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Housing, Collateral Constraints, and Fiscal Policy

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Abstract

This paper studies the preferential tax treatment of housing that can be observed in many industrialized countries. It provides a rationale for it by means of an optimal taxation approach, taking into account an important feature of housing, namely its usage as collateral. In a borrower-lender framework, where private loans are assumed to be non-enforceable and have to be collateralized by housing, optimal fiscal policy should disburden constrained borrowers by subsidizing their housing.

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1. Introduction

Housing is subject to a preferential tax treatment in many industrialized countries. In the US, total housing subsidies added up to $220 billion in 2011, corresponding to 1.5% of GDP (US Budget, 2011). Also, in various European countries, the values of total housing subsidies expressed in percent of GDP were in that range: e.g., 0.9% in Germany, 1.1% in France, and 1.4% in Spain in 2000 (ECB, 2003).

The two most important housing subsidies are the deductibility of mortgage interest payments from income and the tax exemption of imputed rents on owner-occupied housing. In the US, the former amounted to $105 billion while the latter added up to $38 billion in 2011 (US Budget, 2011). These two subsidies accounted for 65% of total housing subsidies.

However, among economists, this preferential tax treatment of housing is controversial. On the one hand, it is criticized by researchers like Poterba (1992) and Gervais (2002), among others, who argue that this favoritism leads to a welfare loss, since it distorts investment decisions of individuals towards housing. These studies are in line with Rosen, who argues that “paternalism and political considerations seem to be the sources of this policy” (1985, p. 380).

On the other hand, there are proponents of this treatment who argue that homeownership is accompanied by externalities that are internalized through these subsidies. For instance, Green and White (1997) stress the positive impact of homeownership on the education of children, and DiPasquale and Glaeser (1999) state that homeowners are better citizens in the sense that they are more involved in local organizations.

In contrast to these papers, this work gives a rationale for housing subsidies based on market imperfections. We assume that private loans are not enforceable and therefore have to be collateralized by housing. Furthermore, the data make clear the importance of housing as a component of wealth and the relevance of its usage as collateral. First, housing makes up a large part of total household wealth as well as total national wealth. In the US, the value of housing accounts for half of total household wealth and is larger than annual GDP, with an average ratio of housing wealth to GDP of about 1.5, from 1952 to 2008 (Iacoviello, 2009). Secondly, in 2010, residential mortgage debt amounted to 77% of GDP in the US and to 47% in Germany, to 41% in France, and to 64% in Spain (Hypostat, 2010). To the best of our knowledge, this paper is the first one that studies optimal taxation of housing in the presence of collateral constraints.

The structure of the model is as follows. We consider a household sector that relates to Kiyotaki and Moore's (1997) model with two types of agents who differ in their discount factors, patient and impatient ones. Due to this difference in patience, we get lenders, the patient agents, and borrowers, the impatient ones, in equilibrium. While for the former the collateral constraint is irrelevant in equilibrium, it is of importance for the latter. As in Iacoviello (2005), housing plays a dual role for households. First, it delivers utility together with consumption and leisure, and, secondly, private loans are collateralized by housing. The government is assumed to have exogenous expenditures that have to be financed by two taxes, a housing property tax (which can differ for the two types of agents) and a labor income tax. The different housing tax rates for the two types can be understood as follows. The patient households for whom the collateral constraint is irrelevant will always own larger houses than the impatient ones and therefore are taxed at another, higher, rate than the impatient and hence wealth-poor agents.

The main thrust of this paper is to provide a rationale for housing subsidies. In the presence of collateral constraints, optimal fiscal policy should subsidize the housing of impatient households, for whom the collateral constraint is relevant, in order to disburden them. This subsidy has to be financed to the largest extent by a housing tax on patient households and to a smaller part by a labor income tax. In other words, this can be interpreted as redistribution from wealth-rich patient households owning larger houses to wealth-poor impatient households who own smaller houses.

The main result of housing subsidies for impatient households is robust for several parameter variations and can be attributed for the most part to the collateral constraint. To illustrate this point, we analyze the effects of the discount-rate difference between the types of agents on housing subsidies in comparison to the effects of the collateral constraint, and we find that the former plays a minor role.

We also consider a representative-agent version of the model as a reference case. We thereby understand how the inclusion of a durable good, housing *per se*, affects optimal fiscal policy compared to standard models. Furthermore, this allows us to compare the results of the representative-agent version to existing literature. These results are, in fact, quite intuitive and in line with the principle of optimal taxation: namely, goods with lower elasticities should be taxed at a higher rate. For the benchmark calibration, the housing tax rate is positive in the representative-agent version, as it is for patient households in the model with two types of agents.

The paper further relates to the work of Eerola and Määttänen (2009), which considers optimal taxation of housing in a dynamic representative-agent model with fairly general preferences and an extended tax system, compared to the model of this paper. However, the results of the representative-agent version of our model are compatible with their results. Another closely related paper is that of Monacelli (2008), who considers a model with two types of agents with different patience rates and collateral constraints similar to the one of this paper. While Monacelli analyzes optimal *monetary* policy in that framework, he also points out that the analysis of optimal fiscal policy in such a model would be of interest, which is done in this paper.

The rest of the paper is organized as follows. In Section 2, the model with two types of agents, firms, and the government is described, the Ramsey problem is set up, and the equilibrium conditions for the steady state are derived. In Section 3, the results for the full as well as the representative-agent version are presented, and a sensitivity analysis is given. The fourth and last section presents the conclusion.

2. The Model

In this section, we present the model with a household sector consisting of two types of agents, a production sector consisting of two types of firms, and the government. Concerning the household sector, we follow Kiyotaki and Moore (1997), who pioneered the models with two types of agents, patient and impatient ones, resulting in an equilibrium with lenders and borrowers. We assume that private debt contracts are not enforceable and have to be collateralized by housing, as in Iacoviello (2005). Therefore, a head of household can only borrow up to a fraction m of his expected end-of-period housing wealth. Additionally to its usage as collateral, housing delivers utility together with consumption and leisure.

As in Favilukis et al. (2012), we consider a two-sector production side, such that both housing demand and supply are modeled explicitly. There are two types of firms, one of which produces non-durable consumption goods and the other durable housing.

The government levies a flat-rate tax on labor income and a housing property tax that can differ for the two types of agents and issues one-period bonds to finance an exogenous stream of government expenditures. It has no access to lump-sum taxes. The reason why housing tax rates can differ is that a patient household will own a larger house than an impatient one. Hence, rather than taxing degrees of patience differently, we can understand this as taxing the ones with a larger house at a higher rate than the ones with a smaller house. Due to the usage of housing as collateral, which is only relevant for the borrowers, who will be the impatient agents in equilibrium, we will see that the housing tax rates will differ markedly.

2.1. Households

There is a continuum of households consisting of two types, patient and impatient ones. They differ in their discount factors $1 > \beta > \beta' > 0$, with β being the discount factor of patient and β' of impatient households. Henceforth, variables of patient (impatient) households are denoted without (with) a prime, while aggregate variables are denoted with a superscript T (e.g. c_t^T, for total consumption). The population share of patient households is s. Borrowing between the two types of households is modeled as follows. A household can borrow an amount $-\frac{b_t}{1+r_{t-1}}$ in period $t-1$ and has to pay back $-b_t$ in period t, where r_{t-1} is the real interest rate on loans between $t-1$ and t. Since we assume that private debt contracts are not enforceable, there is a limit on private debt, given by a fraction m of the expected end-of-period housing wealth

$$b_{t+1}^{(')} \geq -mp_{h,t+1}h_t^{(')}, \tag{1}$$

where m denotes the exogenous pledgeable fraction of housing. As we will see below, this constraint will become relevant for impatient households, while it will be irrelevant for patient ones.

