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STATISTICAL APPROACH TO QUALITY IMPROVEMENT and SIX SIGMA IMPROVEMENT MODEL (DMAIC)

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ABSTRACT

Six Sigma has been a combination of various statistical methods for organizations to analyze the deviation sources in their processes, products or services since 1990's. For Six Sigma programs it is clear that the target 3.4 DPMO is only achievable with the use of statistical methods. The system based on the performance measurement dealing with the deviation of the output has a development model called DMAIC as a guide.

In this study, the nature and the relationship with the process performance and capability of Six Sigma, calculation principles of sigma level are concerned. Also DMAIC is examined with the know how of the steps and the belonging statistical techniques.

Key Words: Six Sigma, DPMO, DMAIC, Process Capability, Process Performance

ÖZET

Altı Sigma 1990'lardan itibaren tüm dünya üzerinde pek çok kuruluşun ürün, hizmet veya süreçlerindeki değişkenliği analiz etmede kullandıkları ve istatistik tabanlı bir yöntemler bütünüdür. Pek çok Altı Sigma programında 3,4 DPMO seviyesinin ancak istatistiksel araçlarla ulaşılabilecek bir hedef olduğu sonucuna varılmıştır. Kuruluşların performanslarının, çıktılarının değişkenliği ile ifade edildiği bu sistem istatistiksel problem çözme mantığından hareketle oluşturulan bir modele-DMAIC-sahiptir.

Bu çalışmada Altı Sigma'nın temel bileşenleri, yapısı ve kalite geliştirmede kullanılan Süreç performansı ve Süreç yeterliliği gibi istatistiksel süreç kontrol teknikleri ile ilişkisi incelenmektedir. DMAIC iyileştirme modeli ise her aşamada hangi istatistiksel tekniklerin kullanılabileceği bakımından incelenmiştir.

Anahtar Kelimeler: Altı Sigma, DPMO, DMAIC, Süreç Yeterliliği, Süreç Performansı

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INTRODUCTION

For many years, the Greek letter sigma (σ) has been the universally accepted symbol for standard deviation. Standard deviation is, of course, a measure of dispersion, variation or spread. To anyone with an elementary knowledge of the normal distribution, six sigma is the spread about the mean that includes 99.74% of the population.

However, to many employees of Motorola, General Electric, Allied Signal (now part of Honeywell), Bombardier, Black and Decker, ABB and Polaroid, Six Sigma is much more. To these people it is a company-wide transformation that has helped them to become very successful. The companies listed above have publicized their success and have publicly emphasized the part played by Six Sigma in the achievement of this success. Here is an example from the General Electric (GE) Annual Report 1998:

"... we plunged into Six Sigma with a company-consuming vengeance just over three years ago. We have invested more than a billion dollars in the effort, and the financial returns have now entered the exponential phase— more than three quarters of a billion dollars saving beyond our investment in 1998, with a billion and a half in sight for 1999" (Caulcutt R., 2001).

In this study, the meaning of Six Sigma excellence model, some important definitions of Six Sigma, statistical basis of Six Sigma, the calculation mechanism of DPMO and sigma level and their brief definitions, the relationship between process capability and performance with variation via Six Sigma philosophy, and Six Sigma development model namely DMAIC are given. In conclusion section, some comment on understanding the nature of Six Sigma and implementation are given. The main purpose of the study is to understand the nature of Six Sigma and its components like DPMO, DMAIC and sigma level.

WHAT IS SIX SIGMA?

There are many different perspectives on what 'Six Sigma' is. The most well-known description is that "Six Sigma is a highly technical method used by engineers and statisticians to fine-tune products and processes" (Pande, P. S., 2001). Measures and statistics are a key ingredient of Six Sigma improvement- but other perspectives can not be omitted.

