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Moment-curvature and effective section stiffness of reinforced concrete beams

Betonarme kirişlerin moment-eğrilik ve etkin kesit rijitlkleri

Authors (Yazarlar): *Saeid Foroughi [1], S. Bahadır Yüksel [2]*

ORCID[1]: 0000-0002-7556-2118

ORCID[2]: 0000-0002-4175-1156

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Moment-Curvature and Effective Section Stiffness of Reinforced Concrete Beams

Highlights

- *Seismic performance*
- *Effective stiffness*
- *Moment-curvature*
- *Structural elements*

Graphical Abstract

In this study, RC beam models with different parameters were designed to investigate nonlinear moment-curvature relationships and effective stiffness coefficients. Analytically investigated parameters were calculated from TBEC (2018), ACI318 (2014), ASCE/SEI41 (2017), Eurocode2 (2004) and Eurocode8 (2004, 2005) regulations and moment-curvature relationships. The effective section stiffness coefficient obtained from the analyses were compared with the effective section stiffness coefficient given for RC members in different regulations.

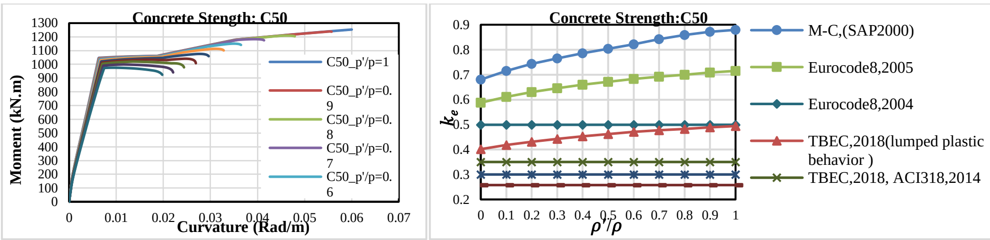

Figure. *Comparative moment-curvature and $k_e - \rho'/\rho$ relations for doubly RC beam sections with different parameters*

Aim

Earthquake-resistant structure design has three main principles: stiffness, strength and ductility. In this study, RC beam models with different concrete strength, tensile reinforcement ratio and compression reinforcement ratio were designed to investigate moment-curvature and effective section stiffness.

Design & Methodology

Analytically investigated parameters were calculated from TBEC (2018), ACI318 (2014), ASCE/SEI41 (2017), Eurocode2 (2004) and Eurocode8 (2004, 2005) regulations and moment-curvature relationships.

Originality

The effective stiffness coefficient obtained from the analyses were compared with the effective section stiffness coefficient given for RC members in different regulations. The results obtained at the end were examined by comparing them according to different parameters and models.

Findings

For constant concrete strength and tensile reinforcement ratio in RC beams, with increasing compression reinforcement ratio, effective stiffness coefficient values increase. In RC beams with constant compression and tensile strength ratio, effective stiffness coefficient values increase with increasing concrete strength. The compression reinforcement ratio has been proved to be effective on the maximum moment bearing capacity of the RC beams, effective flexural stiffness and effective stiffness coefficient and ductility of the sections.

Conclusion

For constant concrete strength and tensile reinforcement ratio in RC beams, with increasing compression reinforcement ratio, effective stiffness coefficient values increase. In RC beams with constant compression and tensile strength ratio, effective stiffness coefficient values increase with increasing concrete strength. The compression reinforcement ratio has been proved to be effective on the maximum moment bearing capacity of the RC beams, effective flexural stiffness and effective stiffness coefficient and ductility of the sections.

Declaration of Ethical Standards

The authors of this article declares that the materials and methods used in this study do not require an ethical committee permission and/or legal-special permission.

Moment-Curvature and Effective Section Stiffness of Reinforced Concrete Beams

Araştırma Makalesi / Research Article

Saeid Foroughi *, S. Bahadır Yüksel

Konya Technical University, Department of Civil Engineering, Faculty of Engineering and Natural Sciences, Konya, Turkey

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ABSTRACT

In determining the seismic performance of reinforced concrete (RC) structures in national and international earthquake regulations, it is desired to use effective section stiffness of the cracked section in RC structural elements during the design phase. Although the effective section stiffness of the cracked section is not constant, it depends on parameters such as the concrete strength and reinforcement ratio. In this study, RC beam models with different concrete strength, tensile and compression reinforcement ratios were designed to investigate nonlinear moment-curvature relationships and effective stiffness coefficients. Analytically investigated parameters were calculated from TBEC (2018), ACI318 (2014), ASCE/SEI41 (2017), Eurocode2 (2004) and Eurocode8 (2004, 2005) regulations and moment-curvature relationships. The effective section stiffness coefficient obtained from the analyses were compared with the effective section stiffness coefficient given for RC members in different regulations. The results obtained at the end were examined by comparing them according to different parameters and models. For constant concrete strength and tensile reinforcement ratio in RC beams, with increasing compression reinforcement ratio, effective stiffness coefficient values increase. In RC beams with constant compression and tensile strength ratio, effective stiffness coefficient values increase with increasing concrete strength. The compression reinforcement ratio has been proved to be effective on the maximum moment bearing capacity of the RC beams, effective flexural stiffness and effective stiffness coefficient and ductility of the sections.

