HEURISTIC METHODS FOR THE BATTLEFIELD DISTRIBUTION NETWORK

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ABSTRACT

In this paper, we study a logistics problem in which ammunition requirements of the combat units, which are located in the battlefield and engaged with the enemy, are to be satisfied in the right amount when and where they are needed. A mathematical programming model was already given for this problem in the literature. Based on this mathematical model we developed two model-based three-phase heuristic methods of which the first one is a "routing first-location second method", and the second a "location first-routing second" heuristic method. The computational results show that the first method outperforms the second one and complex real world problems can be solved in reasonable times.

Keywords: Network design, Location routing problem, Logistics, Distribution, Heuristic methods

MUHAREBE SAHASI DAĞITIM SİSTEMİ İÇİN SEZGİSEL YÖNTEMLER

ÖZET

Biz bu çalışmada muharebe alanında düşmanla temas hâlinde bulunan birliklerin mühimmat ihtiyaçlarının ihtiyaç duyulan yer ve zamanda, doğru miktarlarda karşılanmasını ihtiva eden bir lojistik problem üzerinde çalıştık. Bu problemle ilgili bir matematiksel model literatürde mevcuttur. Biz bu modeli temel alarak birincisi "önce rotalama-sonra yerleşim" ve ikincisi "önce yerleşim-sonra rotalama" olan modele özel ve üç safhadan oluşan iki sezgisel yöntem geliştirdik. Test sonuçları birinci metodun ikinci metoddan daha başarılı olduğunu ve karmaşık gerçek yaşam problemlerinin makul zamanlar içerisinde çözülebileceğini göstermektedir.

Anahtar Kelimeler: Ağ tasarımı, Yer seçimi ve yol atama problemi, Lojistik, Dağıtım, Sezgisel yöntemler

1. INTRODUCTION

Toyoglu et al. (2010) define a military logistics problem in which ammunition (henceforth called ammo) requirements of the combat units, which are located in the battlefield and engaged with the enemy, are to be satisfied in the right amount when and where they are needed. To do so they propose a continuous replenishment system which is called Mobile

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Ammunition Distribution System (Mobile-ADS) where ammo flows from depots to combat units via some transfer points.

In detail, ammo that is produced or procured is first received by main depots from where it is moved forward with rail network by trains to Fixed Transfer Points (Fixed-TP). From Fixed-TPs ammo is moved to Mobile Transfer Points (Mobile-TP) by commercial trucks on road networks. Then Mobile-TPs issue ammo to its attached combat units with special ammo trucks, which have the capability to move on terrain. A Mobile-ADS on the battlefield is presented in Figure 1. In this figure FTP, MTP and CU stands for Fixed-TP, Mobile-TP and combat unit respectively.

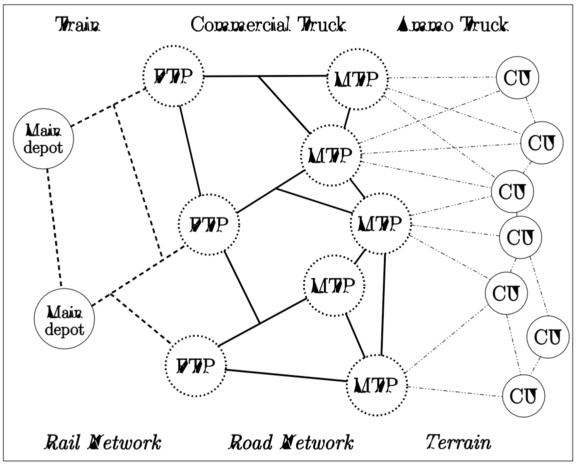

Figure 1. Mobile Ammo Distribution System on the battlefield

The decisions that must be made are; (1) the locations of Fixed-TPs and Mobile-TPs and (2) routes and schedules of commercial and ammo trucks to distribute ammo among Fixed-TPs, Mobile-TPs and combat units. Since these decisions must be made simultaneously, Mobile-ADS problem is a Location Routing Problem (LRP).