Both types of households derive utility from consumption $c_t^{(')}$ and housing $h_t^{(')}$ and disutility from labor $n_t^{(')}$ and maximize the infinite sum of expected utility. Their objective is given by

$$\sum_{t=0}^{\infty} \beta^{(')t} u(c_t^{(')}, h_t^{(')}, n_t^{(')}). \tag{2}$$

We consider the following CRRA-specification of the utility function

$$u(c_t, h_t, n_t) = \frac{c_t^{1-\mu^c}}{1-\mu^c} + \frac{h_t^{1-\mu^h}}{1-\mu^h} - \frac{n_t^{1+\mu^n}}{1+\mu^n}, \tag{3}$$

where $\mu^{c(h)}$ denotes the inverse of the intertemporal elasticity of substitution in consumption (housing) and μ^n the inverse of the Frisch elasticity of the labor supply.

2.1.1 Patient households

The representative patient household generates income from working $w_t n_t$, with w_t being the real wage rate and the return of bond holdings b_t^g. Labor income is taxed at the rate τ_t^n. Every period the household can adjust its stock of housing according to $h_t - (1-\delta_h)h_{t-1}$ at the price of housing $p_{h,t}$, with δ_h being the depreciation rate of housing. The value of the housing stock owned by the household is taxed at the rate τ_t^h. Thus, we consider a housing property tax that is proportional to the value of the current housing stock and is paid every period. The budget constraint of the patient households is given by

$$\begin{aligned} & c_t + p_{h,t}\left[\left(1+\tau_t^h\right)h_t - (1-\delta_h)h_{t-1}\right] + \frac{b_{t+1}^g}{R_t^g} + \frac{b_{t+1}}{R_t} \\ & = \left(1-\tau_t^n\right)w_t n_t + b_t^g + b_t, \end{aligned} \tag{4}$$

where c_t denotes consumption spending, $\frac{b_{t+1}^g}{R_t^g}$ investment in new government bonds with the relating gross interest rate $R_t^g = 1 + r_t^g$ and b_t privately issued debt with the gross interest rate $R_t = 1 + r_t$. The patient household will hold positive amounts of $b_t^g > 0$ and $b_t > 0$ and hence will be the lender in equilibrium. That's why the collateral constraint (1) will be irrelevant for patient households: $b_{t+1} > 0 > -mp_{h,t+1}h_t$.

2.1.2 Impatient households

The budget constraint of the representative impatient household analogously reads

$$c_t' + p_{h,t}\left[\left(1+\tau_t'^h\right)h_t' - (1-\delta_h)h_{t-1}'\right] + \frac{b_{t+1}'^g}{R_t^g} + \frac{b_{t+1}'}{R_t} \\ = \left(1-\tau_t^n\right)w_t n_t' + b_t'^g + b_t'. \tag{5}$$

Since we rule out short sales in government bonds, the impatient households will set $b_{t+1}'^g = b_t'^g = 0$. Furthermore, this type will be the private borrower in equilibrium, i.e., $b_{t+1}' = -\frac{s}{1-s} b_{t+1} < 0$, following from the market-clearing condition for private debt $(1-s)b_{t+1}' + sb_{t+1} = 0$. Hence, the collateral constraint (1) will become relevant here. Therefore, there is a limit on the obligations of impatient households, which is given by $b_{t+1}' \geq -mp_{h,t+1}h_t'$.

2.2 Government

The government levies a flat-rate tax on labor income τ_t^n and a housing property tax $\tau_t^{(')h}$ and issues one-period bonds ($b_t^{(')g} \geq 0 \quad \forall t \geq 0$) to finance an exogenous stream of government expenditures (g_t):

$$g_t - \frac{b_{t+1}^g}{R_t^g} + b_t^g = s\tau_t^h p_{h,t} h_t + (1-s)\tau_t'^h p_{h,t} h_t' + \tau_t^n w_t n_t^T, \tag{6}$$

where $n_t^T = sn_t + (1-s)n_t'$ denotes total labor supply. As mentioned before, the different housing tax rates τ_t^h and $\tau_t'^h$ can be understood as taxing the wealthier agents, which will be the patient households in equilibrium, at a rate that differs from the one for the wealth-poor impatient households, which will own smaller houses in equilibrium.

2.3 Firms

The production side of the economy is characterized by two sectors, one of which produces consumption goods y_c and the other housing y_h. In both sectors, there is a continuum of firms, which are assumed to produce with the same technology for simplicity's sake. The representative firm of each sector produces its output with labor according to $y_{c,t} = n_{c,t}^T$ and $y_{h,t} = n_{h,t}^T$, where total labor input in each sector is given by the weighted sum of labor input of the patient and impatient households in this sector $n_{c,t}^T = sn_{c,t} + (1-s)n_{c,t}'$

and $n_{h,t}^T = sn_{h,t} + (1-s)n_{h,t}^{'}$. On the other hand, total labor supply $n_t^T = sn_t + (1-s)n_t^{'} = n_{c,t}^T + n_{h,t}^T$ is divided between the two types of firms. Labor is assumed to be totally mobile between the two sectors, leading to a wage rate that is the same for both sectors.

2.4 Competitive Equilibrium

We now describe the competitive equilibrium of the private sector and then set up the Ramsey problem.

Patient households

A patient household chooses the values of c_t, h_t, n_t, b_{t+1}^g and b_{t+1} to maximize (2), subject to the budget constraint (4), leading to the first-order conditions

$$h_t^{-\mu^h} = (1+\tau_t^h) p_{h,t} c_t^{-\mu^c} - \beta c_{t+1}^{-\mu^c} (1-\delta_h) p_{h,t+1} \tag{7}$$

$$n_t^{\mu^n} = (1-\tau_t^n) w_t c_t^{-\mu^c} \tag{8}$$

$$c_t^{-\mu^c} = \beta R_t^g c_{t+1}^{-\mu^c} \tag{9}$$

$$c_t^{-\mu^c} = \beta R_t c_{t+1}^{-\mu^c} . \tag{10}$$

Equation (7) describes housing demand. In the optimum, the marginal utility of current housing $h_t^{-\mu^h}$ equals the marginal utility of foregone consumption $c_t^{-\mu^c}$ at the gross price of housing $(1+\tau_t^h) p_{h,t}$ less the discounted marginal utility of next period's consumption $\beta c_{t+1}^{-\mu^c}$ achieved from selling the house after depreciation $(1-\delta_h)$ at the price $p_{h,t+1}$. Equation (8), which is fairly standard, describes the labor supply of a patient household and equates the marginal rate of substitution between consumption and leisure $\frac{n_t^{\mu^n}}{c_t^{-\mu^c}}$ to the net real wage rate $(1-\tau_t^n) w_t$. Equations (9) and (10) are Euler equations with respect to public and private lending.

