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Araştırma Makalesi / Research Article

Analysis of wave solutions of (2+1)-dimensional Nizhnik-Novikov-Veselov equation

Tolga AKTÜRK[1*] Çağlar KUBAL[2]

[1]Department of Mathematics and Science Education, Faculty of Education, Ordu University, Ordu, Turkey
[2]Faculty of Arts and Sciences, Department of Mathematics, Ordu University, Ordu, Turkey

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Abstract

In this article, the wave solutions of the (2+1)-dimensional Nizhnik-Novikov-Veselov equation, which is investigated as a mathematical model, are get by using the modified exponential function method (MEFM). When the solution functions found are analyzed, it is determined that hyperbolic and trigonometric functions, which are periodic functions, are also obtained in their rational functions. Two-dimensional, three-dimensional, contour and density graphs representing the wave solutions of the mathematical model found were plotted by determining appropriate parameters.

Keywords: the (2+1)-dimensional Nizhnik-Novikov-Veselov equation; the modified exponential function method; the wave solutions

(2+1)-boyutlu Nizhnik-Novikov-Veselov denkleminin dalga çözümlerinin analizi

Öz

Bu makalede matematiksel model olarak incelenen (2+1)-boyutlu Nizhnik-Novikov-Veselov denkleminin dalga çözümleri modifiye edilmiş üstel fonksiyon metodu (MEFM) kullanılarak elde edilmiştir. Bulunan çözüm fonksiyonları incelendiğinde periyodik fonksiyonlar olan hiperbolik ve trigonometrik fonksiyonların ayrıca rasyonel fonksiyonların da elde edildiği belirlenmiştir. Bulunan matematiksel modelin dalga çözümlerini temsil eden iki boyutlu, üç boyutlu, kontur ve yoğunluk grafikleri uygun parametreler belirlenerek çizilmiştir.

Anahtar Kelimeler: (2+1)-boyutlu Nizhnik-Novikov-Veselov denklemi; modifiye edilmiş üstel fonksiyon metodu; dalga çözümleri

1. Introduction

Throughout the centuries, humanity has witnessed many events that concern different branches of science encountered in nature. With the creation of mathematical models representing them and obtaining their solutions, the opportunity to comment on the behavior of these events has emerged. However, nonlinear partial differential equations, which are mathematical models, are used to understand events in many fields such as physics,

**Sorumlu Yazar / Corresponding Author: tolgaakturkk@gmail.com, https://orcid.org/0000-0002-8873-0424; Çağlar KUBAL: https://orcid.org/0000-0003-2958-7514*

chemistry, biology, engineering, and health. By obtaining solutions of these equations depending on different parameters, it is possible to comment on the behavior of these events. A mathematical model is needed to observe the spread rate of the Covid-19 epidemic, which affects the whole world, depending on which social interaction parameters, and to comment on the course of the epidemic. Thus, comments can be made about the effect of the mutations that may arise from the spreading rate due to social interaction on the epidemic. There are various studies related to this that have been brought to the literature recently. There are various methods in the literature to find solutions for nonlinear partial differential equations: The trial equation method (Du 2010), the new function methods (Bulut et al 2015), the extended trial equation method (Gurefe et al 2013), Kudryashov method (Nuruddeen et al 2018), the sine-Gordon expansion method (Chen & Yan 2005), the sub-equation method (Yokus et al 2021), the generalized Bernoulli sub-equation function method (Baskonus & Bulut 2015), Hirota bilinear method (Wazwaz 2007), two variables (G'/G, 1/G) expansion method (Duran 2020), the exponential function method (Bulut 2017; Duran et al 2017; He & Wu 2006), the generalized exponential rational function method (Duran 2021), the finite difference method (Erdogan et al 2020), the variational iteration method (Sakar et al 2020), direct algebraic method (Duran 2020), and so on. In the solution of the (2+1)-dimensional Nizhnik-Novikov-Veselov equation that we discussed in this study, using the modified exponential function method, graphs of the wave solutions of this mathematical model are included. The many scientist's studies on the (2+1)-dimensional Nizhnik-Novikov-Veselov equation via various methods can be seen at (Peng 2005; Ren & Zhang 2006; Wazwaz 2010; Xu & Deng 2016).
(2+1)-dimensional Nizhnik-Novikov-Veselov equation is as follows (Manukure et al 2019).

$$u_t + au_{xxx} + bu_{yyy} + du_x + eu_y = 3a(uv)_x + 3b(uw)_y, \tag{1}$$

$$u_x = v_y,\ u_y = w_x.$$

In the second part of the study, information about the modified exponential function method is given.
In the third part, two-dimensional, three-dimensional, contour, and density graphs representing wave solutions were drawn by determining the appropriate parameters by applying the modified exponential function method developed to (2+1)-dimensional Nizhnik-Novikov-Veselov equation.
In the last part, comment of the solutions obtained for the (2+1)-dimensional Nizhnik-Novikov-Veselov equation is included.