Toyoglu et al. (2010) show that their Mobile-ADS problem possesses characteristics of which some are different from the majority of the literature and some are rarely included in the previous models. To be more specific their model; locates multiple capacitated facilities, uses a capacitated heterogeneous vehicle fleet, has three layers (Fixed-TPs, Mobile-TPs and customers), considers multiple planning periods, has two-sided time windows, locates facilities at two layers, distributes multiple products and allows customers to be supplied by multiple vehicles and depots.

To solve this realistic problem, they develop two models, namely static and dynamic models. The static model solves Mobile-ADS problem for a fixed period, whereas the dynamic model solves the same problem for successive multiple periods. In other words, the dynamic model can be used for long-term strategic planning and the static model is for short-term tactical planning.

They emphasize that even the static model is the first attempt to model such a comprehensive real world problem. They also assert that solution time is not a major issue during long-term planning; however, it is of great importance during short-term tactical planning. Hence, their focus is on computationally more viable static model.

Short term tactical decisions must be made in a very short time. They especially state in their conclusion that unplanned contingencies must be urgently answered by using the static model. Hence, they try to speed up the solution time of the static model by using valid inequalities. In this study, realizing the need of a fast solution time, we aim to solve the static Mobile-ADS problem in a shorter amount of time by developing two heuristic solution methods. We believe that the static model, which has a faster solution time with the help of our heuristics, can better assist logistics planners on the battlefield.

2. MATHEMATICAL FORMULATION

We consider the Mobile-ADS design problem that is defined by Toyoglu et al. (2010) and we present their specifications and model below. We also utilize all the assumptions they made in their model development.

Table 1. Model Specifications

	SETS
N	: set of all nodes such that $N = N_F \cup N_M \cup N_C$.
N_F	: set of potential Fixed-TP nodes such that $N_F \subset N$.
N_M	: set of potential Mobile-TP nodes such that $N_M \subset N$.
N_C	: set of combat unit nodes such that $N_C \subset N$.

V	: set of all vehicles such that $V = V_F \bigcup V_M$.
V_F	: set of commercial trucks such that $V_F \subset V$.
V_M	: set of ammo trucks such that $V_M \subset V$.
P	: set of ammo types.
	PARAMETERS
Q_{ip}	: demand of combat unit i for ammo p .
CD_{ip}	: non-negative capacity of transfer point i for ammo p .
CV_{vp}	: non-negative capacity of vehicle v for ammo p .
CT_v	: non-negative total capacity of vehicle.
TI_{ij}	: travel time between nodes i and j .
TE_{ip}	: earliest time combat unit i can receive supplies of ammo p .
TL_{ip}	: latest time combat unit i can receive supplies of ammo p .
TM_p	: maximum latest arrival time of ammo p among units.
TM	: maximum of the latest arrival times of all ammo types.
TC_{vp}	: cost of transporting one unit of ammo p on vehicle v per hour.
VC_v	: cost of acquiring vehicle v .
DC_v	: cost of driving vehicle v per hour.
FC_i	: fixed cost of opening transfer point i .
	DECISION VARIABLES
f_{ijvp}	: flow of ammo p carried from node i to j by vehicle v .
tp_{ip}	: arrival time of ammo p at node i .
tv_{iv}	: arrival time of vehicle v at node i .
y_i	: 1, if transfer point i is opened; 0 otherwise.
x_{ijv}	: 1, if vehicle v travels from node i to j ; 0 otherwise.
w_{ijp}	: 1, if ammo p travels from node i to j ; 0 otherwise.

