there exists a labeling $\left|f(u) - f(v)\right|$ such that $\left|\Delta_{f_1} - \Delta_{f_1^c}\right| \le 1$, where Δ_{f_1} and $\Delta_{f_1^c}$ respectively denote the number of edges labeled with 1 and number of edges not labeled with 1. A graph G for which there exists a pair difference cordial labeling is called a pair difference cordial graph. In this paper we investigate the pair difference cordial labeling behavior of $P_n \odot K_1$,$P_n \odot K_2$,$C_n \odot K_1$,$P_n \odot 2K_1$,$L_n \odot K_1$,$G_n \odot K_1$, where G_n is a gear graph and etc.

Subject Classification (2020): 05C78.

1. Introduction

In this paper we consider only finite, undirected and simple graphs. Cordial labeling was introduced in [1] and more cordial related labeling was studied in [2, 3]. Corona product operations used in several areas of graph theory [4–6]. In [7] the notion of pair difference cordial labeling of a graph was introduced and also in the same article pair difference cordial labeling behaviour of path, cycle, star, ladder have been studied. The pair difference cordial labeling behavior of snake related graph and butterfly graph have been investigated in [8]. In this paper we have study about the pair difference cordiality of some graphs using corona product operations like $P_n \odot K_1$,$P_n \odot K_2$,$C_n \odot K_1$,$P_n \odot 2K_1$,$L_n \odot K_1$,$G_n \odot K_1$, where G_n is a gear graph.

2. Preliminaries

Definition 2.1. [9] Let $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ be two graphs. The join $G_1 + G_2$ as $G_1 \cup G_2$ together with all the edges joining vertices of V_1 to the vertices of V_2.

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Definition 2.2. [9] The corona graph $G_1 \odot G_2$ is the graph obtained by taking one copy of G_1 and n copies of G_2 and joining the i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy G_2, where G_1 is graph of order n.

Definition 2.3. [10] The graph $W_n = C_n + K_1$ is called the wheel graph.

Definition 2.4. [10] The ladder L_n is the product graph $P_n X K_2$ with $2n$ vertices and $3n-2$ edges.Let $V(L_n) = \{a_i, b_i : 1 \le i \le n\}$ and $E(L_n) = \{a_i b_i : 1 \le i \le n\} \cup \{a_i a_{i+1}, b_i b_{i+1} : 1 \le i \le n\}$.

Definition 2.5. [5] The gear graph G_n is obtained from the wheel W_n by adding a vertex between every pair of adjacent vertices of the cycle C_n.Let $V(G_n) = \{x, x_i, y_i : 1 \le i \le n\}$ and $E(G_n) = \{xx_i : 1 \le i \le n\} \cup \{x_i y_i : 1 \le i \le n\} \cup \{y_i x_{i+1} : 1 \le i \le n-1\} \cup \{y_n x_1\}$.

3. Main Results

$P_n \odot K_1$ is pair difference cordial for all values of n [7]. Now we investigate the pair difference cordiality of $P_n \odot K_2$.

Theorem 3.1. $P_n \odot K_2$ is pair difference cordial for all values of n.

Proof.
Let $V(P_n \odot K_2) = \{x_i, y_i, z_i : 1 \le i \le n\}$ and $E(P_n \odot K_2) = \{x_i x_{i+1} : 1 \le i \le n-1\} \cup \{y_i z_i, x_i y_i, x_i z_i : 1 \le i \le n\}$.Clearly $P_n \odot K_2$ has $3n$ vertices and $4n-1$ edges. There are two cases arises.

Case 1. n is even.

First assign the labels $\frac{3n}{2}, \frac{3n-2}{2}, \frac{3n-4}{2}, \cdots, n+1$ to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n}{2}}$ and assign the labels $-\frac{3n}{2}$, $-\frac{3n-2}{2}$, $-\frac{3n-4}{2}$, $\cdots$,$-(n+1)$ to the vertices $x_{\frac{n+2}{2}}, x_{\frac{n+4}{2}}, x_{\frac{n+6}{2}}, \cdots, x_n$. Next assign the labels $1, 3, 5, \cdots, n-1$ to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n}{2}}$ and assign the labels $-1, -3, -5, \cdots, -(n-1)$ to the vertices $y_{\frac{n+2}{2}}, y_{\frac{n+4}{2}}, y_{\frac{n+6}{2}}, \cdots, y_n$. Now assign the labels $2, 4, 6, \cdots, n$ to the vertices $z_1, z_2, z_3, \cdots, z_{\frac{n}{2}}$ and assign the labels $-2, -4, -6, \cdots, -n$ to the vertices $z_{\frac{n+2}{2}}, z_{\frac{n+4}{2}}, z_{\frac{n+6}{2}}, \cdots, z_n$.