Some other definitions are about its goal of near-perfection in meeting customer requirements based on the normality assumptions. ‘Six Sigma’ itself refers to a statistically derived performance target of operating with only 3.4 defects for every million activities or ‘opportunities.’ It’s a goal few companies or processes can claim to have achieved. Motorola- one of the world leaders in the world- is still striving to reach the target.

On the other hand, another definition can be made on its stunning cultural change affect. Considering the companywide commitment to Six Sigma at places like General Electric or Motorola, ‘culture change’ is certainly a valid way to describe Six Sigma.

All these perspectives can be gathered in one definition for Six Sigma. Pande, Neumann and Cavanagh (2000) defined Six Sigma as “a comprehensive and flexible system for achieving, sustaining and maximizing business success”. Six Sigma is uniquely driven by close understanding of customer needs, disciplined use of facts, data, and statistical analysis, and diligent attention to managing, improving, and reinventing business processes.

In Six Sigma, standard deviation measures two things: how much one thing varies from a specific point or target and how much one thing varies from another. In business terms it measures the capability of any given process to perform defect free work.

Sigma—or standard deviation—is used to quantify how good or bad a process is performing their ideal functions. In other words, how many mistakes a company makes, doing whatever it does, from manufacturing steel to delivering the newspaper. Six is the Sigma level of perfection the companies are shooting for.

THE STATISTICAL BASIS OF SIX SIGMA

From a measurement perspective, ‘Six Sigma’ represents a quality level of at most 3.4 defects per million opportunities. The concept arises from the notion of design specifications in manufacturing and the ability of a process to achieve the specifications.

A six sigma quality level corresponds to half of the design tolerance while allowing the mean to shift as much as 1.5 standard deviations from the target.

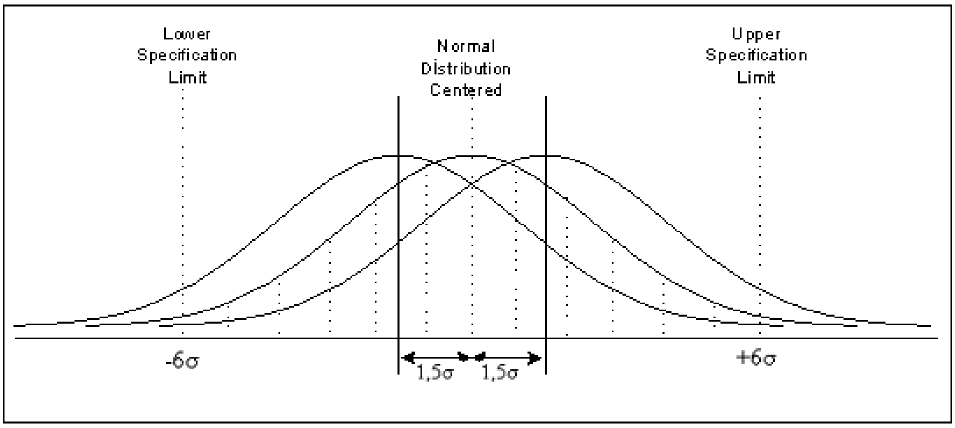

Figure 1: Normal Distribution with 1.5 Standard Deviation Shift

Motorola choose this figure because field failure data suggested that Motorola's processes drifted by this amount on average (Swiney Z., 2005). The allowance of a shift in the distribution is important, because no process can be maintained in perfect control. The impact of six sigma development to the variation of the process is given in Figure 2 with parts-per-million(ppm) defect rate outputs.