Impatient households

An impatient household chooses the values of $c_t^{'}$, $h_t^{'}$, $n_t^{'}$ and $b_{t+1}^{'}$ to maximize (2), subject to the budget constraint (5) *and* the collateral constraint (1), leading to the first-order conditions

$$h_t^{'-\mu^h} = \left(1+\tau_t^h\right) p_{h,t} c_t^{'-\mu^c} - \beta' c_{t+1}^{'-\mu^c} \left(1-\delta_h\right) p_{h,t+1} + \omega_t m p_{h,t+1} \quad (11)$$

$$n_t^{'\mu^n} = \left(1-\tau_t^n\right) w_t c_t^{'-\mu^c} \quad (12)$$

$$\omega_t = \frac{c_t^{'-\mu^c} - \beta' c_{t+1}^{'-\mu^c} R_t}{R_t} \quad (13)$$

and the complementary slackness conditions

$$\omega_t \left(b_{t+1}^{'} + m p_{h,t+1} h_t^{'}\right) = 0,\ b_{t+1}^{'} + m p_{h,t+1} h_t^{'} \geq 0,\ \omega_t \geq 0.$$

Equation (11) describes the housing demand of an impatient household. The term $\omega_t m p_{h,t+1}$ stems from the collateral constraint, with ω_t being the multiplier on this constraint. Equation (12) is the labor-supply function of an impatient household. Equation (13) is the modified Euler equation resulting from the fact that the impatient household is borrowing constrained. In the steady state, the collateral constraint will be binding, as we can see from (10), which becomes $\frac{1}{R} = \beta$ and (13), leading to $\omega = c^{'-\mu^c}\left(1/R - \beta'\right) = c^{'-\mu^c}\left(\beta - \beta'\right) > 0$. Finally, from the complementary slackness conditions, we get $b' + m p_h h' = 0 \Leftrightarrow b' = -m p_h h'$.

Furthermore, the transversality conditions $\lim_{t\to\infty} \beta^t u_t^c \frac{-b_{t+1}^g}{R_t^g} = 0$ and $\lim_{t\to\infty} \beta^t u_t^c \frac{b_{t+1}^{'}}{R_t} = 0$ must hold, of which the latter is redundant due to the collateral constraint that is more restrictive.

Firms

In both sectors, the representative firm maximizes profits according to $\max_{n_{c,t}^T} \Pi_{c,t} = \max_{n_{c,t}^T} \left(n_{c,t}^T - w_t n_{c,t}^T\right)$ in the final consumption goods sector and $\max_{n_{h,t}^T} \Pi_{h,t} = \max_{n_{h,t}^T} \left(p_{h,t} n_{h,t}^T - w_t n_{h,t}^T\right)$ in the housing sector, leading to the first-order conditions

$w_t = 1$ and $p_{h,t} = 1$.

Aggregate resource constraint

Finally, due to identical production technologies and perfect mobility of labor between the two sectors, the aggregate resource constraint is given by (see Appendix A.1)

$$c_t^T + g_t + p_{h,t}h_t^T = y_{c,t} + p_{h,t}y_{h,t} + (1-\delta_h)p_{h,t}h_{t-1}^T. \quad (14)$$

2.5 The Ramsey Problem

We assume that the government has access to a commitment technology and is able to bind itself to its policy. The government chooses the values of h_t, c_t, n_t, $h_t^{'}$, $c_t^{'}$, $n_t^{'}$ and the tax rates τ_t^h, $\tau_t^{'h}$ and τ_t^n in order to maximize social welfare, subject to the private-sector equilibrium conditions, the resource and the implementability constraint, while financing an exogenous stream of government expenditures $\{g_t\}_{t=0}^{\infty}$. Following Monacelli (2008), in this economy with two types of agents, social welfare is measured by the weighted sum of utility of the two types

$$\sum_{t=0}^{\infty} \beta^t su(c_t, h_t, n_t) + \beta^{'t}(1-s)u(c_t^{'}, h_t^{'}, n_t^{'})$$

and the aggregate discount rate is defined as $\beta^A = \beta^s \beta^{'(1-s)}$ to be used as the discount rate for the constraints. For the mathematical formulation of the Ramsey problem, see Appendix A.2.1. The first-order conditions of the Ramsey problem and the steady state are derived in Appendix A.2.3.

3. Results

This section presents and discusses optimal taxation results of the model. First, as a natural starting point of the analysis, results for the representative-agent version, which can be derived analytically, will be given. The relation of these results to existing literature on optimal taxation will be discussed. Afterwards, numerical results for the full version of the model will be given and compared with the results of the representative-agent version in order to point out the role of the collateral constraint. Finally, we will compare the role of the difference in discount rates against the role of the collateral constraint and present sensitivity analyses.

3.1 Representative-Agent Version

By setting the discount rate of the impatient agents equal to that of the patient agents, $\beta' = \beta$, the model collapses to a representative-agent version. For this version, we can derive analytical solutions for the steady-state tax rates, which are the labor income tax τ^n and the housing property tax τ^h. For the derivation of the analytical solutions for the representative-agent version, the interested reader is referred to Polattimur (2013).

The optimal steady-state tax rate on labor income is given by

$$\tau^n = \frac{\phi(\mu^n + \mu^c)}{1+\phi(1+\mu^n)} > 0 \text{ for } \phi > 0,$$

and is positive for $\phi > 0$. It only depends on the multiplier on the implementability constraint $\phi \geq 0$ and the parameters μ^c and μ^h.

The optimal steady-state tax rate on housing is given by

$$\tau^h = \frac{\phi}{1-\phi} \underbrace{\frac{\mu^h - \mu^c}{\mu^h - 1}}_{(i)} \underbrace{(1-\beta(1-\delta_h))}_{(ii)}. \tag{15}$$

This equation reflects two features of housing: *(i)* can be attributed to the fact that housing delivers utility like consumption and *(ii)* to the durability of housing.

For $\phi > 0$, the sign of the tax rate (related to the question of whether housing should be taxed or subsidized) depends on the parameters μ^h and μ^c. For the sign of τ^h, the term *(ii)* in (15) can be discarded, since $1 - \underbrace{\beta(1-\delta_h)}_{\in(0,1)}$ is positive. Here, the analysis has to be restricted to values of $\phi < \phi^* = \frac{1}{\mu^h - 1}$, since for larger values the second derivatives become positive, resulting in minima (see Polattimur (2013)).

As mentioned before, the sign of τ^h only depends on the term *(i)* in (15). From principles of optimal taxation, we know that goods with lower elasticities should be taxed at a higher rate. Since we do not consider a consumption tax at all, whether housing should be taxed or subsidized depends on whether its intertemporal elasticity of substitution is lower or higher than the one of consumption. There are three cases:

1) For $\mu^c = \mu^h$ housing and consumption should be treated identically due to identical intertemporal elasticities of substitution, leading to an optimal tax rate on housing of zero.

2) If the elasticity of housing is smaller than the one of consumption, i.e., $\frac{1}{\mu^c} > \frac{1}{\mu^h} \Leftrightarrow \mu^c < \mu^h$, the optimal housing tax rate is positive.

3) For $\mu^c > \mu^h$ the optimal housing tax rate is negative, since the elasticity of consumption is smaller than the one of housing.

