A Branch-and-Price Algorithm for the Robust Airline Crew Pairing Problem

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Abstract

Robust Airline Crew Pairing Problem is a proactive planning approach, which includes considering delays and disruptions that could happen in the operations. The main objective is to create crew pairings that are less prone to disruptions or easier to reschedule once disrupted. We model the problem as a Bi-Objective General Set Covering Problem to minimize the estimated propagated delay while at the same time maintaining a cost effective solution. We exploit historical information on disruptions and separate dataset into two subsets. We optimize the expected propagated delays based on the first portion of the dataset, and limit the price of robustness using ε-constraint method. We develop a branch-and-price based solution algorithm that column generation is applied at each node of the branch-and-bound tree. A field-collected actual schedule dataset for a small-scale Turkish airline which operates short-haul domestic flights on a hub-and-spoke network, is used for the experiments in evaluating the model and the proposed solution method. We also conduct simulation experiments to verify optimization results and to determine the robustness of the obtained solutions, where the second part of the historical data is used as an input. The computational results obtained are encouraging, which show that on average the proposed methodology attains optimal solutions for the obtained dataset and solution times are reasonable enough to conduct several different scenarios for a final decision.

Keywords: Robust Crew Pairing Optimization, Branch-and-Price Algorithm, Column Generation, Robustness, Delay Propagation.

Aksaklıklara Karşı Dayanıklı Ekip Eşleme Problemi İçin Bir Dal-Ücret Algoritması

Öz

Aksaklıklara Karşı Dayanıklı Ekip Eşleme Problemi, klasik ekip eşleme probleminin çizelgelerin uygulanması safhasında meydana gelebilecek gecikme ve aksaklıkları dikkate alarak yapılan proaktif bir planlama yaklaşımı ile ele alınmış halidir. Yaklaşımda esas amaç, aksaklıklara daha az maruz kalabilecek veya aksaklıklara maruz kalındığı zaman tekrar çizelgelenmesi daha kolay ekip eşlemelerinin üretilmesidir. Söz konusu problem, yayılan gecikmelerin beklenen değerinin enazlanması ve aynı zamanda maliyet-etkin bir çözümün muhafaza edilmesini amaçlayan Çift-Amaçlı Genel Küme Kapsama modeli olarak formüle edilmiştir. Çalışmada, aksaklık bilgilerini içeren geçmiş veri setlerinin kullanılması ve bu veri setinin iki alt parçaya ayrılması önerilmiştir. Veri setinin birinci parçasının yayılan gecikmelerin beklenen değerinin eniyilenmesinde kullanılması öngörülmüş ve dayanıklılığın bedeli, ε-yöntemi kullanılarak sınırlandırılmaya çalışılmıştır. Çözüm yaklaşımı olarak dal-sınır ağacının her bir düğümünde sütun oluşturma yöntemi uygulanan Dal-Ücret algoritması esaslı bir algoritma geliştirilmiştir. Önerilen model ve çözüm yaklaşımının değerlendirilmesi için yapılan deneylerde; Türkiye'de yerleşik, ana dağıtım üssü-kenar üs uçuş ağ yapısını uygulayan, küçük ölçekli bir havayolu şirketine ait gerçek veriler kullanılmıştır. Ayrıca, geçmiş veri setinin ikinci parçası girdi olarak kullanılarak elde edilen eniyileme sonuçlarının geçerlemesi ve çözümlerin dayanıklılığının

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belirlenmesi için çeşitli benzetim deneyleri yapılmıştır. Gerçek veri seti kullanılarak elde edilen deneysel sonuçlar umut verici olup önerilen yaklaşımın ortalamada eniyi sonuçlar üretebildiği ve son karar öncesi birçok değişik senaryonun değerlendirilmesine imkân verecek ölçüde, kabul edilebilir çözüm zamanlarında çözümlerin elde edilebildiği gözlenmiştir.

Anahtar Kelimeler: Dayanıklı Ekip Eşleme Eniyilemesi, Dal-Ücret Algoritması, Sütun Oluşturma Yöntemi, Dayanıklılık, Gecikme Yayılımı.

Introduction

Since 1950s, operations research techniques have been applied in the airline industry, especially in airline scheduling problems, which belong to a broad class of large-scale, network-based resource allocation problems and form a decision making process that plays a crucial role in the industry. Airline scheduling problems consist of constructing schedules for the airline companies' scarce resources in such a way as to use that resources available in an efficient manner, both in schedule planning and execution (operations) stages. The most expensive resources for airlines are the aircrafts and crew members, and effective scheduling of these two resources can lead to huge amount of savings.

Airline Schedule Planning process is too large and complex to be solved as a whole, single problem; so it is typically divided into four sequential sub-problems – *flight schedule design, fleet assignment, aircraft routing* and *crew scheduling*. In *flight schedule design*, considering the marketing needs, a flight schedule, which consists of a set of flight legs having specific origin/destination airports and departure/arrival times, is determined that offers the highest potential revenue. In *fleet assignment*, each fleet, an aircraft type such as Boeing 777 or Airbus 380, is assigned to each flight leg. The specific aircraft in the pre-determined fleet, which will actually operate that flight leg is decided in the *aircraft routing* stage. Also in this stage the maintenance requirements of each aircraft is taken into account. Small-scale and sometimes middle-scale airlines might merge fleet assignment and aircraft routing into a single problem.

Lastly, in *crew scheduling*, crew members are assigned to flights in the least expensive way possible, assuring that each flight is covered and that government/safety regulations regarding crew scheduling are met (Barnhart, Cohn, Johnson, Klabjan, Nemhauser, & Vance, 2003). Crew scheduling forms a critical phase in this schedule planning process, since crew costs are the second largest component of direct operating costs for airlines, amounting approximately 15-20% of the total costs.

Crew scheduling is traditionally divided into two stages: *crew pairing* and *crew rostering*, and solved with a sequential approach. *Crew*

pairing (Tour-of-duty planning) is done in order to construct anonymous pairings, which are the sequences of flight legs that start and end at a crew base, to partition the given flight schedule in a cost-effective way. In *crew rostering* the fixed pairings are assigned to individual crew members, considering the leave times, training periods etc. in order to construct work schedules, usually for a month. *Crew pairing* is generally accepted as the most significant stage of crew scheduling, because the most of the cost improvements can be achieved by optimizing pairings (Barnhart, Cohn, Johnson, Klabjan, Nemhauser, & Vance, 2003). In this paper, only the crew pairing problem is dealt with.

In the airline crew scheduling context, there are substantial differences between the way that domestic and international operations are scheduled. International flight networks have a tendency to be relatively sparse with a limited number of flight legs (Barnhart, Cohn, Johnson, Klabjan, Nemhauser, & Vance, 2003). On the other hand, domestic flight networks are characterized by *hub-and-spoke* networks, which are a system of flight connections arranged like a wheel, in which all air traffic moves along low activity airports (spokes), connected to the high activity airports (hubs). Most of the airlines use the hub-and-spoke network structure for their domestic flights to develop a wider network and to increase profitability. However, in this kind of network the flight connections are highly increased, so inbound and outbound flights are tightly timed and coordinated to maximize flying time (McShan & Windle, 1989). However, this aspect leads to explosion in the number of probable pairings and exacerbate the sensibility of the schedules to disruptions, and as a consequence the problem becomes more intractable and prone to delays.

Previously mentioned *flight schedule design* step typically begins one year prior to operations of the flight schedule and lasts nearly nine months (Lohatepanont & Barnhart, 2004). *Fleet assignment* step is performed as early as three months in advance, *aircraft routing* and *crew scheduling* stages are conducted two weeks to one month in advance. Approximately two or three weeks before the day of operations all the schedules are passed to the unit that is responsible for the operations, airline's Operations Control Center (OCC) or Flight Dispatch Office.

OCC monitors and analyses all irregular events, which is defined as disruption that is unavoidable to result in rescheduling. Also OCC conducts coordination with the other internal/external entities and rescheduling actions to ensure seamless execution of the planned schedules. To resolve the crew-related disruptions in operations, OCC implement essential changes to the planned crew schedules, in order to return to original crew

schedule as soon as possible. In airline operations when disruptions occur, crew rescheduling typically functions as a bottleneck for the whole airline rescheduling process, because of convoluted and tight crew schedules, and the fact that most crew members are operating at near-capacity. Most of the time this tight connections lead to brittle schedules that can be easily destroyed even by a minor delay.

Flight delays' negative impacts are comprehensive, increasing flight delays pose a major threat on the whole air travel system and cost airlines at billions of dollars each year. As stated in a research made by Airlines for America (A4A), in 2012 it is estimated that delays driven $7.2 billion in direct operating costs for scheduled U.S. passenger airlines, and considering that crew costs are estimated as $16.26 per minute, the total crew related cost is approximately $1.5 billion, see Table 1.

Table 1: Cost of Delays to U.S. Airlines (Airlines for America, 2013)

Calendar Year 2012	Direct Aircraft Operating Cost per Block Minute	Δ vs. 2011	2012 Delay Costs ($ millions)
Fuel	$39.26	5.3%	$3,603
Crew	$16.26	1.7%	$1,492
Maintenance	$12.02	3.1%	$1,103
Aircraft Ownership	$7.92	-1.1%	$727
Other	$2.71	5.0%	$249
Total Direct Operating Costs	$78.17	3.5%	$7,175

With every passing year, flight disruptions become more prevalent in airline operations. Basically there are two approaches to deal with the uncertainties related to flight disruptions. Rescheduling crews in operations is a *reactive approach* to handle uncertainty in real time after disruptions occur. Main drawback of this approach is the fact that solution is expected immediately. Additionally, rescheduling requires considering several complex constraints and these aspects leads to intractability issues. Because of the need of immediate response, airlines currently employ manual or heuristic methods that yield sub-optimal solutions. Unlike the reactive approach, which includes dealing with disruptions on the day of operations; robust airline scheduling is a *proactive approach*, which involves considering disruptions and delays that could happen in operations. The main objective in robust scheduling is to reduce the total realized costs,

including both planning and rescheduling costs. Thus, schedules that are less susceptible to disturbances or easier to repair once disrupted, could be created. Considering the crew scheduling stage, robustness is often considered in the pairing problem instead of rostering problem.