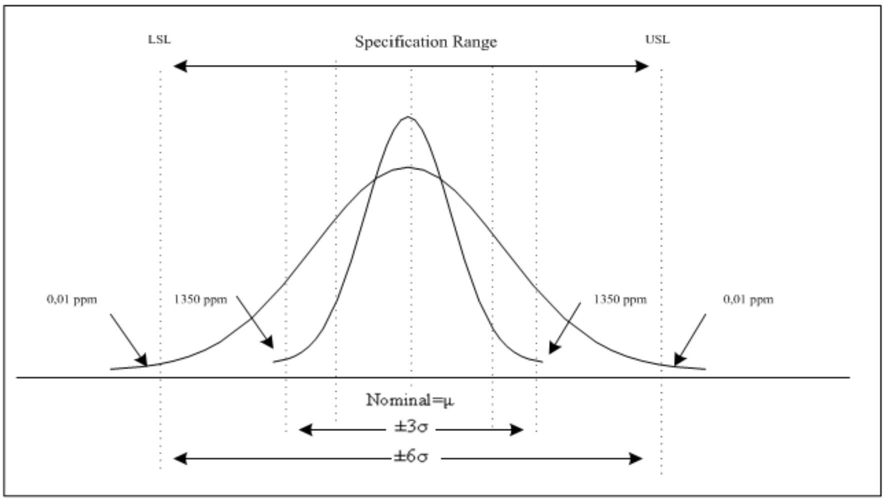

Figure 2: The Impact of Six Sigma Development to the Shape of a Distribution

In Figure 1, the area under the tail of one of the shifted curves beyond the six sigma range (that is, either above or below the tolerance limit) is only 0.0000034, or 3.4 parts per million. Thus if the process mean can be controlled to allow a one-way shift of at most 1.5 standard deviations of the target, then a maximum of 3.4 defects per million can be expected. If it is held exactly on target as it is seen in Figure 2, only one defect per billion would be expected beyond the six sigma range in either tail. If shifts can occur in both directions, then the defect rate at a six sigma level would be at most 6.8 per million; and if held exactly on target, only two billion. (Evans J. R. & Lindsay W.M., 2005)

Many common statistical process control (SPC) plans are based on sample sizes that only allow detection of shifts of about two standard deviations. Thus it would not be unusual for a process to drift this much and not be noticed. To illustrate this concept a data set is obtained from two normal distributions for a fastener manufacturing process with means μ_1=10 micrometer and μ_2=11 micrometer and constant variance (σ=1 micrometer). The upper specification limit of the process is 14 mm and lower specification limit is 8 mm. As it is seen the shift is one standart deviation in the mean of the distribution. There is aproximately no evidence to indicate this shift in the mean. (Figure 3)

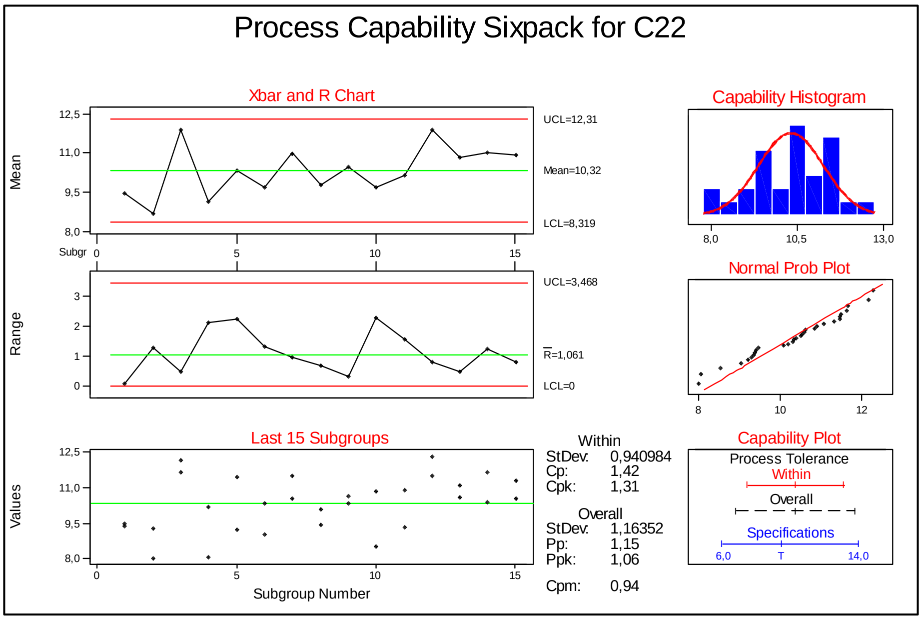

Figure 3: Statistical Process Analysis with one Standard Deviation Shift

From an earlier poll of a huge web portal for six sigma professionals, greater than 50% of polled quality professionals indicated that they are not aware of why a process may shift 1.5 sigma.

