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Generalized fermi derivative on the hypersurfaces

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ABSTRACT

In this paper, generalized Fermi derivative, generalized Fermi parallelism, and generalized non-rotating frame concepts are given along any curve on any hypersurface in E^{n+1} Euclidean space. The generalized Fermi derivative of a vector field and being generalized non-rotating conditions are analyzed along the curve on the surface in Euclidean 3-space. Then a correlation is found between generalized Fermi derivative, Fermi derivative, and Levi-Civita derivative in E^3. Then we examine generalized Fermi parallel vector fields and conditions of being generalized non-rotating frame with the tensor field in E^4. Generalizations have been made in E^n.

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1. Introduction

To interpret the universe, it needs to be observed. An observer needs an appropriate frame construction for the definition of its location and its geometric analysis at a proper time. Rest spaces of an observer γ are transported through Levi-Civita parallelism when γ is freely falling, so a fix direction has a null covariant derivative. If γ is not freely falling, the rest space also is not transported by Levi-Civita parallelism anymore. So in order to define "constant" directions a new connection was defined for accelerated observers [5, 7, 15, 19, 20]. This connection which is called Fermi-Walker connection is an isometry between tangent spaces along the curve [5, 7, 9, 13]. But the Fermi-Walker connection is only relevant for accelerating observers. Also this connection and Levi-Civita connection coincide along γ if and only if γ is geodesic. Starting from this point, many scientists have given extensions for the Fermi-Walker transport with several physical motivations [8, 11]. Then in [13] and [14] Pripoae enlarged the context by defining a rich class of generalized Fermi-Walker connections which are relevant for both accelerating and non-accelerating observers. According to the new connection, γ must be able to choose between several parallel transports and not resume itself to the Fermi-Walker one. In [17], we have shown the conditions of generalized Fermi-Walker parallelism along any curve and generalized non-rotating frame in Euclidean 3-space.

In this study, we enlarge the concepts of Fermi derivative to generalized Fermi derivative by using the definition of generalized Fermi-Walker derivative along any curve which was given by Pripoae [13, 14]. Generalized Fermi derivative and generalized Fermi parallelism concepts are considered for Frenet and Darboux frames along curves. Also the generalized non-rotating frame is defined by using the definition of non-rotating frame and the conditions of being generalized non-rotating frame are analyzed along the curve in Euclidean 3-space. Similarly we investigate generalized Fermi-derivative along any curve on hypersurface in E^4 and generalized Fermi parallelism conditions has been obtained. Generalizations have been made in Euclidean $n-$space.

2. Preliminaries

We now recall some basic concepts of Fermi derivative and give the definition generalized Fermi derivative.

Definition 2.1. Let M be a n-hypersurface, $\alpha : I \rightarrow M$ be a unit speed curve and X be a vector field, which is tangent to M along α and also orthogonal to α everywhere. $\frac{\delta X}{\delta s}$ Fermi derivative is defined as

$$\frac{\delta X}{\delta s} = \nabla_T X - \langle \nabla_T X, T \rangle T.$$

Here, ∇ is the Levi-Civita connection of M [2, 10].

Definition 2.2. Let M be a n-hypersurface, $\alpha : I \rightarrow M$ be a unit speed curve and X be a vector field, which is tangent to M along α and also orthogonal to α everywhere. If the Fermi derivative of the vector field X vanishes, i.e., $\frac{\delta X}{\delta s} = 0$, then the vector field X is called Fermi parallel along the curve [2, 10].

Definition 2.3. Let $\alpha : I \rightarrow M$ be unit-speed curve in a n-hypersurface and U, V, W are orthonormal vectors along α. If the Fermi derivative of the vector fields vanish, then $\{U, V, W\}$ is called non-rotating frame [2, 10].

We can give the definition below by using definition 2.1. and definition of generalized Fermi-Walker derivative by Pripoae [13, 14].

Definition 2.4. Let M be a n-hypersurface, $\alpha : I \rightarrow M$ be a unit speed curve and X be a vector field, which is tangent to M along α and also orthogonal to α everywhere. Then there exists a unique $(1, 1)$ −tensor field A along the curve such that

$$\frac{\widetilde{\delta} X}{\widetilde{\delta} s} = \frac{\delta X}{\delta s} + A(X) \tag{1}$$

and

$$\langle A(Z), T \rangle = 0, Z \in \chi^{\perp}(M). \tag{2}$$

Thus, the formulaes 1 and 2 are defined as generalized Fermi derivative. Here, $\frac{\delta X}{\delta s}$ is the Fermi derivative of the vector field X [18].