The root causes of the airline disruptions may be inclement weather, equipment failure, crew sickness, late passengers, airport congestion etc. Such disruptions most probably may result with flight delays and cancellations that could not only considerably increase the operational crew costs, but also may negatively impact airline's goodwill. In recent years importance of robust crew pairing problem is highly understood, because even a little improvement in solution can save millions of dollars for large-scale airlines.

Main challenge of constructing robust schedules is to define and find abstract measures of robustness that can make possible to formulize mathematically the scheduling problems. In airline schedule planning context, robustness has two components; flexibility, the ability to recover easily and timely in the face of disruptions; and resilience, the ability to continue to operate as planned by absorbing minor disruptions.

In case of delays it is essential to identify if it is a primary delay or if it is a propagated delay, which is a consequence of another earlier event. Primary delay is the one that is not affected by any earlier or accumulated delay. However, a propagated delay is the accumulated one and it is identified as the largest delay cause (Guest, 2007). Because of higher additional operational costs incurred by delay propagation, nowadays airlines are interested beyond just crew schedules with optimal planned crew cost, they are interested in robust schedules with low operational crew cost, in which optimal planned cost is not considered as necessary.

In this paper, we attempt to approach the robust crew pairing problem from both ends, including methodology, mathematical model and solution algorithm. The contributions of this paper are threefold: First, we develop a new bi-objective formulation for robust ACP, which aims to minimize propagated delays in case of disruptions while at the same time maintaining a cost effective solution. Second, we build a Branch-And-Price solution algorithm to deal with the additional complexity introduced by the consideration of robustness to the already *NP*-hard problem. In order to construct operationally robust crew schedules, the proposed model computes optimal schedules for flight schedules depending on enough and accurate historical data related to propagated delays. Third, we partition the historical data into two separate sets, and both for optimization and

evaluation, and we employ Sample Average Approximation method, where the expected value is approximated by the average over samples. This mitigates the side effects of using the same data or distribution functions while building robust schedules and evaluating the performance of them.

The remainder of the paper is organized as follows. Section 2 presents a brief literature review of related work in the area. Section 3 defines basic terminology and contains a detailed description of the robust ACP problem, including the concept of robustness and methods for incorporating a measure of robustness into the crew scheduling. Section 4 presents the proposed methodology, bi-objective formulation as well as Branch-and-Price based solution approach. The results of computational experiments for the proposed model based on real data from a Turkish airline are discussed in Section 5. Finally, the concluding remarks and outlining possible research extensions are presented in Section 6.

Literature Survey

Well-designed airline crew scheduling algorithms can lower the crew cost of an airline company, enabling the company to stay competitive. In the 1960s most researchers tried to develop efficient exact methods that are essentially non polynomial-time algorithms. With the advent of the complexity theory, researchers began to realize that many of these problems may be inherently difficult to solve. In the 1970s, many scheduling problems including the ACP problem were shown to be *NP*-hard and researchers started to focus on heuristic or meta-heuristic methods. But for the last ten years, thanks to improvements on computational technology, exact mathematical methods such as Column Generation (CG), decomposition and approximation algorithms, have been come into use heavily.

The ACP problem has a set of specific terms (Barnhart, Cohn, Johnson, Klabjan, Nemhauser, & Vance, 2003). A *flight schedule* is the list of flight legs including the origin/destination airports and departure and arrival times. A *flight* or a *flight leg* is a non-sop direct flight from an origin airport to a destination airport with specified departure and arrival times. *Block time* is the time between arrival time to the gate and departure time from the gate. *Crew base* is the home base where crew members must start and end their job. Most of the airlines have several crew bases.

A *crew pairing* (*tour-of-duty*), similar to aircraft routing, is a sequence of connected flight legs, which start and end at the same crew base that could be conducted by the same crew. In order to be legal, a crew

pairing must satisfy Directorate of Civil Aviation, labor union, contractual rules and some safety requirements (e.g. the 8-in-24 rule), and airline's own constraints. Some of these restrictions include, but are not limited to, maximum daily working hours, minimum overnight rest period, maximum number of operating flights, and maximum time away from base. For crews that fly in domestic flights, pairings tend to last one to three days depending on those legality constraints. Normally, airlines heuristically opt for short pairings, because it is generally believed that short pairings are more robust to disruptions than long ones.

Crew pairings are usually separated into *duty periods* which are divided by *layover* or *rest period*. Similarly duty periods consist of one or more flight legs separated by *ground times*. *Deadheading* is the practice of positioning crew members as a passenger to a particular destination to assume a flight or back to a crew base. The total duration of a pairing, that is the total time from departure to return back to crew base is called *Time Away From Base (TAFB). Minimum Turn Time* is the minimum time interval between two consecutive flights that belong to the same pairing. *Flying time* is the total duration of a pairing spent in flying.

One of the important factors that require to be considered when constructing a pairing is the cost of it. The cost of a pairing is typically calculated as a maximum value of flying time, total elapsed time of the duty periods and TAFB, which is a nonlinear function. Usually, the cost of a crew pairing is not measured in monetary terms, but it is determined in terms of time.

Existing crew pairing formulations in literature can be classified into two main categories, namely the set partitioning (covering) and the network flow models. Desaulniers et al. formulate the problem as an integer, nonlinear multi-commodity network flow model with additional resource variables, and pairing generation is performed by the approach of resource constrained shortest path sub-problem (G. Desaulniers, J. Desrosiers, et al. 1997). Hoffman and Padberg propose both pure set partitioning and set partitioning formulation with side constraints, and present a Branch-and-Cut solution approach to optimality (Hoffman & Padberg, 1993). Because of the difficulty in dealing with complex and various constraints, set partitioning (covering) models are much preferred by the researchers (Barnhart and Shenoi 1998).

The Set Partitioning Problem (SPP) is to partition elements into a number of subsets and each binary variable (column) represents a subset of elements defined by the coefficients. In ACP problem each variable

(column) is a pairing with a given cost and the set partitioning constraints represent the flight legs that must be flown. Set Covering Problem is the enhanced version of the SPP to allow the possibility of deadheading. The number of variables in the problem is usually very large in practice. Thus, a special method such as dynamic (delayed) CG is typically employed to solve the Linear Programming (LP) relaxation of the problem (Vance, et al. 1997) (Anbil, Forrest, & Pulleyblank, 1998).

The ACP problem is generally enlarged with additional crew base balancing constraints, which restricts the number of crews involved from each crew base. Because of specified lower and upper bounds, this form of constraints is known as *two-sided knapsack constraints*. Since these two-sided knapsack constraints have non-unit right hand side values, the ACP problem is often modelled as a Generalized Set Covering Problem (GSCP). GSCP model can be formulated as;

$$minimize \quad c^T x \tag{1}$$

$$subject\ to \quad \boldsymbol{A}x \geq 1 \tag{2}$$

$$\boldsymbol{M}x \begin{Bmatrix} \geq \\ = \\ \leq \end{Bmatrix} b \tag{3}$$

$$x \in \{0,1\}^n \tag{4}$$

In this GSCP formulation, each decision variable (column) *x* refers to a feasible pairing and *c* is the cost of each pairing in terms of time. The objective function (1) minimizes the cost of the required crews. The decision variable *x* is equal to 1 if the pairing is included in the solution and 0 otherwise. The set of constraints in (2) referred to as *flight leg constraints* that guarantee flight coverage that each flight leg is included in at least one pairing, which means that deadheading is allowed. The set of constraints in (3) referred to as *crew base balancing constraints* and confirm that crews contained in the solution which start/end at a crew base must be between stated lower and upper bounds. And (4) ensures that the decision variables are integral.

In order to enumerate variables in other words generate pairings, two main types of network structures are used in the literature, *duty period network* and *flight (time-space) network*. The *duty period network* has an arc for each possible duty period and arcs demonstrating possible layover

connections between the duties. The *flight (time-space) network* has an arc for each flight leg in the given flight schedule and arcs representing possible connections between the flights. In domestic or short-haul international flights, the flight network is preferred, because the flight times are tend to be short and there are large number of flight legs in duty periods. On the other hand, in long-haul international flights, duty period network is preferred, because the flight times are long and there are only several flights in duty periods (Barnhart, Cohn, Johnson, Klabjan, Nemhauser, & Vance, 2003).

Usually CG technique is employed to solve large-scale ACP, because of the extremely large number of variables, complexity of the constraints that must be satisfied for each pairing, and non-linear cost structure of the pairings. In CG technique the master problem is the LP relaxation of the original integer programming model and the pricing sub-problem is typically formulated as a resource constrained shortest path problem in which the resource constraints ensure that only legal pairings that satisfy all legality constraints are generated.

CG technique is one of the most common methods for solving large-scale integer programming problems, in which only a restricted version of the problem is solved with only a subset of the columns. A general reference on CG is the book edited by Desaulniers et al. (Desaulniers, Desrosiers, & Solomon, 2005). The basic idea on CG is to formulate the problem as an integer programming problem, solve the underlying LP relaxation to obtain an optimal fractional solution, and then round the fractional solution to a feasible integer solution in such a way that the error can be bounded.

In CG approach only a subset of pairings needs to be dealt with in order to solve the problem, because in the optimal solution most of the columns will be non-basic with a value of zero. In CG process, only the columns which have the potential to improve the objective function are used. When implementing CG first, the problem is divided into two sub-problems, the Restricted Master Problem (RMP) and the Pricing Sub-Problem (PSP). RMP is the LP relaxation of the original problem and consist of only the initial columns to start with. Then, LP relaxation of the RMP is solved so as to find the dual variables and these duals are passed to PSP. The objective of the PSP is to find potential new columns relating to the current dual variables obtained from RMP. And then the new columns with the reduced cost value are passed to the RMP until no such column is found.