Using the tables for sigma levels one can easily find that 6 sigma actually translates to about 2 defects per billion opportunities, and 3.4 defects per million opportunities, which normally defined as six sigma, really corresponds to a sigma value of 4.5. Motorola has determined, through years of process and data collection, that processes vary and drift over time - what they call the ‘Long-Term Dynamic Mean Variation’. This variation typically falls between 1.4 and 1.6. Also it is obvious that for many stuations controlling the process is less expensive than reducing the process variability. Tadikamalla (2005) produced the table for different sigma shift levels from the nominal and showed the sigma levels associated as it is seen below.

Table 1: Numbers of Defectives (Parts Per Million) for Specified Off-Centering of the Process and Quality Levels (One Tail Only)

	Quality Level						
Off Centering	**3 sigma**	**3,5 sigma**	**4 sigma**	**4.5 sigma**	**5 sigma**	**5,5 sigma**	**6 sigma**
0,00 sigma	1350	233	32	**3,4**	0,29	0,017	0,001
0,25 sigma	3577	666	99	12,8	1,02	0,1056	0,0063
0,50 sigma	6440	1382	236	32	**3,4**	0,71	0,019
0,75 sigma	12288	3011	665	88,5	11	1,02	0,1
1,00 sigma	22832	6433	1350	233	32	**3,4**	0,39
1,25 sigma	40111	12201	3000	577	88,5	10,7	1
1,50 sigma	66803	22800	6200	1350	233	32	**3,4**
1,75 sigma	105601	40100	12200	3000	577	88,4	11
2,00 sigma	158700	66800	22800	6200	1300	233	32

It is important here to say that a quality level of 3.4 defects per million can be achieved in several ways, for instance:

- With centered data and 4.5 sigma level of quality
- With 1.00 sigma shift and 5.5 sigma level of quality
- With 1.50 sigma shift and 6.0 sigma level of quality

The difference between a 4 and 6 sigma quality level can be surprising. To put it in practical terms, If your cell phone system operated at a 4 sigma level, it is expected that the customers will be out of service for more than 4 hours each month, on the other hand, a six sigma level of quality means in this process that the customers will be out of service at about 9 seconds a month. Figure 4 indicates the surprising nature of improvement gained from Six Sigma. In its 1990 results as stuckted in 5.4 sigma level of quality over all, it is obvious that the target is not easy to reach and Motorola decided to establish the Six Sigma Research Institute to achieve 'Six Sigma and Beyond' (Barney, M. & McCarty, T. 2003)