2. Analysis of the Method

In this section, information about the process of the modified exponential function method will be given. Let us assume that the general form of the nonlinear partial differential equation analyzed in the study is as follows:

$$P\left(u, u_x, u_y, u_t, (uv)_x, (uw)_y, u_{xxx}, u_{yyy}\right) = 0, \tag{2}$$

where $u = u(x, y, t)$ is an unknown function and its partial derivatives with respect to x and t.

Step 1. Let's assume that the traveling wave transformation is as follows,

$$u(x, y, t) = u(\xi), \qquad \xi = k \cdot (x + y - ct), \tag{3}$$

k, c are constants not equal to zero and which will be determined later.

If the partial derivative terms from Eq. (3) are used in Eq. (2), Eq. (3) is converted into a nonlinear ordinary differential equation:

$$N(u, u^2, u'', \dots) = 0, \tag{4}$$

where N is a polynomial dependent on the function u.

Step 2. The solution function we assume is as follows:

$$u(\xi) = \frac{\sum_{i=0}^{n} A_i . [e^{-\vartheta(\xi)}]^i}{\sum_{j=0}^{m} B_j . [e^{-\vartheta(\xi)}]^j} = \frac{A_0 + A_1 . e^{-\vartheta(\xi)} + \dots + A_n . e^{-n\vartheta(\xi)}}{B_0 + B_1 . e^{-\vartheta(\xi)} + \dots + B_m . e^{-m\vartheta(\xi)}}, \tag{5}$$

$A_i, B_j, (0 \le i \le n, 0 \le j \le m)$ constants are then determined and the term $\vartheta = \vartheta(\eta)$ provides the following ordinary differential equation:

$$\vartheta'(\eta) = e^{-\vartheta(\eta)} + \mu e^{\vartheta(\eta)} + \lambda. \tag{6}$$

If Eq. (6) is solved, we get the following solution families (Naher & Abdullah 2013):

Family 1: When $\mu \neq 0, \ \lambda^2 - 4\mu > 0,$

$$\vartheta(\eta) = \ln\left(\frac{-\sqrt{\lambda^2 - 4\mu}}{2\mu} \tanh\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\eta + EE)\right) - \frac{\lambda}{2\mu}\right). \tag{7}$$

Family 2: If $\mu \neq 0, \ \lambda^2 - 4\mu < 0,$

$$\vartheta(\eta) = \ln\left(\frac{\sqrt{-\lambda^2 + 4\mu}}{2\mu} \tan\left(\frac{\sqrt{\lambda^2 - 4\mu}}{2}(\eta + EE)\right) - \frac{\lambda}{2\mu}\right). \tag{8}$$

Family 3: When $\mu = 0, \ \lambda \neq 0, \ \lambda^2 - 4\mu > 0,$

$$\vartheta(\eta) = -\ln\left(\frac{\lambda}{e^{\lambda(\eta + EE)} - 1}\right). \tag{9}$$

Family 4: $\mu \neq 0, \ \lambda \neq 0, \ \lambda^2 - 4\mu = 0,$

$$\vartheta(\eta) = \ln\left(-\frac{2\lambda(\eta + EE) + 4}{\lambda^2(\eta + EE)}\right). \tag{10}$$

Family 5: If $\mu = 0,\ \lambda = 0,\ \lambda^2 - 4\mu = 0,$

$$\vartheta(\eta) = ln(\eta + EE). \tag{11}$$

The constants $A_0, A_1, \ldots, A_n, B_0, B_1, \ldots, B_m, EE, \lambda, \mu$ can be determined later. In Eq. (5), the relationship between n and m can be found using the balance principle between the highest order derivative term of u and the highest nonlinear term.

Step 3. In the statement of Eq. (5), polynomials of $\vartheta(\eta)$ are obtained by using the solution families in Equation (6). The algebraic equation system consisting of $A_0, A_1, \ldots, A_n, B_0, B_1, \ldots, B_m, EE, \lambda, \mu$ is obtained. Then, the traveling wave solutions of Eq. (2) are get by determining the coefficients obtained from Eq. (5).