$$\begin{aligned}\min z_1 = &\sum_{i \in N_{FM}} FC_i \cdot y_i \\ &+ \sum_{i \in N_F} \sum_{j \in N_M} \sum_{v \in V_F} VC_v \cdot x_{ijv} + \sum_{i \in N_M} \sum_{j \in N_C} \sum_{v \in V_M} VC_v \cdot x_{ijv} \\ &+ \sum_{i \in N} \sum_{j \in N} \sum_{v \in V} \sum_{p \in P} TC_{vp} \cdot TI_{ij} \cdot f_{ijvp}\end{aligned}$$

or

$$\begin{aligned}\min z_2 = &\sum_{i \in N_{FM}} FC_i \cdot y_i \\ &+ \sum_{i \in N_F} \sum_{j \in N_M} \sum_{v \in V_F} VC_v \cdot x_{ijv} + \sum_{i \in N_M} \sum_{j \in N_C} \sum_{v \in V_M} VC_v \cdot x_{ijv} \\ &+ \sum_{i \in N} \sum_{j \in N} \sum_{v \in V} DC_v \cdot TI_{ij} \cdot x_{ijv}\end{aligned}$$

subject to;

$$\sum_{v \in V_F} \left(\sum_{\substack{j \in N_{FM} \\ j \neq i}} f_{jivp} - \sum_{\substack{j \in N_M \\ j \neq i}} f_{ijvp} \right) = \sum_{v \in V_M} \sum_{j \in N_C} f_{ijvp}; \forall i \in N_M, p \in P \tag{1a}$$

$$\sum_{v \in V_M} \left(\sum_{\substack{j \in N_{MC} \\ j \neq i}} f_{jivp} - \sum_{\substack{j \in N_C \\ j \neq i}} f_{ijvp} \right) = Q_{ip}; \forall i \in N_C, p \in P \tag{1b}$$

$$\sum_{\substack{j \in N_{FM} \\ j \neq i}} f_{jivp} \geq \sum_{\substack{j \in N_M \\ j \neq i}} f_{ijvp}; \forall i \in N_M, v \in V_F, p \in P \tag{2a}$$

$$\sum_{\substack{j \in N_{MC} \\ j \neq i}} f_{jivp} \geq \sum_{\substack{j \in N_C \\ j \neq i}} f_{ijvp}; \forall i \in N_C, v \in V_M, p \in P \tag{2b}$$

$$\sum_{i \in N_F} \sum_{j \in N_M} x_{ijv} \leq 1; \forall v \in V_F \tag{3a}$$

$$\sum_{i \in N_M} \sum_{j \in N_C} x_{ijv} \leq 1; \forall v \in V_M \tag{3b}$$

$$\sum_{j \in N_M} x_{jiv} = \sum_{j \in N_M} x_{ijv}; \forall i \in N_F, v \in V_F \tag{4a}$$

$$\sum_{j \in N_C} x_{jiv} = \sum_{j \in N_C} x_{ijv}; \forall i \in N_M, v \in V_M \tag{4b}$$

$$\sum_{\substack{j \in N_{FM} \\ j \neq i}} x_{jiv} = \sum_{\substack{j \in N_{FM} \\ j \neq i}} x_{ijv}; \forall i \in N_M, v \in V_F \tag{5a}$$

$$\sum_{\substack{j \in N_{MC} \\ j \neq i}} x_{jiv} = \sum_{\substack{j \in N_{MC} \\ j \neq i}} x_{ijv}; \forall i \in N_C, v \in V_M \tag{5b}$$

$$\sum_{v\in V_F}\sum_{j\in N_M} f_{ijvp} \le CD_{ip}\cdot y_i ;\forall i\in N_F, p\in P \tag{6a}$$

$$\sum_{v\in V_M}\sum_{j\in N_C} f_{ijvp} \le CD_{ip}\cdot y_i ;\forall i\in N_M, p\in P \tag{6b}$$

$$\sum_{v\in V_F}\sum_{\substack{j\in N_M\\ j\neq i}} f_{ijvp} \le \left(\sum_{l\in N_C} Q_{lp}\right)\cdot y_i ;\forall i\in N_M, p\in P \tag{6c}$$

$$\begin{aligned}&f_{ijvp} \le CV_{vp}\cdot x_{ijv}\\ &;\forall i\in N_F, j\in N_M, v\in V_F, p\in P;\forall i,j\in N_M, i\neq j, v\in V_F, p\in P\\ &;\forall i\in N_M, j\in N_C, v\in V_M, p\in P;\forall i,j\in N_C, i\neq j, v\in V_M, p\in P\end{aligned} \tag{7}$$