Case 2. n is odd.

Assign the labels $\frac{3n-3}{2}$, $\frac{3n-5}{2}, \frac{3n-7}{2}, \cdots, n$ to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n-1}{2}}$ and assign the labels $-\frac{3n-3}{2}$,$-\frac{3n-5}{2}$, $-\frac{3n-7}{2}$,$\cdots, -n$ to the vertices $x_{\frac{n+1}{2}}$, $x_{\frac{n+3}{2}}$, $x_{\frac{n+5}{2}}$, $\cdots$,x_{n-1}. Now assign the labels $1, 3, 5, \cdots, n-2$ to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-1}{2}}$ and assign the labels $-1, -3, -5, \cdots, -(n-2)$ to the vertices $y_{\frac{n+1}{2}}, y_{\frac{n+3}{2}}, y_{\frac{n+5}{2}}, \cdots, y_{n-1}$. Next assign the labels $2, 4, 6, \cdots, n-1$ to the vertices $z_1, z_2, z_3, \cdots, z_{\frac{n-1}{2}}$ and assign the labels $-2, -4, -6, \cdots, -(n-1)$ to the vertices $z_{\frac{n+1}{2}}$, $z_{\frac{n+3}{2}}$, $z_{\frac{n+5}{2}}$,$\cdots$, z_{n-1}. Finally assign the labels $\frac{3n-1}{2}$, $-(\frac{3n-1}{2})$, $\frac{3n-3}{2}$ to the vertices x_n, y_n, z_n. The Table 1 given below establish that this vertex labeling f is a pair difference cordial of $P_n \odot K_2$.

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
n is odd	$2n-1$	$2n$
n is even	$2n$	$2n-1$

Table 1

$C_n \odot K_1$ is pair difference cordial for all values of $n \geq 3$ [7]. We now investigate the pair difference cordiality of $C_n \odot K_2, n \geq 3$.

Theorem 3.2. $C_n \odot K_2$ is pair difference cordial for all values of $n \geq 3$.

Proof.

Let $V(C_n \odot K_2) = \{x_i, y_i, z_i : 1 \leq i \leq n\}$ and $E(C_n \odot K_2) = \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{x_1 x_n\} \cup \{y_i z_i, x_i y_i, x_i z_i : 1 \leq i \leq n\}$. Obviously $C_n \odot K_2$ has $3n$ vertices and $4n$ edges. As in theorem 3.2, Assign the labels to the vertices $x_i, y_i, z_i (1 \leq i \leq n)$ of $C_n \odot K_2$. This vertex labeling f yields that $\Delta_{f_1} = \Delta_{f_1^c} = 2n$.

Theorem 3.3. $P_n \odot 2K_1$ is pair difference cordial for all values of n.

Proof.

Let $V(P_n \odot 2K_1) = \{x_i, y_i, z_i : 1 \leq i \leq n\}$ and $E(P_n \odot 2K_1) = \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{x_i y_i, x_i z_i : 1 \leq i \leq n\}$. Note that $P_n \odot K_2$ has $3n$ vertices and $3n-1$ edges.There are two cases arises.

Case 1. n is even.

Assign the labels $2, 5, 8, \cdots, \frac{3n-2}{2}$ respectively to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n}{2}}$ and assign the labels $-1, -4, -7, \cdots,$ -3n-4$_{2}$ to the vertices $x_{\frac{n+2}{2}}, x_{\frac{n+4}{2}}, x_{\frac{n+6}{2}}, \cdots, x_n$ respectively. Next we assign the labels $1, 4, 7, \cdots, \frac{3n-4}{2}$ to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n}{2}}$ respectively and assign the labels $-2, -5, -8, \cdots, -\frac{3n-2}{2}$ respectively to the vertices $y_{\frac{n+2}{2}}, y_{\frac{n+4}{2}}, y_{\frac{n+6}{2}}, \cdots, y_n$. We now assign the labels $3, 6, 9, \cdots, \frac{3n}{2}$ respectively to the vertices $z_1, z_2, z_3, \cdots, z_{\frac{n}{2}}$ and assign the labels $-3, -6, -9, \cdots, -\frac{3n}{2}$ to the vertices $z_{\frac{n+2}{2}}, z_{\frac{n+4}{2}}, z_{\frac{n+6}{2}}, \cdots, z_n$ respectively.