These results are compatible with those of Eerola and Määttänen (2009), who consider a more general representative-agent framework with capital and optimal taxation of capital in addition to housing.

While the term *(ii)* in (15) is irrelevant for the sign of τ^h, it has a large effect on the size of it. For the baseline calibration (see Table 1), for instance, it reduces the housing tax by more than 97%. However, the higher δ_h is, i.e., the lower the durability of housing is, the smaller the impact is of *(ii)* on the size of τ^h. Notice that *(ii)* disappears for the case $\delta_h = 1$, where durability of housing is assumed away and housing fully depreciates within one period.

3.2 Results of the Full Version

Since analytical results are not available for the full version, we consider numerical results for the steady state, where the collateral constraint is binding, as we have seen before in Section 2.4. For comparison, we also give numerical results for the representative-agent version and the baseline calibration.

3.2.1 Calibration

In this section, the baseline calibration of the model is described. Following Iacoviello (2005), one time period is set to one quarter and the discount factor of patient households to $\beta = 0.99$, leading to a steady-state gross real interest rate of *R = 1.01*, which is equivalent to an annual real interest rate of 4%. The discount factor of impatient households is set to $\beta' = 0.95$ by Iacoviello (2005) as a compromise of the estimates given in the literature, which is adopted here. However, in Section 3.3, we will consider a variation in β'

between 0.95 and 0.97 to see how this affects the result. In order to get a wage share of patient households equal to $\frac{swn}{swn+(1-s)wn'}=0.64$ as in Iacoviello (2005), we set *s = 0.62*, while we will also show in the sensitivity analyses how a variation in population shares alters the results. Moreover, we set the pledgeable fraction of housing to *m = 0.55*, resulting from an estimation of Iacoviello (2005). Hence, an impatient agent can only borrow up to 55% of the value of his house. We will also consider in Section 3.3 how a variation in *m* between 0 and 1, which covers all relevant values for *m*, affects the results. The depreciation rate of housing is set according to Davis and Heathcote (2005), who estimate an annual rate of 1.41%. We thus set $\delta_h = 0.0035$ for a quarter.

In the calibration of the utility parameters μ^c and μ^n , we follow King and Rebelo (1999), who say that the basic RBC model with log utility in consumption implies a labor supply elasticity of 4. Hence, we set $\mu^c = 1$ and $\mu^n = 1/4$, while we will also conduct robustness checks for both of these parameters in Section 3.4.

Since the aim of this paper is to evaluate optimal taxation of housing, the utility parameter of housing μ^h is calibrated in order to match an empirical fact about housing. According to Iacoviello (2009), where some stylized facts about housing are listed and should be matched when calibrating models of housing, total housing wealth was on average 1.5 times as large as annual GDP in the US between 1952 and 2008. Therefore, we set the parameter μ^h in order to match this value. Since in the model one time period is one quarter, and therefore *y* in the notation of the model denotes quarterly GDP, we have to multiply this value by four in order to match the ratio of total housing stock to quarterly GDP of $\frac{h^T}{y} = 6$. This is achieved by setting $\mu^h = 1.75$, leading to an elasticity of $\frac{1}{\mu^h} = 4/7$. In addition, we will also give sensitivity results concerning the parameter μ^h in Section 3.4.

For the calibration of governmental variables *g* and b^g , we use data from the World Bank (2012a, 2012b). In 2010, US general government final consumption expenditures amounted to 17% of annual GDP. Since both government expenditures and GDP are flow variables, the ratio is the same for a time period of one quarter, $\frac{g}{y} = 0.17$. Moreover, US total central government debt made up 76.8% of annual GDP in 2010. Since government debt is a stock variable, this value again has to be multiplied by four. Hence, the ratio that we have to match in terms of quarterly GDP is given by $\frac{b^g}{y} = 3$. These values of the governmental variables are achieved by setting $g = 0.172$ and $b^g = 3.1$. The baseline parameter calibration is summarized in Table 1.

Given this parameter calibration, we compute the steady state numerically, which delivers the optimal values of consumption, housing, and labor for both types of agents as well as the optimal tax rates τ^h, τ'^h and τ^n.

Table 1. Baseline Parameter Calibration

Description	Source/Target	Parameter	Value
Discount factor patient households	Iaco. 2005	β	0.99
Disc. factor impatient households	Iaco. 2005	β'	0.95
Pledgeable fraction of housing	Iaco. 2005	m	0.55
Depreciation rate of housing	D&H 2005	δ_h	0.0035
Share of patient households	Wage share = 0.64	s	0.62
Inverse of Frisch elasticity	K&R 1999	μ^n	1/4
Inverse of IES in consumption	K&R 1999	μ^c	1
Inverse of IES in housing	$h^T / y = 6$	μ^h	1.75
Government expenditures	$g / y = 0.17$	g	0.172
Government debt	$b^g / y = 3$	b^g	3.1

3.2.2 Numerical results

The results of the full and the representative-agent version for the baseline calibration are summarized in Table 2. Notice that the optimal tax rate on housing in the representative-agent version is close to zero but still positive ($\tau^h = 0.2\%$), while for the full model we get two housing tax rates that both differ markedly from zero. The optimal housing tax rate for patient households is $\tau^h = 1.65\%$, and the one for impatient households $\tau'^h = -2.72\%$. Thus, for the baseline calibration, it is optimal to subsidize housing of impatient/constrained households and to tax patient ones in the full version, while in the representative-agent version housing is taxed at a rate close to zero. The subsidy for impatient households results from the heterogeneity in patience rates and the collateral constraint, which are absent in the representative-agent version.

To see how this subsidy optimally is financed, we consider the government budget (6) in the steady state

$$g+(1-\beta)b^{g}=\tau^{n}n^{T}+s\tau^{h}h+(1-s)\tau^{'h}h'. \tag{16}$$

Expenditures are given by $g+(1-\beta)b^{g}=0.203$ and revenues by $\tau^{n}n^{T}+s\tau^{h}h+(1-s)\tau^{'h}h'=0.1887+0.0668-0.0526=0.203$. We see that the labor income tax finances government expenditures, while the housing subsidy for impatient households is financed for the most part by a housing tax on the patient households. Therefore, the housing tax rate on the patient households is much larger than the tax rate on housing in the representative-agent version. This point becomes clearer when we consider the case $g=b^{g}=0$ (last column of Table 2). For this case, the left-hand side of the government budget (16) is zero, $g+(1-\beta)b^{g}=0$ and there is a large decline in the labor income tax rate. On the right hand of (16), we have revenues from taxing labor income equal to $\tau^{n}n^{T}=0.029$, revenues from taxing housing of patient households given by $s\tau^{h}h=0.069$, and housing subsidies for impatient households equal to $(1-s)\tau^{'h}h'=-0.098$. Once again, we see that the largest part, more than 70%, of housing subsidies are financed by taxing the housing of patient households. This can be interpreted as a redistribution from wealthy i.e., patient, households with higher housing stocks (*h = 6.5)* to poorer households with lower housing stocks (*h' = 5.1*).