Keywords: Seismic performance, effective stiffness, moment-curvature, structural elements, reinforced concrete beam.

Betonarme Kirişlerin Moment-Eğrilik ve Etkin Kesit Rijitlkleri

ÖZ

Ulusal ve uluslararası deprem yönetmeliklerinde betonarme yapıların sismik performansının belirlenmesinde, tasarım aşamasında betonarme yapı elemanlarında çatlamış kesite ait etkin kesit rijitliğinin kullanılması istenmektedir. Çatlamış kesitin etkin kesit rijitliği sabit olmamakla birlikte beton dayanımı ve donatı oranı gibi parametrelere bağlıdır. Bu çalışmada, doğrusal olmayan moment-eğrilik ilişkileri ve etkin rijitlik katsayılarını araştırmak için farklı beton dayanımı, çekme ve basınç donatı oranlarına sahip betonarme kiriş modelleri tasarlanmıştır. Analitik olarak incelenen parametreler TBEC (2018), ACI318 (2014), ASCE/SEI41 (2017), Eurocode2 (2004) ve Eurocode8 (2004, 2005) yönetmeliklerinden ve moment-eğrilik ilişkilerinden hesaplanmıştır. Sonunda elde edilen etkin rijitlik katsayısı, farklı yönetmeliklerde betonarme elemanlar için öngörülen rijitlik katsayısı ile karşılaştırılmıştır. Elde edilen sonuçlar farklı parametre ve modellere göre karşılaştırılarak incelenmiştir. Betonarme kirişlerde sabit çekme donatı oranı ve beton dayanımı için, basınç donatı oranının artması ile etkin rijitlik katsayısı değerleri artar. Sabit çekme ve basınç donatı oranına sahip betonarme kirişlerde beton dayanımı arttıkça etkin rijitlik katsayı değerleri de artmaktadır. Basınç donatı oranının betonarme kirişlerin maksimum moment taşıma kapasitesi, etkin eğilme rijitliği ve etkin rijitlik katsayısı ve kesitlerin sünekliği üzerinde etkili olduğu kanıtlanmıştır.

Anahtar Kelimeler: Sismik performans, etkin rijitlik, moment-eğrilik, yapı elemanları, betonarme kiriş.

1. INTRODUCTION

Cross-section analysis have an important place in nonlinear analysis of building systems. Analysis of RC sections is carried out by complex iteration methods, depending on a large number of variables and nonlinear material models [1]. Seismic design and analysis of RC buildings are performed based on the linear response however, it is universally accepted that under severe seismic inelastic response and cracking is accepted [2]. In order to understand the behavior of RC elements, the cross-sectional behavior must be well known. The cross-sectional behavior can best be evaluated with the moment-curvature relationship [3]. The bending response is usually calculated by numerically based moment-curvature relations of the base section and the equivalent plastic hinge length [4]. Curvature ductility would provide to the structure an increased chance of survival against accidental impact and seismic attack [5]. Cracks in concrete, which reduce the stiffness of RC elements, occur at loads much smaller than those that correspond to the yielding of reinforcement and bearing capacity [6]. The level of accuracy in estimating the section stiffness plays a very important role in determining realistic values for the structural stiffness and therefore the applied seismic forces [7].

**Sorumlu Yazar (Corresponding Author)*
e-posta : saeid.foroughi@yahoo.com

Paulay and Priestley [8] investigated various factors affecting the bending stiffness of RC elements and suggested average values. Mehanny et al, [9] proposed simple formulas for determining the effective bending and shear stiffness coefficients of beams and columns, taking into account the axial load level. Panagiotakos and Fardis [10] developed expressions for the yield and ultimate deformation capacities of RC elements, which are necessary to predict the effective elastic stiffness of cracked RC elements. Kumar and Singh [11] proposed two different effective stiffness models for normal-strength and high-strength concrete members to be used in the design of buildings. Pan et al., [12], investigated the reduction factor and proposed different methods to improve the accuracy of the stiffness prediction [13].

The seismic codes and guidelines, the effective stiffness of RC elements is expressed as a proportion of their stiffness, calculated on the basis of the cross-section properties. Several procedures are suggested to considered effective rigidity: Turkish Building Earthquake Code (TBEC) [14], American Concrete Institute (ACI 318) [15], Seismic Evaluation and Retrofit of Existing Buildings (ASCE/SEI41) [16], Design of Structures for Earthquake Resistance (Eurocode 8) [17], Eurocode 8-Part 3: Assessment and Retrofitting of Buildings (Eurocode 8-3) [18] and Design of Concrete Structures (Eurocode 2) [19]. The effective section stiffness and effective stiffness coefficient of the cracked section of the RC beam designed in different parameters were obtained analytically. Analytically investigated parameters were calculated from different seismic codes and moment-curvature relationships. The effective section stiffness coefficient obtained from the analyzes were compared with the effective section stiffness coefficient given for RC members in different regulations. Events such as how the stiffness and strength of the section change, the ductility state of the cross-section behavior can also be observed through the moment-curvature relationship. The moment-curvature relationship is essential for the inelastic analysis of structures to predict section strength, flexural stiffness, and section ductility. Effective stiffness reflects not only the effect of cracking but also the behavior of reinforced concrete elements determined from moment-curvature analyses. For the moment-curvature relations, SAP2000 Software [20] was used considering the different parameters.