3. Application

The nonlinear partial differential Eq. (1) analyzed in this study is reduced to the following nonlinear ordinary differential equation model by applying the wave transformation (3).

$$(d + e - c).u + k^2(a + b).u'' - (3a + 3b).u^2 + R = 0. \tag{12}$$

Where R is integral constant. The following equation is obtained by balancing the highest order nonlinear term in Eq. (12) with the term containing the highest order derivative.

$$n = m + 2.$$

If $m = 1$ in the above equation, then $n = 3$ is obtained. In this case, Eq. (5) is as follows,

$$u(\xi) = \frac{A_0 + A_1.e^{-\vartheta(\xi)} + A_2.e^{-2\vartheta(\xi)} + A_3.e^{-3\vartheta(\xi)}}{B_0 + B_1.e^{-\vartheta(\xi)}}. \tag{13}$$

When the derivative concepts in Eq. (12) are obtained and replaced in Eq. (13), an algebraic equation system consisting of coefficients is found. When this system was solved, the following cases were obtained by selecting the coefficients suitable for the method.

CASE 1:

$$A_0 = \frac{1}{6}\left(\frac{\sqrt{-12R + (a+b)k^4(\lambda^2 - 4\mu)^2}}{\sqrt{a+b}} + k^2(\lambda^2 + 8\mu)\right)B_0,$$

$$A_1 = 2k^2\lambda B_0 + \frac{1}{6}\left(\frac{\sqrt{-12R + (a+b)k^4(\lambda^2 - 4\mu)^2}}{\sqrt{a+b}} + k^2(\lambda^2 + 8\mu)\right)B_1,$$

$$A_2 = 2k^2(B_0 + \lambda B_1),$$

$A_3 = 2k^2 B_1,$

$c = d + e - \sqrt{a+b}\sqrt{-12R + (a+b)k^4(\lambda^2 - 4\mu)^2}.$

Using the coefficients determined according to the method, the following exact traveling wave solutions have been found.

Family 1: If, $\mu \neq 0,\ \lambda^2 - 4\mu > 0,$

$$u_{1,1}(x,y,t) = \frac{1}{6}\left(\frac{\sqrt{-12R+(a+b)k^4(\lambda^2-4\mu)^2}}{\sqrt{a+b}} + k^2\left(\lambda^2 - \frac{24\lambda\mu}{\varphi}\right) + 8\mu\left(1 + \frac{6\mu}{\varphi^2}\right)\right). \tag{14}$$

$\left(\omega = \left[\frac{1}{2}(EE + x + y - ct)\sqrt{\lambda^2 - 4\mu}\right], \varphi = \lambda + \sqrt{\lambda^2 - 4\mu}Tanh[\omega]\right).$

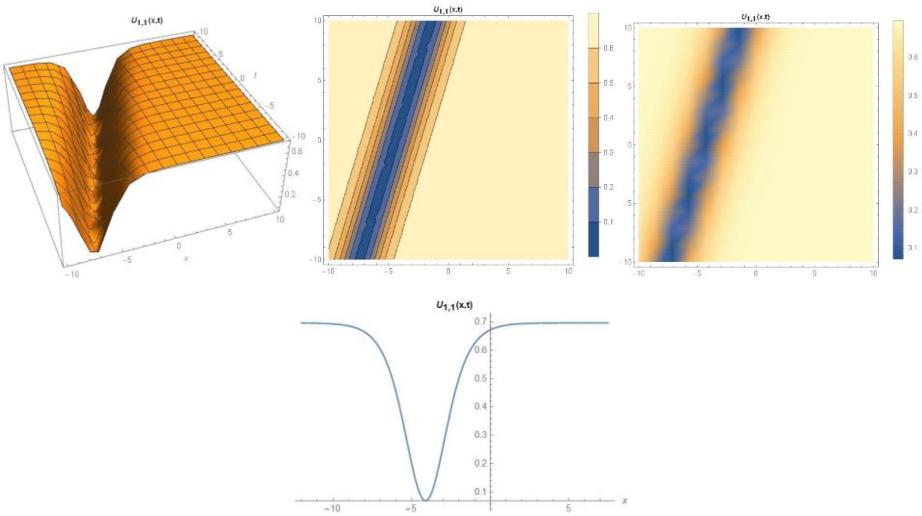

Figure 1. The 3D, contour, and density graphs and 2D graph of Eq.(14) in $k = 0.5, a = 1.2, b = 0.1, d = 2.5, R = -0.75637, e = 1.6, \lambda = 3, \mu = 1, c = 0.3, y = 1, EE = 0.85$ and $t = 1$.

