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Sensitivity analysis of a three-dimensional subsurface irrigation hydrology model

Üç boyutlu bir toprakaltı sulama hidrolojisi modelinin duyarlılık analizi

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ABSTRACT

In this study, a sensitivity analysis was performed and presented for a three-dimensional subsurface irrigation hydrology model. Input parameters such as number of nodes, van Genuchten's soil hydraulic parameters (K_s, α, and n), the longitudinal and transverse dispersivity (α_L, α_T), the sorption distribution coefficient (K_d), initial solute concentration distribution, water table depth from soil surface and root water uptake parameters were changed by fixed amounts around a base value and the resulting changes in the outputs were analyzed. Results showed that output was sensitive to van Genuchten's soil hydraulic parameters (K_s, α, and n), initial solute concentration distribution, water table depth and root water uptake parameters. The cumulative solute load was insensitive to the changes in the α_L, and K_d, while the solute concentration distribution was quite sensitive to these parameters. The simulated outputs were not sensitive to the changes in the number of nodes and transverse dispersivity coefficient. Results could be used by modelers not only for this model but also for similar models.

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***Anahtar kelimeler*:**

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Toprakaltı hidrolojisi

ÖZ

Bu çalışmada, üç boyutlu bir toprakaltı sulama modelinin duyarlılık analizi yapılarak sonuçları sunulmuştur. Düğüm sayısı, van Genuchten toprak hidrolik parametreleri (K_s, α, ve n), boyuna ve enine dispersivite (α_L, α_T), soğurma dağılım katsayısı (K_d), başlangıç çözünmüş madde konsantrasyonu, su tablası derinliği ve kök su alım parametreleri gibi modelin giriş parametreleri bir baz değere göre belirli oranlarda değiştirilmiş ve bu değişikliğe bağlı olarak elde edilen sonuçlar analiz edilmiştir. Model sonuçlarının van Genuchten toprak hidrolik parametreleri, başlangıç çözünmüş madde konsantrasyonu, su tablası derinliği ve kök su alım parametrelerine karşı duyarlı olduğu belirlenmiştir. Kümülatif çözünmüş madde yükünün α_L, ve K_d'deki değişikliklere duyarsız olduğu gözlenirken, toprak profilindeki çözünmüş madde konsantrasyonu dağılımının söz konusu parametrelere karşı duyarlı olduğu gözlenmiştir. Simülasyon sonuçlarının düğüm sayısı ve enine dispersiviteye karşı duyarsız olduğu belirlenmiştir. Sonuçlar, sadece bu model için değil buna benzer model çalışmalarında da kullanıcılar için yararlı olacaktır.

1. Introduction

The use of mathematical models to predict water flow and solute transport in field soils is rapidly spreading. The basic reasons of using a model for soil systems are: (1) models enable easy evaluation of many different potential environmental scenarios with little cost and time, (2) simulations are repeatable and nondestructive, and (3) results are often easier to interpret (Corwin et al. 1999).

A generic procedure for deterministic model development consists of (1) formulation of a simplified conceptual model characterized by integrated processes, (2) representation of each individual processes by an algorithm, (3) verification of the algorithm to ascertain that the conceptual model is truly represented, (4) sensitivity analysis to determine the relative importance of the variables and parameters, (5) model calibration, (6) model validation, and (7) application of the model (Corwin 1996).

Corwin et al. (1999) reviewed the deterministic and stochastic models published in mainstream journals over the past decade. He reported that only five of 44 models included a sensitivity analysis. As pointed out by Anderson and Woessner (1992), a sensitivity analysis is an essential step in all modeling applications.

Table 1. Values of selected model input parameters for sensitivity analysis.

Parameter (1)	Values Lower value (2)	 Base (3)	 Upper value (4)
Number of nodes	3094 4186	5382 (Base)	7774
Van Genuchten (1980) soil hydraulic parameters			
K_s, m day^{-1}	0.025	0.25	2.5
alpha (α), m^{-1}	0.5	1.0	2.0
n	1.5	2.0	3.0
Longitudinal dispersivity, (α_L), m	0.1	1.0	10.0
Transverse dispersivity, (α_T), m	0.01	0.1	1.0
Sorption distribution coefficient, (K_d), m^3 kg^{-1}	0.0001	0.0005	0.001
Initial solute concentration distribution, kg m^{-}	Case I in Fig. 1	Case II in Fig. 1	Case III in Fig. 1
Water table depth from soil surface, m	0.9	1.2	1.5
Root water uptake	Fig. 2b	Fig.2c	Fig. 2d

Even though models are useful tools, they have limitations. The sensitivity of model output to input parameters or measurement inaccuracies in model-input parameters is among these limitations.

Sensitivity analysis is used to measure the impact of changing one input factor on another output factor. In other words, it is a methodical study of the model response to variations in input parameters. Such an analysis can provide valuable information to users, guiding their parameterization effort, e.g. resource allocation for data collection (Ferreira et al. 1995). Sensitivity analysis is also useful for trial-and-error calibration because it displays the importance of different parameters in calculating dependent variables (Zheng and Bennett 1995). It also provides a means of identifying those parameters with the greatest influence on the simulations, thereby indicating which parameter should be more accurately measured (Corwin 1995).

For model calibration and validation, it is necessary to know the degree to which the simulated water and solute distribution is affected by inherent model limitations and by errors estimating or measuring input parameters. This information can aid in determining which input parameters must be accurately known and which parameters can be estimated.