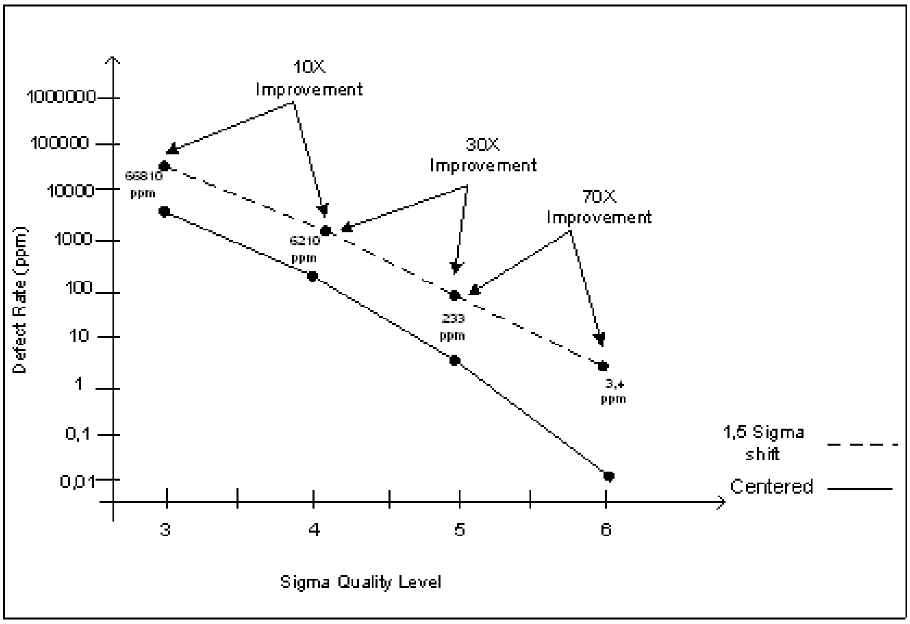

Figure 4: Implication of sigma quality level from Breyfogle (2001)
Part per million (ppm)rate for part or process step

Sigma level can be easily calculated from the formula below:

Sigma Level=NORMSINV(1-Number of Defects/Number of Opportunities)+SHIFT

In the Sigma level formula one of the Six Sigma Metrics is given called DPMO. The discussion above showed the sigma level relationship from a normal distribution to a parts-per-million (ppm) defect rate. Sometimes organizations calculate a ppm defect rate or defects per million opportunities (DPMO) rate and then convert this to a six sigma measurement unit that considers this 1.5 standard deviation shift. DPMO is calculated from the formula below:

$$\text{DPMO} = \frac{\text{Total Defects}}{\text{Total Opportunities}} * 1{,}000{,}000$$

A process DPMO calculation is quite easy, with starting to identify the opportunities and counting defects then finding its proportion in one million opportunities. But there is a confusion and discussion in organizations to find overall DPMO and thus overall sigma level.

Six Sigma quality levels have a strong relationship with process capability and process performance metrics. Actually in Six Sigma organizations process performance is reported to the organization as a sigma level. The higher the sigma level, the higher the process performance obviously from the calculation of DPMO. Another way to report process capability and process performance is through the statistical measurements of C_p, C_{pk}, P_p, and P_{pk}.

$$C_p = (USL - LSL)/6s \text{ within}$$

$$CPU = (USL - \mu)/3s \text{ within},\ CPL = (\mu - LSL)/3s \text{ within},\ C_{pk} = \min(CPU, CPL)$$

$$P_p = (USL - LSL)/6s \text{ overall}$$

$$PPU = (USL - \mu)/3s \text{ overall},\ PPL = (\mu - LSL)/3s \text{ overall},\ P_{pk} = \min(PPU, PPL)$$

Six Sigma development closely effects the process capability and performance indices. An experiment to show the relationship between these indices, sigma level and the standard deviation of a process is conducted. The real data is from a fastener manufacturers manufacturing process, measurements are for head height of the fasteners and collected in two shifts for two different operators.

Table 2: The Relationship of Standard Deviation with Process Capability and Performance

Centered Data						
Standart Deviation	**Cp**	**Cpk**	**Sigma Level**	**PP**	**PPK**	**Expected ppm**
0,088	0,79	0,78	2,8	0,76	0,75	22572
0,055	1,21	1,17	3,54	1,18	1,14	424
0,048	1,5	1,47	4,3	1,47	1,44	12
0,038	1,63	1,62	4,8	1,61	1,6	1,45
0,032	1,89	1,86	5,6	1,87	1,84	0,02
Non Central Data(1.5 st dev shift)						
0,088	0,79	0,28	2,35	0,76	0,27	207631
0,055	1,21	0,67	3,45	1,18	0,65	24712
0,048	1,5	0,99	4,4	1,47	0,97	1811
0,038	1,63	1,16	4,95	1,61	1,14	304
0,032	1,89	1,47	5,85	1,87	1,45	6,36

The target value and the specifications are given from the customer. The original data is randomly generated with differantiating the standard deviation suggested in six sigma development. The data also generated for non central situation.