2. EFFECTIVE STIFFNESS of RC BEAM

2.1. Effective Stiffness Coefficient According to Moment-Curvature Relations

The behavior of reinforced concrete structural elements is determined by the cross-sectional behavior of the element. Section behavior depends on the material used in the section, the geometry of the section, and the loads acting on the section. Behaviors of reinforced concrete sections such as stiffness, strength and ductility can be observed through the nonlinear moment-curvature relationship [21]. Effective section stiffness (EI_e) of the cracked section in RC sections is determined by the ratio corresponding to the yield moment (M_y) and the yield curve ($\emptyset_y$), taking into account the moment-curvature relationship. The gross moment of inertia (I) of the RC members and the uncracked section stiffness (EI) of the concrete according to the modulus of elasticity ($E_c = 3250\sqrt{f_{ck}} + 14000\ MPa$) were calculated. Effective stiffness coefficient of reinforced concrete elements; it is calculated as $k_e = EI_e/EI$.

2.2. Effective Section Stiffness Coefficient Proposed in Different Seismic Codes

Effective section stiffness coefficients will be used in modeling the sectional properties of RC structural members within the scope of Strength by Design. Effective cross-sectional stiffness multipliers will only be applied to calculations that are included in earthquake-effect load combinations and under loads entered into these combinations. In the seismic evaluation and retrofit of existing buildings [14-17], the stiffness reduction coefficient values for RC beams are given in Table 1.

Table 1. Flexural Stiffness coefficient values for RC Beams

RC Beams	Different Seismic Codes			
	TBEC, 2018	ACI318, 2014	ASCE/SEI-41, 2017	Eurocode8, 2004
Flexural Stiffness Coefficient	0.35	0.35	0.30	0.50

2.3. Effective Stiffness Coefficient Proposed in TBEC (2018)

EI of the RC members designed according to the lumped plastic behavior will be determined according to Eq. 1. M_y is the yield moment, θ_y is the chord rotation at the yielding end and L_s is the shear span. Shear span can be taken as approximately half of the opening in and beams.

$$(EI)_e = \frac{M_y}{\theta_y}\frac{L_s}{3} \tag{1}$$

For the nonlinear calculation, chord rotation at the yielding (θ_y) of the RC members is calculated by Eq. 2. f_{ye} is the expected yield strength of transverse reinforcement (f_{ye}=1.2f_{yk}) and f_{ce} ; expected compressive strength of concrete (f_{ce}=1.3f_{ck}), h is the section height and d_b is the longitudinal reinforcement diameter, $\eta = 1$ in beams [14].

$$\theta_y = \frac{\emptyset_y L_s}{3} + 0.0015\eta\left(1 + 1.5\frac{h}{L_S}\right) + \frac{\emptyset_y d_b f_{ye}}{8\sqrt{f_{ce}}} \tag{2}$$

2.4. Effective Stiffness Coefficient Proposed in Eurocode 8 (2005)

Part 3 of Eurocode 8 [18] provides a relation based on moment-shear ratio and yield rotation, which can be used for the determination of a more accurate effective stiffness ($M_y L_v/3\theta_y$). The chord rotation at yielding θ_y, calculated at Eq 3.

$$\theta_y = \Phi_y \frac{L_V + a_V z}{3} + 0.0014\left(1 + 1.5\frac{h}{L_V}\right) + \Phi_y \frac{d_{bL} f_y}{8\sqrt{f_c}} \quad (3)$$

$a_V z$: is the tension shift of the bending moment diagram, $z = d - d'$ in beam section, $\varepsilon_y = f_y/E_s$, d ve d': are the depths to the tension and compression reinforcement respectively and d_{bL}: is the diameter of the reinforcement. $a_V = 0$, if $V_{Rc} > M_y/L_s$ and $a_V = 1$, if $V_{Rc} \leq M_y/L_s$. $V_{R,c}$, taken in accordance with 1992-1-1 [19].

2.5. Effective Stiffness Coefficient Proposed in Eurocode 2 (2004)

The following model can be used to estimate the nominal stiffness of RC compression elements:

$$EI = K_c E_{cd} I_c + K_s E_s I_s \quad (4)$$

E_{cd} and E_s is the modulus of elasticity of concrete and reinforcement, I_c is the moment of inertia of concrete cross-section, I_s is the second moment of area of reinforcement, K_c is a factor for effects of cracking, creep etc and K_s is a factor for contribution of reinforcement. The following factors may be used in Eq. 5, provided $\rho \geq 0.002$:

$$K_s = 1 \quad , \quad K_c = \frac{K_1 K_2}{(1 + \varphi_{\mathrm{ef}})} \quad (5)$$

As a simplified alternative, provided $\rho \geq 0.01$, the following factors may be used in Eq. 6.

$$K_s = 0 \quad , \quad K_c = \frac{0.3}{(1 + 0.5\varphi_{\mathrm{ef}})} \quad (6)$$