After CG process, in order to find integer solutions, a Branch-and-Bound (B&B) or Branch-and-Cut (B&C) method must be applied to the

original problem which has only the columns generated in the previous step. However, this process does not guarantee a globally optimal solution, because at the B&B or B&C stage, there may be a pairing that would have more reduced cost value than the current one but is not present in the master problem. Thus, to find a globally optimal solution, columns have to be generated after branching and this leads to Branch-and-Price (B&P) method (Barnhart, Johnson, Nemhauser, Savelsbergh, & Vance, 1998). B&P is similar to the B&B technique, but CG is used at each node of the B&B tree dynamically.

The traditional ACP approach requires that all the data related to flights and resources be fully known and deterministic. But airline operations are prone to suffer disruptions caused by several reasons, and in case of disruptions optimal crew pairings easily become sub-optimal. Traditional approaches have proved to be not capable to address such disruptions adequately, and furthermore tight schedules created by deterministic approaches lead to propagation of the delays, because globally optimized deterministic solutions have short buffer times between flights and also easily rescheduling is not taken into account.

Even though usually external factors, such as inclement weather or airport congestion, are held responsible for delays, certain delays especially propagated delays are predictable and avoidable with incorporating robustness to the schedules at the planning stage. Most of the flight delays are caused by propagated delay, which converts a minor flight disruption to a major one. This translates into that decrease in propagated delay leads to an operationally robust schedule (M. B. Tam 2011). Propagated delay often occurs in the domestic or short-haul international flights because of relatively short flight times. In operations stage the effect of an individual primary delay may be negligible, but its effect on the overall schedule, namely the propagated delay may lead significant negative complications. In order to reduce operational costs and improve punctuality, robustness has to be integrated into the models by decreasing the propagated delay.

Despite the fact that there are extensive publications dealing with traditional ACP problem, there is small fraction of research related specifically to robust ACP problem. Some different robust crew scheduling approaches are as follows;

Schaefer et al. propose an approach in which the planned cost of pairings are substituted with expected operational cost of pairings, which is calculated by the simulation of individual pairings. In the simulation they use only push-back recovery, which assume that departure of each flight leg is delayed until the crew and the aircraft are available. Hence, the main

drawbacks of this approach are the lack of consideration of the interactions between pairings, especially the delay propagation, and the complex estimation of the expected operational cost of each pairing (Schaefer, Johnson, Kleywegt, & Nemhauser, 2005).

Ehrgott and Ryan present a bi-objective optimization approach which attempts to generate Pareto-optimal solutions. The bi-objective is to minimize both the planned cost and the non-robustness penalty, in which non-robustness penalty is used to penalize the aircraft changes when the crew ground time is short. They develop a model based on ε-constraint method and a set partitioning algorithm to solve the model (Ehrgott & Ryan, 2002).

Yen and Birge introduce a two-stage stochastic programming model with recourse, in which recourse problem is employed to calculate the cost of delays. In this two-stage approach non-robust flight connections are discarded on at a time until the operational crew cost is minimized. The calculation of the operational crew cost is done by adding expected recovery cost to planned crew cost. The robustness measure is similar to Ehrgott and Ryan's, except the assumption that the flight time is a random variable. Main shortcoming of the proposed approach is the computationally intractability of the solution method (Yen & Birge, 2006).

Shebalov and Klabjan propose a bi-objective optimization approach to generate crew schedules that has increased flexibility in case of crew recovery. One of the two objective functions is to maximize the number of move-up crews, which are the crews that can be possibly swapped during operations. The other objective is to minimize the crew costs and it is incorporated as a constraint into the model. Also, a solution method based on the combination of CG and Lagrangian relaxation for the computationally difficult problem is introduced (Shebalov & Klabjan, 2006).

Tam and Ehrgott suggest a bi-objective optimization approach with an additional objective to maximize unit crewing connection with minimizing the planned crew cost. The unit crewing is to construct crew schedules to facilitate crew members operate the same sequence of flight legs within the duty periods as much as possible. Robustness is considered as the utilization of unit crewing (Tam & Ehrgott, 2007).

Lucian and Natalia formulate the problem as a two-stage stochastic programming model with recourse with regard to swapping opportunities of crews. As a solution method a CG based framework with a constraint generation extension is proposed. So as to evaluate the robustness of the

schedules, a scenario based simulation model with improved recovery actions is used (Lucian & Natalia, 2011).

However, in most of the researches mentioned above, the interaction and interdependence between pairings and propagated delays have not been captured literally; there are still opportunities for further enhancements. Additionally, to our knowledge, there is no crew pairing optimization approach available yet which uses separate sets of historical data during robust schedule optimization and evaluation.

Robust Airline Crew Pairing Problem

Disruptions are inevitable in airline operations due to irregular events such as inclement weather, equipment failure, crew sickness, late passengers, airport congestion etc. In case of these irregular operations, flight legs, aircrafts and crews required to be rescheduled. In practice, it is done in an iterative way similar to schedule planning process. Possible course of actions for flight rescheduling are delaying or cancelling flights, swapping or ferrying aircrafts, using reserve crew or swapping crew. Disruptions effect crew schedules in different ways depending on the robustness of the planned crew schedule.

Traditional airline scheduling models usually end up with highly utilized aircrafts and crews. Solutions to these models include short turn and ground times in order to maximize aircraft utilization and crew flying hours. But in the face of operations when disruptions occur, these solutions could render operational schedules fragile. Even short delays in flights may cause a snowball effect on the schedules and lack of slack times between flights may cause propagation of delays across the whole schedule. This snowball effect usually can be described by delay propagation.

So taking into consideration the disruptions, which may occur in operations, it is of paramount importance to construct robust schedules, because they can cause to suffer large recovery and rescheduling costs. Hence, most of the descriptions of robust schedule involve minimizing the operational costs rather than the planned costs. However, there is still no a clear and common definition of robustness in airline scheduling.

Robustness

There are several difficulties related to robustness. First of all, it is really hard to define robustness in the problem context and incorporate it into the model, and unfortunately there is no methodological way in the literature to define and incorporate robustness for the airline schedule

planning problems. Robustness is generally distinguished into two characteristics; flexibility, the ability to recover easily and timely in the face of disruptions; and resilience (stability), the ability to continue to operate as planned by absorbing minor disruptions. Flexibility is generally achieved by providing recovery opportunities such as maximizing swapping opportunities of resources. Resilience is largely attained by reducing delay propagation reducing aircraft and crew changes per pairing and adding slack (buffer) times into ground and turn times. With incorporating these aspects of robustness into the schedules, a delay on a flight leg may be confined to that pairing and have no overall impact on the whole schedule.

Secondly, measuring the robustness of a schedule in advance of operations is really challenging. Because it is hard to estimate what kind of disruptions may happen and when will happen. This also depends on the airline's disruption recovery strategy. A realistic way to measure the robustness of schedules is the On-Time Performance (OTP) of operated flights in a real or simulation environment (Lucian & Natalia, 2011). OTP is the most practically valid and well-known indicator of punctuality of the schedules, which is the fraction of flights arriving on-time at their destination airport. OTP is a key measure for all stakeholders especially for airlines, because they are related to direct costs due to the loss of efficiency additionally to indirect costs due to the loss of time and goodwill (Wu & Caves, 2003). Also it must be noted that propagated delays have a great impact on OTP.

Finally, it is difficult to compromise between cost and robustness, because of quantifying the additional cost of robustness and uncertainty. Thus, most of the research in the literature related to robust scheduling consider using bi-objective optimization approach to obtain a schedule that operates relatively well concerning different robustness measures.

Propagated Delay

Flight delays can be separated into two components: *primary delay*, which is the delay which originates at the destination or during the course of the fly, and *propagated delay*, which is the delay that is caused by snowball effect from a primary delay and generally referred to be caused by late arriving flight. Propagated delay, which has a minimum value of 0, is calculated as the difference between delay time and the slack time, is the difference between provided ground time and minimum turn time. Crew schedules that have reduced level of propagated delay appear to be more robust when confronted with unforeseen volatility in the operations, largely due to their capability to adapt better and more quickly.

According to European Organization for the Safety of Air Navigation (EUROCONTROL) records, average delay per flight in Europe increased 134 % from 12 minutes in 2006 to 28 minutes in 2009. And more than 24% of arrivals were over 15 minutes late in 2010. Propagated delays have shown a general upward trend since 2003 and constitute nearly half of all delay minutes in Europe (Cook & Tanner, 2011).

In the past it was limited to better comprehend and handle propagated delay in practice and usually primary delays were taken into account as the main factor for better OTP (Guest, 2007). Propagated delays create significantly greater impact than the primary delays and an individual primary delay can create snowball effect through the entire schedule, affecting a large number of subsequent flights (AhmadBeygi, Cohn, Guan, & Belobaba, 2008).

Modeling Approaches

At present, there are two common modeling approaches that consider robustness in ACP. First one is the bi-objective optimization approaches, in which a deterministic measure is defined for the propagation of delays in the schedule. Considering the conflicting nature of the cost and robustness objective functions, constraint scalarisation techniques are usually employed, in which one of the objective functions is transformed into a constraint (Tam, Ehrgott, Ryan, & Zakeri, 2011).

The other one is the two-stage stochastic integer programming approaches with recourse, in which sampling from the determined delay distributions are used to evaluate the propagation of delays. This is captured with a recourse cost added to the crew cost. Therefore, the intention of employing the recourse cost is similar to the robustness objective in the bi-objective approaches (Tam, Ehrgott, Ryan, & Zakeri, 2011).

However, in stochastic programming approaches, usually unrealistic assumptions are made in order to reduce the complexity of the scheduling problem, such as flight delays are considered independent or flights are delayed until all resources (crews and aircraft) are available. Additionally, it is unlikely to determine a single probability density function with certain parameters that can reflect the flight delay distribution completely, and they require significantly higher solution times compared to bi-objective models.

Methodology and Solution Approach

We develop a bi-objective optimization model, in which we attempt to minimize propagated delay while a certain level of cost is preserved with

the help of a given set of historical data. In the proposed model, there is a compromise between the crew cost and delay propagation minimization, because in order to obtain low level delay propagation, crew costs becomes higher and vice versa. Thus, we employ ε-constraint method to transform one of the constraints into a constraint. As an initial stage for our methodology, deterministic ACP problem must be solved in order to find the minimum crew cost level, and propagated delays must be calculated from the given historical data.