The original case has 0.088 micrometer standard deviation; the generated samples as it is seen above are generated with step wise improvement in standard deviation. The improvement in standard deviation is effected strongly the capability and performance indices and also expected parts per million. Figure 5 shows the levels of the metrics with the standard deviation.

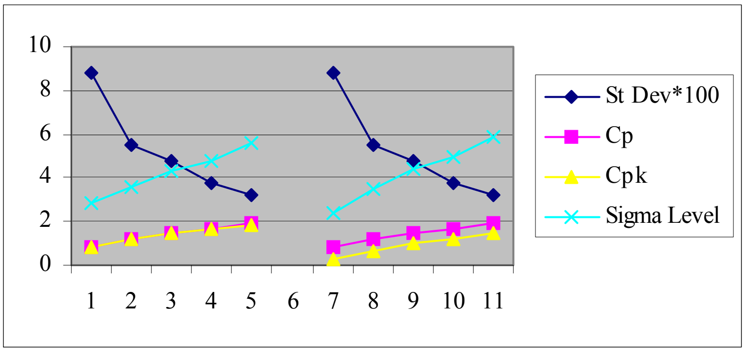

Figure 5: Plot of Standard Deviation and Process Capability and Performance Indices for Centered and Non Central Data

Six Sigma has a nature based on the Normal Distribution. All the statistical metrics dealed above have the same assumption “Normality”. But what if your data is not normally distributed? Most processes, particularly those involving life data and reliability, are not normally distributed. Some examples of non-normal distributions would include: Cycle time, Calls per hour, Customer waiting time, Shrinkage etc. As a result normality test is vital for six sigma applications. After the verification of non-normality, few methods are suggested to the six sigma practitioners. These are;

- Using Non-Parametric Statistics
- Searching the Original Distribution
- Transforming Data
- Segmenting Data

All these statistical metrics are parts and basis of Six Sigma development, but there is beyond these. Six Sigma suggests a model for improvement called DMAIC.

SIX SIGMA DEVELOPMENT MODEL-DMAIC

'Process Improvement' refers to a strategy of finding solutions to eliminate the root causes of performance problems in processes that already exist in the companies. Process Improvement efforts seek to fix problems by eliminating the causes of variation in the process while leaving the basic process intact. In Six Sigma terms, Process Improvement teams find the critical Xs (causes) that create the Ys (defects) that the companies do not prefer to face of produced by the process (Pande, P. S., Neuman, R. P., Cavanagh, R. R., 2002). This process—often called DMAIC. The steps are explained below.

D (Define) the goals of the improvement activity. The most important goals are obtained from customers. At the top level the goals will be the strategic objectives of the organization, such as greater customer loyalty, or increased market share, or greater employee satisfaction. At the operations level, a goal might be to increase the throughput of a production department. At the project level goals might be to reduce the defect level and increase throughput for a particular process. The development team obtains goals from direct communication with customers, shareholders, and employees.

M (Measure) the existing system. The development team establishes valid and reliable metrics to help monitor progress towards the goal(s) defined at the previous step.

A (Analyze) the system to identify ways to eliminate the gap between the current performance of the system or process and the desired goal. Analyze begins by determining the current baseline. Here exploratory and descriptive data analyses are used to help the development team to understand the data.

