SOLVING SOLID TRANSPORTATION PROBLEMS UNDER UNCERTAIN ENVIRONMENT USING GOAL PROGRAMMING

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Keywords

Solid transportation problem, multi-attribute, multi-objective, goal programming

Abstract

A transportation problem can be involving multiple objectives, multiple products, and multiple conveyances. These kinds of transportation problems are named multi-objective multi-attributes solid transportation problems (MMSTP). In this study, a solution based on goal programming has been proposed for MMSTP, in which the supply and demand are uncertain. Moreover, to handle uncertainty, different uncertainty parameters, which are between 0.6 and 0.9, have been used. Then, the results with obtained these parameters are compared by using cost function values. The results indicate that when the uncertainty parameter decreases, the cost increases. Finally, it is shown that an optimal solution can be found using this model through an example.

BELİRSİZ BİR ORTAMDA SAĞLAM ULAŞTIRMA PROBLEMLERİNİN HEDEF PROGRAMLAMA İLE ÇÖZÜLMESİ

Kelimeler

Sağlam ulaşım problemi, çok özellikli, çok amaçlı, hedef programlama

Öz

Bir nakliye sorunu, birden çok hedefi, birden çok ürünü ve birden çok nakliyeyi içerebilir. Bu tür ulaşım sorunları, çok amaçlı çok öz nitelikli sağlam (solid) ulaştırma problemleri (MMSTP) olarak adlandırılır. Bu çalışmada arz ve talebin belirsiz olduğu MMSTP için hedef programlamaya dayalı bir çözüm önerilmiştir. Ayrıca belirsizliği ele almak için 0.6 ile 0.9 arasında değişen farklı belirsizlik parametreleri kullanılmıştır. Daha sonra elde edilen bu parametrelerle elde edilen sonuçlar maliyet fonksiyonu değerleri kullanılarak karşılaştırılmıştır. Sonuçlar, belirsizlik parametresi azaldığında maliyetin arttığını göstermektedir. Son olarak, bir örnek aracılığıyla bu model kullanılarak optimal bir çözüm bulunabileceği gösterilmiştir.

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1. Introduction

The *solid transportation problem* (STP) can be formulated as an extension of the traditional transportation problem (TP) where constraints are sources and destinations. The STP deals with 3 types of constraints: source, destination and another constraints about product type and/or transportation mode. If more than one objective is to be optimized in an STP then the problem is called *multi objective solid transportation problem* (MOSTP). If problem is considered more than one attribute, then it is called *multi-item solid transportation problem* (MSTP). If we consider more than one item and more than one objective at a time in an STP, then it is called a *multi-objective multi-attribute solid transportation problem* (MMSTP). In the classical form, the transport problem minimizes the cost of shipping for some product that is available at some sources (*m*) and required at some destination (*n*). However, in most real world problems the complications of the economic, business and social environment require to be taken into account both the other objective functions and the cost objective function. Therefore, since the classical TP cannot fully meet these needs of the real world problems, MMSTP is encountered. In MMSTP are given on three attribute properties that are supply, demand, and type of product or mode of transport (conveyance) and T homogenous product that will be transported and L objective functions that are conflicting in nature and will be optimized simultaneously. For this reason, MMSTP is an important planning tool for many business and industrial problems and problems that *p* homogeneous products are delivered from an origin to a destination by means of different modes of transport such as cargo flights, ships, goods trains, trucks, etc.

The STP was proposed by Shell (1955). The solution of The STP which is an extension of modified distribution method was introduced by Haley (1962). Bit, Biswal, and Alam (1993) presented an application of fuzzy linear programming to the linear MOSTP. Gen, Ida, Li, and Kubzota (1995) presented an implementation of genetic algorithm (GA) to solve the fuzzy bi-criteria STP (FSTP). Li, Ida, Gen, and Kobuchi (1997) designed a neural network approach to formulate bi-criteria STP. Again, Yang, Liu, Li, Gao, and Ralescu (2015), Das and Bera (2015) and Chen, Peng, and Zhang (2017) as an uncertain theory investigated to solving the bi-criteria STP. Li, Ida, and Gen (1997) also presented an improved GA to solve the MOSTP with fuzzy numbers. Jiménez and Verdegay (1998; 1999a; 1999b) obtained a solution procedure for uncertain STP and developed a parametric approach for solving Fuzzy STP by an evolutionary algorithm.