To link these results to the empirical findings described in the introduction, we compute the ratio of total housing subsidies to GDP given by $\frac{-(1-s)\tau^{'h}h'}{swn+(1-s)wn'}$. For the baseline calibration, we get a ratio of 5.24%. As a result, according to the model, the subsidies granted in the US that added up to 1.5% of GDP in 2011 seem to have been lower than what would have been optimal. On the other hand, the model is likely to overestimate housing subsidies, since it does not incorporate physical capital. Housing is the only component of wealth in the model, while in the US it accounts for half of total household wealth (see e.g., Iacoviello (2009)).

Moreover, the resulting labor income tax of 19% for the baseline calibration is in the range of the effective average labor income tax estimates in the literature. For instance, Carey and Rabesona (2003) estimate an average effective labor income tax of 23% for the US between 1990 and 2000, while Mendoza et al. (1994) put it at 25% between 1965 and 1988.

Table 2. Numerical Results – Comparison

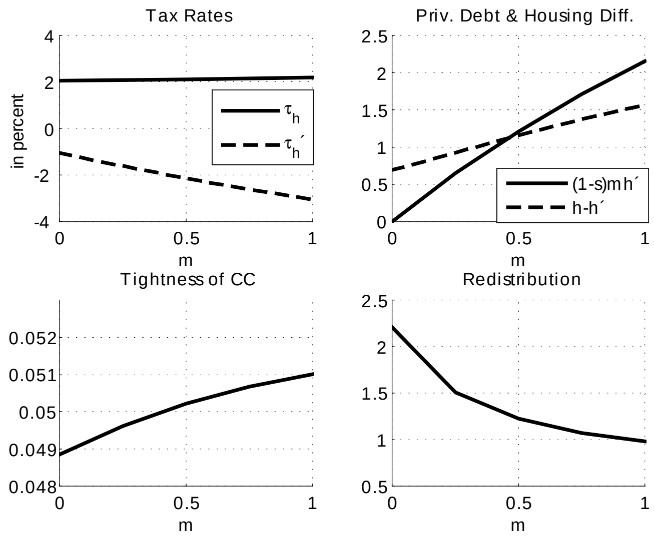
Version	Repr. Agent	Full Version	
Calibration	Baseline	Baseline	$g = b^g = 0$
c	0.8161	0.7999	0.9485
h	9.6310	6.5323	7.4249
n	1.0218	1.0630	1.0954
c'	-	0.8316	1.0136
h'	-	5.0929	6.6759
n'	-	0.9100	0.8398
τ^n	0.1795	0.1878	0.0296
τ^h	0.0020	0.0165	0.0149
τ'^h	-	-0.0272	-0.0388

Finally, our quantitative results can be linked to the recent work of Jacob and Ludwig (2012), who study how housing assistance programs affect labor supplied by the assisted households and provide empirical evidence of a negative effect. In line with their results, our model also predicts that the labor supply of impatient households declines with subsidies. The mechanism is as follows. The higher housing subsidies are, the lower the effective costs of housing for impatient households are; at the same time, the labor income tax is higher. Both lead to a reduction in labor supply, in line with the empirical evidence.

3.3 Discounting vs. Collateral Constraint

The result of subsidizing impatient agents' housing stems from two features of the model, as we have seen in the previous section: the different discount rates of the two types and the collateral constraint, with the former being necessary for the latter. Without different discount rates, the model collapses to become the representative-agent version, where private borrowing and, hence, the collateral constraint are irrelevant.

Figure 1. Effects of varying the pledgeable fraction of housing m for the baseline calibration with s=0.5.



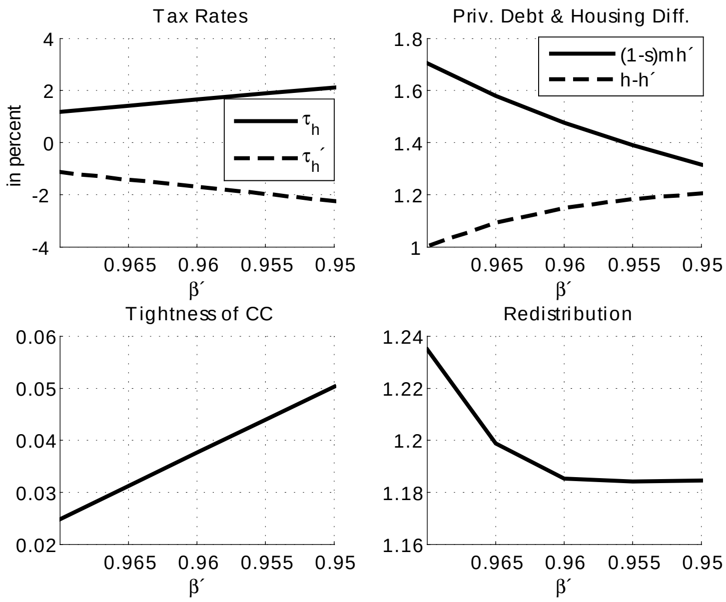
The aim of this section is to analyze how these two features affect housing subsidies. Therefore, we first define the two effects related to these two features. Housing subsidies stemming from the collateral constraint as described by the Ramsey model (in order to soften the constraint and thus can be said to originate from the market friction) are attributed to the *collateral effect*, whereas housing subsidies that purely arise from the difference in discounting (and are accordingly based on preferences) are attributed to the *discount-rate effect.* To identify how housing subsidies are influenced by these two effects, we conduct the following experiment. Let us consider a variation in the pledgeable fraction of housing, *m*, reaching from 0 to 1 and illustrate in Figure 1 how this affects the housing tax rates τ^h and $\tau^{'h}$, private debt given by $(1-s)mh'$, the difference in housing stocks of the two agents, $h-h'$, the tightness of the collateral constraint measured by $\omega = c^{'-\mu^c}(\beta-\beta')$ (see (13)) , and redistribution as measured by the ratio of revenues from taxing housing of the patient agents to the subsidies that impatient agents receive, $red = -\frac{sh\tau^h}{(1-s)h'\tau^{'h}}$. The plots are given for the benchmark calibration but with equal shares, *s = 0.5*, for convenience in aggregation. Then we do the same for a variation in the borrowers' discount rate, between $\beta' = 0.95$ and $\beta' = 0.97$.

First, consider the lower limit $m = 0$, where private borrowing and hence the collateral effect are shut down (see Iacoviello (2005) for a similar experiment). Since the link between borrowing and housing of the impatient household is cut off, in this case the resulting level of subsidies is only due to the discount-rate effect. Then the variation in m between the lower and upper limit $m = 1$, where housing is fully pledgeable, illustrates the role of the collateral effect compared to the discount-rate effect for a given $\beta' = 0.95$. Figure 1 shows that a higher pledgeable fraction of housing leads to a larger amount of private debt (Panel 2) and later to a tighter collateral constraint (Panel 3), resulting in a higher level of housing subsidies for the constrained households (Panel 1, dashed line), whereas the tax rate on the patient agents does not change much (Panel 1, solid line). This is explained by the collateral constraint and with it the parameter m not being directly relevant for the patient agents. Thus, the level of redistribution (Panel 4), as measured here, decreases in m, since housing subsidies to impatient agents rise faster than housing tax revenues from patient ones do.