Family 2: When, $\mu \neq 0,\ \lambda^2 - 4\mu < 0,$

$$u_{1,2}(x,y,t) = \frac{1}{6}\left(\frac{\sqrt{-12R+(a+b)k^4(\lambda^2-4\mu)^2}}{\sqrt{a+b}} + k^2\left(\lambda^2 - \frac{24\lambda\mu}{\tau} + 8\mu\left(1 + \frac{6\mu}{\tau^2}\right)\right)\right). \tag{15}$$

$\left(\sigma = \left[\frac{1}{2}(EE + x + y - ct)\sqrt{-\lambda^2 + 4\mu}\right], \tau = \lambda - \sqrt{-\lambda^2 + 4\mu}Tan[\sigma]\right).$

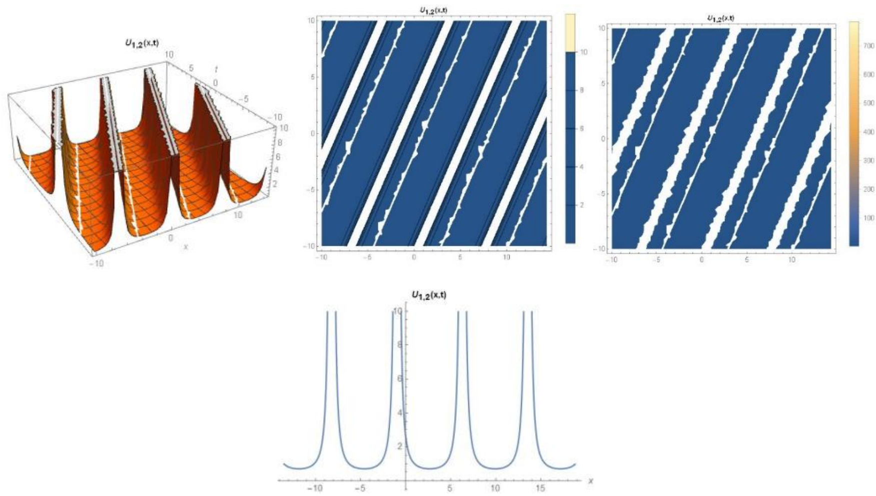

Figure 2. The 3D, contour, and density graphs and 2D graph of Eq.(15) in $k = 0.5, a = 1.2, b = 0.1, d = 2.5, R = -0.75637, e = 1.6, \lambda = 1, \mu = 1, c = 0.529286, y = 1, EE = 0.85$ and $t = 1$.

Family 3: $\mu = 0, \lambda \neq 0,\ \lambda^2 - 4\mu > 0,$

$$u_{1,3}(x,y,t) = \frac{1}{6}\left(\frac{\sqrt{-12R+(a+b)k^4\lambda^4}}{\sqrt{a+b}} + k^2\lambda^2\left(1 + 3Csch\left[\frac{1}{2}(EE + x + y - ct)\lambda\right]^2\right)\right). \quad (16)$$

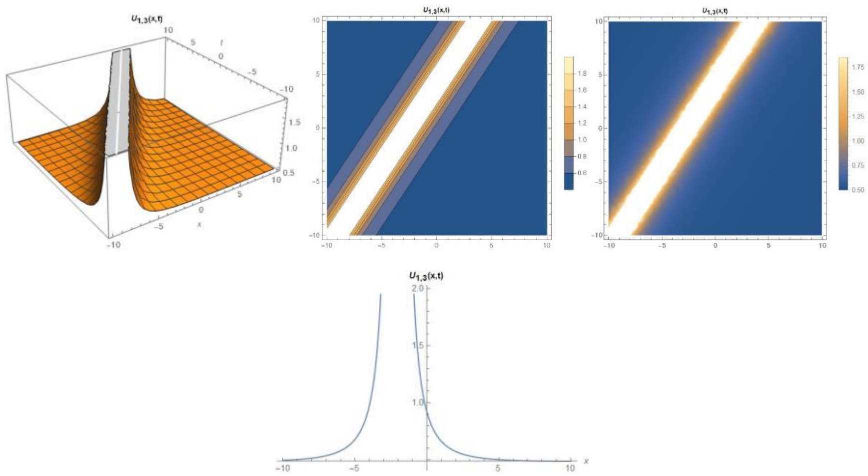

Figure 3. The 3D, contour, and density graphs and 2D graph of Eq.(16) in $k = 0.5, a = 1.2, b = 0.1, d = 2.5, R = -0.75637, e = 1.6, \lambda = 1, \mu = 0, c = 0.649638, y = 1, EE = 0.85$ and $t = 1$.

Family 4: If, $\mu \neq 0, \lambda \neq 0,\ \lambda^2 - 4\mu = 0,$

$$u_{1,4}(x,y,t) = \frac{1}{6}\left(\frac{\sqrt{-12R+(a+b)k^4(\lambda^2-4\mu)^2}}{\sqrt{a+b}} + k^2\left(\lambda^2\left(-2 + \frac{12}{(2+(EE+x+y-ct)\lambda)^2}\right) + 8\mu\right)\right). \quad (17)$$