The model developed by Buyuktas and Wallender (2002) is selected for sensitivity analysis. Briefly, the model is a three-dimensional, deterministic model simulating unsaturated-saturated water flow and solute transport, subject to root water uptake, drainage, and various fluxes at the soil-atmosphere interface due to different irrigation practices. The water movement and solute transport are modeled by numerical solution of Richards' equation and the convection-diffusion equation, respectively, using the Galerkin finite element method, subject to the appropriate initial and boundary conditions. The model can simulate the processes that couple irrigation practices, land use, evaporation, transpiration and soil water extraction by roots, with vadose zone and groundwater flow and transport. The model can also predict changes in water table elevations and water quality due to agricultural management strategies. The details of the model can be found elsewhere (Buyuktas and Wallender 2002).

The purpose of this paper is to perform and present a sensitivity analysis to the variations in the following parameters: number of nodes, van Genuchten's parameters of the water retention curve (K_s, α and n), longitudinal and transverse dispersivity, sorption distribution coefficient, water table depth, initial solute concentration distribution and root water uptake parameters.

2. Method

A traditional form of sensitivity analysis is Independent Parameter Perturbation (IPP) in which parameters are varied individually, usually by a fixed percentage or value, around a base value, while fixing all other parameters at their base value (Fereira et al. 1995). Sensitivity can be also evaluated by observing the magnitude of change in the results due to a parameter change ranging from plus-or-minus one standard deviation. However, it can be problematic to compare model sensitivities determined with standard deviations because such sensitivities reflect both the model effects and the measurement uncertainties. Consequently, to evaluate model effects, the use of a fixed percentage or value change is more reasonable (Corwin 1995). In this study the method used by Fereira et al. (1995) has been adopted.

The values for the parameters investigated in the sensitivity analysis are presented in Table 1. It should be noted that Table 1 is not a complete list of all the input data required by the model, but is rather the selection of critical parameters.

A 60 m x 60 m x 7 m domain with a homogeneous and isotropic soil was used in the simulations. A subsurface tile drain was at a depth of 1.8 m from soil surface. The initial distribution of the pressure head within the domain was assumed to be at hydrostatic equilibrium. At the soil surface, a time-dependent flux boundary condition was applied while a no-flow boundary was used at the bottom and on all other sides. The tile drains in the domain were treated as boundary nodes surrounded by four regular square elements with adjusted hydraulic conductivities using the electric analog approach of Vimoke et al. (1963) and Fipps et al. (1986). For solute transport modeling, salt concentration of the irrigation water (Table 2) was input as a Cauchy boundary condition across the top boundary while Neumann boundary conditions were used

Table 2. Irrigation schedule.

Time of irrigation (Day of year)	Irrigation duration (days)	Irrigation depth (m)	Contaminant concentration (kg m^{-3})
140	1	0.130	1.0
162	1	0.156	1.0
190	1	0.128	1.0
215	1	0.116	1.0

for the bottom and side boundaries. Finer discretizations were used near the soil surface and around the subsurface tile drain to accommodate abrupt changes in local fluxes and hence pressure gradients. Simulations were performed over a 200-day period, starting on day of year 100 and ending on day 300. The simulated crop was planted on day 112 and harvested on day 294 and irrigated with the schedule given in Table 2, with a constant reference evapotranspiration rate of 5 mm day^{-1}.

3. Results and Discussion

The results for the parameters given in Table 1 are presented in terms of changes in cumulative drain outflow, evolution of the water table elevation, and solute mass distribution in the beginning and at the end of the simulation period. Solute mass distributions are plotted on the vertical axis at the midpoint between the drains. The depth of the domain used in the simulations is 7 m. Before discussing the sensitivity analysis, recall that the results are only valid in the context of the chosen parameters.

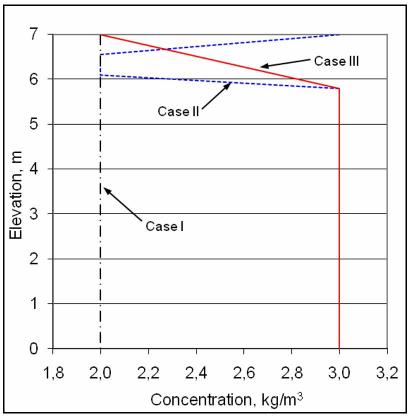

Figure 1. Concentration profiles used in the sensitivity analysis.

3.1. Number of nodes

The number of nodes was varied by changing grid size in one dimension and holding the other two dimensions constant. The grid Peclet number, which characterizes the space dimension of the grids, was used as a criterion to select four appropriate grid sizes. The Peclet number is the ratio of the magnitude of convective term to dispersive term in the convection-diffusion equation and it increases when the convection dominates. Numerical accuracy is maintained by using relatively fine grid spacing resulting in a low Peclet number. According to Simunek et al. (1995), numerical oscillation can be virtually eliminated when the local Peclet numbers do not exceed 5. However, Huyakorn and Pinder (1983) suggest that the oscillation may be acceptably small with Peclet number as high as 10. The maximum grid Peclet number obtained with 3094 and 4186 nodes was in the neighborhood of 15 and it was about 7 with 5382 and 7774 nodes.

Cumulative drain flow, evolution of the water table, and solute mass distribution are apparently insensitive to the different number of nodes (Figure 3). This is a useful result since there are many cases where the computational costs resulting from using very refined grids become excessive. Thus, coarser grids can be used as long as the grid Peclet number is maintained around 10. Because of the above-mentioned criteria, 5382 nodes were selected for further simulations.