For $m = 0$, where the collateral channel is shut down, the resulting subsidy is $\tau'^h = -1.04\%$, whereas for the baseline case of $m = 0.55$, it more than doubles, to $\tau'^h = -2.24\%$. This makes clear that housing subsidies not only result from a difference in preference parameters but are also due to the market friction, the collateral constraint. Regarding the rates just mentioned and taking into account that the discount-rate channel dampens the effect of the collateral channel, which is discussed below, more than half of the resulting subsidies can be attributed to the collateral constraint in the baseline calibration.

Figure 2 plots the results for a variation in β'. Notice that β' decreases, i.e., the difference in discount rates increases from left to right on the abscissa. The higher this difference is, the larger the housing subsidy is for impatient agents τ'^h (Panel 1, dashed line) and the housing tax for patient agents τ^h (Panel 1, solid line). In contrast to the variation in m, the variation in β' affects both rates equally. As for a higher m, the level of redistribution (Panel 4) decreases in the difference in discount rates for the same reason. In contrast, unlike a higher m leading to higher borrowing, a larger discount-rate difference lowers borrowing, since it reduces the housing of the impatient agents. Hence, we can conclude that the discount-rate effect dampens the collateral effect in reducing private borrowing.

Figure 2. Effects of varying the impatient agents' discount rate β' for the baseline calibration with s=0.5.



3.4 Sensitivity Analyses

In the previous section, we have seen that the main result of optimality of housing subsidies to impatient agents is robust for variations in the parameters m and β'. In this section, we will check whether it is also robust for changes in the parameters μ^c, μ^h and s. Two interesting questions come to mind here. The first question is: what happens if the intertemporal elasticities are changed, i.e., if $\mu^h < \mu^c$? Since we have seen that this changed the sign of the housing tax in the representative-agent version, one wonders how this change in the parameters will affect optimal taxation in the full version. Another question we will explore is what happens when the share of lenders s is changed. We will consider the case where both types have equal shares $s = 0.5$. Table 3 summarizes the results.

Table 3. Numerical Results – Robustness

		Baseline Calibration with the exception of		
	-	$\mu^h = 1.5$	$\mu^c = 2$	$s = 0.5$
c	0.7999	0.7933	0.8713	0.7950
h	6.5323	9.2023	6.8915	5.9908
n	1.0630	1.0965	1.1581	1.0595
c'	0.8316	0.8396	0.8945	0.8198
h'	5.0929	6.8126	5.6412	4.7861
n'	0.9100	0.8739	0.9383	0.9370
τ^n	**0.1878**	**0.1882**	**0.2124**	**0.1935**
τ^h	0.0165	0.0150	0.0124	0.0212
τ'^h	**-0.0272**	**-0.0281**	**-0.0366**	**-0.0224**

First of all, we can conclude from Table 3 that for every parameter variation we consider, it remains optimal to subsidize the housing of impatient households and to tax the housing of patient ones.

In the third column, where we lower μ^h, housing demand rises, and both types have higher housing stocks ($\frac{h^T}{y} \approx 8.2$) compared to the baseline calibration in Column 2 of Table 3. Although τ^h is lower, tax revenues from taxing the housing of patient agents are higher due to their higher housing stock $h = 9.2$. Therefore, subsidies for impatient households can increase slightly.

In Column 4, we set $\mu^c = 2 > \mu^h = 1.75$, and we see that, in contrast to the representative-agent version, there is no important change in the tax rates. Moreover, τ'^h becomes larger while τ^h decreases, since households attach a higher value to housing compared to consumption. As a result, both types work more to own a larger house, while the labor income tax rises to finance the subsidies.

In Column 5, the share of lenders in the economy is lower than in the baseline calibration. This means that there are fewer wealth-rich households in the economy bearing the tax burden. Therefore, the tax rates τ^n and τ^h are higher, while the subsidy τ'^h is lower. As a result, both types of households have lower consumption and housing levels.

In summary, in every variation we considered, m, β', μ^h and s, the main principle of this paper holds: it is optimal to disburden the impatient and constrained households by subsidizing their housing.

4. Conclusion

Housing subsidies, which are common in many industrialized countries, have been subject to macroeconomic studies for many years. Nevertheless, no definite conclusion has yet been drawn from all this research. While its opponents highlight the inefficiencies associated with the practice caused by the resulting distortions in investment decisions of agents, its proponents argue that subsidies internalize the externalities brought by homeownership.

This paper, in which we have reported on our study of optimal taxation of housing in a borrower-lender framework with different discount rates and where housing is used as collateral for private loans, provides results in favor of housing subsidies. The main finding of this paper is that in such an economy, optimal fiscal policy should disburden impatient borrowers by subsidizing their housing in the presence of collateral constraints. This subsidy has to be financed to the largest extent possible by a housing tax on the patient and unconstrained households and to a smaller extent by a labor income tax. That being the case, redistribution from patient/unconstrained households to impatient/constrained ones would take place.

In this framework, housing subsidies result from two features of the model, the different discount rates of the two types of agents and the collateral constraint. We have seen that, for the baseline calibration, more than half of the subsidy can be attributed to the collateral constraint. Consequently, housing subsidies not only result from the difference in preference parameters but are also from the market friction in our model. Moreover, the sensitivity analyses show that the main result of housing subsidies for constrained households is robust for several parameter variations.

In addition, we considered a representative-agent version of the model, the results of which bore out our intuition and were in line with the principles of optimal taxation. For the baseline calibration, however, it was not optimal to subsidize housing.

This paper gives a rationale for governments to continue providing housing subsidies that goes beyond the externalities that others have focused on in the literature. As such, it indicates a new path for further research. One extension of the model could be the addition of inter-generational heterogeneity in an overlapping-generations model, as in Gervais (2002). The life-cycle behavior of agents could also have substantial implications and should also be accounted for when trying to measure the effects of housing subsidies on social welfare.

A. Appendix

In this Appendix, only the derivation of the solution of the full version is given. To economize on space, we do not present the analytical solution of the representative-agent version of the model here and refer the interested reader to Polattimur (2013), where this is done.

A.1 Aggregate Resource Constraint

Consolidation of the budget constraints (4), (5) and (6) delivers

$$\begin{aligned} & sc_t + (1-s)c_t' + sp_{h,t}\left[\left(1+\tau_t^h\right)h_t - (1-\delta_h)h_{t-1}\right] \\ & +(1-s)\,p_{h,t}\left[\left(1+\tau_t'^h\right)h_t' - (1-\delta_h)h_{t-1}'\right] + g_t \\ = {} & s\left(1-\tau_t^n\right)w_t n_t + (1-s)\left(1-\tau_t^n\right)w_t n_t' \\ & + s\tau_t^h p_{h,t} h_t + (1-s)\tau_t'^h p_{h,t} h_t', \end{aligned}$$

since the terms b_t, b_t' and b_t^g cancel out. With $x_t^T = sx_t + (1-s)x_t'$ for aggregate variables this becomes

$$\begin{aligned} & c_t^T + p_{h,t}\left[\left(1+\tau_t^h\right)h_t^T - (1-\delta_h)h_{t-1}^T\right] + g_t \\ = {} & \left(1-\tau_t^n\right)w_t n_t^T + \tau_t^h p_{h,t} h_t^T, \end{aligned}$$

which can further be simplified to

$$\begin{aligned} & c_t^T + p_{h,t} h_t^T + g_t \\ = {} & w_t n_t^T + p_{h,t}(1-\delta_h)h_{t-1}^T. \end{aligned}$$

Inserting the production functions, we get (14).