I (Improve) the system. The development team should be creative in finding new ways to do things better, cheaper, or faster. Project management and other planning and management tools are used to implement the new approach. Statistical methods are used to validate the improvement

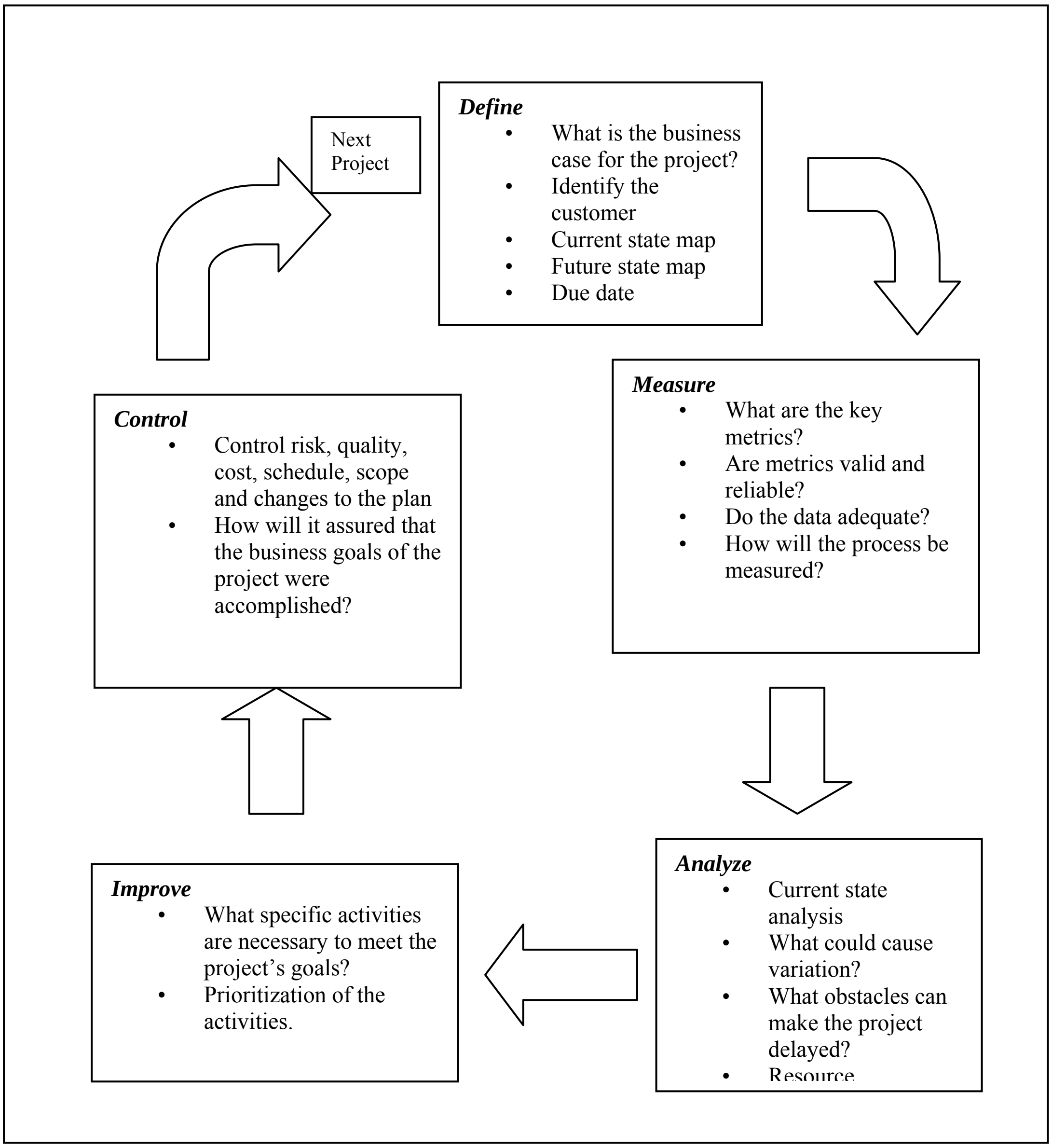

Figure 6: Using DMAIC on a Six Sigma Project from Pyzek (2003)

C (Control) the new system. The development team should institutionalize the improved system by modifying compensation and incentive systems, policies, procedures, budgets, operating instructions and other management systems. Standardization such as ISO 9000 is used to assure that documentation. Statistical tools are used to monitor stability of the new systems.