3.2. Saturated hydraulic conductivity, $K_{s.}$

Both water and solute transport results are extremely sensitive to the changes in the saturated hydraulic conductivity. Figures 4a and 4b show that as the value of the saturated hydraulic conductivity is increased, the soil tends to drain at a faster rate, and hence, the water table falls more rapidly. Wise et al. (1994) also observed that the simulated water table falls more rapidly for high saturated hydraulic conductivity. As K_s is increased, the soil drains quickly resulting in lower solute mass in the soil (Figure 4c). In other words, more salt is leached from root zone. For the smallest value of K_s, salt builds up in the root zone, compared to the initial condition. Here, it should be noted that the sensitivity analysis is based upon the assumption that the saturated hydraulic conductivity, K_s, is independent of the pore sizes parameterized by α and n in the van Genuchten (1980)'s water retention curve (i.e. α and n were held constant at their base values).

3.3. van Genuchten parameter α.

The results are sensitive to the parameter α, as seen in Figure 5 and reflect the combined effect of root water uptake and parameter α because changing α causes changes in the shape of the water retention curve. Recall that root water uptake is also a function of the water retention curve (Figure 2).

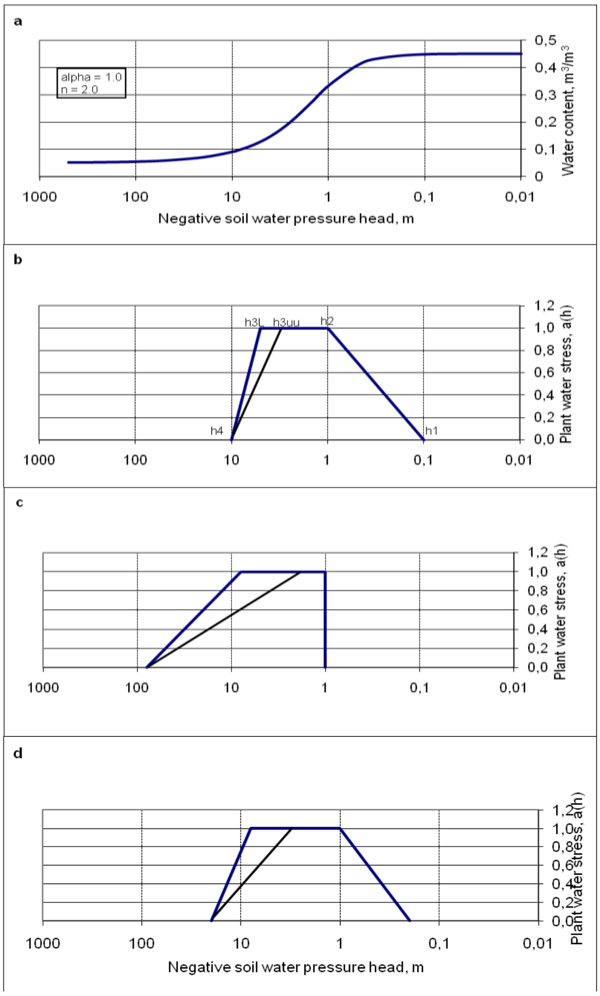

Figure 2. a) Soil-water retention curve for base values of van Genuchten parameters and b, c and d) different plant water response functions used in the sensitivity analysis.

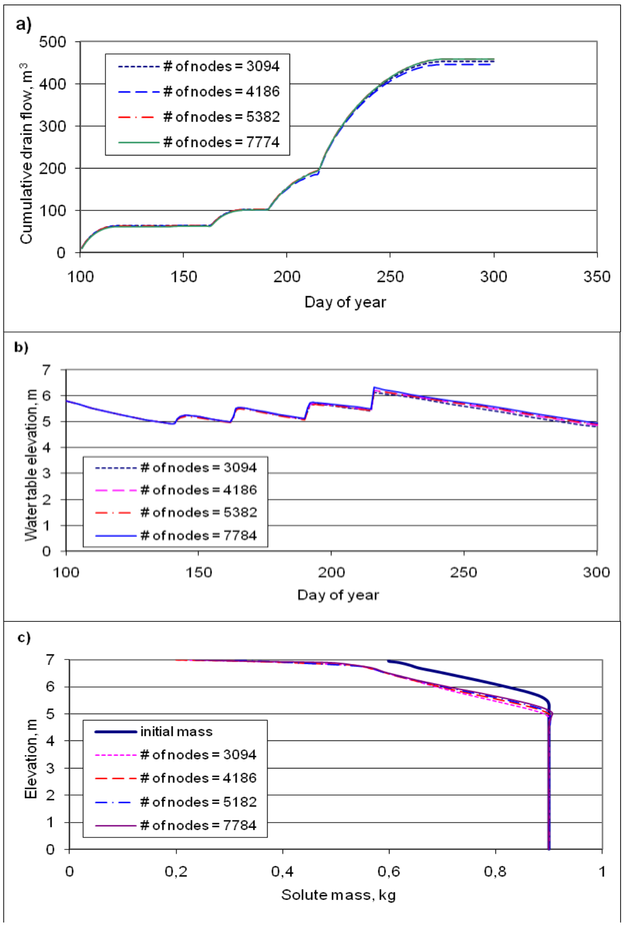

Figure 3. Sensitivity of a) cumulative drain flow, b) water table evolution at the midpoint of the drains, and c) initial and final solute mass profiles at the midpoint of the drains to the number of nodes.