Definition 2.5. Let M be a n-hypersurface, $\alpha : I \rightarrow M$ be a unit speed curve and X be a vector field, which is tangent to M along α and also orthogonal to α everywhere. If the generalized Fermi derivative of vector field X vanishes, then the vector field X is called generalized Fermi parallel along the curve [18].

Definition 2.6. Let $\alpha : I \rightarrow M$ be unit-speed curve in a n-hypersurface and U, V, W are orthonormal vectors along α. If the generalized Fermi derivative of the vector fields vanish, then $\{U, V, W\}$ is called non-rotating frame with the generalized Fermi terms or shortly generalized non-rotating frame [18].

In the light of these concepts let us continue our study the main subject of the paper.

3. Generalized Fermi Derivative on the Surface

In this section, we investigate generalized Fermi derivative along any unit speed curve on the surface in E^3. Let $\alpha : I \subset \mathbb{R} \rightarrow M$ be unit-speed curve and $\{T, Y, n\}$ be the Darboux frame of the surface, and X is a vector field along the curve α with the corresponding connection ∇. According to Darboux frame we will give a lemma of generalized Fermi derivative and investigate the vector field X, whether it is a generalized Fermi parallel vector field along the curve or not.

Lemma 3.1. Let M be a surface in E^3, $\alpha : I \rightarrow M$ be a unit speed curve and X be a vector field, which is tangent to M along α and also orthogonal to α everywhere, A be $(1, 1)$ −tensor field along α. Generalized Fermi derivative of the vector field X can be expressed as

$$\frac{\widetilde{\delta} X}{\widetilde{\delta} s} = \frac{d(\ln \lambda(s))}{ds} X + A(X).$$

Proof In [10] Karakuş and Yaylı obtained the Fermi derivative as

$$\frac{\delta X}{\delta s} = \frac{d(\ln \lambda(s))}{ds} X.$$

Then from the definition of the generalized Fermi derivative, we obtain the result. □

Theorem 3.2. Let M be any surface in E^3 and $\alpha : I \rightarrow M$ be a unit speed curve. $X = \lambda(s) Y$ vector field along the curve, is generalized Fermi parallel if and only if

$$A(X) = -\frac{d(\ln \lambda(s))}{ds} X.$$

Here $\lambda(s)$ is continuously differentiable function of a real parameter s.

Proof In [10], the Fermi derivative of $X = \lambda(s) Y$ was obtained as

$$\frac{\delta X}{\delta s} = \frac{d(\ln \lambda(s))}{ds} X.$$

Then from the equality $\frac{\widetilde{\delta} X}{\widetilde{\delta} s} = \frac{\delta X}{\delta s} + A(X)$,

$$A(X) = -\frac{d(\ln \lambda(s))}{ds} X$$

is obtained. □

Now, by using Theorem 3, we will give more specifical vector field with constants.

Corollary 3.3. If λ is constant, the Fermi derivative of the vector field $X = \lambda Y$ coincides with the generalized Fermi derivative of X.

Corollary 3.4. Let M be any surface in E^3 and $\alpha : I \rightarrow M$ be a unit speed curve. If α is an asymptotic curve, the normal vector field of the curve N is generalized Fermi parallel along the curve. Moreover, if α is a geodesic curve, the binormal vector field of the curve B is generalized Fermi parallel along the curve.

Proof Let α be an asymptotic curve, since the normal curvature of the curve $\kappa_n = 0$,

$$Y = N$$

and then

$$\frac{\widetilde{\delta} N}{\widetilde{\delta} s} = \frac{\widetilde{\delta} Y}{\widetilde{\delta} s} = 0$$

is obtained. On the other hand, let α be a geodesic curve. Since the geodesic curvature of the curve $\kappa_g = 0$,

$$Y = B$$

and

$$\frac{\widetilde{\delta} B}{\widetilde{\delta} s} = \frac{\widetilde{\delta} Y}{\widetilde{\delta} s} = 0$$

is obtained which gives the result. □

Corollary 3.5. Let M be any surface in E^3 and $\alpha : I \longrightarrow M$ be a unit speed geodesic curve. The vector field $X = \lambda Y$ is Levi-Civita parallel along the curve if and only if the vector field is generalized Fermi parallel, also Fermi parallel, along the curve. Here, λ is constant.

Proof Let $X = \lambda Y$ be Levi-Civita parallel along the curve. In that case the generalized Fermi derivative of X is

$$\frac{\widetilde{\delta} X}{\widetilde{\delta} s} = A\left(X\right).$$

Since λ is constant,

$$A\left(X\right) = 0$$

is obtained.