A.2 Solution of the Full Version

A.2.1 The Ramsey Problem

The Ramsey problem reads

$$J=\sum_{t=0}^{\infty}\left\{\begin{array}{c}\beta^t su(c_t,h_t,n_t)+\beta'^t(1-s)u(c_t',h_t',n_t') \\ +\left(\beta^A\right)^t\lambda_{t,1}\left[h_t^{-\mu^h}-\left(1+\tau_t^h\right)c_t^{-\mu^c}+\beta(1-\delta_h)c_{t+1}^{-\mu^c}\right] \\ +\left(\beta^A\right)^t\lambda_{t,2}\left[n_t^{\mu^n}c_t^{\mu^c}-1+\tau_t^n\right]+\left(\beta^A\right)^t\lambda_{t,3}\left[n_t'^{\mu^n}c_t'^{\mu^c}-1+\tau_t^n\right] \\ +\left(\beta^A\right)^t\lambda_{t,4}\left[\begin{array}{c}h_t'^{-\mu^h}-\left(1+\tau_t'^h\right)c_t'^{-\mu^c}+\beta'(1-\delta_h)c_{t+1}'^{-\mu^c} \\ -m\left(c_t'^{-\mu^c}c_t^{\mu^c}\beta c_{t+1}^{-\mu^c}-\beta' c_{t+1}'^{-\mu^c}\right)\end{array}\right] \\ +\left(\beta^A\right)^t\lambda_{t,5}\left[\begin{array}{c}-c_t'-\left(1+\tau_t'^h\right)h_t'+\left(1-\tau_t^n\right)n_t'+(1-\delta_h)h_{t-1}' \\ +\frac{mh_t'}{c_t^{-\mu^c}}\beta c_{t+1}^{-\mu^c}-mh_{t-1}'\end{array}\right] \\ +\left(\beta^A\right)^t\lambda_{t,6}\left[\begin{array}{c}-sc_t-(1-s)c_t'-g_t-sh_t-(1-s)h_t' \\ +sn_t+(1-s)n_t'+(1-\delta_h)\left(sh_{t-1}+(1-s)h_{t-1}'\right)\end{array}\right] \\ +\beta^t\lambda_7\frac{c_t^{-\mu^c}}{c_0^{-\mu^c}}\left[g_t-s\tau_t^h h_t-(1-s)\tau_t'^h h_t'-\tau_t^n\left(sn_t+(1-s)n_t'\right)\right]+\beta^t\lambda_7 b_0^g\end{array}\right\}$$

where $\lambda_{t,i}$ denotes the Langrange multiplier on constraint i in period t, while the multiplier λ_7 on the implementability constraint, which is derived in Appendix A.2.2, has no time index, since it is an intertemporal constraint. The first-order conditions of the Ramsey problem are derived in Appendix A.2.3, where the steady state of the problem is also given.

A.2.2 Intertemporal government budget constraint

The intertemporal government budget constraint is derived as follows. We write the government budget (6) for $t+1$ and solve for

$$b_{t+1}^g=s\tau_{t+1}^h p_{h,t+1}h_{t+1}+(1-s)\tau_{t+1}'^h p_{h,t+1}h_{t+1}'+\tau_{t+1}^n w_{t+1}n_{t+1}^T-g_{t+1}+\frac{b_{t+2}^g}{R_{t+1}^g}$$

and insert this in the one for t

$$g_t-\frac{1}{R_t^g}\left[s\tau_{t+1}^h p_{h,t+1}h_{t+1}+(1-s)\tau_{t+1}'^h p_{h,t+1}h_{t+1}'+\tau_{t+1}^n w_{t+1}n_{t+1}^T-g_{t+1}+\frac{b_{t+2}^g}{R_{t+1}^g}\right]+b_t^g$$
$$=s\tau_t^h p_{h,t}h_t+(1-s)\tau_t'^h p_{h,t}h_t'+\tau_t^n w_t n_t^T.$$

This can be rewritten as

$$g_t + \frac{g_{t+1}}{R_t^g} - \frac{b_{t+2}^g}{R_t^g R_{t+1}^g} + b_t^g = s\tau_t^h p_{h,t} h_t + (1-s)\tau_t'^h p_{h,t} h_t' + \frac{s\tau_{t+1}^h p_{h,t+1} h_{t+1} + (1-s)\tau_{t+1}'^h p_{h,t+1} h_{t+1}'}{R_t^g} + \tau_t^n w_t n_t^T + \frac{\tau_{t+1}^n w_{t+1} n_{t+1}^T}{R_t^g}.$$

Incorporating the transversality condition on government debt yields the intertemporal government budget constraint:

$$\sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}\left(R_i^g\right)^{-1}\right) g_t + b_0^g$$

$$= \sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}\left(R_i^g\right)^{-1}\right) s\tau_t^h p_{h,t} h_t + (1-s)\tau_t'^h p_{h,t} h_t' + \sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}\left(R_i^g\right)^{-1}\right)\tau_t^n w_t n_t^T$$

$$\Leftrightarrow \sum_{t=0}^{\infty}\left(\prod_{i=0}^{t-1}\left(R_i^g\right)^{-1}\right)\left[g_t - s\tau_t^h p_{h,t} h_t - (1-s)\tau_t'^h p_{h,t} h_t' - \tau_t^n w_t n_t^T\right] + b_0^g = 0.$$

A.2.3 First-Order conditions and steady state

The first-order conditions of the Ramsey problem can be summarized by

$$\lambda_{t,1} c_t^{-\mu^c} + \overline{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}} s h_t = 0$$

$$\lambda_{t,2} + \lambda_{t,3} - \lambda_{t,5} n_t' - \overline{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}} n_t^T = 0$$

$$\lambda_{t,4} c_t'^{-\mu^c} + \lambda_{t,5} h_t' + \overline{\beta}^t \lambda_7 \frac{c_t^{-\mu^c}}{c_0^{-\mu^c}} (1-s) h_t' = 0$$

for the tax rates

$$\overline{\beta}^{t}\frac{sc_t}{\mu^c}+\lambda_{t,1}\left(1+\tau_t^h\right)+\lambda_{t,2}n_t^{\mu^n}c_t^{2\mu^c}-\lambda_{t,4}mc_t^{\prime-\mu^c}\beta c_{t+1}^{-\mu^c}c_t^{2\mu^c}$$

$$+\lambda_{t,5}mh_t^{\prime}\beta c_{t+1}^{-\mu^c}c_t^{2\mu^c}-\lambda_{t,6}\frac{sc_t^{\mu^c+1}}{\mu^c}-\overline{\beta}^{t}\frac{\lambda_7}{c_0^{-\mu^c}}\left[g_t-s\tau_t^h h_t-(1-s)\tau_t^{\prime h}h_t^{\prime}-\tau_t^n n_t^T\right]$$

$$-\overline{\beta}\lambda_{t-1,1}(1-\delta_h)+\overline{\beta}\lambda_{t-1,4}mc_{t-1}^{\prime-\mu^c}c_{t-1}^{\mu^c}-\overline{\beta}\lambda_{t-1,5}mh_{t-1}^{\prime}c_{t-1}^{\mu^c}=0$$