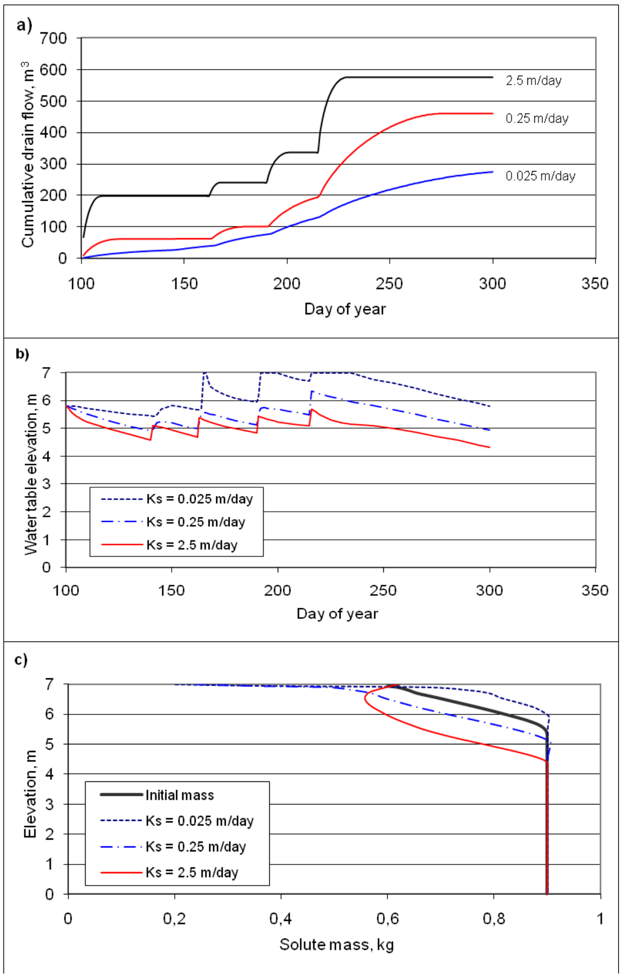

Figure 4. Sensitivity of a) cumulative drain flow, b) water table evolution at the midpoint of the drains, and c) initial and final solute mass profiles at the midpoint of the drains to the saturated hydraulic conductivity, K_s.

The van Genuchten parameter α is a measure of pore size; a porous medium with a large value of α has large pores. The unsaturated hydraulic conductivity of the zone above the water table is greater for soils having large values of α than for soils with smaller pores. However, recall that the saturated hydraulic conductivity is held constant below the water table. In order to conduct water to the drain a larger hydraulic gradient is required through a thinner transmission zone for the soil with large α. The results presented in Figures 5a and 5b are consistent with this interpretation. Because the larger values of α correspond to larger, highly conductive pores just above the water table, one observes higher drain flow (Figure 5a) and higher water table depths (larger difference in water table depth between the midpoint between the drain (Figure 5b) and the drain) as α increases. In addition, as α increases, cumulative drainage (Figure 5a) increases and less water is removed from soil storage and consequently the decline in water table is less as α increases (Figure 5b). Final solute mass distribution followed the distribution of water content because concentration was similar. Higher water tables (Figure 5b) caused higher water content and higher salt mass for large α (Figure 5d).

The initial and final solute mass distribution at the midpoint between drains is also sensitive to the changes in alpha (Figure 5c and d). In the model, hydrostatic pressure head distribution is given initially. Depending on alpha values, for the same negative soil water pressure head, different water content is obtained and that results in different initial solute mass distributions.

3.4. van Genuchten parameter n

The results of the sensitivity analysis to the parameter n are presented in Figure 6. The results are sensitive to the parameter n. As explained in the previous section, both parameter α and n have effect on the water retention curve and the results shown in Figure 6 represent combined effects of the van Genuchten parameter n and root water uptake.

The van Genuchten parameter n is an inverse measure of the breadth of the pore size density function. As n decreases, the width of the pore size density function increases. When n is reduced, the relative abundance of the smaller pores compared to the mean pore size increases (Wise et al. 1994). The larger pores tend to drain first, constraining water flow to the smaller pores. Thus, as the value of n decreases, the unsaturated hydraulic conductivity decreases more rapidly with decreasing water content, which decreases the drain outflow (Figure 6a). The initial and final solute mass distribution at the midpoint between drains is sensitive to the changes in n (Figure 6c and d). As mentioned in the previous section, depending on n values, for the same negative soil water pressure head, different water content is obtained and that results in different initial