In recent years, many models and algorithms in the literature have been investigated in this area. Ojha, Das, Mondal and Maiti (2009) formulated with and without entropy in fuzzy environment for the first time, capacitated MOSTP. Another study, which consists of the entropy environment, took place in the study of Dalman (2019). In this MSTP aims to minimize total cost via maximizing the entropy. Baidya, Bera, and Maiti (2015) used entropy based STP to minimize total cost and to maximize total profit. They modeled STP with and without interval entropy function. Pandian and Anuradha (2010) proposed a new method using the concept of zero point method for finding an optimal solution to a STP. Ojha, Das, Mondal and Maiti (2010) formulated a stochastic STP and optimized using the MOSTP. Cui and Sheng (2012) modeled the cost solid transportation problem (CSTP) based on uncertainty theory. In the study of Das, Bera, and Maiti (2018), the total profit is maximized and the carbon emission is minimized by proposing defuzzification process. Baidya, Bera, and Maiti (2013) developed five models and consider three types of uncertainty (stochastic, fuzzy, and hybrid) in different models to a STP with imprecise unit and safety factor. Das, Bera, and Maiti (2019) proposed two different model: first one has fuzzy variables, time and cost, whereas, in the second model all variables are taken into account as fuzzy variables. Then, they solved these models by using two different solving techniques that are reduction method and constrained programming model. Another fuzzy related study made by Ojha, Das, Mondal, and Maiti (2013). In this study, to find optimal shipment schedule, they minimize the total cost. Dalman, Güzel, and Sivri (2016) modeled MMTSP model by using interval programming model by using transportation, costs, supplies, and demands as fuzzy variables. Then, they proposed an interval fuzzy programming model. Kuiri and Das (2020) studied MMSTP by using fuzzy inequality constraints to maximize profit and minimize the total cost. Sengupta, Das, and Bera (2018) studied STP with carbon emission constraints that are taken into account as fuzzy variables. Chakraborty, Jana, and Roy (2014) modeled a MMSTP with fuzzy inequality constraints. Different uncertainty models of MOSTP and MMSTP are investigated by Baidya and Bera (2014), Chakraborty, Jana, and Roy (2014), Jalil, Javaid, and Muneeb (2018), Kundu, Kar, and Maiti (2014b; 2014a), Narayanamoorthy and Anukokila (2015), Pramanik, Jana, and Maiti (2013), Radhakrishnan and Anukokila (2014), Sarma, Das, and Bera (2020), Tao and Xu (2012), and Yang, Liu, Li, Gao, and Ralescu (2015). **Hata! Başvuru kaynağı bulunamadı.** shows that classification of the literatures, which are given below, according to the number of item and number of objective.

Table 1
Summary of the Literature Review

Author(s)	Year	Number of Attribute		Number of Objective		Methodology
		Single	**Multi**	**Single**	**Multi**	
Gen et al.	1995	✓		✓		GA
Li et al.	1997	✓		✓		Neural network
Ojha et al.	2009				✓	-
Ojha et al.	2010				✓	-
Ojha et al.	2013	✓				GA
Baidya, Bera, and Maiti	2015		✓			-
Yang et al.	2015	✓		✓		-
Dalman, Güzel, and Sivri	2016		✓		✓	Interval Programming Model
Chen, Peng, and Zhang	2017	✓		✓		Goal Programming
Das, Bera, and Maiti	2018				✓	-
Sengupta, Das, and Bera	2018				✓	GA + Particle Swarm Optimization
Dalman	2019		✓		✓	Expected value programming + Expected constraint programming
Sarma, Das, and Bera	2020			✓		Mathematical model
Kuiri and Das	2020		✓		✓	
Current Study	2020		✓		✓	Goal Programming

In this study, a MMSTP is modeled using uncertainty theory. To model this STP problem, we use goal programming approach. Since there are a few study in the literature about the implementation of goal programming approach for MMSTP with uncertainty theory, our motivation is to provide goal programming approach for this problem and handle the uncertainty. Then, to make an implementation of the proposed model, the sample problem is investigated. Uncertainty theory was founded by Liu (2007) and refined by Liu (2011), which is a branch of mathematics based on normality, duality, sub additivity, and product axioms. Since the STP is one of the main study area, to the best of the authors' knowledge, there are few study about MMSTP analyzed with the goal programming approach.

The rest of the paper the study is organized as follow. Section 2 contains mathematical models of the problem. In Section 3, the implementation of the MMSTP and its results are given. We conclude in Section 4.