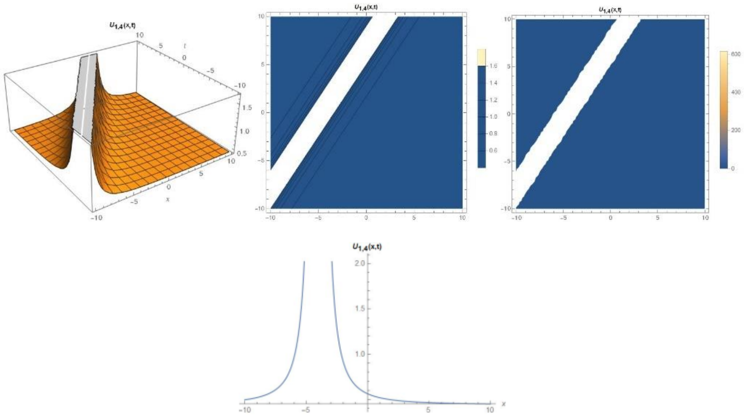

Figure 4. The 3D, contour, and density graphs and 2D graph of Eq.(17) in $k = 0.5, a = 1.2, b = 0.1, d = 2.5, R = -0.75637, e = 1.6, \lambda = 2, \mu = 1, c = 0.664979, y = 1, EE = 0.85$ and $t = 1$.

Family 5: $\mu = 0, \lambda = 0,\ \lambda^2 - 4\mu = 0$,

$$u_{1,5}(x, y, t) = \left(\frac{\sqrt{-R}}{\sqrt{3}\sqrt{a+b}} + \frac{2k^2}{(EE+x+y-ct)^2}\right). \tag{18}$$

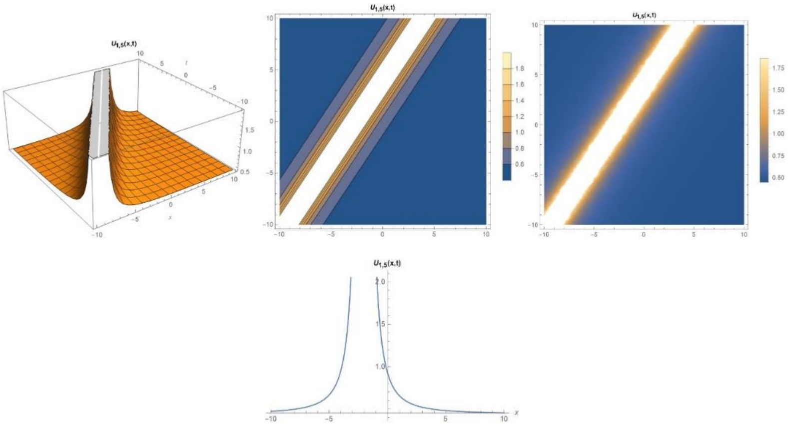

Figure 5. The 3D, contour, and density graphs and 2D graph of Eq.(18) in $k = 0.5, a = 1.2, b = 0.1, d = 2.5, R = -0.75637, e = 1.6, \lambda = 0, \mu = 0, c = 0.664979, y = 1, EE = 0.85$ and $t = 1$.

CASE 2:

$$A_0 = \frac{\left(-c+d+e+(a+b)k^2(\lambda^2+8\mu)\right)B_0}{6(a+b)},$$

$$A_1 = \frac{\left(-c+d+e+(a+b)k^2(\lambda^2+8\mu)\right)A_3}{12(a+b)k^2} + 2k^2\lambda B_0,$$

$$A_2 = \lambda A_3 + 2k^2 B_0,$$

$$B_1 = \frac{A_3}{2k^2},$$

$$R=\frac{-(-c+d+e)^2+(a+b)^2k^4\left(\lambda^2-4\mu\right)^2}{12(a+b)}.$$

Family 1: When, $\mu\neq 0,\ \lambda^2-4\mu>0,$

$$u_{2,1}(x,y,t)=\left(\frac{-c+d+e+(a+b)k^2\left(\lambda^2+8\mu\right)}{6(a+b)}-\frac{4k^2\mu\left(\lambda^2-2\mu+\lambda\,\rho\right)}{(\lambda+\rho)^2}\right). \tag{19}$$

$\left(\omega=\left[\frac{1}{2}(EE+x+y-ct)\sqrt{\lambda^2-4\mu}\right],\rho=\sqrt{\lambda^2-4\mu}Tanh[\omega]\right).$