ρ is the reinforcement ratio, φ_{ef} is the creep ratio, K_1 and K_2 is a factor which depends on concrete grade and axial force (Eq. 7). $n = N_{Ed}/(A_c f_{cd})$, $\lambda = l_0/i$ (l_0 is the effective length and i is the radius of gyration of the uncracked concrete section) [19].

$$K_1 = \sqrt{\frac{f_{ck}}{20}} \quad , \quad K_2 = n\frac{\lambda}{170} \leq 0.2 \quad (7)$$

3. MATERIAL AND METHOD

Factors affecting the non-linear behavior of reinforced concrete beams; tensile and compressive reinforcement ratio, diameter and spacing of transverse reinforcement and compressive strength of concrete. In this study, RC beam models with different concrete strength, tensile reinforcement ratio and compression reinforcement ratio were designed to investigate effective section stiffness. The summary of the designed RC beam cross-section properties is given in Table 2. The beam sections analyzed are rectangular cross-sections. A typical beam section has a width $b_w = 300mm$ and total depth $h = 600mm$ were designed. The compression and tensile reinforcement is provided at depth $d' = 50mm$ and $d = 550mm$ ($d = h - d'$) from the top, respectively. For all RC members, C30, C40 and C50 were chosen as concrete grade and B420C was selected as reinforcement for the reinforcement behavior model. In moment-curvature analyses, material models given in Fig. 1 are used for concrete and reinforcement steel.

The limit values given in Requirements for Design and Construction of RC Structures (TS500) [22] are taken into consideration in this study. In beams the ratio of tensile reinforcement (ρ) should not be less than the minimum values given in Eq. 8. In TS500 [22], the reinforcement ratio is limited to Eq. 9 in order to provide ductile behavior in RC beams. The difference between the tensile and compression reinforcement ratios in beams should not exceed 0.85 of the balanced reinforcement. According to TS500 [22], k_1 value is provided not to be less than 0.70 and not greater than 0.85.

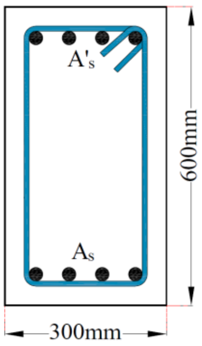
$$\rho = \frac{A_s}{b_w d} \geq \rho_{min} = 0.8\frac{f_{ctd}}{f_{yd}} \quad (8)$$

$$\begin{cases} (\rho - \rho') \leq \rho_{max} = 0.85\rho_b \\ \rho_b = 0.85 k_1 \left(\frac{f_{cd}}{f_{yd}}\right)\left(\frac{700}{700 + f_{yd}}\right) \end{cases} \quad (9)$$

A total of 33 RC beam models with different concrete strength, different tensile and compression reinforcement ratios have been designed. In the design of RC beam models, the provisions in TBEC [14] and TS500 [22] have been taken into consideration. In RC beam models designed in different parameters, as the ratio of tensile reinforcement; $\rho_{max} = 0.85\rho_b$ and as the compression reinforcement ratio; the values of $\rho_s' = 0.0$, $0.1\rho_{max}$, $0.2\rho_{max}$, $0.3\rho_{max}$, $0.4\rho_{max}$, $0.5\rho_{max}$, $0.6\rho_{max}$, $0.7\rho_{max}$, $0.8\rho_{max}$, $0.9\rho_{max}$ ve ρ_{max} are taken into account. In RC beam models, six different concrete grades are taken into account as C30, C40 and C50. In beam models, the tensile reinforcement ratio for each concrete strength was kept constant with $\rho_{max} = 0.85\rho_b$. By changing the compression reinforcement ratios, the moment-curvature relation of RC beams were obtained.

Analytically investigated parameters were calculated from different standards and moment-curvature relationships of sections. Analytically investigated parameters were calculated from seismic regulations [14-19] and moment-curvature relationships of sections. The major factors affecting the effective stiffness of a doubly RC beam section are investigated. The effective section stiffness coefficient obtained from the analyzes were compared with the effective section stiffness coefficient given for RC beams in different regulations. The results of the comparison are examined in detail. The effective stiffness values obtained from the analysis results are presented in detail in the Research Findings and Discussion section.

Table 2. Details for the designed RC beam cross-sections

Cross-Section Dimensions	ρ'/ρ	Material: C30		Material: C40		Material: C50	
		ρ_{max}	ρ'	ρ_{max}	ρ'	ρ_{max}	ρ'
	0.0		0.0		0.0		0.0
	0.1		0.0021		0.0026		0.0030
	0.2		0.0043		0.0053		0.0061
	0.3		0.0064		0.0079		0.0091
	0.4		0.0085		0.0105		0.0121
	0.5	0.0213	0.0107	0.0263	0.0132	0.0303	0.0152
	0.6		0.0128		0.0158		0.0182
	0.7		0.0149		0.0184		0.0212
	0.8		0.0171		0.0211		0.0243
	0.9		0.0192		0.0237		0.0273
	1.0		0.0213		0.0263		0.0303