2. Mathematical Model

This section devoted into four subsections: (i) STP model, (ii) goal programming model, (iii) MOSTP with uncertain supplies and demands, (iv) proposed MOSTP with uncertain supplies and demands using goal programming. In these subsections, we will give the mathematical formulations of the models.

2.1 Solid Transportation Model

A type of linear programming known as transportation problems uses very frequently in practical applications. The transportation problem determines optimum shipping pattern (Gakhar 2012).

The STP may be considered as a special case of linear programming problem (Pandian and Anuradha 2010). Assume that there are m sources, n destinations and k conveyances. The mathematical model STP can be given as follows,

$$\min z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{K} c_{ijk} * x_{ijk} \qquad (1)$$

Subject to

$$\sum_{j=1}^{n} \sum_{k=1}^{K} x_{ijk} \leq a_i \qquad i = 1,2,\dots,m \qquad (2)$$

$$\sum_{i=1}^{m} \sum_{k=1}^{K} x_{ijk} \geq b_j \qquad j = 1,2,\dots,n \qquad (3)$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} \geq e_k \qquad k = 1,2,\dots,K \qquad (4)$$

where

$$x_{ijk} \geq 0 \qquad \begin{matrix} (i = 1,2,\dots,m) \\ (j = 1,2,\dots,n) \\ (k = 1,2,\dots,K) \end{matrix} \qquad (5)$$

Constraint (2) ensures that transported product from source i to destination j by using conveyance k cannot exceed the production amount of source i. Constraint (3) provides the required amount of the product from source i to destination j by using conveyance k. Constraint (4), on the other hand, illustrates the transported product with conveyance k. The non-negativity condition of the decision variable is met with constraint (5). The definition of the parameters and variables are given in Table 2. In this mathematical formulation, there is only objective aim that is the minimization of the cost. The objective function is given in the constraint (1)

Table 2
Sets and Variables

Set / Variable	Definition
a_i	Amount of products which are transported from source i
b_j	Amount of products which are transported to destination j
e_k	Amount of products which can be carried by conveyance k
c_{ijk}	A cost of associated with transportation of a unit of the product source i to destination j by conveyance k
x_{ijk}	The decision variable which refer to product than transported from source i to destination j by conveyance k (Kocken and Sivri 2016)

The multi objective TP involves multiple objective functions. A MOSTP model may be written as:

$$z^l_{\min/max} = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} c^l_{ijk} * x_{ijk} \qquad l = 1,2,\dots,L \qquad (6)$$

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \le a_i \qquad i = 1,2,\dots,m \qquad (7)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ge b_j \qquad j = 1,2,\dots,n \qquad (8)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \ge e_k \qquad k = 1,2,\dots,K \qquad (9)$$

where

$$x_{ijk} \ge 0 \qquad (i = 1,2,\dots,m) \quad (j = 1,2,\dots,n) \quad (k = 1,2,\dots,K) \qquad (10)$$

The difference between the STP and MOSTP is observed in the number of objectives. The former has only objective function, whereas the latter has more than one objectives that are provided by using l index. This index shows that the mathematical model has at least two objective functions.

2.2 Goal Programming Model

Multi-criteria decision making (MCDM) refers to making decisions in the presence of multiple, usually conflicting and incommensurable criteria. The goal programming techniques have become a widely used approach in Operations Research. The classical goal programming model have been used to solve MCDM problems (Jayaraman, Colapinto, La Torre, and Malik 2017). One of the example of the goal programming implementation is made by Ozmutlu and Chandra (2001). In this study, they tried to find computation of the mean for a better quality characteristic. The goals in this study are minimizing the cost and maximizing the process mean. The other study about goal programming is presents by Chen, Li, Chen, and Huang (2009). The subject of the study is that machine purchasing for flexible manufacturing cell. Since the machine configurations require multi-objective planning, they goal programming approach. Moreover, they met the goals with four conflict, they used fuzzy structure in their model. Here, the goal programming model was introduced by Charnes and Cooper (1961; 1962) and in the classical formulation it takes the following form:

$$z_{min} = \sum_{l=1}^{L}(d^+_l + d^-_l) \qquad l = 1,2,\dots,L \qquad (11)$$

Subject to

$$\sum_{j=1}^{n} c_{ij}x_j + d^-_l - d^+_l = G_l \qquad l = 1,2,\dots,L \qquad (12)$$

$$\sum_{j=1}^{n} a_{ij}x_j \le b_j \qquad i = 1,2,\dots,m \qquad (13)$$

where

$$x_j \ge 0 \quad j = 1,2,\dots,n; \qquad d^-_l, d^+_l \ge 0 \quad l = 1,2,\dots,L \qquad (14)$$

2.3 Multi-objective STP with Uncertain Supplies and Demands

Uncertainty theory is a branch of axiomatic mathematics for modelling the uncertain quantities using the uncertain variables in uncertain environment. Uncertainty theory was founded by Liu (2007). In order to model an uncertain solid transportation problem (USTP) we shall firstly give definition of uncertainty variables by founded by Liu (2007).