Calculating the Propagated Delays

Flight delay decomposition that is demonstrated in Figure 1 illustrates the relationship between departures, arrivals, and delays of two flights in the same pairing. The solid lines with arrows show the planned flight and the dotted ones display the actual flight, for flight legs i and j. *PDT, ADT, PAT* and *AAT* are the Planned Departure Time, Actual Departure Time, Planned Arrival Time and Actual Arrival Time, respectively. The Planned Turn Time between flights i and j (PTT_{ij}) is the time between *PAT* of flight i and *PDT* of flight j, see Equation (5). *ATT* (Actual Turn Time) is the *ADT* of flight leg j minus the *AAT* of flight leg i. If *ATT* is smaller than Minimum Turn Time (*MMT*) the crew is considered disrupted. It is assumed that the same crew is assigned to flight leg i and j. As shown in Figure 1, when flight leg i is delayed, the Slack time ($Slack_{ij}$) between two flights completely and some part of the *MMT* is consumed, and this causes Propagated Delay (*PD*). Hence, the Total Departure Delay (*TDD*) of flight j is consist of the PD_{ij} and the Independent Departure Delay (*IDD*) of flight j itself. Likewise, the Total Arrival Delay (*TAD*) of flight j comprises PD_{ij} and the Independent Arrival Delay (*IAD*). Although *IDD* includes only the independent delay before a flight departs, *IAD* includes both *IDD* and the additional independent delay in the air or at the origin and destination airports.

$$PTT_{ij} = PDT_j - PAT_i \qquad (5)$$

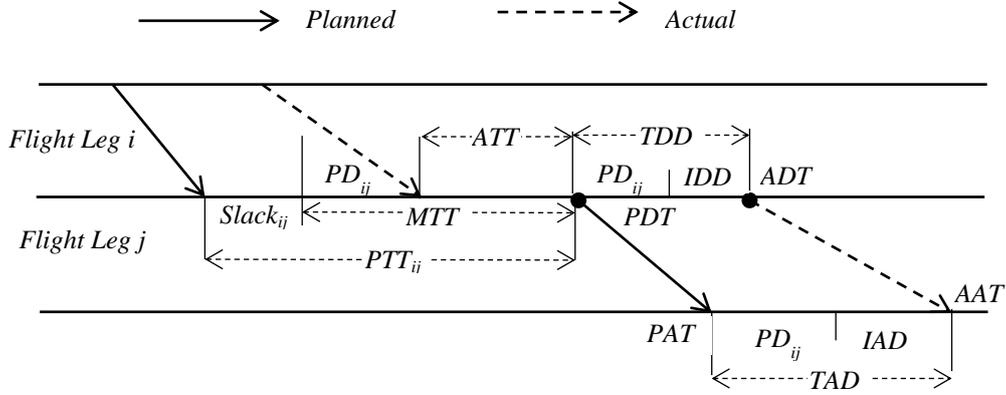$$Slack_{ij} = PTT_{ij} - MTT_{ij} \qquad (6)$$

Figure 1: Flight Delay Decomposition

As shown in Figure 1, when flight leg i is delayed, the Slack time ($Slack_{ij}$) between two flights is consumed completely and also some part of the MMT is consumed, and this causes Propagated Delay (PD). Hence, the Total Departure Delay (TDD) of flight j is consist of the PD_{ij} and the Independent Departure Delay (IDD) of flight j itself. Likewise, the Total Arrival Delay (TAD) of flight j comprises PD_{ij} and the Independent Arrival Delay (IAD). Although IDD includes only the independent delay before a flight departs, IAD includes both IDD and the additional independent delay in the air or at the origin and destination airports. We can calculate the following relationships in order to find the propagated delay:

$$TDD_j = \max\left(ADT_j - PDT_j, 0\right) = \max(PD_{ij} + IDD_j, 0) \qquad (\,7\,)$$

$$TAD_j = \max\left(AAT_j - PAT_j, 0\right) = \max(PD_{ij} + IAT_j, 0) \qquad (\,8\,)$$

$$PD_{ij} = \max\left(TAD_i - Slack_{ij}, 0\right) \qquad\qquad\qquad (\,9\,)$$

We split our historical dataset into two separate parts as *past data* and *future data*. We use the Sample Average Approximation method, which is a non-parametric approach, in order to make use of past data to generate the solution, and then the future data is used to evaluate the solution. We consider each day of operation in the historical data as one instance of a delay scenario (ω), assuming that each delay scenario is equally likely. Therefore, the set of delay scenarios (Ω) cardinality is equal to total day of operations in the set. Each pair of consecutive flight legs i and j in delay scenario (ω) is the *TAD* of flight leg i, which is obtained from historical

dataset, and *Slack$_{ij}$* is the crew ground slack time between flight legs *i* and *j*, which is obtained from the original flight schedule calculated simply by subtracting the *MMT* from the *PTT*. Propagated delay for each pairing must be calculated by taking into account each consecutive flight legs in any given pairing.

For each day of operation, we can obtain *PD*s using *IAD*s, which assumes that the first flight of each day has zero propagated delay. We then solve the proposed robust model over each day of operation to obtain a planned schedule. Next, we use the actual delay information from future data to evaluate performance of the robust schedules. After that, we conduct simulation experiments in order to evaluate the performance of the obtained solutions.

Mathematical Model

All required notations are defined in follow:

Nomenclature
Indices
i,j flight leg index
k pairing index
m crew base index
ω delay scenario index
Sets
F set of flight legs
P set of pairings
M set of crew bases
Ω set of delay scenarios
p^{ω} set of delay scenario probabilities ($\omega \in \Omega$)
Parameters
c_k cost of pairing k
pd^{w}_{ijk} propagated delay from flight i to j in pairing k in delay scenario ω
a_{ik} 1 if flight leg i is covered by pairing k, 0 otherwise
h_{km} 1 if crew base m is the crew base for pairing k, 0 otherwise
l_m minimum number of crew who serve in crew base m
u_m maximum number of crew who serve in crew base m
Z_{opt} optimal solution value of the deterministic problem
ε robustness factor , which measures how much extra crew cost we are ready to compromise
Variable
x_k 1 if pairing k is involved in the solution, 0 otherwise

As mentioned before, we attempt to make crew pairings robust by minimizing expected propagated delay. Using the notation introduced above, we assume that the total propagated delay of each paring is independent. Then, the objective function for minimization of expected propagated delay can be written as;

$$minimize \ \ \mathbb{E}\left[\sum_{k\in P}\left\{\sum_{i,j\in K}(pd_{ijk})\right\}x_k\right] \tag{10}$$

$$minimize \ \ \sum_{k\in P}\left\{x_k\sum_{i,j\in K}\mathbb{E}\left(pd_{ijk}\right)\right\} \tag{11}$$

By assuming that each delay scenario (ω) is equally likely and probable set of delay scenarios (Ω) has finite cardinality, the expected value of the total propagated delay in a pairing can be calculated as follows;

$$\mathbb{E}\left(pd_{ijk}\right) = \sum_{w\in\Omega}p^w pd_{ijk}^w \tag{12}$$

In order to capture the compromise between two conflicting objectives, the problem is formulated as a bi-objective optimization model. Specifically, the objective functions are;

$$minimize \ \ \sum_{k\in P}\left(x_k\sum_{i,j\in K}\sum_{w\in\Omega}p^w\,pd_{ijk}^w\right) \tag{13}$$

$$minimize \ \ \sum_{k\in P}c_k x_k \tag{14}$$

We try to minimize the propagated delay while simultaneously conserving a cost-effective solution. Thus, we employed ε-constraint method that crew minimization objective function is transformed into a constraint limited by an upper bound. The compact formulation after applying ε-constraint method is shown below;

$$minimize \quad \sum_{k\in P}\left(x_k\sum_{i,j\in K}\sum_{w\in\Omega}p^w\,pd_{ijk}^w\right) \tag{15}$$

$$subject\ to \quad \sum_{k\in P}a_{ik}x_k \ \geq \ 1 \qquad \forall i\in F \tag{16}$$

$$\sum_{k\in P}h_{km}x_k \geq l_m \qquad \forall m\in M \tag{17}$$

$$\sum_{k \in P} h_{km} x_k \leq u_m \qquad\qquad \forall m \in M \qquad\qquad (\,18\,)$$

$$\sum_{k \in P} c_k x_k \leq (1 + \varepsilon)\ Z_{Opt} \qquad\qquad (\,19\,)$$

$$x_k \in \{0,1\} \qquad\qquad \forall k \in P \qquad\qquad (\,20\,)$$

$$pd_{ijk}^w \geq 0 \qquad \forall i,j \in F, \forall k \in P, \forall w \in \Omega \qquad (\,21\,)$$

$$l_m \geq 0 \qquad\qquad \forall m \in M \qquad\qquad (\,22\,)$$

$$u_m \geq 0 \qquad\qquad \forall m \in M \qquad\qquad (\,23\,)$$

$$a_{ik} \geq 0 \qquad\qquad \forall i \in F, \forall k \in P \qquad (\,24\,)$$

$$h_{km} \geq 0 \qquad\qquad \forall k \in P, \forall m \in M \qquad (\,25\,)$$

The original compact formulation is a GSCP-based formulation consisting of a binary decision variable. The objective function of the model is to minimize expected propagated delays (15) with the flight leg covering or set covering constraints (16), crew base balancing constraints (17,18) and robustness-cost objective limit constraint (19). There is one flight leg covering constraint for each flight leg and there is one crew base balancing constraint for each crew base.

The flight leg covering constraints (16) ensure that every flight in the given flight schedule is covered by at least one pairing. Thus, a pairing can only be flown by a crew member. In our approach set covering formulation is preferred to set partitioning one, because LP relaxation of set covering is mathematically far steadier and thus easier to solve, and it is easier to construct a feasible integer solution from a LP relaxation solution (Barnhart, Johnson, Nemhauser, Savelsbergh, & Vance, 1998).