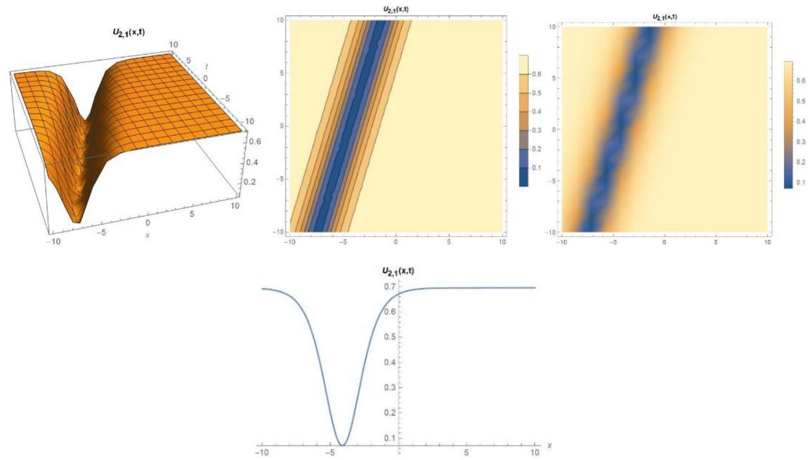

Figure 6. The 3D, contour, and density graphs and 2D graph of Eq.(19) in $k=0.5, a=1.2, b=0.1, d=2.5, R=-0.75637, e=1.6, \lambda=0, \mu=0, c=0.3, y=1, EE=0.85$ and $t=1$.

Family 2: $\mu\neq 0,\ \lambda^2-4\mu<0,$

$$u_{2,2}(x,y,t)=\left(\frac{-c+d+e+(a+b)k^2\left(\lambda^2+8\mu\right)}{6(a+b)}+\frac{4k^2\mu\left(-\lambda^2+2\mu+\lambda\psi\right)}{(\lambda-\psi)^2}\right). \tag{20}$$

$\left(\sigma=\left[\frac{1}{2}(EE+x+y-ct)\sqrt{-\lambda^2+4\mu}\right],\psi=\sqrt{-\lambda^2+4\mu}Tan[\sigma]\right).$

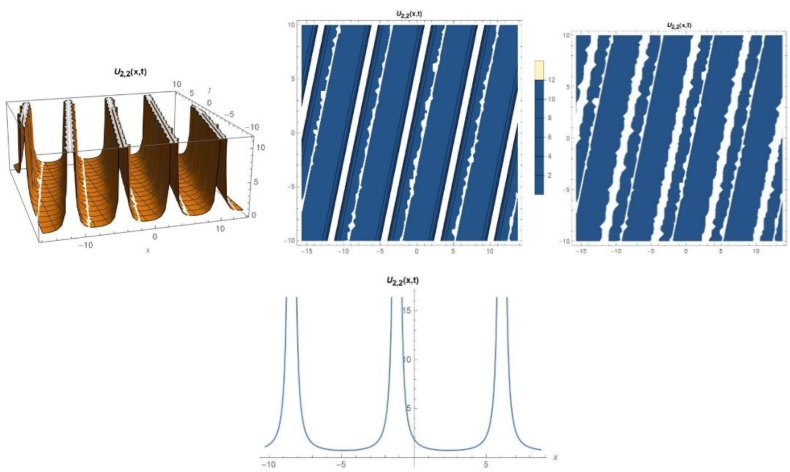

Figure 7. The 3D, contour, and density graphs and 2D graph of Eq.(20) in $k = 0.5, a = 1.2, b = 0.1, d = 2.5, R = -0.86470, e = 1.6, \lambda = 0, \mu = 0, c = 0.3, y = 1, EE = 0.85$ and $t = 1$.

Family 3: If, $\mu = 0, \lambda \neq 0,\ \lambda^2 - 4\mu > 0$,

$$u_{2,3}(x,y,t) = \left(\frac{-c+d+e+(a+b)k^2\lambda^2+3(a+b)k^2\lambda^2 Csch\left[\frac{1}{2}(EE+x+y-ct)\lambda\right]^2}{6(a+b)}\right). \tag{21}$$

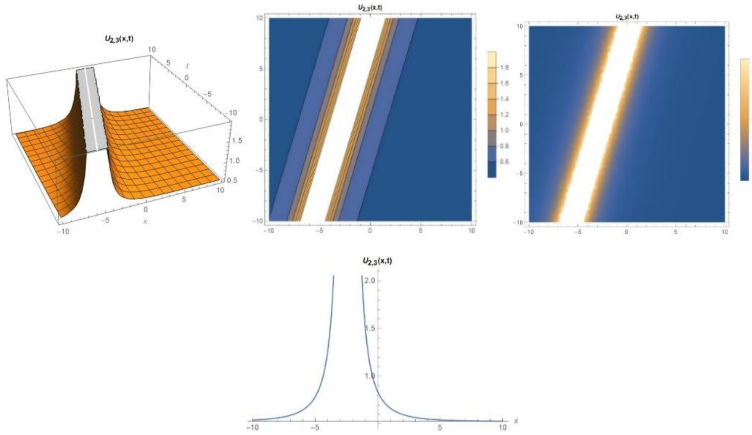

Figure 8. The 3D, contour, and density graphs and 2D graph of Eq.(21) in $k = 0.5, a = 1.2, b = 0.1, d = 2.5, R = -0.91887, e = 1.6, \lambda = 0, \mu = 0, c = 0.3, y = 1, EE = 0.85$ and $t = 1$.