On the other hand, Let X be generalized Fermi parallel along the curve. The Levi-Civita derivative of X is

$$\nabla_T X = -\kappa_g \lambda T,$$

since α is a geodesic curve

$$\nabla_T X = 0$$

is obtained. □

Theorem 3.6. Let λ be a constant and A, $(1,1)$ −tensor field be $A\left(X\right) = t_r \left(X \wedge T\right)$. X vector field is generalized Fermi parallel along the curve if and only if curve α is curvature of line.

Proof Let X vector field be generalized Fermi parallel along the curve. By using the Lemma 3.1 we obtain

$$\frac{\widetilde{\delta} X}{\widetilde{\delta} s} = \frac{d\left(\ln \lambda\left(s\right)\right)}{ds} X + t_r \left(X \wedge T\right).$$

Thus the generalized Fermi derivative along the curve of X vector field is

$$\frac{\widetilde{\delta} X}{\widetilde{\delta} s} = -t_r \lambda n.$$

Since X is generalized Fermi parallel along the curve

$$t_r = 0.$$

On the other hand, let the curve α be a curvature of line. Since λ is a constant

$$\frac{\widetilde{\delta} X}{\widetilde{\delta} s} = 0.$$

□

4. Generalized Fermi Derivative on the Hypersurface

In this section we will get any curve on the hypersurface in E^4. We will analyze generalized Fermi derivative along any curve on the hypersurface. Then we will examine generalized Fermi parallelism of a vector field along the curve by using Frenet frame and also we will investigate which vector fields are generalized parallel along the curve on the hypersurface in E^4.

Lemma 4.1. Let M be a hypersurface in E^4, $\alpha : I \longrightarrow M$ be a unit speed curve and X be a vector field, which is tangent to M along α and also orthogonal to α everywhere, A be $(1,1)$ −tensor field along α. The generalized Fermi derivative $\frac{\widetilde{\delta} X}{\delta s}$ can be expressed as

$$\frac{\widetilde{\delta} X}{\widetilde{\delta} s} = \frac{dX}{ds} - \left\langle \frac{dX}{ds}, n \right\rangle n - \left\langle \frac{dX}{ds}, T \right\rangle T + A\left(X\right).$$

Proof In [10], Karakuş and Yaylı obtained the Fermi derivative as

$$\frac{\delta X}{\delta s} = \frac{dX}{ds} - \left\langle \frac{dX}{ds}, n \right\rangle n - \left\langle \frac{dX}{ds}, T \right\rangle T$$

by using Levi-civita derivative. Thus from the definition of the generalized Fermi derivative, we obtain the result.

□

Theorem 4.2. Let M be a hypersurface in E^4, $\alpha : I \longrightarrow M$ be a unit speed curve. $X = \lambda_1 N + \lambda_2 B$ vector field along the curve, is generalized Fermi parallel if and only if

$$A\left(X\right) = -\left[\left(\frac{d\lambda_1}{ds} - \tau\lambda_2\right) N + \left(\frac{d\lambda_2}{ds} + \tau\lambda_1\right) B\right].$$

Here $\lambda_1, \lambda_2, \lambda_3$ are continuously differentiable functions of a real parameter s and $\{T, N, B\}$ is Frenet frame along the curve α.

Proof In [10], the Fermi derivative of $X = \lambda_1 N + \lambda_2 B$ was obtained as

$$\frac{\delta X}{\delta s} = \left[\left(\frac{d\lambda_1}{ds} - \tau\lambda_2\right) N + \left(\frac{d\lambda_2}{ds} + \tau\lambda_1\right) B\right]$$

by using $\{T, N, B\}$. Thus by using (1)

$$A(X) = -\left[\left(\frac{d\lambda_1}{ds} - \tau\lambda_2\right) N + \left(\frac{d\lambda_2}{ds} + \tau\lambda_1\right) B\right]$$

is obtained. □

In Theorem 4.2 we gave a general choise of $(1,1)-$ tensor field. Now, let us find it with more specific way.

Corollary 4.3. Let M be any hypersurface in E^4, $\alpha : I \longrightarrow M$ be unit-speed on any curve. $X = \lambda_1 N + \lambda_2 B$ vector field along the curve, is generalized Fermi parallel if and only if

$$A(X) = \tau (X \wedge T).$$

Here λ_1, λ_2 are constants and τ is torsion of the curve.