In the next table the most frequently used statistical and management tools are given with the development steps to get used.

Table 3: Six Sigma tools commonly used in each phase of a project.

Project Phase	Meaning of the Phase	Commonly Used Tools
Define	Identify the Problem Define Requirements Set Goals	• Project charter • VOC tools (surveys, focus groups, letters, comment cards) • Through process map • QFD, SIPOC • Benchmarking • Process Map
Measure	Validate Problem/Process Refine Problem/Goal Measure Key Steps/Inputs	• Measurement systems analysis • Exploratory data analysis • Descriptive statistics • Data mining • Run charts • Pareto analysis
Analyze	Develop Causal Hypotheses Identify Root Causes Validate Hypothesis	• Cause-and-effect diagrams • Tree diagrams • Brainstorming • Process behavior charts (SPC) • Process maps & Design of experiments • Enumerative statistics (hypothesis tests) • Inferential statistics (Xs and Ys) • FMEA • Simulation
Improve	Develop Ideas to Remove Root Causes Test Solutions Standardize Solutions Measure Results	• Force field diagrams • 7M tools • Project planning and management tools • Prototype and pilot studies
Control	Establish Standard Measures to Maintain Performance Correct Problems as Needed	• SPC • FMEA • Documentation • Change budgets, bid models, cost estimating models • Reporting system

CONCLUSIONS

In this study, the nature of Six Sigma, its statistical background and Six Sigma improvement model are adressed. The concept of Six Sigma is wholly a matter of discussion. TQM mentality is not completely different from Six Sigma Philosophy. Pande, Neumann and Cavanagh (2000) defined Six Sigma as TQM with steroids that bumpers the TQM activities in a short time with its characteristics properties; Use of Statistics and Data Analysis, Teamwork Support and also Commitment of the Members.

Six Sigma is being a popular icon of statistics and management, a trademark and being a fad all over the globe. The popularity brings some other claims and problems with its fame. Some of us even think of the meaning of 'six' in the name of the methodology.

Also the target 3.4 DPMO is sometimes getting foggy. Concept of 3.4 DPMO is questionable with the thougts of 'Deming' about the danger caused by the numerical goals. Not all products are produced equally when considering a jet engine and a blender. The way to clarify the stuation is to determine the concerning process' baseline on sigma levels from its nature. Some processes are only acceptable to shift 0.5 standard deviation from the mean or less. On the other hand, the goal simplifies the concepts, present numerical targets to management and provides a common communication. 1.5 standard deviation concept is Motorola's own knowledge only on their processes.

As it is clear in the development model there is no new statistical method in any of the steps. Six Sigma is a methodology able to bring the known tools to analyze the variability. Six Sigma is creative rather than innovative.

Six Sigma is popular with the other methodologies already. Six Sigma Fusion or Beyond Six Sigma is bodied first in design activities (Edgeman R. L. & Bigio I. D., 2004). GE suggested a new development model DMADV (Define, Measure, Analyze, Design and Verify) to reach Six Sigma target in design called DFSS (Design for Six Sigma) using QFD and TRIZ (Tennant, G., 2002). Integration between Lean Thinking and Six Sigma anticipated called Lean Six Sigma (George, M. L., 2002).

Edgeman & Bigio (2004) suggested a new route to Six Sigma to other bottom-lines such as the biophysical-environmental, societal, and technological (built environment) called BEST principles and 'BEST Six Sigma'.

A new development model is intruduced called DMAIC with Six Sigma. In fact it is hard to to say it is quite different from PDCA but suggestions of the statistical tools for all the steps and the flaw of monitoring the analysis on variation is more sophisticated.

Six Sigma is a style today; some black belts are preparing six sigma projects on dietary and quality of life. Even though, Six Sigma sometimes claimed to be the same of TQM for many aspects and found suspicious for the metrics it uses, its support of teamwork, motivating power, strong data analysis background can not be ignored.

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