$$\begin{aligned}&\sum_{p\in P} f_{ijvp} \le CT_v\cdot x_{ijv}\\ &;\forall i\in N_F, j\in N_M, v\in V_F;\forall i,j\in N_M, i\neq j, v\in V_F\\ &;\forall i\in N_M, j\in N_C, v\in V_M;\forall i,j\in N_C, i\neq j, v\in V_M\end{aligned} \tag{8}$$

$$\begin{aligned}&\sum_{p\in P} f_{ijvp} \ge x_{ijv}\\ &;\forall i\in N_F, j\in N_M, v\in V_F;\forall i,j\in N_M, i\neq j, v\in V_F\\ &;\forall i\in N_M, j\in N_C, v\in V_M;\forall i,j\in N_C, i\neq j, v\in V_M\end{aligned} \tag{9}$$

$$\begin{aligned}&\left(\sum_{l\in N_C} Q_{lp}\right)\cdot w_{ijp} \ge f_{ijvp}\\ &;\forall i\in N_F, j\in N_M, v\in V_F, p\in P;\forall i,j\in N_M, i\neq j, v\in V_F, p\in P\\ &;\forall i\in N_M, j\in N_C, v\in V_M, p\in P;\forall i,j\in N_C, i\neq j, v\in V_M, p\in P\end{aligned} \tag{10}$$

$$\begin{aligned}&\sum_{v\in V_F} f_{ijvp} \ge w_{ijp}\\ &;\forall i\in N_F, j\in N_M, p\in P;\forall i,j\in N_M, i\neq j, p\in P\end{aligned} \tag{11a}$$

$$\begin{aligned}&\sum_{v\in V_M} f_{ijvp} \ge w_{ijp}\\ &;\forall i\in N_M, j\in N_C, p\in P;\forall i,j\in N_C, i\neq j, p\in P\end{aligned} \tag{11b}$$

$$TE_{ip} \le tp_{ip} \ge TL_{ip};\forall i\in N_C, p\in P \tag{12}$$

$$tp_{ip} = 0;\forall i\in N_F, p\in P \tag{13}$$

$$\begin{aligned}&tp_{ip} + TI_{ij}\cdot w_{ijp} - TM_p\cdot\left(1-w_{ijp}\right) \le tp_{jp}\\ &;\forall i\in N_F, j\in N_M, p\in P;\forall i,j\in N_M, i\neq j, p\in P\\ &;\forall i\in N_M, j\in N_C, p\in P;\forall i,j\in N_C, i\neq j, p\in P\end{aligned} \tag{14}$$

$$\begin{aligned}&tv_{iv} + TI_{ij}\cdot x_{ijv} - TM\cdot\left(1-x_{ijv}\right) \ge tv_{jv}\\ &;\forall i\in N_F, j\in N_M, v\in V_F;\forall i,j\in N_M, i\neq j, v\in V_F\\ &;\forall i\in N_M, j\in N_C, v\in V_M;\forall i,j\in N_C, i\neq j, v\in V_M\end{aligned} \tag{15}$$

The first objective considers the costs of transfer point establishment, vehicle acquisition and ammo distribution. The second one considers again the costs of transfer point establishment and vehicle acquisition plus the cost of truck driving.

Constraints (1) guarantee that inflow to a Mobile-TP or combat unit is equal to the sum of the total outflow from that node and the demand of that node. Constraints (2) maintain that a vehicle cannot leave a node with a heavier load than it was carrying before entering into that node. Constraints (3) ensure that a vehicle start its route from one and only one transfer point. Constraints (4) force each vehicle to turn back to its home transfer point. Constraints (5) ensure that each vehicle leave the node that it enters. Constraints (6), (7) and (8) respect the transfer point and vehicle capacities. Constraints (9) require that a truck carry some amount of ammo if it is dispatched. Constraints (10) and (11) define the correct logical relationships between the decision variables *f* and *w.* Constraints (12) impose the time window requirements. Constraints (13) set the initial condition of arrival times. Constraints (14) compute the arrival times of ammo types at nodes. Constraints (12)–(14) ensure that the latest ammo arrivals respect the time windows. Constraints (15) are the subtour elimination constraints.