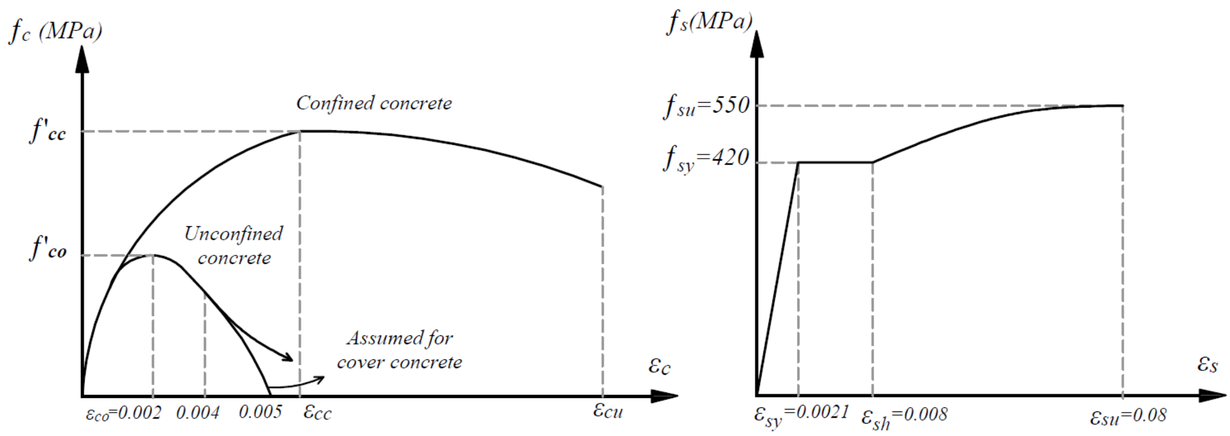


Figure 1. Stress-strain relationships for concrete and reinforcement.

4. RESEARCH FINDINGS AND DISCUSSION

In this part of the study, the moment-curvature relations are obtained by changing the concrete strength, tensile reinforcement ratio and compression reinforcement ratio. A total of 33 different analyzes were performed to determine the moment-curvature relationships of rectangular double RC beam sections with eleven different longitudinal reinforcement (tensile and compression reinforcement) ratios and three different concrete compressive strengths. Comparative moment-curvature curves for doubly RC beam sections with different concrete strength are shown in Fig. 2. Fig. 3 shows the comparative moment-curvature curves for doubly RC beam sections with different compressive reinforcement ratio. The relationship of the effective stiffness factors variation with the ρ'/ρ are given in Fig. 4. Fig. 5 shows the relations of the effective stiffness factor variation with the f_{ck}.