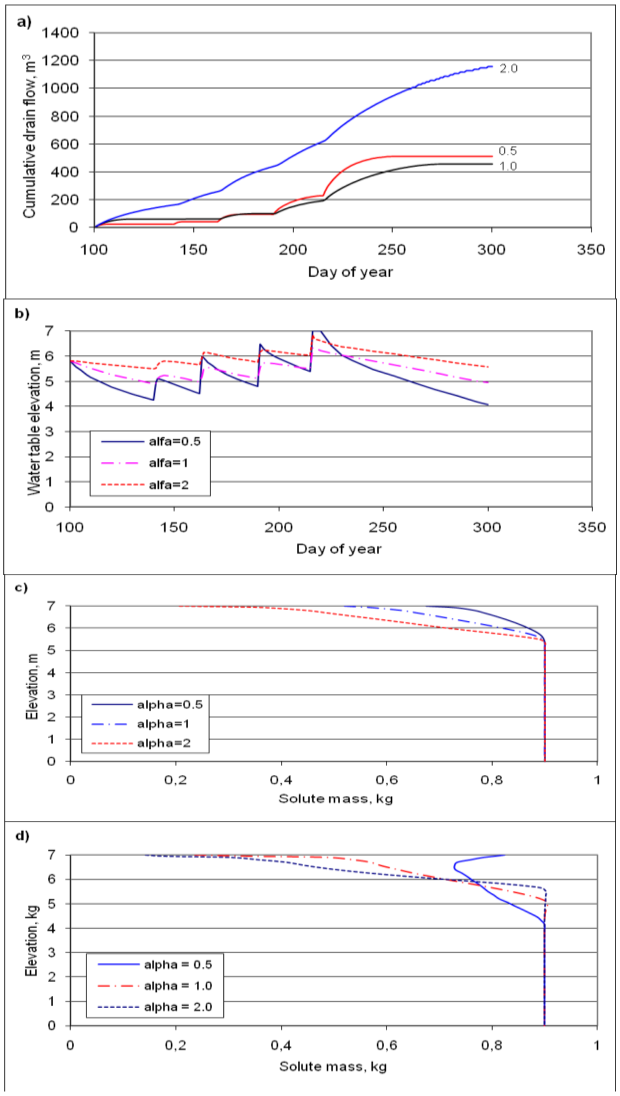

Figure 5. Sensitivity of a) cumulative drain flow, b) water table evolution at the midpoint of the drains, c) initial solute mass, and d) final solute mass profiles at the midpoint of the drains to the van Genuchten parameter α.

solute mass distributions.

Similar to *α*, as n increases unsaturated hydraulic conductivity increases and the thickness of the transmission zone above the water table decreases, cumulative drainage increases and the water table elevation difference between the midpoint and the drain increases.

3.5. Longitudinal dispersivity, α_L.

The cumulative solute load is not sensitive to the changes in the longitudinal dispersivity (Figure 7a). However, the solute mass distributions at the midpoint of the drains are sensitive to different values of α_L, as shown in Figure 7b. As the value of the longitudinal dispersivity is increased, more solute is leached from unsaturated zone to the saturated zone (Figure 7b). However, this does not cause large changes in the solute concentration in the vicinity of the drain. Therefore, the cumulative solute load is not sensitive to the changes in the longitudinal dispersivity. The result found in this study is in qualitative agreement with Forrer et al. (1999). They also

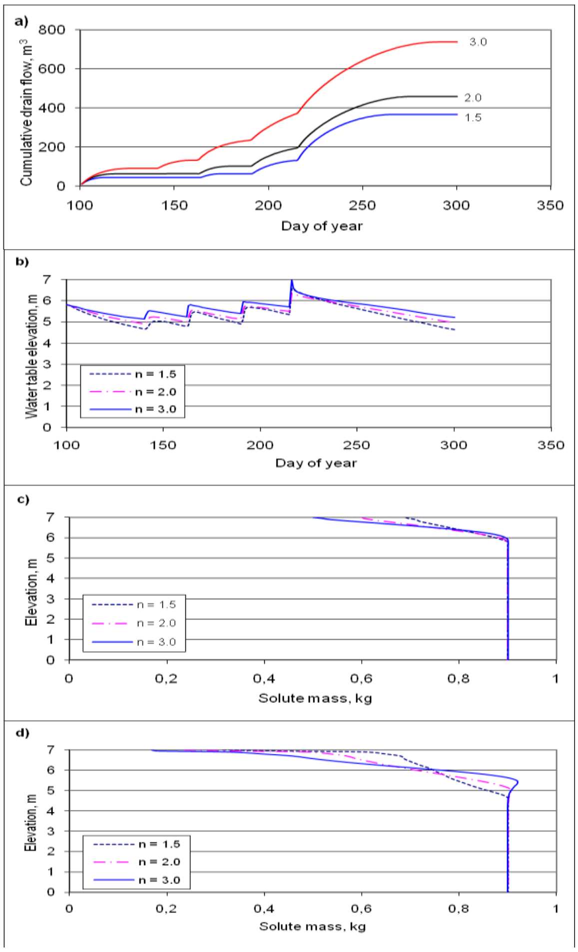

Figure 6. Sensitivity of a) cumulative drain flow, b) water table evolution at the midpoint of the drains, c) initial solute mass, and d) final solute mass profiles at the midpoint of the drains to the van Genuchten parameter *n*.

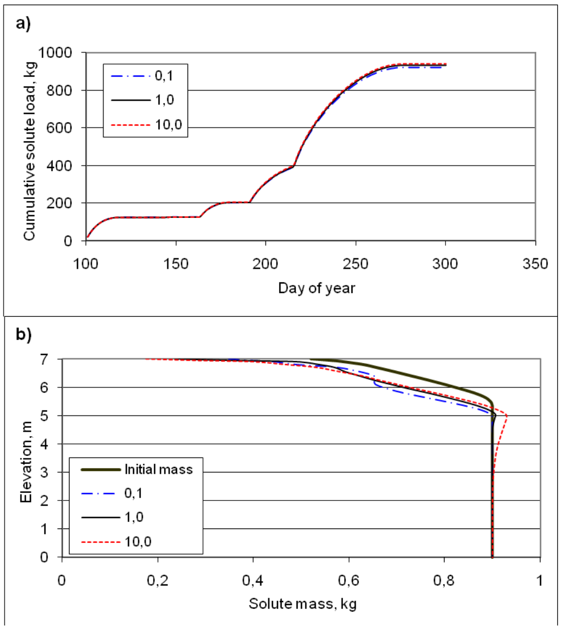

Figure 7. Sensitivity of a) cumulative solute load and b) initial and final solute mass profiles at the midpoint of the drains to the longitudinal dispersivity coefficient.

showed in their experimental study in an unsaturated field soil that concentration distribution along vertical axes is sensitive to the changes in the longitudinal dispersivity. This is a useful result, because the α_L can be used to calibrate the model without affecting the cumulative solute load.