The vector $(x_1, x_2, \dots, x_n)$ is an uncertain vector if and only if $(x_1, x_2, \dots, x_n)$ are uncertain variables. Suppose that $f: \Re^n \to \Re$ is measurable function $x_1, x_2, \dots, x_n$ uncertain variables on the uncertainty space (Γ, L, M). Then $\xi = f(\xi_1, \xi_2, \dots, \xi_n)$ is an uncertain variable defined as $(\gamma) = f\big(\xi_1(\gamma), \xi_2(\gamma), \dots, \xi_n(\gamma)\big)\ \forall\gamma \in \Gamma$.

Let ξ be an n-dimensional uncertain vector, and $f: \Re^n \to \Re$ a measurable function. Then, $f(\xi)$ is an uncertain variable. An uncertain variable ξ is a measurable function from uncertainty space (Γ, L, M) to the set of real numbers, i.e., for any Borel set B of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma | \gamma(\xi) \in B\} \qquad (15)$$

is an event (Liu 2007).

The uncertainty distribution $F(x): \hat{A}$ ® [0,1] of an uncertain variable ξ is defined by

$$F(x) = M\{g\hat{I}\, Gx(g)\ £\ x\} \qquad (16)$$

for any real number *x.*

An uncertain variable ξ is said to be normal if it has an uncertainty distribution

$$F(x) = M\{x£x\} = \left\{1 + \exp\left\{\frac{p(e-x)}{\sqrt{3}s}\right\}\right\} \; x\hat{I}R \qquad (17)$$

denoted by $N(e,\sigma)$, where the parameters e and s^2 the expected value and variance respectively.

An uncertainty distribution $F(x)$ is said to be regular if it has an inverse function $F^{-1}(x)$. The inverse function $F^{-1}(x)$ is called an inverse uncertainty distribution of the uncertain variable ξ. In this case, normal uncertainty distribution $N(e,\sigma)$, is regular and its inverse uncertainty distribution is

$$F^{-1}(r) = e + \frac{\sqrt{3}s}{p} ln\frac{r}{r-1} \qquad (18)$$

In this study, according to the uncertainty theory, the following parameter is used when calculating the uncertainty in cases where the right side values are uncertain in a goal programming.

$$Y = -\left\{\frac{\sqrt{3}s}{p}\ln\frac{r}{r-1}\right\} \qquad (19)$$

Assume that there are m sources, n destinations and k conveyances. The mathematical model of STP can be given as follows,

$$z^l_{min/max} = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} c_{ijk} * x_{ijk} \qquad l = 1,2,\ldots,L \qquad (20)$$

Subject to

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \le a_i + \Psi^1_{\mathrm{i}} \qquad i = 1,2,\ldots,m \qquad (21)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \ge a_i - \Psi^1_{\mathrm{i}} \qquad i = 1,2,\ldots,m \qquad (22)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \le b_j + \Psi^2_{\mathrm{j}} \qquad j = 1,2,\ldots,n \qquad (23)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ge b_j - \Psi^2_{\mathrm{j}} \qquad j = 1,2,\ldots,n \qquad (24)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} \ge e_k \qquad k = 1,2,\ldots,K \qquad (25)$$

where

$$x_{ijk} \ge 0 \qquad (i = 1,2,\ldots,m) \; (j = 1,2,\ldots,n) \; (k = 1,2,\ldots,K) \qquad (26)$$

L is number of objective function,

$\boldsymbol{\Psi^1_{\mathrm{i}}}$ Uncertainty distribution of source *i*,

$\boldsymbol{\Psi^2_{\mathrm{j}}}$ Uncertainty distribution of destination *j*.