The crew base balancing constraints (17, 18) ensure that the number of crews included from each crew base are in the specified limits. Each such constraint represents a crew base limitation for that crew base. The robustness-cost objective limit constraint (19) consists of variables of all the pairings and control the crew cost that it is not exceeding the minimum crew cost too much. But first of all, we have to solve the deterministic ACP problem in order to obtain the minimum crew cost of the deterministic problem. With the help of the robustness factor (ε) it is possible to generate multiple solutions with various degrees of robustness. Constraint (20)

ensures that decision variables have integer value, and constraints (21-25) assures that the parameters are non-negative.

There are two main underlying assumptions in the proposed model. First, the previous planning stages (flight schedule, fleet assignment and aircraft routing) are done and given. Second, because of the fact that any delay caused by an unavailable aircraft is highly likely to cause delay in multiple pairings, it is assumed that the aircrafts are always available.

It is evident that the estimation of the real performance of a crew schedule in advance of operations is really challenging. Also it is difficult to estimate the operational cost and indirect delays that will take place during operations, because they all heavily depend on the strategy an airline uses to recover from disruptions. The decisions made in the recovery process include delaying or cancelling flights, and swapping or re-routing aircraft and crew. These recovery options are required to be taken into consideration when delays are forecast for a planned crew schedule.

Solution Approach

A CG-based approach seems to be the only practical approach to solve the proposed model. Because, first of all, there are numerous numbers of variables (pairings) with propagated delays related to them and majority of the variables are non-basic in the optimal solution, therefore it is not necessary to use all variables which has non-negative reduced costs. Secondly, the storage requirements for the compact formulation are massive, thus optimizing only reduced version of the compact problem would yield higher performance in terms of solution time.

The compact formulation which is introduced above is reformulated via the CG technique. In the reformulation structure, the flight legs covered by pairings and balanced crew bases are maintained. A pairing is a unique path in the flight network as governed by the constraints (16-18) which ensure that at least one pairing covers the flight legs and pairings conform to crew base limitations. Therefore, the decomposition of the compact formulation can be written is as;

Master Problem:

$$minimize \quad \sum_{k \in P} \left(x_k \sum_{i,j \in K} \sum_{w \in \Omega} p^w \, pd_{ijk}^w \right) \qquad (\,26\,)$$

$$subject\ to \quad \sum_{k \in P} a_{ik} x_k \geq 1 \qquad \forall i \in F \qquad (\,27\,)$$

$$\sum_{k \in P} h_{km} x_k \geq l_m \qquad \forall m \in M \qquad (\,28\,)$$

$$\sum_{k \in P} h_{km} x_k \le u_m \qquad \forall m \in M \qquad (\ 29 \ )$$

$$x_k \in [0,1] \qquad \forall k \in P \qquad (\ 30 \ )$$

Pricing Sub-Problem:

$$minimize \quad \sum_{k \in P} \left\{ x_k \left(\sum_{i,j \in K} \sum_{w \in \Omega} p^w \, pd_{ijk}^w - \pi A \right) \right\} \qquad (\ 31 \ )$$

$$\sum_{k \in P} c_k x_k \ \le (1 + \varepsilon) \ Z_{Opt} \qquad (\ 32 \ )$$

$$x_k \in \{0,1\} \qquad \forall k \in P \qquad (\ 33 \ )$$

The master problem for robust ACP problem is defined with (26-30) linear program where the columns of constraints (27-29) are some initial subset of pairings. If we compare the master problem formulation with the compact formulation, the integer restriction on the solution of decision variables has been relaxed, robustness-cost objective limit constraint removed, and the number of variables has been restricted to a reasonable sized subset of all pairings. By applying strong duality theorem, we obtain dual values (π) for the optimal solution of the master problem. These dual values (π) are used in the objective function of the PSP to generate columns which will be added to the RMP to improve solution value (31). The constraint of the PSP ensure that the columns obey the robustness-cost objective limit constraint (32).

Pairing Generation:

The necessity to calculate total propagated delays for each pairing, dictates us to enumerate all the legal pairings. A recursion based Depth-First Search (DFS) procedure is used as the search engine in order to enumerate all feasible pairings explicitly on a flight network. In flight network each flight is represented by an arc and there are also arcs representing legal connections between flights. Two flights will have a connection arc, if the arrival airport of the first flight is the same as the departure of the second one, and the time between them is legal.
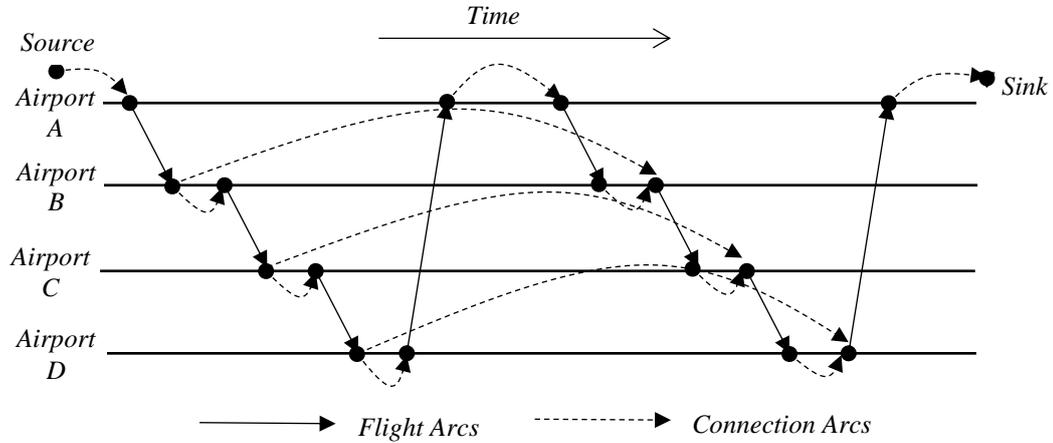
Figure 2: Flight Network Example

In addition to the flight and connection arcs shown in Figure 2, the directed flight network has two additional nodes per crew base representing the source and sink of each complete crew base-to-crew base pairing. For each flight departing from a crew base, there is a connecting arc from the source node to the departing flight, and similarly for each flight arriving at a crew base, there is a connecting arc from the arriving flight to the sink node. By defining the flight network in such a manner that legality constraints are satisfied including Directorate of Civil Aviation, labor union, contractual rules and some safety requirements, and legality constrains related to that a pairing have to start and end at the same crew base and the flights must be sequential in space and time. It assures that every source to sink path in the flight network with the limited length is a legal pairing. All feasible pairings are enumerated for each crew base and in every step of the DFS, the latest generated pairing is checked for legality. Also the cost of each pairing is calculated based on the components of flying time, elapsed time of the duty period and time-away-from-base.

Branch-and-Price Approach:

The problem instances we study are small enough that lets us to generate all pairings before the optimization process begins. Classical CG approach does not guarantee the optimal integer solution to the original problem by optimally solving the LP relaxation of the RMP, because it may not generate the columns needed for an optimal integer solution. However, Branch-and-Price (B&P) approach is capable of yielding optimal integer solutions via applying CG within the B&B search tree.

Actually, B&P is the dual version of the Branch-and-Cut (B&C), while B&C focuses on row generation, B&P focuses on column generation, where valid inequalities (rows) instead of columns are added to the node problems. B&P methods are previously applied to classical ACP problem (Barnhart, Johnson, Nemhauser, Savelsbergh, & Vance, 1998), but now we have to deal with additional complexity introduced by the consideration of robustness in the classical problem. Figure 2 illustrates the flowchart of the proposed B&P algorithm for the robust ACP problem and the main steps of the algorithm are given below:

Step 1 - Compact Formulation and read all input files: Construct the Compact Formulation and read all the data including the flight leg covering, crew base balancing constraints, propagated delays, robustness factor, optimal solution of the deterministic ACP problem, crew rules etc.

Step 2 - Flight network construction: Build flight networks in line with the flight schedule and crew rules.

Step 3 - Pairing generation with Depth-First Search (DFS) procedure: Construct pairings considering the feasibility rules and calculate the related cost for each feasible pairing.

Step 4 - Construct GSPC: Construct the GSPC with the objective of minimizing propagated delay, flight leg coverage constraints, crew balancing constraints and robustness-cost objective constraint. Set Z_{UP} (the upper bound of the tree) to infinity.

Step 5 - Solve the LP relaxed RMP: Solve and find the optimal solution of the LP relaxed RMP.

Step 6 - Get the dual values from the RMP and solve the PSP: Get the dual values from the RMP and solve the PSP with the exact or greedy method in order to generate columns with negative reduced cost. From each PSP solution generate certain amount of columns that satisfies the minimum reduced cost threshold. If no such column exist GOTO Step 8, the optimal solution for the LP relaxed RMP is found.

Step 7 - Add such columns to the RMP: Add new columns generated by the PSP to the RMP and GOTO Step 5.

Step 8 - Check the feasibility of the current solution: If the current solution is not feasible, prune the node and GOTO Step 13.