Case 2. n is odd.

Assign the labels $2, 5, 8, \cdots, \frac{3n-5}{2}$ respectively to the vertices $x_1, x_2, x_3, \cdots, x_{\frac{n-1}{2}}$ and assign the labels $-1, -4,$ $-7, \cdots,$ -3n-7$_{2}$ to the vertices $x_{\frac{n+1}{2}}, x_{\frac{n+3}{2}}, x_{\frac{n+5}{2}}, \cdots, x_{n-1}$ respectively. Next assign the labels $1, 4, 7, \cdots, \frac{3n-7}{2}$ to the vertices $y_1, y_2, y_3, \cdots, y_{\frac{n-1}{2}}$ respectively and assign the labels $-2, -5, -8, \cdots, -\frac{3n-5}{2}$ respectively to the vertices $y_{\frac{n+1}{2}}, y_{\frac{n+3}{2}}, y_{\frac{n+5}{2}}, \cdots, y_{n-1}$. Now assign the labels $3, 6, 9, \cdots, \frac{3n-3}{2}$ respectively to the vertices $z_1, z_2,$ $z_3 \cdots, z_{\frac{n-1}{2}}$ and assign the labels $-3, -6, -9, \cdots, -\frac{3n-3}{2}$ to the vertices $z_{\frac{n+1}{2}}, z_{\frac{n+3}{2}}, z_{\frac{n+5}{2}}, \cdots, z_{n-1}$ respectively. Finally assign the labels $\frac{3n-1}{2}, -\frac{3n-1}{2}, \frac{3n-3}{2}$ to the vertices x_n, y_n, z_n.

The Table 2 given below establish that this vertex labeling f is a pair difference cordial of $P_n \odot 2K_1$.

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
n is odd	$\frac{3n-1}{2}$	$\frac{3n-1}{2}$
n is even	$\frac{3n-2}{2}$	$\frac{3n}{2}$

Table 2

Theorem 3.4. $C_n \odot 2K_1$ is pair difference cordial for all values of $n \geq 3$.

Proof.
Let $V(C_n \odot 2K_1) = \{x_i, y_i, z_i : 1 \leq i \leq n\}$ and $E(P_n \odot 2K_1) = \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{x_1 x_n\} \cup \{x_i y_i, x_i z_i : 1 \leq i \leq n\}$. Clearly $C_n \odot 2K_1$ has $3n$ vertices and $3n$ edges. As in theorem 3.4, Assign the labels to the vertices $x_i, y_i, z_i (1 \leq i \leq n)$ of $C_n \odot 2K_1$.
The Table 3 given below establish that this vertex labeling f is a pair difference cordial of $C_n \odot 2K_1$.

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
n is odd	$\frac{3n-1}{2}$	$\frac{3n+1}{2}$
n is even	$\frac{3n}{2}$	$\frac{3n}{2}$

Table 3

Theorem 3.5. $L_n \odot K_1$ is pair difference cordial for all values of n.

Proof.
Let $V(L_n \odot K_1) = V(L_n) \cup \{x_i, y_i : 1 \leq i \leq n\}$ and $E(L_n \odot K_1) = E(L_n) \cup \{a_i x_i, b_i y_i : 1 \leq i \leq n\}$. Note that $L_n \odot K_1$ has $4n$ vertices and $4n-2$ edges. Assign the labels $1, 2, 3, \cdots, n$ to the vertices $a_1, a_2, a_3, \cdots, a_n$ and assign the labels $-1, -2, -3, \cdots, -n$ to the vertices $b_1, b_2, b_3, \cdots, b_n$. Next assign the labels $2n, 2n-1, 2n-2, \cdots, n+1$ to the vertices $x_1, x_2, x_3, \cdots, x_n$ and assign the labels $-2n, -2n+1, -2n+2, \cdots, -n-1$ to the vertices $y_1, y_2, y_3, \cdots, y_n$. This vertex labeling gives that, $\Delta_{f_1} = 2n, \Delta_{f_1^c} = 2n-1$.