2.4 Proposed Multi-objective STP with Uncertain Supplies and Demands Using Goal Programming

The goal programming is minimize the deviations from the objectives. In this condition, the mathematical model is

$$z_{min} = \sum_{l=1}^{L}(d_l^+ + d_l^-) \qquad l = 1,2,\ldots,L \qquad (27)$$

Subject to

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} c_{ijk} * x_{ijk} - d_l^+ + d_l^- = G_l \qquad l = 1 \qquad (28)$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} - d_l^+ + d_l^- = G_l \qquad l = 2,3,\ldots,L \qquad (29)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \le a_i - \Psi^1_{\mathrm{ik}} \qquad i = 1,2,\ldots,m \qquad (30)$$

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} \ge a_i + \Psi^1_{\mathrm{ik}} \qquad i = 1,2,\ldots,m \qquad (31)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \le b_j - \Psi^2_{\mathrm{jk}} \qquad j = 1,2,\ldots,n \qquad (32)$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} \ge b_j + \Psi^2_{\mathrm{jk}} \qquad j = 1,2,\ldots,n \qquad (33)$$

where

$$x_{ijk} \ge 0 \qquad (i = 1,2,\ldots,m) \; (j = 1,2,\ldots,n) \; (k = 1,2,\ldots,K) \qquad (34)$$

G is used to express the target value of each goal set by decision makers.

3. Implementation and Results

This part of the study, there are two subsections. First, the sample problem is defined and then the obtained results of this sample problem are given.

Before we give the results, first we present the feature of computer which is used. In this study, the instances are run on an Intel Core i5-4210 CPU computer with 8 GB RAM. Note that research and publication ethics are complied with in this study.

3.1 Problem

In this part of the study, to give an implementation of the proposed model, we address a sample problem. In the sample problem, there are three sources, four destinations, two different paths, and two products. The parameters and their definitions are given in the .

Table 3.

Table 3
Parameters

Parameters	Definition
$i = 1, \ldots, 3$	Number of sources
$j = 1, \ldots, 4$	Number of destinations
$k = 1,2$	Number of conveyance
$l = a, b$	Number of product

For the unit of the product a, cost values (c_{ijka}), when are incurred from source *i* to destination *j* by using k conveyance, mean (μ) and standard deviation (σ) of supply of sources, and demands for destinations are given in Table 4. For the product b that uses *k* conveyance from source *i* to destination *j*, the values are given in

Table **5**, which has the cost value of the product b (c_{ijka}), mean and standard deviation of supply of sources, and demands for destinations.

Table 4
Cost Values of Product a

		Destinations				Supply of Sources (a_i)	
		1	**2**	**3**	**4**	μ	σ
Sources	**1**	c_{111a}=16 c_{112a}=30	c_{121a}=16 c_{122a}=30	c_{131a}=16 c_{132a}=30	c_{141a}=14 c_{142a}=29	35	1.5
	2	c_{211a}=16 c_{212a}=10	c_{221a}=7 c_{222a}=21	c_{231a}=13 c_{232a}=17	c_{241a}=16 c_{242a}=30	30	1.5
	3	c_{311a}=16 c_{312a}=10	c_{321a}=14 c_{322a}=28	c_{331a}=14 c_{332a}=18	c_{341a}=8 c_{342a}=22	35	2.0
Demand for Destination (b_j)	μ	25	25	20	30	100	
	σ	1.5	1.5	1.5	2.0		

Table 5
Cost Values of Product b

		Destinations				Supply of Sources (a_i)	
		1	**2**	**3**	**4**	μ	σ
Sources	**1**	c_{111b}=14 c_{112b}=12	c_{121b}=14 c_{122b}=18	c_{131b}=20 c_{132b}=6	c_{141b}=8 c_{142b}=12	40	1.5
	2	c_{211b}=14 c_{212b}=4	c_{221b}=14 c_{222b}=8	c_{231b}=18 c_{232b}=20	c_{241b}=12 c_{242b}=18	30	1.5
	3	c_{311b}=20 c_{312b}=10	c_{321b}=8 c_{322b}=10	c_{331b}=8 c_{332b}=4	c_{341b}=16 c_{342b}=20	30	2.0
Demand for Destination (b_j)	μ	20	30	20	30	100	
	σ	1.5	2.0	1.5	2.0		

The problem, which is taken into account and solved in this study, has three objective functions. Decision makers define target values for these objective functions. Cost minimization expressing the first objective function (cost) is determined for three different values. These values were determined as 1700, 1750 and 1800 respectively.