Step 9 - Compare current objective with the upper bound: If the current objective is not lower than Z_{UP}, prune the node and GO TO Step 13.
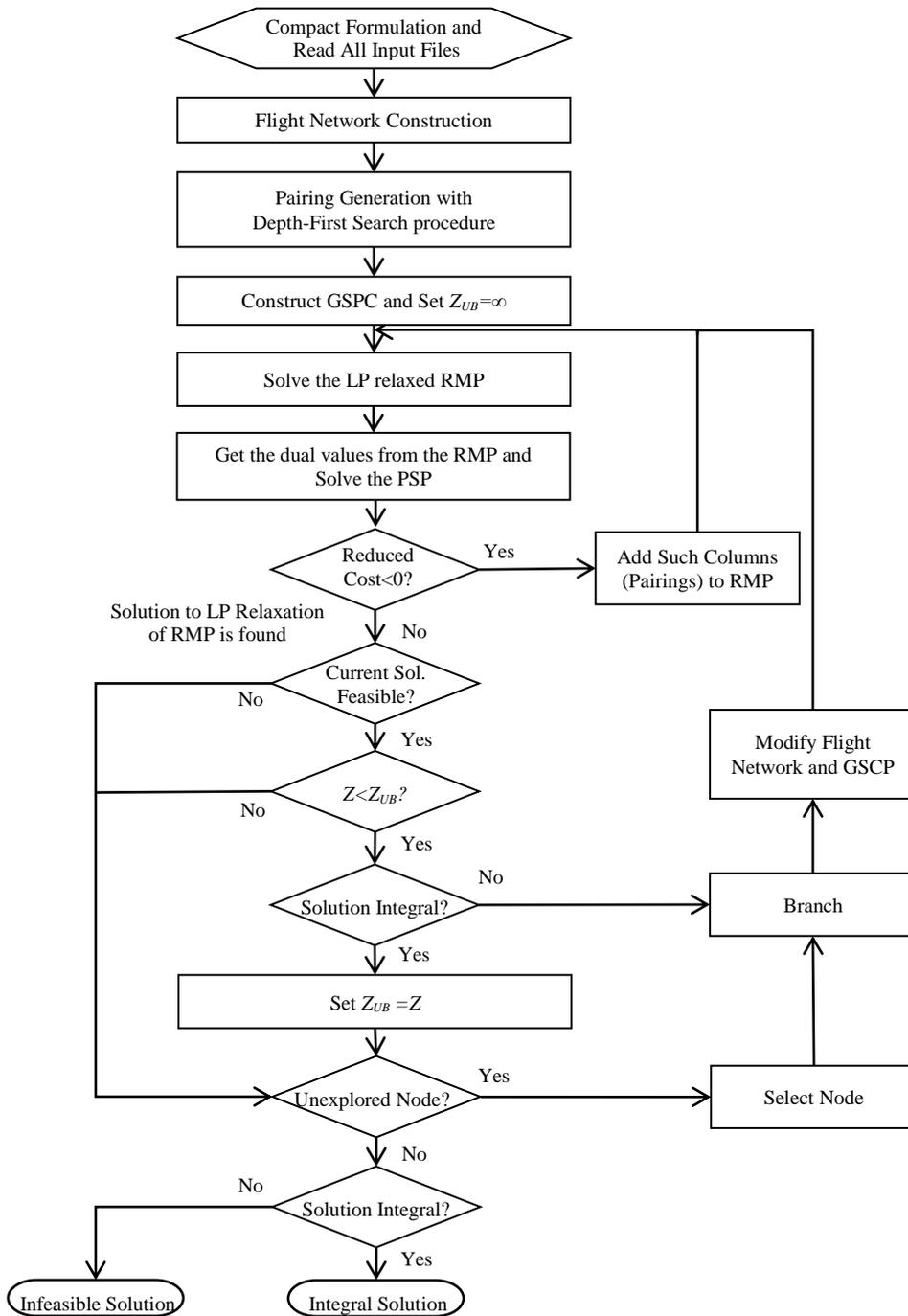
Figure 3: Solution Algorithm Flowchart

Step 10 - Check if the current solution is integral: If the current solution is integral, switch Z_{UP} with the current solution objective value, prune any node that has objective value higher than this upper bound and GO TO Step 13. Otherwise, record the current node as the parent node.

Step 11 - Branch: Implement constraint (follow-on) branching.

Step 12 - Modify the flight network and GSCP: Modify the flight network and GSCP by removing the related arcs and columns in line with the branch selected and GOTO Step 5.

Step 13 - Check if there is unexplored node: If there is unexplored node, select a recently created unexplored node and GOTO Step 12.

Step 14 - Check if the current solution is integral: If the current solution is integral, all tree is explored and the latest solution is the optimal solution. Otherwise, the problem is infeasible.

As an improvement to the B&P technique we apply a heuristic pricing, particularly a greedy algorithm as an acceleration technique, because most of the time in CG process is spent in pricing. This heuristic pricing repeatedly executes a procedure, which initially sorts all the pairings by the reduced costs in ascending order and then in the previously determined order, assigns each pairing until the robustness-cost objective limit constraint is violated. At each pricing step, we stop the pricing algorithm as soon as a fixed number of negative reduced cost columns is found. Also at every specified number of CG iterations, all columns with reduced cost above a given threshold are eliminated to improve the solution time.

In B&P process, branching rule must be compatible with the PSP so as to prevent columns that have been branched on from being regenerated. The main difficulty in determining a branching strategy is to find one that excludes the current solution, partitions the solution space of the problem reliably and provides a PSP that is still easy to solve. Therefore, on branching step we use constraint branching, particularly follow-on branching, in which certain two fights are fixed or deleted that could operate as a consecutive connection in a pairing (Ryan & Falkner, 1988). For example, if two flight legs i and j are assigned to same pairing with no other flights between these two flights. On one branch, all pairings that include only one of the flights i and j are removed, and on the zero branch, all pairings that include both of the flights i and j are removed.

Simulation Model

Due to the complex nature of airline operations it is not practical to verify optimization results, and assess the robustness of the obtained

solutions and the performance of the proposed approach in an analytical way. Thus, we employed an open-source discrete-event simulation model, SIMAIR (**Sim**ulation of **Air**line Operations), which is a C++ based research tool to simulate airline operations and various unexpected disruptions that are faced during daily airline operations (Lee, et al., 2003). The simulation model is used as a trace-driven simulation, where the second part of the historical (future) data is used as an input.

After inputting the schedule to SIMAIR, we specify empirical distributions that constructed from future data to describe propagated delays as random elements. In order to convert historical data into empirical distribution, the data is transformed into a cumulative probability distribution function. In order to make this transformation, first of all data is sorted into ascending order and the identical values are grouped into classes, and then the relative frequency of each class and cumulative probability of each class is calculated.

The main assumption underlying the simulation model is that real rescheduling methods are not able to be integrated into the simulation model. In the model when a flight is delayed, either a push-back recovery is applied, in which flights are kept waiting until all the resources (aircrafts, crews etc.) are available regardless of their lateness or reserve crews are used, in which infinite number of reserve crews are assumed. Even though push-back recovery heuristic is easier to analyze than other recovery methods, it is not practical except in delays of short duration. Also the infinite number of reserve crew assumption is not realistic. Hence, results of the simulation study are descriptive and should be interpreted cautiously.

Computational Results

The proposed approach is tested on actual historical data provided by a small-scale Turkish airline, which operates short-haul domestic flights on a hub-and-spoke network with one hub. The provided historical data consists of flight data (the flight number, origin, destination, scheduled/actual date and time of arrival) from 1st of January 2013 to 30th of July 2013, with 22,680 flights in total. All domestic flights serve 23 airports with two of them as the crew bases.

We implement the solution algorithm using the SCIP (Solving Constraint Integer Programs) libraries, in which these specific libraries provide the basic framework for the B&P algorithm (Achterberg, 2009) along with IBM ILOG CPLEX 12.2 version as a solver. Because the commercial solvers are not yet capable of implementing B&P approach, we

opted for a specialized approach and coded the algorithm in a high level programming language, namely in C++. All computational experiments are performed on a standard PC machine with a 64 bit 2.50 GHz Intel(R) Core(TM) i5-3210 CPU and 6 GB of RAM running on Windows 8.1 operating system.

Initially the historical dataset is divided into two parts, first 3 months representing past data and the remaining 3 months is considered as future data. In order to approximate the expected value of propagated delay, we use the Sample Average Approximation method and consider each day of operation in the historical data as one instance of a delay scenario (ω). It is assumed that each delay scenario is equally likely, and therefore the set of delay scenarios (Ω) has cardinality 90. When a flight is not included in some scenarios, we use the average historical propagated delay over the scenarios containing the flight. In order to calculate propagated delays, 428 flights which does not have any predecessor flight (first flight of the day, flights after cancellation etc.) are excluded. 10,326 number of flights delayed with total time of 134,238 minutes and out of these delayed flights 6,723 flights include propagated delay with total time of 62,059 minutes, which is approximately the % 46 of total delay time. And we can conclude from the data that the major contribution to total arrival delay is from propagated delays from preceding flights.

Major rules mandated by both Directorate of Civil Aviation and labor union related to crews are listed below:

- The maximum duty time in any given 24-hour period is eight hours (480 minutes),
- The turnover time between two flights in a non-crew base airport is minimum 10 minutes,
- The turnover time between two flights in a crew base airport is minimum 20 minutes,
- The turnover time cannot exceed 60 minutes,
- Crews have a 120 minutes minimum guarantee pay,
- Crews have a guaranteed 0.5 percentage of duty time or full percentage of flying time.

The performance of the B&P algorithm is evaluated according to the solution times and the quality of the final solution attained from the classical Column Generation and Branch-and-Cut (CG and B&C) algorithm, that is to say CPLEX defaults are used. In CG and B&C approach CG is used to solve the LP relaxation of the RMP and then the generated columns are used to formulate an integer problem. In the next phase B&C algorithm is

applied to obtain integer solution and solved by the IBM ILOG CPLEX 12.2 commercial solver.

The flight schedule available for the optimization tests consists of 376 flight legs. For both of the algorithms (the benchmark and the proposed algorithm) we executed 5 runs and generated 3 million pairings in the 1st run, and in each additional run 1 million extra pairings are generated until the 5th run, in which nearly 6.9 million pairings is generated. This number is the maximum number of pairings that could be generated for the given flight schedule. The results are summarized in Table 2.