Proof By using Lemma 4.1, we get

$$\frac{\widetilde{\delta} X}{\widetilde{\delta} s} = -\lambda_2 \tau N + \lambda_1 \tau B + A(X).$$

Let $\frac{\widetilde{\delta} X}{\widetilde{\delta} s} = 0$. Then

$$\begin{aligned} A(X) &= \tau(\lambda_2 N - \lambda_1 B), \\ A(X) &= \tau(\lambda_2 (B \wedge T) + \lambda_1 (N \wedge T)), \\ A(X) &= \tau (X \wedge T) \end{aligned}$$

is obtained.

On the other hand let $(1,1)-$ tensor field be $A(X) = \tau (X \wedge T)$. From the generalized Fermi derivative

$$\begin{aligned} \frac{\widetilde{\delta} X}{\widetilde{\delta} s} &= -\lambda_2 \tau N + \lambda_1 \tau B + A(X), \\ \frac{\widetilde{\delta} X}{\widetilde{\delta} s} &= -\lambda_2 \tau N + \lambda_1 \tau B + \tau (X \wedge T), \\ \frac{\widetilde{\delta} X}{\widetilde{\delta} s} &= 0 \end{aligned}$$

is obtained which gives the result. □

So, if we choose $(1,1)-$ tensor field as $A(X) = \tau (X \wedge T)$, we can give some results about the vector field.

Corollary 4.4. Let M be any hypersurface in E^4, $\alpha : I \longrightarrow M$ be unit-speed on any curve. If $(1,1)-$ tensor field is $A(X) = t_r (X \wedge T)$, the vector fields $\{N, B\}$ are generalized Fermi parallel along the curve α. Here N is the normal vector field and B is the binormal vector field.

Proof From the Corollary 4 the generalized Fermi derivative of the vector field N is

$$\begin{aligned} \frac{\widetilde{\delta} N}{\widetilde{\delta} s} &= \tau B + A(N), \\ \frac{\widetilde{\delta} N}{\widetilde{\delta} s} &= \tau B + \tau (N \wedge T), \\ \frac{\widetilde{\delta} N}{\widetilde{\delta} s} &= 0. \end{aligned}$$

Similarly it can be shown that the vector field B is also generalized Fermi parallel along the curve. □

5. Generalized Fermi Derivative in Euclidean $n-$Space

In this section, we investigate generalized Fermi derivative along the curve in M which is a Riemannian manifold in E^n $(n \geq 4)$. We examine X generalized Fermi parallelism of any vector field along the curve $\alpha : I \longrightarrow M$ which is a unit speed W-curve in M by considering the tangent space $\{V_1, V_2, ..., V_{n-1}\}$ of M and the Levi-Civita connection ∇ of M.

Theorem 5.1. Let $X = \sum_{i=1}^{n-2} \lambda_i V_{i+1}$ be a vector field along the W-curve α. X is generalized Fermi parallel along the curve if and only if

$$\begin{aligned} A(X) = -\Bigg[&\left(\frac{d\lambda_1}{ds} - k_2\lambda_2\right) V_2 \\ &+ \sum_{i=3}^{n-2}\left(\frac{d\lambda_{i-1}}{ds} + k_{i-1}\lambda_{i-2} - k_i\lambda_i\right) V_i \\ &+ \left(\frac{d\lambda_{n-2}}{ds} + k_{n-2}\lambda_{n-3}\right) V_{n-1}. \end{aligned}$$

Here λ_i $(1 \leq i \leq n-2)$ are continuously differentiable functions of a real parameter s and k_i are curvatures of the curve α according to the Levi-Civita connection..

Proof In [9] the Fermi derivative of $X = \sum_{i=1}^{n-2} \lambda_i V_{i+1}$ was obtained as

$$\frac{\delta X}{\delta s} = \left(\frac{d\lambda_1}{ds} - k_2\lambda_2\right) V_2 + \sum_{i=3}^{n-2}\left(\frac{d\lambda_{i-1}}{ds} + k_{i-1}\lambda_{i-2} - k_i\lambda_i\right) V_i + \left(\frac{d\lambda_{n-2}}{ds} + k_{n-2}\lambda_{n-3}\right) V_{n-1},$$

Here, if $3 > n-2$, then $\sum_{i=3}^{n-2}\left(\frac{d\lambda_{i-1}}{ds} + k_{i-1}\lambda_{i-2} - k_i\lambda_i\right) V_i = 0$.