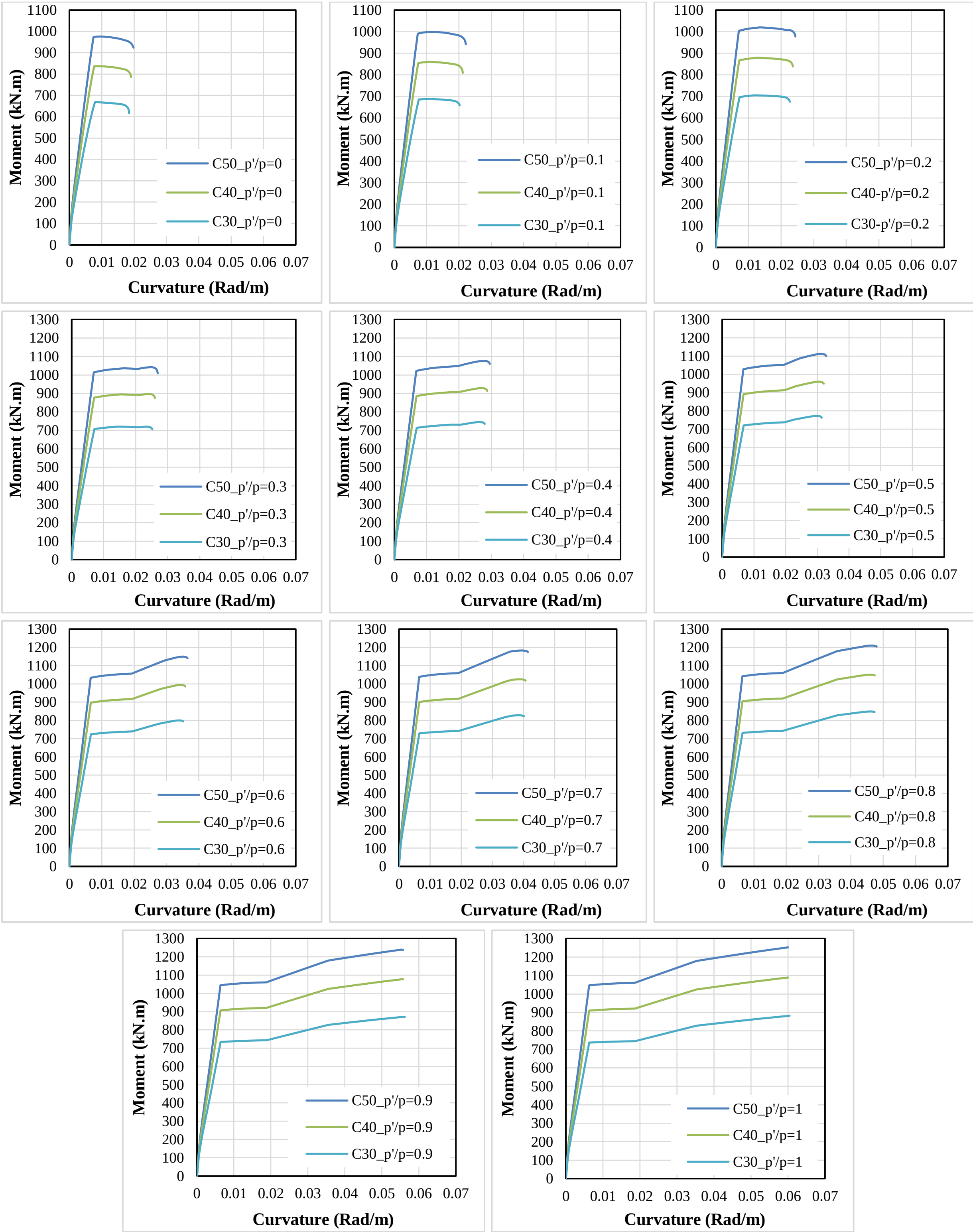


Figure 2. Comparative moment-curvature relations for doubly RC beam with different concrete grade

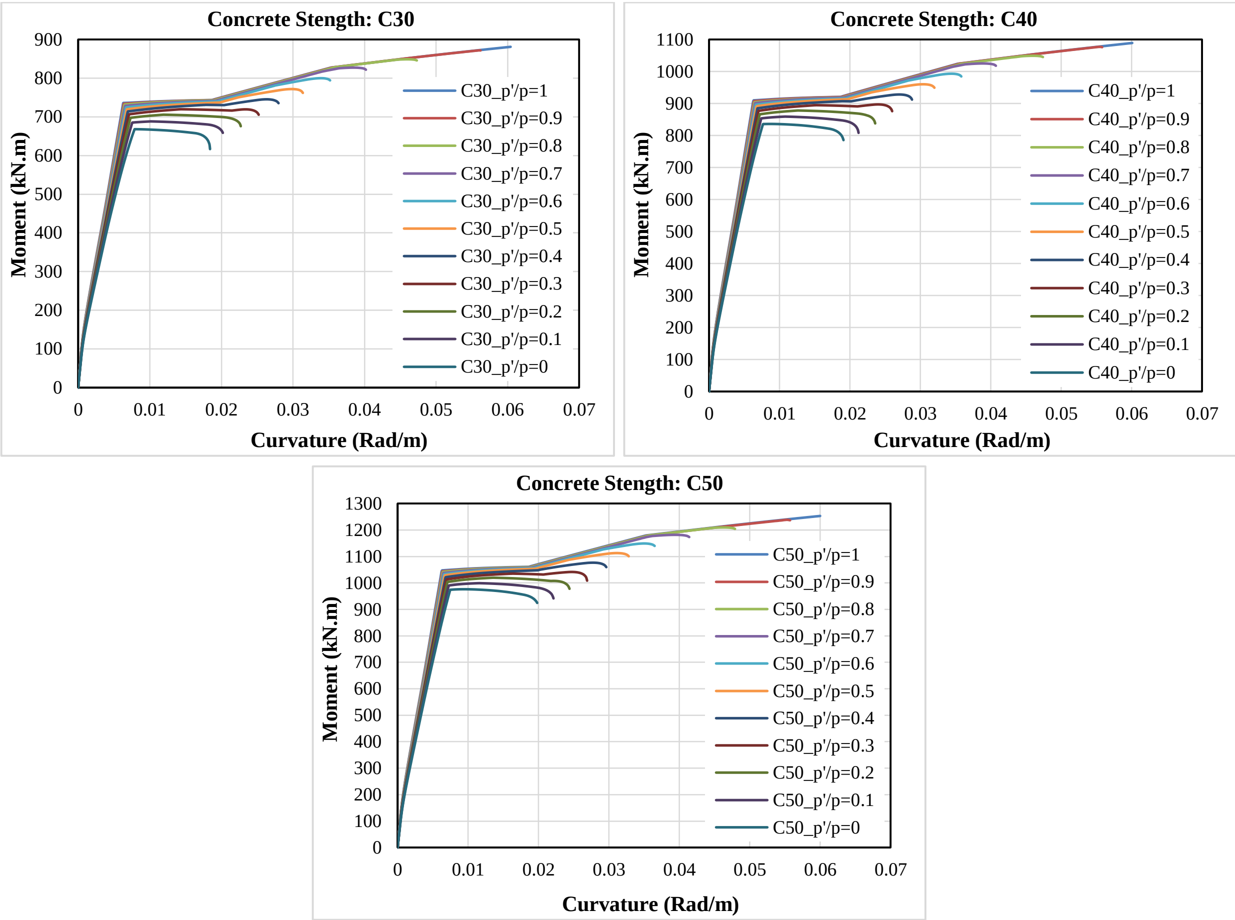


Figure 3. Comparative moment-curvature relations for doubly RC beam sections with different compressive reinforcement ratio