Theorem 3.6. $L_n \odot 2K_1$ is pair difference cordial for all values of n.

Proof.
Let $V(L_n \odot 2K_1) = V(L_n) \cup \{x_i, y_i, u_i, v_i : 1 \leq i \leq n\}, E(L_n \odot 2K_1) = E(L_n) \cup \{a_i x_i, a_i u_i, b_i v_i, b_i y_i : 1 \leq i \leq n\}$. Obviously $L_n \odot 2K_1$ has $6n$ vertices and $7n-2$ edges.

Case 1. n is even.

Define the map $f : V(L_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm 3n\}$ by

$$
\begin{aligned}
&f(a_1) = 2, && f(b_1) = -2, \\
&f(x_1) = 1, && f(y_1) = -1, \\
&f(u_1) = 3, && f(v_1) = -3, \\
&f(a_i) = f(a_{i-1}) + 3, && 2 \leq i \leq \frac{n}{2}, \\
&f(b_i) = f(b_{i-1}) - 3, && 2 \leq i \leq n-1, \\
&f(x_i) = f(x_{i-1}) + 3, && 2 \leq i \leq \frac{n}{2}, \\
&f(u_i) = f(u_{i-1}) + 3, && 2 \leq i \leq n, \\
&f(y_i) = f(y_{i-1}) - 3, && 2 \leq i \leq n-1, \\
&f(v_i) = f(v_{i-1}) - 3, && 2 \leq i \leq n-1,
\end{aligned}
$$

$$f(a_{\frac{n+2}{2}}) = f(a_{\frac{n}{2}}) + 2,$$
$$f(x_{\frac{n+2}{2}}) = f(x_{\frac{n}{2}}) + 4,$$
$$f(a_{\frac{n+2i}{2}}) = f(a_{\frac{n+2}{2}}) + 3i - 3, \qquad 2 \le i \le \frac{n}{2},$$
$$f(u_{\frac{n+2i}{2}}) = f(u_{\frac{n+2}{2}}) + 3i - 3, \qquad 2 \le i \le \frac{n}{2},$$
$$f(x_{\frac{n+2i}{2}}) = f(x_{\frac{n+2}{2}}) + 3i - 3, \qquad 2 \le i \le \frac{n}{2},$$
$$f(b_n) = -f(a_n),$$
$$f(v_n) = -f(u_n),$$
$$f(x_n) = -f(y_n).$$

Case 2. n is odd.
Define the map $f : V(L_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm 3n\}$ by

$$f(a_1) = 2, \qquad f(b_1) = -2,$$
$$f(x_1) = 1, \qquad f(y_1) = -1,$$
$$f(u_1) = 3, \qquad f(v_1) = -3,$$
$$f(a_i) = f(a_{i-1}) + 3, \qquad 2 \le i \le \frac{n+1}{2},$$
$$f(x_i) = f(x_{i-1}) + 3, \qquad 2 \le i \le \frac{n+1}{2},$$
$$f(u_i) = f(u_{i-1}) + 3, \qquad 1 \le i \le n,$$

$$f(a_{\frac{n+3}{2}}) = f(a_{\frac{n+1}{2}}) + 2,$$
$$f(x_{\frac{n+3}{2}}) = f(x_{\frac{n+1}{2}}) + 4,$$
$$f(a_{\frac{n+2i+1}{2}}) = f(a_{\frac{n+1}{2}}) + 3i - 3, \qquad 2 \le i \le \frac{n-1}{2},$$
$$f(u_{\frac{n+2i+1}{2}}) = f(u_{\frac{n+1}{2}}) + 3i - 3, \qquad 2 \le i \le \frac{n-1}{2},$$
$$f(x_{\frac{n+2i+1}{2}}) = f(x_{\frac{n+1}{2}}) + 3i - 3, \qquad 2 \le i \le \frac{n-1}{2},$$
$$f(b_i) = -f(a_i), \qquad 1 \le i \le n,$$
$$f(v_i) = -f(u_i), \qquad 1 \le i \le n,$$
$$f(x_i) = -f(y_i), \qquad 1 \le i \le n.$$

The Table 3 given below establish that this vertex labeling f is a pair difference cordial of $L_n \odot 2K_1$.