Furthermore, the total product transported from the first conveyance is 120, the total number of products transported from the second conveyance is 80. These objective functions are used by adding positive and negative deviation variables within the scope of the goal programming approach. Then, they are utilized in the model as constraints. The equations (19)-(21) show the constraint forms of the objectives.

$$\left(\sum_{i=1}^{3}\sum_{j=1}^{4}\sum_{k=1}^{2}\sum_{l=a}^{b} c_{ijkl} * x_{ijkl}\right) - d_1^+ + d_1^- = cost \quad (35)$$

$$\left(\sum_{i=1}^{3}\sum_{j=1}^{4}(x_{ij1a} + x_{ij1b})\right) - d_2^+ + d_2^- = 120 \quad (36)$$

$$\left(\sum_{i=1}^{3}\sum_{j=1}^{4}(x_{ij2a} + x_{ij2b})\right) - d_3^+ + d_3^- = 80 \quad (37)$$

To make a new objective function for the problem, deviation variables are used. That is, in this situation, objective function is constructed as follows.

$$z_{min} = d_1^+ + d_1^- + d_2^+ + d_2^- + d_3^+ + d_3^- \quad (28)$$

By considering the uncertainty values, which are calculated for each source and target of each product, two constraints are constructed. In the sample problem, since there are two different products, three different sources, and four destinations, 28 constraints ***((Number of product*(number of sources + number of destinations))*2)*** are created.

While these constraints were established, the uncertainty parameter, that is the r parameter, was accepted as 0.9, 0.8, 0.7 and 0.6, respectively. Moreover, the non-negativity constraint of the transportation quantities of *a* and *b* products is added into the model. The prepared model for the sample problem is given in the Appendix 1.

3.2 Results

When the problems in Appendix 1 are solved, the number of products distributed using for each transport from source *i* to target *j* is given in Table 6.

Table 6
Distribution of Product *a* and Product *b*

Cost				1700				1750				1800
r	0.9	0.8	0.7	0.6	0.9	0.8	0.7	0.6	0.9	0.8	0.7	0.6
x111a	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	19.67	18.60	18.60
x131a	16.82	16.15	15.00	15.00	16.24	16.24	16.24	15.34	18.18	16.15	17.10	17.10
x141a	16.36	17.70	19.30	19.66	18.38	18.38	18.38	19.32	16.73	0.00	0.00	0.00
x211a	0.00	0.00	0.00	0.00	0.47	0.47	0.47	0.00	5.00	0.00	0.00	0.00
x221a	26.82	26.15	25.00	25.00	25.70	25.70	25.70	25.34	26.82	26.15	25.70	25.70
x231a	1.36	2.70	4.30	4.66	0.00	0.00	0.00	4.32	0.00	0.00	3.60	3.60
x232a	0.00	0.00	0.00	0.00	3.14	3.14	3.14	0.00	0.00	2.70	0.00	0.00
x341a	11.46	10.77	9.77	9.89	10.70	10.70	10.70	10.23	15.60	29.29	30.23	30.23
x312a	23.18	23.85	24.30	24.66	25.24	25.24	25.24	25.11	21.82	4.19	5.70	5.70
x131b	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	0.00	0.00
x141b	32.42	29.25	29.07	29.55	29.69	29.69	29.69	30.37	18.80	28.47	24.77	24.77
x132b	6.98	9.22	10.00	10.00	9.38	9.38	9.38	9.18	12.51	12.78	10.00	10.00
x142b	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	8.78	0.00	4.30	4.30
x212b	21.82	21.15	19.30	19.66	20.70	20.70	20.70	20.08	18.87	21.15	20.23	20.23
x222b	10.60	10.38	9.77	9.89	10.23	10.23	10.23	10.37	8.71	8.04	10.47	10.47
x321b	14.76	17.28	19.30	19.66	18.84	18.84	18.84	15.08	18.87	0.00	0.00	0.00
x322b	2.22	0.81	0.00	0.00	0.00	0.00	0.00	4.10	0.00	23.06	18.60	18.60
x332b	14.84	11.93	10.00	10.00	11.32	11.32	11.32	11.16	9.31	8.09	10.70	10.70

The variables that took the value of zero in twelve different solutions were deleted from the table, and the values of the variables that got a value other than zero are given in Table 6. As a result, supply values of sources and demand values of destinations are given in Table 6 for product *a* and product *b*. The values of the deviation variables of each solution and the values of the objective function are given in table 7.

As can be seen, solutions were calculated for 3 different cost values and 4 different r values. The different 12 solutions made are important for decision makers in terms of observing the solutions that can be applied in different situations.