Let X be generalized Fermi parallel. Then from the definition of generalized Fermi derivative

$$\frac{\widetilde{\delta} X}{\widetilde{\delta} s} = 0$$

is obtained. The rest is obvious. □

If we take $n = 4$ in Theorem 5.1 we get Theorem 4.2.

Corollary 5.2. Let $(1,1)-$ tensor field be

$$A(X) = -\left[\left(\frac{d\lambda_1}{ds} - k_2\lambda_2\right)V_2 + \sum_{i=3}^{n-2}\left(\frac{d\lambda_{i-1}}{ds} + k_{i-1}\lambda_{i-2} - k_i\lambda_i\right)V_i + \left(\frac{d\lambda_{n-2}}{ds} + k_{n-2}\lambda_{n-3}\right)V_{n-1}.\right.$$

$\{V_2, V_3, ..., V_{n-1}\}$ is generalized non-rotating along the curve.

Proof Let $(1,1)-$ tensor field be

$$A(X) = -\left[\left(\frac{d\lambda_1}{ds} - k_2\lambda_2\right)V_2 + \sum_{i=3}^{n-2}\left(\frac{d\lambda_{i-1}}{ds} + k_{i-1}\lambda_{i-2} - k_i\lambda_i\right)V_i + \left(\frac{d\lambda_{n-2}}{ds} + k_{n-2}\lambda_{n-3}\right)V_{n-1}.\right.$$

Then

$$\begin{aligned} \frac{\widetilde{\delta}V_2}{\widetilde{\delta}s} &= k_2V_3 + A(V_2), \\ \frac{\widetilde{\delta}V_i}{\widetilde{\delta}s} &= -k_{i-1}V_{i-1} + k_iV_{i+1} + A(V_i),\ 3 \le i \le n-2, \\ \frac{\widetilde{\delta}V_{n-1}}{\widetilde{\delta}s} &= -k_{n-2}V_{n-2} + A(V_{n-1}) \end{aligned}$$

are obtained. By considering $A(X)$,

$$\frac{\widetilde{\delta}V_i}{\widetilde{\delta}s} = 0$$

for all $2 \le i \le n-1$. □

6. Conclusions

In this paper, we have explained the concepts of generalized Fermi derivative, generalized Fermi parallelism, the generalized non-rotating frame along the curve in Euclidean space.

By recalling the concept of Fermi-Walker derivative, which is used for defining "constant" directions and shows us one strict method, that may contain lots of condition to have Fermi-Walker parallelism or non-rotating frame. For example, the condition of Femi-Walker parallelism depends on a solution that contains differential equation system which is not always easy to find a solution [9]. But the generalized case has more flexible way and depends on only one condition which contains choice of (1,1)-tensor field. Moreover, generalized Fermi-Walker derivative is more suitable than the Fermi-Walker one in the terms of the movement of the observer and qualifying conditions [13, 14]. Therefore it is important to analyse this concept.

From this point of view in [17], we analysed the generalized Fermi-Walker derivative and the conditions of being generalized non-rotating frame along any curve in Euclidean space. We have shown that generalized Fermi-Walker derivative has more options. The conditions of generalized Fermi-Walker parallelism and non-rotating frame along the curves weaker than the Fermi-Walker derivative. For example unlike the Fermi-Walker case, Frenet frame is generalized non-rotating frames along all types of curves.

In this study, we enlarged the definition of Fermi derivative to generalized Fermi derivative by using the concept of generalized Fermi-Walker derivative, which has defined by Pripoae [13, 14] before. And also we defined generalized non-rotating frame by using the concept of generalized Fermi derivative. Thus a derivative has obtained which is relevant for both accelerating and non-accelerating observers on the hypersurfaces.

By using these concepts, initially we got a curve on the surface in Euclidean 3-space. We have shown which tensor field is necessary for generalized Fermi parallel vector fields. We proved that if the curve is asymptotic curve, the normal vector field of the curve is generalized Fermi parallel along the curve. Also if the curve is geodesic, its binormal vector field is generalized Fermi parallel vector field along the curve. We have shown the conditions of coinciding Levi-Civita parallelism, Fermi paralellism and generalized Fermi parallelism.

Then we examined any curve on the hypersurface in Euclidean 4-space. We have shown the choise of the tensor field to establish whether the vector field is generalized Fermi parallel along the curve or not. We proved that the normal vector field and the binormal vector field of the curve are generalized Fermi parallel along the curve.

Finally, generalizations have been made in Euclidean n-space. We gave the conditions which are necessary in order that any vector field is Fermi parallel along the curve and the tangent space of the manifold is generalized non-rotating along the curve.

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