$$\overline{\beta}^{t}h_t^{-\mu^h}-\lambda_{t,1}\frac{\mu^h}{s}h_t^{-\mu^h-1}-\lambda_{t,6}-\overline{\beta}^{t}\lambda_7\frac{c_t^{-\mu^c}}{c_0^{-\mu^c}}\tau_t^h+\lambda_{t+1,6}\beta^A(1-\delta_h)=0$$

$$-\overline{\beta}^{t}n_t^{\mu^n}+\lambda_{t,2}\frac{\mu^n}{s}c_t^{\mu^c}n_t^{\mu^n-1}+\lambda_{t,6}-\overline{\beta}^{t}\lambda_7\frac{c_t^{-\mu^c}}{c_0^{-\mu^c}}\tau_t^n=0$$

for the patient agents, and

$$\underline{\beta}^{t}\frac{(1-s)c_t^{\prime}}{\mu^c}+\lambda_{t,3}n_t^{\prime\mu^n}c_t^{\prime2\mu^c}+\lambda_{t,4}\left[\left(1+\tau_t^{\prime h}\right)+mc_t^{\mu^c}\beta c_{t+1}^{-\mu^c}\right]-\frac{\lambda_{t,5}}{\mu^c c_t^{\prime-\mu^c-1}}$$

$$-\frac{\lambda_{t,6}(1-s)}{\mu^c c_t^{\prime-\mu^c-1}}-\underline{\beta}\lambda_{t-1,4}\left[1-\delta_h+m\right]=0$$

$$\underline{\beta}^{t}h_t^{\prime-\mu^h}-\lambda_{t,4}\frac{\mu^h}{(1-s)}h_t^{\prime-\mu^h-1}-\frac{\lambda_{t,5}}{(1-s)}\left[\left(1+\tau_t^{\prime h}\right)-mc_t^{\mu^c}\beta c_{t+1}^{-\mu^c}\right]-\lambda_{t,6}$$

$$-\overline{\beta}^{t}\lambda_7\frac{c_t^{-\mu^c}}{c_0^{-\mu^c}}\tau_t^{\prime h}+\beta^A\lambda_{t+1,5}\frac{(1-\delta_h-m)}{(1-s)}+\beta^A\lambda_{t+1,6}(1-\delta_h)=0$$

$$-\underline{\beta}^{t}n_t^{\prime\mu^n}+\lambda_{t,3}\frac{\mu^n}{(1-s)}c_t^{\prime\mu^c}n_t^{\prime\mu^n-1}+\lambda_{t,5}\frac{\left(1-\tau_t^n\right)}{(1-s)}+\lambda_{t,6}-\overline{\beta}^{t}\lambda_7\frac{c_t^{-\mu^c}}{c_0^{-\mu^c}}\tau_t^n=0,$$

for the impatient agents, with $\overline{\beta}^{t}=\frac{\beta^t}{\beta^{At}}=\left(\frac{\beta}{\beta^s\beta^{\prime(1-s)}}\right)^t=\left[\left(\frac{\beta}{\beta^{\prime}}\right)^{1-s}\right]^t$ and $\underline{\beta}^{t}=\frac{\beta^{\prime t}}{\beta^{At}}=\left(\frac{\beta^{\prime}}{\beta^s\beta^{\prime(1-s)}}\right)^t=\left[\left(\frac{\beta^{\prime}}{\beta}\right)^{s}\right]^t$.

Assuming that we are initially in the steady state ($c_0 = c$ for $t = 0$), where variables without subscript henceforth denote steady-state values, we read these conditions as being in the steady state.

$$\lambda_1 c^{-\mu^c} + \lambda_7 sh = 0$$

$$\lambda_2 + \lambda_3 - \lambda_5 n' - \lambda_7 n^T = 0$$

$$\lambda_4 c'^{-\mu^c} + \lambda_5 h' + \lambda_7 (1-s) h' = 0$$

$$\frac{sc}{\mu^c} + \lambda_1 \left[1 + \tau^h - \overline{\beta}(1-\delta_h)\right] + \lambda_2 (1-\tau^n) c^{\mu^c}$$

$$+ \lambda_4 m c'^{-\mu^c} c^{\mu^c} \left(\overline{\beta} - \beta\right) + \lambda_5 m h' c^{\mu^c} \left(\beta - \overline{\beta}\right) - \lambda_6 \frac{sc^{\mu^c+1}}{\mu^c}$$

$$- \lambda_7 c^{\mu^c} \left[g - s\tau^h h - (1-s)\tau'^h h' - \tau^n n^T\right] = 0$$

$$h^{-\mu^h} \left(1 - \lambda_1 \frac{\mu^h}{sh}\right) + \lambda_6 \left[\beta^A (1-\delta_h) - 1\right] - \lambda_7 \tau^h = 0$$

$$- n^{\mu^n} + \lambda_2 \frac{\mu^n (1-\tau^n)}{sn} + \lambda_6 - \lambda_7 \tau^n = 0$$

$$\frac{(1-s)c'}{\mu^c} + \lambda_3 (1-\tau^n) c'^{\mu^c} + \lambda_4 \left[1 + \tau'^h + m\beta - \underline{\beta}(1-\delta_h + m)\right]$$

$$- \frac{\lambda_5}{\mu^c c'^{-\mu^c - 1}} - \frac{\lambda_6 (1-s)}{\mu^c c'^{-\mu^c - 1}} = 0$$

$$h'^{-\mu^h} \left(1 - \lambda_4 \frac{\mu^h}{(1-s)h'}\right) + \lambda_5 \left[\frac{\beta^A (1-\delta_h - m) - 1 - \tau'^h + m\beta}{1-s}\right]$$

$$+ \lambda_6 \left[\beta^A (1-\delta_h) - 1\right] - \lambda_7 \tau'^h = 0$$

$$- n'^{\mu^n} + \lambda_3 \frac{\mu^n (1-\tau^n)}{(1-s)n'} + \lambda_5 \frac{(1-\tau^n)}{(1-s)} + \lambda_6 - \lambda_7 \tau^n = 0.$$

The private-sector equilibrium conditions, which determine the steady state together with the first-order conditions of the Ramsey problem, are given by

$$
\begin{aligned}
h^{-\mu^h} &= c^{-\mu^c}\left[\left(1+\tau^h\right)-\beta\left(1-\delta_h\right)\right] \\
n^{\mu^n} c^{\mu^c} &= \left(1-\tau^n\right) \\
R^g &= R = \frac{1}{\beta} \\
h'^{-\mu^h} &= c'^{-\mu^c}\left[\left(1+\tau'^h\right)-\beta'\left(1-\delta_h\right)+m\left(\beta-\beta'\right)\right] \\
n'^{\mu^n} c'^{\mu^c} &= \left(1-\tau^n\right) \\
c' &= n'\left(1-\tau^n\right)+h'\left[m\left(\beta-1\right)-\delta_h-\tau'^h\right] \\
g+\left(1-\beta\right)b^g &= s\tau^h h+\left(1-s\right)\tau'^h h'+\tau^n\left(sn+\left(1-s\right)n'\right) \\
sc+\left(1-s\right)c'+g &= sn+\left(1-s\right)n'-\delta_h sh-\delta_h\left(1-s\right)h'.
\end{aligned}
$$

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