Table 7
Deviation Variables of Solutions

Cost	1700				1750				1800			
r	0.9	0.8	0.7	0.6	0.9	0.8	0.7	0.6	0.9	0.8	0.7	0.6
d1+	0.00	0.00	4.58	27.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
d1-	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
d2+	0.00	0.00	1.74	3.42	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
d2-	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
d3+	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
d3-	0.36	2.66	6.63	5.79	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Optimal Value (Z)	0.36	2.66	12.95	37.19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

The total cost values of the solutions obtained for each r value and objective cost values are given in Table 8.

Table 8
Cost of Solutions /Result of Cost

	Cost		
r	1700	1750	1800
0.9	1700	1750	1800
0.8	1700	1750	1800
0.7	1704.58	1750	1800
0.6	1727.98	1750	1800

4. Conclusion

In this study, a MMSTP, uncertain supplies, uncertain demands and two conveyance under uncertain environment has been investigated. The proposed solution of MMSTP model has been constructed according to some definitions and theorems based on uncertainty theory. In order to solve the proposed programming model under uncertainty theory, this model can be transformed to its deterministic form, and then using goal programming problem its efficient solution can be find. Finally, as an application of the model has been given an example.

Contribution of Researchers

In this research, Nuran GÜZEL, contributed to the examining the problem, literature review and data acquisition and analysis; Selçuk ALP contributed to the examining the problem, literature review and writing; Ebru GEÇİCİ contributed to the literature review, writing and editing of the article according to the spelling rules.

Conflict of Interest

Conflict of interest was not declared by authors.

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Appendix 1

Open form of the Goal Programming Model

$z_{min} = d_1^+ + d_1^- + d_2^+ + d_2^- + d_3^+ + d_3^-$

Subject to

$$16x_{111a} + 30x_{112a} + 16x_{121a} + 30x_{122a} + 16x_{131a} + 30x_{132a} + 14x_{141a} + 29x_{142a} +$$
$$16x_{211a} + 10x_{212a} + 7x_{221a} + 21x_{222a} + 13x_{231a} + 17x_{232a} + 16x_{241a} + 30x_{242a} +$$
$$16x_{311a} + 10x_{312a} + 14x_{321a} + 28x_{322a} + 14x_{331a} + 18x_{332a} + 8x_{341a} + 22x_{342a} +$$
$$14x_{111b} + 12x_{112b} + 14x_{121b} + 18x_{122b} + 20x_{131b} + 6x_{132b} + 8x_{141b} + 12x_{142b} +$$
$$14x_{211b} + 4x_{212b} + 14x_{221b} + 18x_{222b} + 18x_{231b} + 20x_{232b} + 12x_{241b} + 18x_{242b} +$$
$$20x_{311b} + 10x_{312b} + 8x_{321b} + 10x_{322b} + 8x_{331b} + 4x_{332b} + 16x_{341b} + 20x_{342b} - d_1^+ + d_1^- = \text{cost}$$

$$x_{111a} + x_{121a} + x_{131a} + x_{141a} + x_{211a} + x_{221a} + x_{231a} + x_{241a} + x_{311a} + x_{321a} + x_{331a} + x_{341a} +$$
$$x_{111b} + x_{121b} + x_{131b} + x_{141b} + x_{211b} + x_{221b} + x_{231b} + x_{241b} + x_{311b} + x_{321b} + x_{331b} + x_{341b} - d_2^+ + d_2^- = 120$$

$$x_{112a} + x_{122a} + x_{132a} + x_{142a} + x_{212a} + x_{222a} + x_{232a} + x_{242a} + x_{312a} + x_{322a} + x_{332a} + x_{342a} +$$
$$x_{112b} + x_{122b} + x_{132b} + x_{142b} + x_{212b} + x_{222b} + x_{232b} + x_{242b} + x_{312b} + x_{322b} + x_{332b} + x_{342b} - d_3^+ + d_3^- = 80$$

$$x_{111a} + x_{112a} + x_{121a} + x_{122a} + x_{131a} + x_{132a} + x_{141a} + x_{142a} \geq 35 - \psi_{1a}^1$$
$$x_{111a} + x_{112a} + x_{121a} + x_{122a} + x_{131a} + x_{132a} + x_{141a} + x_{142a} \leq 35 + \psi_{1a}^1$$

$$x_{211a} + x_{212a} + x_{221a} + x_{222a} + x_{231a} + x_{232a} + x_{241a} + x_{242a} \geq 30 - \psi_{2a}^1$$
$$x_{211a} + x_{212a} + x_{221a} + x_{222a} + x_{231a} + x_{232a} + x_{241a} + x_{242a} \leq 30 + \psi_{2a}^1$$