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
n is odd	$\frac{7n-3}{2}$	$\frac{7n-1}{2}$
n is even	$\frac{7n-2}{2}$	$\frac{7n-2}{2}$

Table 4

Theorem 3.7. $W_n \odot 2K_1$ is pair difference cordial for all values of $n \geq 3$.

Proof.
Let $V(W_n \odot 2K_1) = \{x_i, y_i, z_i : 1 \leq i \leq n\} \cup \{x, w_1, w_2\}$ and $E(W_n \odot 2K_1) = \{x_i x_{i+1} : 1 \leq i \leq n-1\} \cup \{xx_i, x_i y_i, x_i z_i : 1 \leq i \leq n\} \cup \{x_1 x_n, xw_1, xw_2\}$. Note that $W_n \odot 2K_1$ has $3n+3$ vertices and $4n+2$ edges.
Case 1. n is even.
Define the map $f : V(W_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm \frac{3n+2}{2}\}$ by

$$
\begin{aligned}
f(x_1) &= 2, & f(y_1) &= 1, \\
f(z_1) &= 3, & f(w_1) &= \frac{3n+2}{2}, \\
f(w_2) &= -\frac{3n+2}{2}, & f(x) &= \frac{3n}{2}, \\
f(x_i) &= f(x_{i-1}) + 3, & 2 &\leq i \leq \frac{n}{2}, \\
f(y_i) &= f(y_{i-1}) + 3, & 2 &\leq i \leq \frac{n}{2}, \\
f(z_i) &= f(z_{i-1}) + 3, & 2 &\leq i \leq \frac{n}{2}, \\
f(x_{\frac{n+2i}{2}}) &= -f(x_i), & 1 &\leq i \leq \frac{n}{2}, \\
f(y_{\frac{n+2i}{2}}) &= -f(y_i), & 1 &\leq i \leq \frac{n}{2}, \\
f(z_{\frac{n+2i}{2}}) &= -f(z_i), & 1 &\leq i \leq \frac{n}{2},
\end{aligned}
$$

Case 2. n is odd.
Define the map $f : V(W_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm \frac{3n+3}{2}\}$ by

$$
\begin{aligned}
f(x_1) &= 2, & f(y_1) &= 1, \\
f(z_1) &= 3, & f(w_1) &= -\frac{3n+1}{2}, \\
f(w_2) &= -\frac{3n+3}{2}, & f(x_n) &= \frac{3n+1}{2}, \\
f(y_n) &= \frac{3n-1}{2}, & f(z_n) &= \frac{3n+3}{2}, \\
f(x) &= -\frac{3n-1}{2}, & & \\
f(x_i) &= f(x_{i-1}) + 3, & 2 &\leq i \leq \frac{n-1}{2}, \\
f(y_i) &= f(y_{i-1}) + 3, & 2 &\leq i \leq \frac{n-1}{2}, \\
f(z_i) &= f(z_{i-1}) + 3, & 2 &\leq i \leq \frac{n-1}{2}, \\
f(x_{\frac{n+2i-1}{2}}) &= -f(x_i), & 1 &\leq i \leq \frac{n-1}{2}, \\
f(y_{\frac{n+2i-1}{2}}) &= -f(y_i), & 1 &\leq i \leq \frac{n-1}{2}, \\
f(z_{\frac{n+2i-1}{2}}) &= -f(z_i), & 1 &\leq i \leq \frac{n-1}{2},
\end{aligned}
$$

In both cases $\Delta_{f_1} = \Delta_{f_1^c} = 2n+1$. Therefore $W_n \odot 2K_1$ is pair difference cordial for all values of $n \geq 3$.

Theorem 3.8. $G_n \odot 2K_1$ is pair difference cordial for all values of $n \geq 3$.