Family 4: $\mu \neq 0, \lambda \neq 0,\ \lambda^2 - 4\mu = 0$,

$$u_{2,4}(x,y,t) = \left(\frac{1}{6}\left(\frac{-c+d+e}{a+b} + 2k^2\left(\lambda^2\left(-1+\frac{6}{(2+EE\lambda+x+y-ct)^2}\right)+4\mu\right)\right)\right). \tag{22}$$

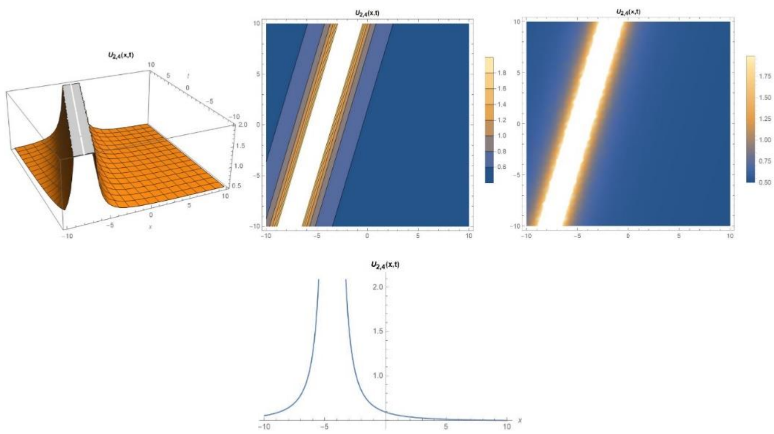

Figure 9. The 3D, contour, and density graphs and 2D graph of Eq.(22) in $k = 0.5, a = 1.2, b = 0.1, d = 2.5, R = -0.92564, e = 1.6, \lambda = 0, \mu = 0, c = 0.3, y = 1, EE = 0.85$ and $t = 1$.

Family 5: $\mu = 0, \lambda = 0, \ \lambda^2 - 4\mu = 0,$

$$u_{2,5}(x, y, t) = \left(\frac{-c+d+e}{6(a+b)} + \frac{2k^2}{(EE+x+y-ct)^2}\right). \tag{23}$$

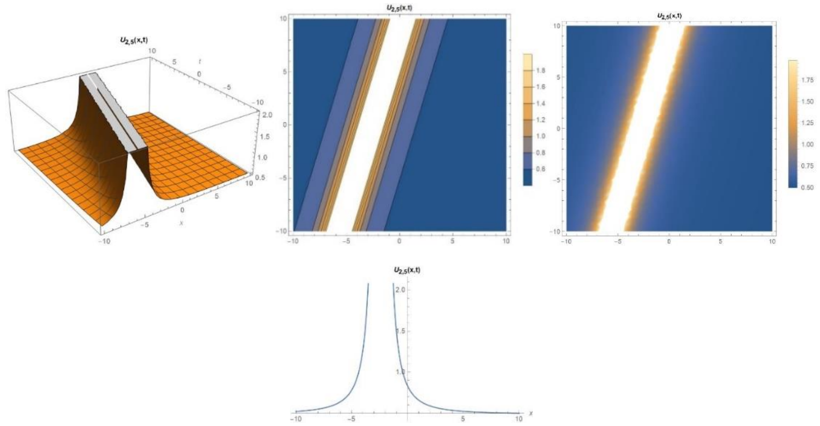

Figure 10. The 3D, contour, and density graphs and 2D graph of Eq.(23) in $k = 0.5, a = 1.2, b = 0.1, d = 2.5, R = -0.75637, e = 1.6, \lambda = 0, \mu = 0, c = 0.3, y = 1, EE = 0.85$ and $t = 1$.

4. Conclusion

In this article, the exact traveling wave solutions of the (2+1)-dimensional Nizhnik-Novikov-Veselov equation have been reached with the help of the modified exponential function method. Two-three dimensional, contour and density graphs representing all exact wave solutions were drawn by determining appropriate parameters. Manukure et al. 2019 related to the solutions of the (2+1)-dimensional Nizhnik-Novikov-Veselov equation examined in the study. In this study, the solutions of the nonlinear partial differential equation have been obtained by using the Hirota bilinear method. The solution functions obtained by the authors are named complexion solutions. In this study, it has been observed that the solution functions obtained by the method we apply to the equation include hyperbolic and trigonometric functions, as well as rational functions, which have periodic function properties. It is especially advantageous to have periodic functions as a solution function because it is elementary to comment on the behavioral models represented by this type of function. When all these results and graphs are commented, it is seen that the MEFM is an effective method to obtain exact solutions of nonlinear partial differential equations.