$$x_{311a} + x_{312a} + x_{321a} + x_{322a} + x_{331a} + x_{332a} + x_{341a} + x_{342a} \geq 35 - \psi_{3a}^1$$
$$x_{311a} + x_{312a} + x_{321a} + x_{322a} + x_{331a} + x_{332a} + x_{341a} + x_{342a} \leq 35 + \psi_{3a}^1$$

$$x_{111b} + x_{112b} + x_{121b} + x_{122b} + x_{131b} + x_{132b} + x_{141b} + x_{142b} \geq 40 - \psi_{1b}^1$$
$$x_{111b} + x_{112b} + x_{121b} + x_{122b} + x_{131b} + x_{132b} + x_{141b} + x_{142b} \leq 40 + \psi_{1b}^1$$

$$x_{211b} + x_{212b} + x_{221b} + x_{222b} + x_{231b} + x_{232b} + x_{241b} + x_{242b} \geq 30 - \psi_{2b}^1$$
$$x_{211b} + x_{212b} + x_{221b} + x_{222b} + x_{231b} + x_{232b} + x_{241b} + x_{242b} \leq 30 + \psi_{2b}^1$$

$$x_{311b} + x_{312b} + x_{321b} + x_{322b} + x_{331b} + x_{332b} + x_{341b} + x_{342b} \geq 30 - \psi_{3b}^1$$
$$x_{311b} + x_{312b} + x_{321b} + x_{322b} + x_{331b} + x_{332b} + x_{341b} + x_{342b} \leq 30 + \psi_{3b}^1$$

$$x_{111a}+x_{112a}+x_{211a}+x_{212a}+x_{311a}+x_{312a}\geq 25-\psi_{1a}^{2}$$
$$x_{111a}+x_{112a}+x_{211a}+x_{212a}+x_{311a}+x_{312a}\leq 25+\psi_{1a}^{2}$$

$$x_{121a}+x_{122a}+x_{221a}+x_{222a}+x_{321a}+x_{322a}\geq 25-\psi_{2a}^{2}$$
$$x_{121a}+x_{122a}+x_{221a}+x_{222a}+x_{321a}+x_{322a}\leq 25+\psi_{2a}^{2}$$

$$x_{131a}+x_{132a}+x_{231a}+x_{232a}+x_{331a}+x_{332a}\geq 20-\psi_{3a}^{2}$$
$$x_{131a}+x_{132a}+x_{231a}+x_{232a}+x_{331a}+x_{332a}\leq 20+\psi_{3a}^{2}$$

$$x_{141a}+x_{142a}+x_{241a}+x_{242a}+x_{341a}+x_{342a}\geq 30-\psi_{4a}^{2}$$
$$x_{141a}+x_{142a}+x_{241a}+x_{242a}+x_{341a}+x_{342a}\leq 30+\psi_{4a}^{2}$$

$$x_{111b}+x_{112b}+x_{211b}+x_{212b}+x_{311b}+x_{312b}\geq 20-\psi_{1b}^{2}$$
$$x_{111b}+x_{112b}+x_{211b}+x_{212b}+x_{311b}+x_{312b}\leq 20+\psi_{1b}^{2}$$

$$x_{121b}+x_{122b}+x_{221b}+x_{222b}+x_{321b}+x_{322b}\geq 30-\psi_{3b}^{2}$$
$$x_{121b}+x_{122b}+x_{221b}+x_{222b}+x_{321b}+x_{322b}\leq 30+\psi_{3b}^{2}$$

$$x_{131b}+x_{132b}+x_{231b}+x_{232b}+x_{331b}+x_{332b}\geq 20-\psi_{3b}^{2}$$
$$x_{131b}+x_{132b}+x_{231b}+x_{232b}+x_{331b}+x_{332b}\leq 20+\psi_{3b}^{2}$$

$$x_{141b}+x_{142b}+x_{241b}+x_{242b}+x_{341b}+x_{342b}\geq 30-\psi_{4b}^{2}$$
$$x_{141b}+x_{142b}+x_{241b}+x_{242b}+x_{341b}+x_{342b}\leq 30+\psi_{4b}^{2}$$

where

$$\left(x_{ijka}\right)\geq 0 \qquad (k=1,2)(j=1,..,4)(i=1,..,3)$$

$$\left(x_{ijkb}\right)\geq 0 \qquad (k=1,2)(j=1,..,4)(i=1,..,3)$$