Table 2: Results of the Computational Experiments

| Run ID | DFS Phase | | CG Phase (Root Node CG Phase for B&P) | | | Column Generation and Branch-and-Cut (CG and B&C) | | | Branch-and-Price (B&P) | | |
| | | | | | | B&C Phase | Total | | | | |
	Pairings Gener.	CPU Time	CG Iter.	CG Columns Gener.	CG Solve CPU Time	B&C Iter.	CG and B&C Solve CPU Time	CG and B&C Obj.	B&P Columns Gener.	B&P Solve CPU Time	B&P Obj.
Run1	3000000	2700	166	2475	710	613	3458	523.35	493	718	523.35
Run2	4000000	3071	148	2191	1040	826	4185	462.15	608	1053	462.15
Run3	5000000	3398	149	2207	1218	840	4996	458.55	762	1488	458.1
Run4	6000000	3820	150	2221	1559	763	5576	453.6	1252	1646	453.6
Run5	6893365	5068	242	3615	2461	1077	7847	445.95	1308	2600	445.95

In Table 2, all *CPU Times* are in seconds and *Objective (Obj.)* values are in minutes. Under *DFS Phase* column, *Pairings Gener.* and *CPU Time* show the number of parings generated and the CPU time for this phase, respectively. *CG Phase* column corresponds to the Column Generation phase for the CG and B&C algorithm, and root node Column Generation phase for the B&P algorithm. *CG Iter.* represents the number of iterations executed in CG phase, *CG Columns Gener.* displays the total number of columns that were added by solving the pricing problem from all iterations. Under *CG and B&C* column, *B&C Iter.* represents the number of iterations executed in *B&C phase*. *CG and B&C Solve CPU Time* and *Obj.* show the total consumed CPU time and the objective function value. Under *B&P* column, *B&P Columns Gener.* shows the number of columns that are added in the non-root nodes of the B&P tree. *B&P Total Solve CPU Time* and *Obj.* represent the solution CPU times and the globally optimal values obtained

by the B&P method. Both *CG and B&C Solve CPU Time* and *B&P Total Solve CPU Time* do not include *DFS Phase* as well as *CG Phase CPU Time*. The performance of the algorithm is evaluated according to the solution times and the quality of the final solution attained. With respect to solve times, the algorithm achieved up to 4.8 times improvement over the CG and B&C algorithm. Objective function values are same except for the 3rd run. The experimental results show that the proposed B&P algorithm is able to find globally optimal solutions in lesser solution times. Thus, the proposed approach performs fine on the real-world instance.

Next, we employ a simulation model in SIMAIR, in which the actual data that represents future operations to evaluate the performance of the robust crew schedules is used. First we input the flight schedule to the simulation model and specify empirical distributions that constructed from future data. We execute one week (7 days) simulation of the flight schedule and perform 1000 runs. The results are shown in Table 3.

Table 3: Simulation Results

Parameter	Result
Flight cancellation	5 out of 376
8-in-24 rule violation	0
Flight delay statistics	
Total propagated delay (min)	4327
15-Min On-Time Performance (%)	78.6
60-Min On-Time Performance (%)	92.71

The results of this experiment show that the solution times obtained from the proposed B&P approach are reasonable enough to conduct several different scenarios for a final decision. Thus, decision maker will have enough options and time choose the best course of action. They also show that the enhancements for B&P approach is crucial for finding solutions in reasonable solution times.

Conclusions

The whole airline schedule planning process is traditionally decomposed into several optimization problems, not only due to its computationally intractability but also in practice each phase is dealt by different sections of an airline. ACP problem is of the most important one, because the fact that crew costs are the second largest component of direct

operating costs for the airlines. Considering the large-scale airlines even small amount of improvement in crew scheduling could result in huge savings. Due to the fact that ACP problem is solved at the planning stage, all flights are assumed to have departure/arrival times that are both fixed and known. And solutions to ACP problem usually lead to schedules that have tight connections which makes the whole schedule fragile in case of disruptions. A minor delay may have a significant impact on the entire crew schedule due to the snowball effects.

In our proposed approach, the deterministic ACP problem must be solved first in order to get the minimum planned crew cost. Then the propagated delay objective is minimized with an additional constraint to control the crew cost. This additional constraint form an upper bound to the planned crew cost and force the crew cost not exceed the minimum crew cost to an extent. For the solution of the model we propose a solution algorithm based on the B&P approach.

For all the experiments a field-collected actual schedule dataset for a Turkish domestic airline is employed. The experimental results indicate that B&P performs better on classical CG and B&C (using CPLEX defaults) approach. Also, we conduct experiments to simulate the robustness performance of the solutions. The simulation results show that the proposed approach provides the ability to decrease delay propagation of a crew schedule.

Consequently, it is apparent that a robust crew schedule could lead to fewer delays and disruptions, providing smaller operational costs. We believe that cost savings from reduced propagated delay could compensate the probable increase in planned crew costs and result in smaller operational crew costs, compared to those of non-robust schedules. Nevertheless, the advantages of robust crew schedules attained from the proposed model can differ from airline to airline, since each one practice different levels of disruptions in their operations and have different crew cost structures.

This paper attempt to point out the importance of taking historical analysis into consideration in developing robust schedules and also it could provide general conceptual framework for further robust optimization studies in similar problem areas. The next step could be to conduct research on integration of robust crew pairing and aircraft routing problems, such as a sub-problem with the objective of maximizing the flexibility of the schedules or rescheduling procedures could be integrated into scheduling.

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Genişletilmiş Özet

Aksaklıklara Karşı Dayanıklı Ekip Eşleme Problemi İçin Bir Dal-Ücret Algoritması

Giriş

Yöneylem araştırması tekniklerini kullanılması, son 20-30 yıl içerisinde havayolu endüstrisinde daha önce görülmemiş bir etki yaratmıştır. Havayolu şirketlerinin yüksek kalitede, düşük maliyetli hizmet üretebilmesi için eniyileme tabanlı karar destek sistemleri kullanarak uçuş tarifeleri, filo planları, uçak rotaları, ekip çizelgeleri ve atamaları gibi kaynak planlamalarını maliyet-etkin ve kâr sağlayacak şekilde yapabilmesini gerekli kılmaktadır.

Yolcu taşımacılığı yapan bir havayolu şirketinin uçak ve uçuş ekibi olmak üzere iki temel kaynağı bulunmaktadır. Havayolu şirketlerinin rekabete dayalı ortamda hayatta kalabilmesi için bu iki kaynağı çok iyi planlaması gerekir. Yakıt maliyetlerinden sonra en büyük ikinci maliyet kalemi olan ekip maliyetleri, yıllık şirket giderlerinin yaklaşık %20-25'ini oluşturmaktadır. Yakıt maliyetlerinin aksine kontrol edilebilir bir maliyet kalemi olduğundan, küçük bir iyileştirme dâhi önemli büyüklükte kazançlar sağlayabilmektedir. Ayrıca, ekiplerin etkili olarak çizelgelenmesi sadece maliyet değil, uçuş emniyeti açısından da çok önemlidir.

Gerçek dünya eniyileme problemlerinden olan havayolu ekip çizelgeleme problemi; en basit tanımı ile uçuş ekiplerinin, bir havayolu şirketinin belirli bir dönem için belirlediği uçuş tarifesinde yer alan tüm uçuşlara atanması problemidir. Problem, ilk önce uçuş ayağı dizilerinin (isimsiz eşlemelerin) oluşturulması için çözülen *ekip eşleme* ve oluşturulan eşlemelere her bir ekip üyesinin atanması için çözülen *ekip atama* problemi olmak üzere genellikle iki alt probleme ayrılarak çözülür.

Ekip üyesinin ikamet ettiği ana üsten başlayıp uçuş tarifesinde yer alan uçuşların oluşturduğu şebekede bir yol izleyerek tekrar ana üsse dönen uçuş dizileri *ekip eşlemesi* olarak adlandırılmaktadır. Bir ekip eşlemesinin uygulanabilir olabilmesi için havacılık otoritelerinin koyduğu uçuş emniyetine yönelik kurallara, toplu sözleşmelerin getirdiği yükümlülüklere ve havayolu şirketinin uyguladığı özel uygulamalardan gelen tüm kısıtlamalara uygun olması gerekir. Her bir eşlemeye bir maliyet tanımlanarak, ekiplerin uçuş tarifesinde yer alan tüm uçuşlara en uygun olarak atanması, tüm uçuşların eşlemeler tarafından en az maliyetle kapsanması ile gerçekleştirilebilir.

Ancak, kötü hava koşulları, mekanik uçak arızaları, havaalanı ve hava sahasındaki sıkışıklık, ekibin geç kalması gibi kontrol dışı aksaklıklar nedeniyle uçuşların gecikmesi kaçınılmazdır. Gecikmelerin en olumsuz etkisi, havayolu operasyonlarının birbirine bağlı birçok faaliyetten oluşması sebebiyle, gecikmelerin çığ etkisi yaratarak sonraki uçuşların gecikme veya iptal edilmesi ile sonuçlanmasıdır.

Bu çalışmada; büyük boyutlu, şebeke esaslı kaynak planlama problemlerinden olan havayolu ekip eşleme probleminin, geçmiş veri setlerini kullanarak havayolu operasyonları esnasında oluşabilecek aksaklıklara karşı dayanıklı olarak modellenmesi ve çözümü amaçlanmıştır. Bu maksatla; Genel Küme Kapsama Modeli olarak formüle edilen problem için dal-ücret esaslı bir çözüm algoritması geliştirilmiştir. Ayrıca, geçmiş veri setleri kullanılarak elde edilen çözümlerin dayanıklılık performansını ölçmek için bir benzetim modeli geliştirilmiştir.

Aksaklıklara Karşı Dayanıklı Havayolu Ekip Eşleme Problemi

Havayolu planlama süreci, çoğunlukla *uçuş planı tasarımı*, *filo atama*, *uçak rotalama* ve *ekip çizelgeleme* olmak üzere dört aşamaya ayrılarak çözülür. Birbiri ardı sıra gelen aşamaların her birinin çıktısı müteakip aşamanın girdisini oluşturur.

Havayolu ekip eşleme problemi; girdi olarak verilen uçuş tarifesinde yer alan tüm uçuş ayaklarının, ekibin yaşadığı ana üste başlayıp yine aynı üste biten ve yasal kurallara uyan ekip eşlemeleri tarafından en az maliyet ile kapsanmasını içerir. Problem, doğrusal olmayan maliyet yapısı, çözüm uzayının büyüklüğü, karmaşık kısıtların varlığı, çeşitli kombinatoriyel alt problemden oluşması, tamsayılı çözüm bulunması zorunluluğu ve işletmelerin uyguladığı Ana Dağıtım Üssü-Kenar Üs (ADÜ-KÜ) uçuş ağı yapısına sahip olması nedenleriyle *NP*-Hard yapıdadır.