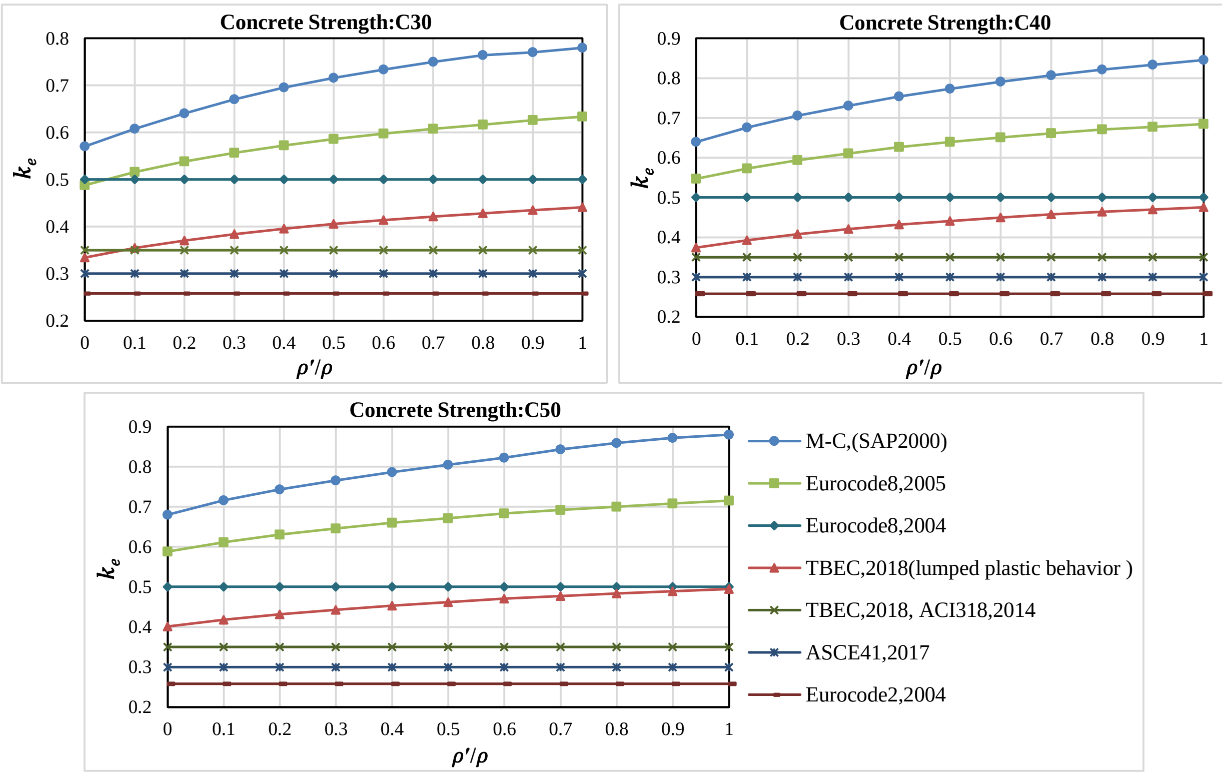


Figure 4. Comparative $k_e - \rho'/\rho$ relations according to different seismic codes and moment-curvature relations.