3.6. Transverse dispersivity, α_T.

Neither the cumulative solute load (Figure 8a) nor the solute mass distributions on the vertical axis at the midpoint of the drains (Figure 8b) are sensitive to the changes in the transverse dispersivity. Apparently, the velocity component in the transverse direction is too small. Forrer et al. (1999) also report in their experimental study in an unsaturated field soil that concentration distributions were not sensitive to the changes in the transverse dispersivity.

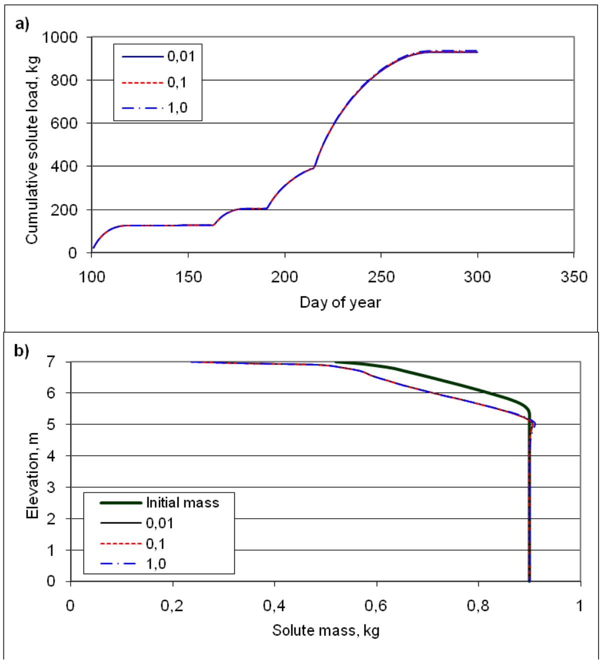

Figure 8. Sensitivity of a) cumulative solute load and b) initial and final solute mass profiles at the midpoint of the drains to the transverse dispersivity coefficient.

3.7. Sorption distribution coefficient, K_d.

The cumulative solute load is not sensitive to the changes in the sorption distribution coefficient (Figure 9a). However, the solute mass distributions on the vertical axis at the midpoint of the drains in the unsaturated zone are sensitive to different values of the sorption coefficient (Figure 9b). Compared to the initial solute mass distribution, the amount of solute mass in the profile decreased for K_d values of 0.001 and 0.0005 m^3 kg^{-1} whereas some solute accumulates in at the upper portion of the profile for K_d value of 0.0001 m^3 kg^{-1}. The amount of solute adsorbed on the soil matrix of the soil increases with K_d. In the saturated zone the effect of the adsorption coefficient is not noticeable. The adsorption coefficient is linearly proportional to the retardation factor, which is inversely proportional to the water content (Fetter 1999). In the saturated zone, the retardation factor is smaller than in the unsaturated zone because the volumetric water content is lower. Similar results are reported by Kamra et al. (1994).

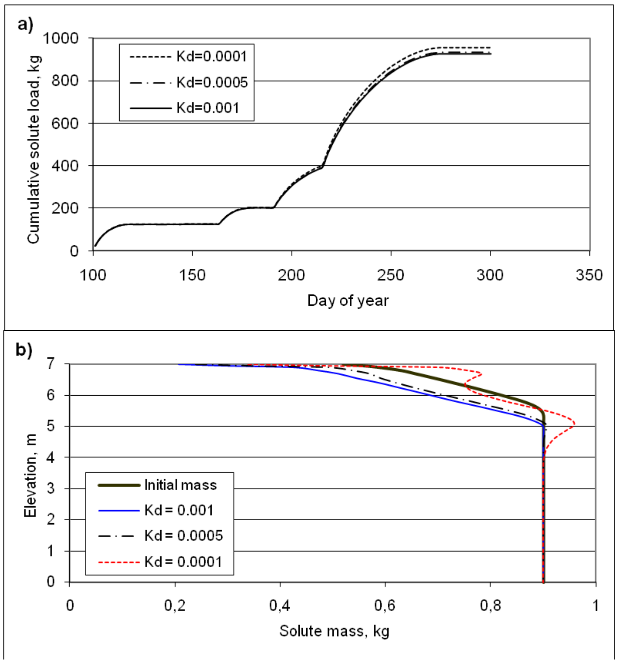

Figure 9. Sensitivity of a) cumulative solute load and b) initial and final solute mass profiles at the midpoint of the drains to the sorption distribution coefficient.

3.8. Initial solute concentration distribution

The results of the sensitivity analysis to the initial solute concentration distributions given in Figure 1 are shown in Figure 10. The cumulative solute load and the final solute mass distributions are sensitive to different initial concentration distributions (Figure 10a and 10c). Cumulative solute load is more sensitive to the concentration in the vicinity of the drain, as seen in Figure 10a. Even though the initial concentration distributions between case II and case III (Figure 1) above the water table are completely different, the difference between those two cases in the cumulative solute load and final concentration distributions are insignificant. This clearly shows that solute load is affected only by the concentration in the neighborhood of the drain and that salt in the unsaturated zone is transported to the saturated zone.