Ekip çizelgelerinin planlandığı şekilde uygulanacağı varsayımı altında çözülen klasik ekip eşleme modeli, kısa bağlantı süreleri (mola ve dinlenme süreleri) kullanarak düşük planlama maliyetlerini amaçlar; ancak bu yaşanabilecek gecikmelere karşı hassasiyet oluşturur. Klasik modelde sıkı olarak hazırlanan ekip eşlemeleri, herhangi bir uçuşta gecikme olduğunda bu gecikmenin sonraki tüm uçuşlara artarak yayılmasına sebep olur. Uçuş gecikmelerinin yayılma nedenlerinin başında, bağlantılı uçuşun geç kalması sebebiyle oluşan ekip eksikliği gelmektedir. İşletmeler açısından gecikmeler, ek maliyetlere ve müşteri memnuniyetsizliğinden dolayı itibar kayıplarına neden olmaktadır.

Pratikte aksaklık ve gecikme maliyetlerinin enazlanması için iki temel yaklaşım uygulanmaktadır. Birincisi; çizelgelerin uygulanması safhasında, aksaklıklar oluştuktan sonra etkili yeniden çizelgeleme yapılmasıdır (reaktif yaklaşım). İkincisi ise; planlama safhasında, aksaklıklara karşı dayanıklı çizelgelerin oluşturulmasıdır (proaktif yaklaşım). Dayanıklı planlama yönteminde, sadece planlama maliyetleri değil operasyonel maliyetlerin beklenen değeri de enazlanmaya çalışılır.

Bir uçuş gecikmesi, bağımsız ve yayılan gecikme olmak üzere iki parçaya ayrıştırılabilir. Bağımsız gecikme, daha önce meydana gelen veya birikmiş bir gecikmenin sebep olmadığı, önceki uçuşlardan kaynaklanmayan gecikmedir. Yayılan gecikme ise, önceki (bağlantılı) uçuşun geç kalması ve gecikmeyi absorbe edecek kadar ek süre olmaması nedeniyle, maruz kalınan gecikmedir. Ekip eşlemeleri ile aksaklık ve gecikmeler arasındaki ilişki, yayılan gecikme ile açıklanabilir.

Gecikmelerin çığ etkisi yaratarak sonraki uçuşlara yayılması durumunda, gecikmelerin havayolu operasyonlarına olan olumsuz etkisi çok daha artar. Bu nedenle; gecikmenin yayılması, bir havayolu çizelgesinin aksaklıklara karşı dayanıklılığının en iyi göstergelerinden birisidir.

Konu ile ilgili literatürde yer alan çalışmalar çoğunlukla geçmiş verileri dikkate almadan çizelge dayanıklılığını artırmaya yönelik metodolojiler içermektedir. Çalışmaların çoğu, aksaklıklar ile ilgili belirsizliğin modellenmesinden ziyade aksaklıklara karşı koruma imkanlarının artırılması veya aksaklıkların etkilerinin azaltılmasını amaçlamaktadır. Yapılan literatür taramasında, dayanaklı ekip çizelgeleme problemlerinde geçmiş verileri kullanarak belirsizliği modelleyen bir çalışmaya rastlanmamıştır. Bu çalışma ile literatürde yer alan bu boşlukların doldurulması amaçlanmıştır.

Matematiksel Model ve Çözüm Yaklaşımı

Aksaklıklara karşı dayanıklı ekip eşleme problemi, çift-amaçlı eniyileme problemi olarak ele alınmıştır. Çift-amaçlı eniyileme, birbiri ile çelişen iki amaç fonksiyonu değerinin kısıtları da gözeterek eşzamanlı olarak eniyilenmesi süreci olarak tanımlanabilir. Söz konusu amaçlar; yayılan gecikmelerin ve ekip maliyetlerinin enazlanmasıdır. Birbiri ile çelişen amaçlardan maliyet enazlanması, ε-kısıt yöntemi ile kısıt haline dönüştürülmüştür. Tüm uçuş ayaklarında oluşan yayılan gecikmenin enazlanması ise havayolu işletmesinin Tam-Zamanında Performasının iyileşmesini sağlamaya yöneliktir.

Tüm uçuş ayaklarındaki yayılan gecikmelerin beklenen değeri; mümkün gecikme senaryo kümesi (Ω) sonlu kardinaliteye sahip olduğu için senaryolardan elde edilen uçuşlar arası yayılan gecikme değerlerinin, senaryo olasılıkları ile çarpılması ve bu işlemin bütün değerler üzerinden toplanmasıyla elde edilebilir. Model kesikli olduğundan ve kısıtlarda rassal değişken olmadığından, yayılan gecikmelerin beklenen değerinin senaryo olasılıkları üzerinde toplanmış hali, amaç fonksiyonuna doğrudan eklenebilir.

Dayanıklı Havayolu Ekip Eşleme Problemi, tek amaçlı eniyileme problem olarak ele alınmasını müteakip Genel Küme Kapsama modeli olarak formüle edilmiştir. Model genel olarak; yayılan gecikmelerin enazlanmasını amaçlayan, klasik küme kapsama kısıtları olan uçuş kapsama kısıtları, ekip dengeleme kısıtları ve dayanıklılık-maliyet sınırlama kısıtından oluşan bir Tamsayılı Programlama modelidir. Söz konusu model bütün halde çok fazla değişkenden oluşması sebebi ile Sütun Oluşturma yöntemi ile Sınırlı Ana Problem ve Ücretlendirme Alt Problemi olarak ayrıştırılmıştır.

Model oluşturulurken verilen bir uçuş tarifesi üzerinde filo atama ve uçak rotalama aşamalarının tamamlandığı ve değişmeyeceği; ayrıca, uçakların her zaman planlandığı şekilde hazır olduğu varsayım olarak kabul edilmiştir.

Matematiksel modelin çözümü için dal-ücret esaslı bir algoritma önerilmiştir. Önerilen algoritma kısaca, başlangıçta tüm eşlemelerin tamsayımı için özyinelemeli Derin Öncelikli Arama sezgiseli ile tüm eşlemeleri üretmekte ve müteakiben dal-ücret algoritması uygulanmaktadır. Dal-ücret algoritması ise, her bir düğümde Doğrusal Programlama gevşetmesi olan Sınırlı Ana Problemi çözmekte ve amaç fonksiyonu değerini iyileştirme potansiyeli olan yeni değişken/değişkenleri ücretlendirerek dal-ücret ağacına eklemektedir.

Deneysel Çalışmalar

Önerilen yaklaşım, Türkiye'de faaliyet gösteren ve ADÜ-KÜ ağ yapısında yurt içi uçuşlar gerçekleştiren küçük boyutlu bir havayolu şirketine ait veriler kullanılarak test edilmiştir. Elde edilen geçmiş veri seti, planlanan ve gerçekleşen toplam 6 aylık uçuş verisini içermektedir. Uçuş ağ yapısı, iki tanesi ekip üssü olmak üzere toplam 23 havaalanından oluşmaktadır. Çözüm algoritması, dal-ücret algoritması için temel bir çerçeve sağlayan SCIP (Solving Constraint Integer Programs) kütüphanelerinden faydalanılarak uygulanmış ve IBM ILOG CPLEX 12.2

sürümü çözücü olarak kullanılmıştır. Ticari çözücülerin dal-ücret algoritmasını uygulama yeteneği olmadığı için algoritmanın yüksek seviye programlama dili olan C++ ile kodlanması tercih edilmiştir.

Öncelikle gerçek veri seti, ilk üç ay geçmiş verileri ve geriye kalan üç ay gelecek verileri temsil etmek üzere iki parçaya ayrılmıştır. Gerçek verilerde yer alan her bir gün, bir gecikme senaryosu (ω) olarak ele alınmıştır. Eğer bir uçuş ayağı bazı senaryolarda yer almıyorsa, o uçuş ayağı için ortalama geçmiş yayılan gecikme değerleri kullanılmıştır. Gerçek verilerin tümü üzerinde yapılan analiz sonucunda, toplam varış gecikmesine en büyük katkının bir önceki uçuştan yayılan gecikmelerden kaynakladığı sonucuna varılmıştır.

Önerilen dal-ücret algoritmasının performansı, klasik Sütun Oluşturma ile dal-kesme algoritmasından elde edilen çözüm zamanları ve çözüm kalitesi ile karşılaştırarak elde edilmiştir. Elde edilen çözümlerin kalitesi hemen hemen aynı iken çözüm zamanı açısından önerilen algoritma daha iyi bir performans göstermiştir.

Sonuç

Bu çalışmada; havayolu ekip eşlemelerinin uygulanması esnasında oluşabilecek yayılan (tepkisel) gecikmelerin enazlanması ve planlama aşamasında ekip çizelgesinin istikrar ve esnekliğinin düşük bir maliyet ile artırılarak aksaklıklara karşı daha dayanıklı ekip eşlemelerinin oluşturulması üzerinde durulmuştur. Bu maksatla; problem çift amaçlı eniyileme problemi olarak modellenmiş ve çözümü için dal-ücret esaslı bir algoritma önerilmiştir. Ayrıca, ekip eşlemelerinin dayanıklılık performasının değerlendirilmesi için bir benzetim modeli geliştirilmiştir. Problemin çözümünde ve sonuçların değerlendirilmesinde uçuş gecikmeleri arasındaki korelasyonu da dikkate almak için geçmiş veri setleri kullanan ve parametrik olmayan bir yöntem kullanılmıştır.

Yapılan deneysel çalışma sonucunda; sadece maliyet enazlanması ile elde edilen eniyi maliyette çok az bir artışa izin verilerek tüm ekip eşlemelerinin aksaklıklara karşı daha dayanıklı olarak oluşturulabileceği gözlenmiştir.