Proof.
Let $V(G_n \odot 2K_1) = V(G_n) \cup \{a_i, a_i', b_i, b_i' : 1 \le i \le n\} \cup \{a, a'\}$ and $V(G_n \odot 2K_1) = E(G_n) \cup \{x_i a_i, x_i a_i', y_i b_i, y_i b_i',: 1 \le i \le n\} \cup \{xa, xa'\}$. Clearly $G_n \odot 2K_1$ has $6n+3$ vertices and $7n+2$ edges. **Case 1.** $n \equiv 0 \pmod 4$.
Define the map $f : V(G_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm \frac{6n+2}{2}\}$ by

$$
\begin{aligned}
f(x_1) &= 2, & f(a_1) &= 1,\\
f(a_1') &= 3, & f(a) &= 3n+1,\\
f(a') &= -(3n+1), & f(x) &= 3n,\\
f(x_i) &= f(x_{i-1}) + 3, & 2 &\le i \le \frac{3n}{4},\\
f(a_i) &= f(a_{i-1}) + 3, & 2 &\le i \le \frac{3n}{4},\\
f(a_i') &= f(a_{i-1}') + 3, & 2 &\le i \le \frac{3n}{4},\\
f(x_{\frac{3n+4}{4}}) &= f(x_{\frac{3n}{4}}) + 2, & &\\
f(a_{\frac{3n+4}{4}}) &= f(a_{\frac{3n}{4}}) + 4, & &\\
f(a'_{\frac{3n+4}{4}}) &= f(a'_{\frac{3n}{4}}) + 3, & &\\
f(x_{\frac{3n+4i}{4}}) &= f(x_{\frac{3n+4i-4}{4}}) + 3, & 2 &\le i \le \frac{n-4}{4},\\
f(a_{\frac{3n+4i}{4}}) &= f(a_{\frac{3n+4i-4}{4}}) + 3, & 2 &\le i \le \frac{n-4}{4},\\
f(a'_{\frac{3n+4i}{4}}) &= f(a'_{\frac{3n+4i-4}{4}}) + 3, & 2 &\le i \le \frac{n-4}{4},\\
f(y_i) &= -f(x_i), & 1 &\le i \le n,\\
f(b_i) &= -f(a_i), & 1 &\le i \le n,\\
f(b_i') &= -f(a_i'), & 1 &\le i \le n.
\end{aligned}
$$

Case 2. $n \equiv 1 \pmod 4$.
Define the map $f : V(G_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm \frac{6n+3}{2}\}$ by

$$
\begin{aligned}
f(x_1) &= 2, & f(a_1) &= 1,\\
f(a_1') &= 3, & f(a) &= 3n+1,\\
f(a') &= -(3n+1), & f(x) &= 3n,\\
f(x_i) &= f(x_{i-1}) + 3, & 2 &\le i \le \frac{3n+1}{4},\\
f(a_i) &= f(a_{i-1}) + 3, & 2 &\le i \le \frac{3n+1}{4},\\
f(a_i') &= f(a_{i-1}') + 3, & 2 &\le i \le \frac{3n+1}{4},\\
f(x_{\frac{3n+5}{4}}) &= f(x_{\frac{3n+1}{4}}) + 2, & &\\
f(a_{\frac{3n+5}{4}}) &= f(a_{\frac{3n+1}{4}}) + 4, & &\\
f(a'_{\frac{3n+5}{4}}) &= f(a'_{\frac{3n+1}{4}}) + 3, & &
\end{aligned}
$$

$$f(x_{\frac{3n+4i+1}{4}}) = f(x_{\frac{3n+4i-3}{4}}) + 3, \qquad 2 \le i \le \frac{n-3}{4},$$
$$f(a_{\frac{3n+4i+1}{4}}) = f(a_{\frac{3n+4i-3}{4}}) + 3, \qquad 2 \le i \le \frac{n-3}{4},$$
$$f(a'_{\frac{3n+4i+1}{4}}) = f(a'_{\frac{3n+4i-3}{4}}) + 3, \qquad 2 \le i \le \frac{n-3}{4},$$
$$f(y_i) = -f(x_i), \qquad 1 \le i \le n,$$
$$f(b_i) = -f(a_i), \qquad 1 \le i \le n,$$
$$f(b'_i) = -f(a'_i), \qquad 1 \le i \le n.$$