3.9. Water table depth

The results of the sensitivity analysis to initial water table depth are shown in Figure 11. The initial saturated profile was hydrostatic, while the unsaturated profile was linear from a negative soil water head on the surface to zero at the water table. Cumulative drain flow is very sensitive to the changes in water table depth (Figure 11a) whereas evolution of water table elevation is less sensitive to the changes in the water table depth (Figure 11b) than to parameters mentioned above. Although initial solute mass distribution is sensitive to initial water table depth, solute mass distributions at the end of the simulation were not sensitive to the water table elevation (Figure 11c). Again the saturated zone salts are transported to the groundwater system. As seen from Figures 11a and 11b, drain flow is proportional to the midpoint water table elevation. These results observed here are in agreement with those of Fipps et al. (1991). During model calibration drain flow, solute load in the drain flow or water table elevations can be adjusted without inducing changes in the final solute mass distributions.

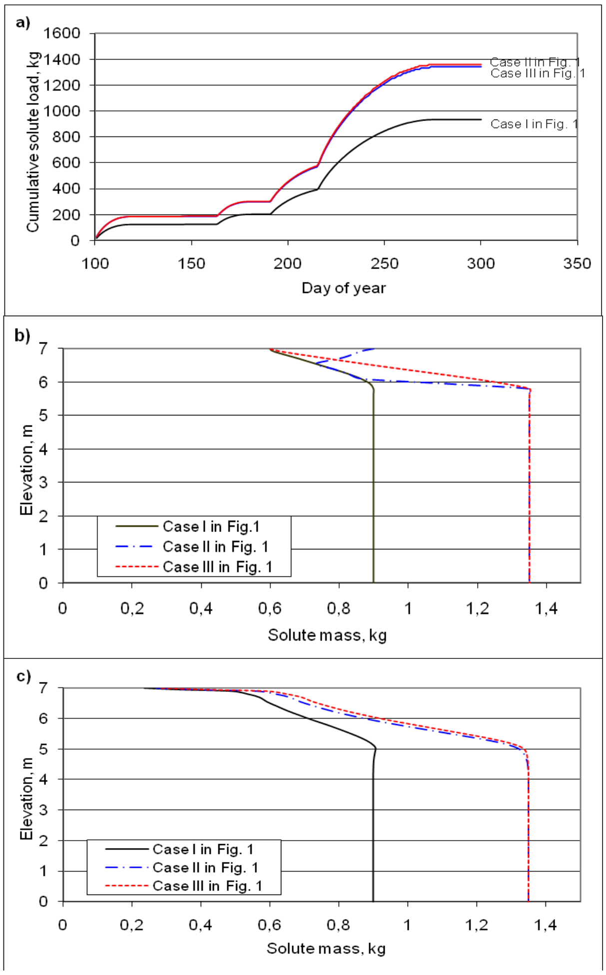

Figure 10. Sensitivity of a) cumulative solute load, b) initial solute mass and c) final solute mass profiles at the midpoint of the drains to the different initial concentration distribution shown in Figure 1.

3.10. Root water uptake

The results of the sensitivity analysis to the root water uptake parameters are illustrated in Figure 12. The results correspond to the plant water response functions given in Figure 2. Root water uptake is computed in the model using the approach given by Feddes et al. (1978). In this approach, root water uptake is a function of potential water uptake rate and a prescribed, dimensionless water stress function of the soil water pressure head (Figure 2b, c, d). Water uptake is assumed to be zero near saturation ($h>h1$) and wilting point pressure head ($h<h4$). Water uptake is considered optimal between $h2$ and $h3L$ whereas for pressure head $h3L$ and $h4$ or $h1$ and $h2$, water uptake changes linearly with soil water pressure head, h. In Figure 2b, water uptake is restricted to relatively wet conditions, while in Figure 2c, relatively dry conditions have been chosen.

Model outputs are sensitive to the root water uptake parameters, as seen from Figures 12 and 13. The dry condition case (Figure 2c) resulted in highest cumulative drain flow (Figure 12a), and the highest water table (Figure 12b) and lowest root water uptake (Figure 13). By contrast, the wet condition cases (Figures 2b and c) give lower cumulative drain flow, the lower water table and higher root water uptake (Figure 13). The elevated water table for the dry case also raised the salt profile (Figure 12c).