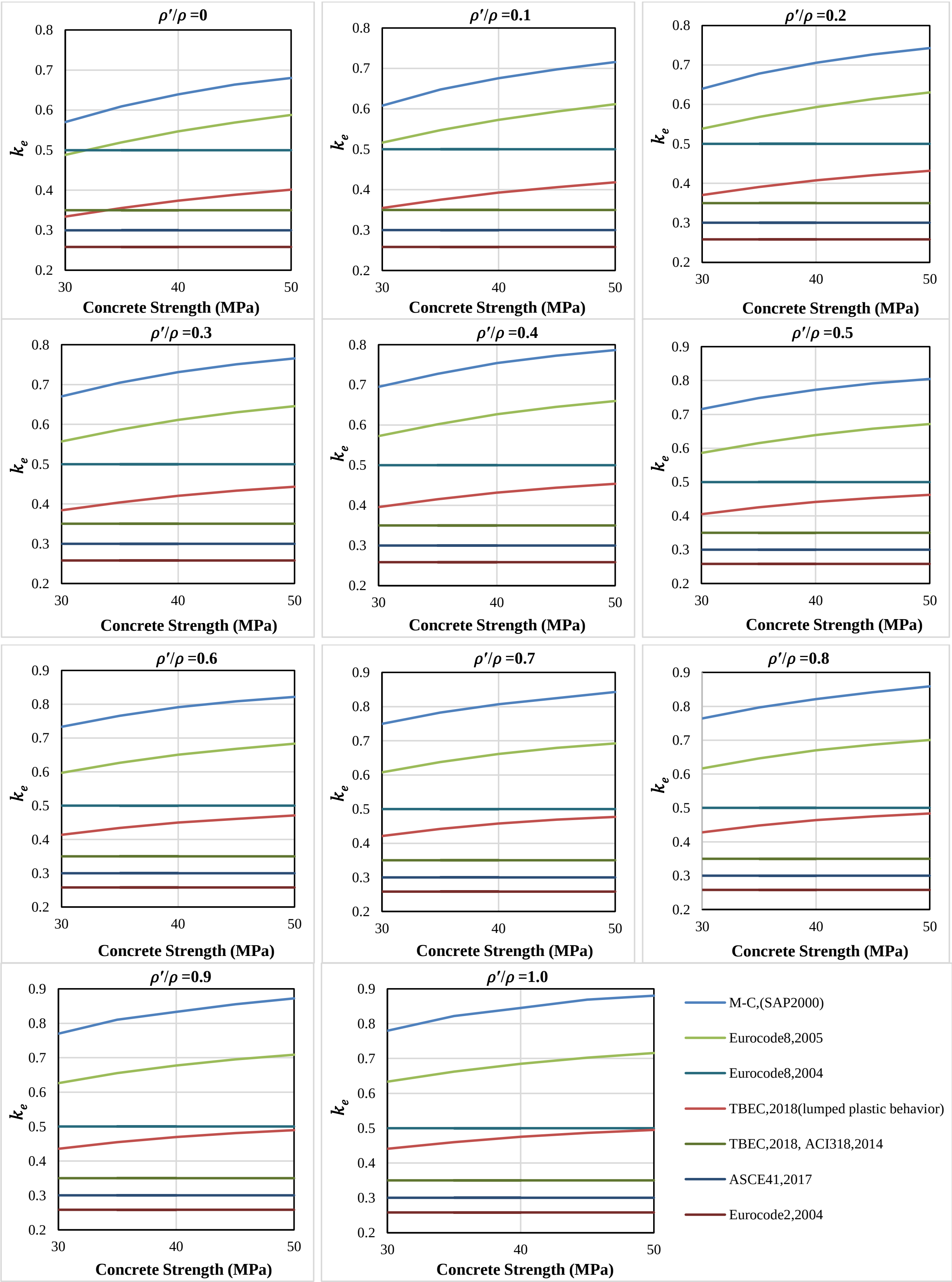


Figure 5. Comparative $k_e - f_{ck}$ relations according to different seismic codes, different researchers and proposed equation for effective stiffness factor.

In ACI318, the effective stiffness to flexural stiffness of an RC beam is estimated by the cross-sectional moment of inertia with a reduction factor to account for the effect of bending and shear cracking. In ASCE/SEI-41, EI_{eff} for nonprestressed beams is specified as $0.3E_cI_g$. The effective section stiffness value of the cracked section given in Eurocode 8 is fixed. The effective section stiffness of the cracked section is considered to be half of the initial stiffness. In Eurocode 8, features such as concrete strength, cross-section geometry, longitudinal and transverse reinforcement ratio and axial force acting on the section are not taken into consideration. Part 3 of Eurocode 8 provides a relation based on moment-shear ratio and yield rotation that can be used to more accurately determine effective stiffness. According to Eurocode 2, nominal values of bending stiffness should be used, taking into account the effects of non-linearity of the material and creep on the overall behavior. Similarly, in TBEC, effective section stiffness is assumed to be constant and effective stiffness coefficient values of the cracked section is 0.35 for beams.

5. CONCLUSIONS

Effective cross-sectional stiffness of reinforced beams modeled according to the lumped plastic behavior in TBEC can be calculated depending on the effective yield moments, yield rotation and the shear span. As can be seen from the comparison results, the effective cross-section stiffness values calculated from the SAP2000 program [19] and different regulations are different from each other. Taking the effective cross-section stiffness higher than the required value will cause the structural stiffness to be overestimated. As a result, problems will arise in the calculation and evaluation of structures.

When the analysis results are examined, it is observed that the variation of the tensile reinforcement ratio, compression reinforcement ratio and concrete grade have an important effect on the moment-curvature behavior of the RC beams. With the increase of the compression reinforcement ratio in RC beams, M_y, M_u and k_u values obtained from moment-curvature relationships increase and k_y values decrease. With the increase of the compression reinforcement ratio, the maximum moment bearing capacity and ductility of the sections increase. With the increase of concrete grade, the maximum moment bearing capacity of RC beams increases.

As can be seen from the comparison of effective section stiffness coefficient values obtained from the moment-curvature for RC beams designed in different parameters; for constant concrete strength and tensile reinforcement ratio in RC beams, with increasing compression reinforcement ratio, effective stiffness coefficient values increase. In RC beams with constant compression and tensile strength ratio, effective stiffness coefficient values increase with increasing concrete strength. The compression reinforcement ratio has been proved to be effective on the maximum moment bearing capacity of the RC beams, effective flexural stiffness and effective stiffness coefficient and ductility of the sections.

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DECLARATION OF ETHICAL STANDARDS

The authors of this article declare that the materials and methods used in this study do not require ethical committee permission and/or legal-special permission.

CONFLICT OF INTEREST

There is no conflict of interest in this study.

AUTHORS CONTRIBUTIONS

Saeid FOROUGHİ: The analysis plan for the study, the design of the reinforced concrete beam models, the data collection, analysis, evaluation of the results and the writing of the article.

Süleyman Bahadır YÜKSEL: The analysis plan for the study, the design of the reinforced concrete beam models and analyzed the numerical results in terms of absolute error.

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