References

1. Baskonus H M & Bulut H (2015). On the complex structures of Kundu-Eckhaus equation via improved Bernoulli sub-equation function method. *Waves in Random and Complex Media* **25**(4): 720-728
2. Bulut H, Akturk T & Gurefe Y (2015). An application of the new function method to the generalized double sinh-Gordon equation. *AIP Conference Proceedings* **1648**(1): 370014
3. Bulut H (2017). Application of the modified exponential function method to the Cahn-Allen equation. *AIP Conference Proceedings* **1798**(1): 020033
4. Chen Y & Yan Z (2005). New exact solutions of (2+1)-dimensional Gardner equation via the new sine-Gordon equation expansion method. *Chaos, Solitons & Fractals* **26**(2): 399-406

5. Du X-H (2010). An irrational trial equation method and its applications. *Pramana - Journal of Physics* **75**(3): 415-422

6. Duran S (2020). Exact Solutions for Time-Fractional Ramani and Jimbo—Miwa Equations by Direct Algebraic Method. *Advanced Science, Engineering and Medicine* **12**(7): 982-988

7. Duran S (2020). Solitary Wave Solutions of the Coupled Konno-Oono Equation by using the Functional Variable Method and the Two Variables (G'/G, 1/G)-Expansion Method. *Adıyaman Üniversitesi Fen Bilimleri Dergisi* **10**(2): 585-594

8. Duran S (2021). Breaking theory of solitary waves for the Riemann wave equation in fluid dynamics. *International Journal of Modern Physics B* 2150130

9. Duran S, Askin M, & Sulaiman T A (2017). New soliton properties to the ill-posed Boussinesq equation arising in nonlinear physical science. *An International Journal of Optimization and Control: Theories & Applications (IJOCTA)* **7**(3): 240-247.

10. Erdogan F, Sakar M G & Saldır O (2020). A finite difference method on layer-adapted mesh for singularly perturbed delay differential equations. *Applied Mathematics and Nonlinear Sciences* **5**(1): 425-436

11. Gurefe Y, Misirli E, Sonmezoglu A & Ekici M (2013). Extended trial equation method to generalized nonlinear partial differential equations. *Applied Mathematics and Computation* **219**(10): 5253-5260

12. He J-H & Wu X-H (2006). Exp-function method for nonlinear wave equations. *Chaos, Solitons & Fractals* **30**(3): 700-708

13. Manukure S, Chowdhury A & Zhou Y (2019). Complexiton solutions to the asymmetric Nizhnik-Novikov-Veselov equation. *International Journal of Modern Physics B* **33**(11): 1950098

14. Naher H & Abdullah F A (2013). New approach of (G′G)-expansion method and new approach of generalized (G′G)-expansion method for nonlinear evolution equation. *AIP Advances* **3**(3): 032116

15. Nuruddeen R I & Nass A M (2018). Exact solitary wave solution for the fractional and classical GEW-Burgers equations: an application of Kudryashov method. *Journal of Taibah University for Science* **12**(3): 309-314

16. Peng Y-Z (2005). A class of doubly periodic wave solutions for the generalized Nizhnik–Novikov–Veselov equation. *Physics Letters A* **337**(1-2): 55-60

17. Ren Y-J & Zhang H-Q (2006). A generalized F-expansion method to find abundant families of Jacobi Elliptic Function solutions of the (2+1)-dimensional Nizhnik–Novikov–Veselov equation. *Chaos, Solitons and Fractals* **27**: 959-979

18. Sakar M, Yeskindirova M & Saldir O (2020). Numerical investigations to design a novel model based on the fifth order system of Emden-Fowler equations. *Theoretical and Applied Mechanics Letters* **10**(5): 333-342

19. Wazwaz A-M (2007). Multiple-soliton solutions for the KP equation by Hirota's bilinear method and by the tanh–coth method. *Applied Mathematics and Computation* **190**(1): 633-640

20. Wazwaz A-M (2010). Multiple soliton solutions for the (2+ 1)-dimensional asymmetric Nizhnik-Novikov-Veselov equation. *Nonlinear Analysis: Theory, Methods & Applications* **72**(3-4): 1314-1318

21. Xu G-Q & Deng S-F (2016). Painlevé analysis, integrability and exact solutions for a (2 + 1)-dimensional generalized Nizhnik-Novikov-Veselov equation. *The European Physical Journal Plus* **131**: 385

22. Yokus A, Durur H, Kaya D, Ahmad H & Nofal T A (2021). Numerical Comparison of Caputo and Conformable Derivatives of Time Fractional Burgers-Fisher Equation. *Results in Physics*: 104247