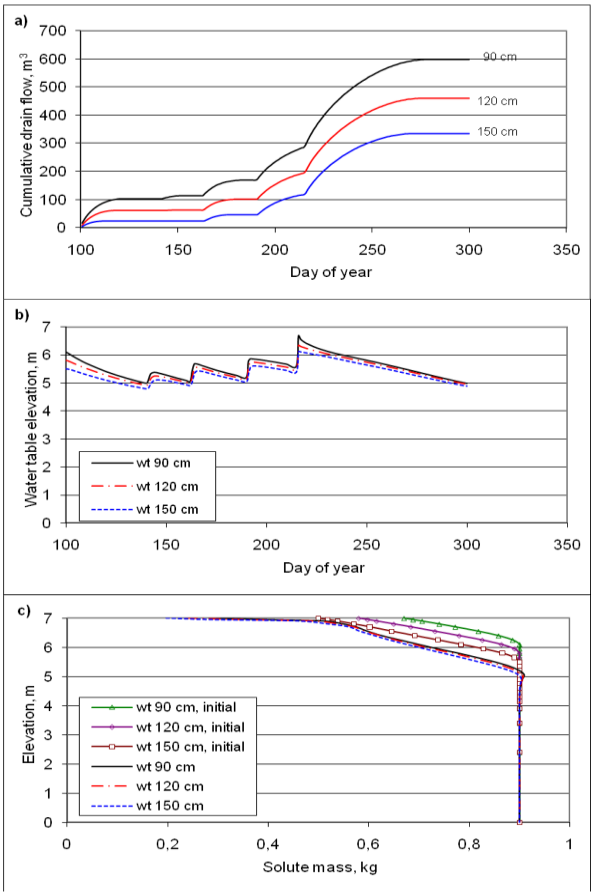

Figure 11. Sensitivity of a) cumulative drain flow, b) water table evolution and c) initial and final solute mass profiles at the midpoint of the drains to the different water table depths.

4. Summary and Conclusions

Numerical models are useful tools for assessing different management and/or research scenarios. Their use is increasing for study of variably saturated flow and solute transport processes in the soil. Effective use of these models depends to a large extent on the accuracy of the available input data. Even though models are useful tools, they have some limitations. The sensitivity of the model to input parameters or measurement inaccuracies is among these limitations. Sensitivity is defined as the degree to which the model results are affected by changes in a selected input parameter.

The sensitivity of the model developed by Buyuktas and Wallender (2002) to the variations in number of nodes, van Genuchten's parameters of the soil water retention curve (K_s, α and n), longitudinal and transverse dispersivity (α_L, α_T), sorption distribution coefficient, (K_d), water table depth, initial solute concentration distribution and root water uptake parameters were performed. Results were presented in terms of the cumulative drain flow, evolution of water table elevation and solute mass distribution at the midpoint between drains.

Model results were found to be insensitive to the number of nodes and the transverse dispersivity coefficient. This means that a coarser grid can be used in numerical simulations as long as the Peclet number does not exceed a certain threshold value. The cumulative drain flow and water table elevation were sensitive to the changes in the van Genuchten soil water retention parameters (K_s, α and n), water table depth and root

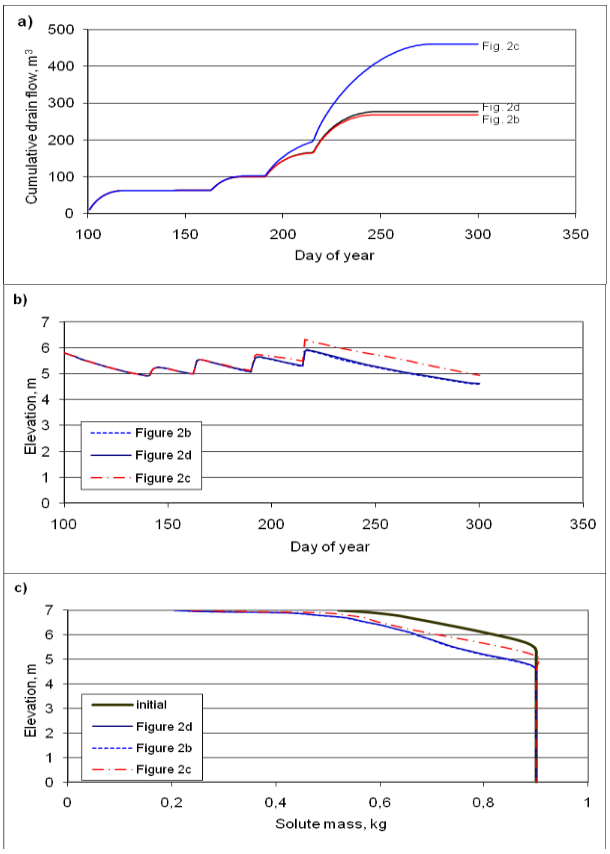

Figure 12. Sensitivity of a) cumulative drain flow, b) water table evolution, and c) initial and final solute mass profiles at the midpoint of the drains to different plant water response functions shown in Figure 2.

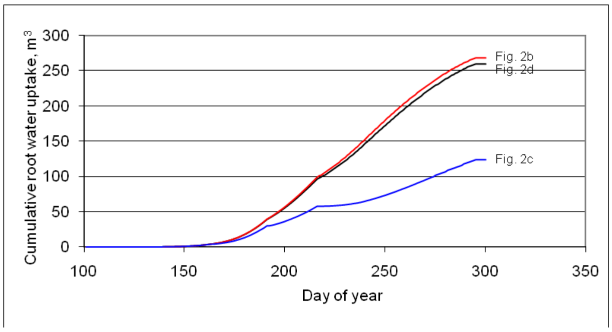

Figure 13. Sensitivity of cumulative root water uptake to different plant water response functions shown in Figure 2.

water uptake parameters. It should be noted that sensitivity to the soil water retention parameters represents an integrated effect of root water uptake and van Genuchten parameters. This implies that for model calibration these parameters need to be measured accurately. The cumulative solute load was found to be insensitive to the changes in the longitudinal dispersivity coefficient and sorption distribution coefficient, K_d, while the solute mass distribution was quite sensitive to these parameters. Cumulative solute load, initial and final solute mass distributions were sensitive to the initial concentration distributions.

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