Case 3. $n \equiv 2 \pmod 4$.
Define the map $f : V(G_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm \frac{6n+2}{2}\}$ by

$$f(x_1) = 2, \qquad f(a_1) = 1,$$
$$f(a'_1) = 3, \qquad f(a) = 3n+1,$$
$$f(a') = -(3n+1), \qquad f(x) = 3n,$$
$$f(x_i) = f(x_{i-1}) + 3, \qquad 2 \le i \le \frac{3n+2}{4},$$
$$f(a_i) = f(a_{i-1}) + 3, \qquad 2 \le i \le \frac{3n+2}{4},$$
$$f(a'_i) = f(a'_{i-1}) + 3, \qquad 2 \le i \le \frac{3n+2}{4},$$
$$f(x_{\frac{3n+6}{4}}) = f(x_{\frac{3n+2}{4}}) + 2,$$
$$f(a_{\frac{3n+6}{4}}) = f(a_{\frac{3n+2}{4}}) + 4,$$
$$f(a'_{\frac{3n+6}{4}}) = f(a'_{\frac{3n+2}{4}}) + 3,$$
$$f(x_{\frac{3n+4i+2}{4}}) = f(x_{\frac{3n+4i-2}{4}}) + 3, \qquad 2 \le i \le \frac{n-2}{4},$$
$$f(a_{\frac{3n+4i+2}{4}}) = f(a_{\frac{3n+4i-2}{4}}) + 3, \qquad 2 \le i \le \frac{n-2}{4},$$
$$f(a'_{\frac{3n+4i+2}{4}}) = f(a'_{\frac{3n+4i-2}{4}}) + 3, \qquad 2 \le i \le \frac{n-2}{4},$$
$$f(y_i) = -f(x_i), \qquad 1 \le i \le n,$$
$$f(b_i) = -f(a_i), \qquad 1 \le i \le n,$$
$$f(b'_i) = -f(a'_i), \qquad 1 \le i \le n.$$

Case 4. $n \equiv 3 \pmod 4$.
Define the map $f : V(G_n \odot 2K_1) \to \{\pm 1, \pm 2, \cdots, \pm \frac{6n+3}{2}\}$ by

$$f(x_1) = 2, \qquad f(a_1) = 1,$$
$$f(a'_1) = 3, \qquad f(a) = 3n+1,$$
$$f(a') = -(3n+1), \qquad f(x) = 3n,$$

$$
\begin{aligned}
f(x_i) &= f(x_{i-1}) + 3, & 2 \le i \le \frac{3n+3}{4},\\
f(a_i) &= f(a_{i-1}) + 3, & 2 \le i \le \frac{3n+3}{4},\\
f(a'_i) &= f(a'_{i-1}) + 3, & 2 \le i \le \frac{3n+3}{4},\\
f(x_{\frac{3n+7}{4}}) &= f(x_{\frac{3n+3}{4}}) + 2, &\\
f(a_{\frac{3n+7}{4}}) &= f(a_{\frac{3n+3}{4}}) + 4, &\\
f(a'_{\frac{3n+7}{4}}) &= f(a'_{\frac{3n+3}{4}}) + 3, &\\
f(x_{\frac{3n+4i+3}{4}}) &= f(x_{\frac{3n+4i-1}{4}}) + 3, & 2 \le i \le \frac{n-1}{4},\\
f(a_{\frac{3n+4i+3}{4}}) &= f(a_{\frac{3n+4i-1}{4}}) + 3, & 2 \le i \le \frac{n-1}{4},\\
f(a'_{\frac{3n+4i+3}{4}}) &= f(a'_{\frac{3n+4i-1}{4}}) + 3, & 2 \le i \le \frac{n-1}{4},\\
f(y_i) &= -f(x_i), & 1 \le i \le n,\\
f(b_i) &= -f(a_i), & 1 \le i \le n,\\
f(b'_i) &= -f(a'_i), & 1 \le i \le n.
\end{aligned}
$$

The Table 4 given below establish that this vertex labeling f is a pair difference cordial of $G_n \odot 2K_1, n \ge 3$.

Nature of n	$\Delta_{f_1^c}$	Δ_{f_1}
n is odd	$\frac{7n+1}{2}$	$\frac{7n+3}{2}$
n is even	$\frac{7n+2}{2}$	$\frac{7n+2}{2}$

Table 5

Author Contributions

All authors contributed equally to this work. They all read and approved the final version of the manuscript.

Conflicts of Interest

The authors declare no conflict of interest.

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