3. VRP FIRST-LRP SECOND HEURISTIC

Nagy and Salhi (2007: 649-672) state that LRP is NP-hard. In this section, we present a ``VRP first-LRP second" type heuristic to solve the Mobile-ADS design problem in a shorter amount time.

There exist several exact solution methodologies for LRPs (Laporte and Nobert (1981: 224-226), Laporte et al. (1986: 293-310), Laporte et al. (1988: 161-172) and Laporte and Dejax (1989: 471-482)). However, most of the LRP studies in the literature resort to heuristics due to the complexity of the problem.

Nagy and Salhi (2007: 649-672) classify LRP heuristics into four groups, namely; sequential, clustering-based, iterative, and hierarchical methods. Sequential methods (Or and Pierskalla (1979: 86-95), Nambiar et. al (1989: 14-26)) first solve a location problem to decide which depots to open and to allocate customers to open depots. Then, given the locations of the open depots a vehicle routing problem (VRP) is solved.

Clustering-based methods (Schwardt and Dethloff (2005: 390-408), Barreto et. al (2007: 968-977) first group the customers into some clusters such that each cluster contains one potential depot or vehicle. Then, for each cluster a VRP is solved either after or before locating a depot.

Iterative methods (Perl and Daskin (1985: 381-396), Salhi and Fraser (1996: 3-21)) usually construct two or more subproblems each one including one or two of the major components. Then, these subproblems are solved repeatedly such that a subproblem provides some input to the next subproblem in an iterative manner.

Hierarchical methods (Sambola et. al (2005: 407-428), Melechovsky et. al (2005: 375-391)) solve the location problem while in each step of the location problem solving a routing problem which in turn provides information to the location problem.

Our solution approaches in this study are model based clustering-based heuristics according to this categorization. In general, we first partition all units into some clusters such that there is at least one potential Mobile-TP site in each cluster. Then we solve VRPs in clusters and an LRP for the whole problem.

`VRP first-LRP second" heuristic consists of three phases; Phase 1 is the clustering, Phase 2 is the VRP and Phase 3 is the LRP part.

3.1 Phase 1. Clustering

In this phase, we group all combat units into clusters such that each cluster includes at least one potential Mobile-TP.

3.1.1 Step 1. Form the clusters: In Mobile-ADS problem each brigade is a cluster. Furthermore, each brigade opens a single Mobile-TP and the units of that brigade can be served by only that transfer point. Let K be the cluster set and proceed to Step 2.

3.1.2 Step 2. Modify ammo truck costs: We modify the acquisition costs of ammo trucks slightly such that every truck has a different cost. With these modified costs, each VRP of a cluster will start to dispatch the ammo trucks starting from the least expensive one. Hence, Mobile-TPs of the same cluster will use the same ammo trucks that eventually will prevent the unnecessary reservation of more than enough trucks. Proceed to Step 3.

3.2 Phase 2. Vehicle routing problem (VRP)

In this phase, we solve a VRP for each potential Mobile-TP.

3.2.1 Step 3. Select a cluster: If set K is empty this means that all clusters have been processed already, and we are ready to proceed to the next phase, hence go to Step 10. Otherwise, select a cluster, i, remove it from K and proceed to Step 4.

3.2.2 Step 4.Update ammo truck set V_M: To increase the computational efficiency, we make an assumption that all combat units require less than truck loads. Hence, we modify the ammo truck set for each cluster such that it includes exactly the same number of trucks equal to the number of units in the cluster.

3.2.3 Step 5.Check infeasibility: If set V_{M_i} is empty this means that there is no unused ammo truck left for that cluster to dispatch and the problem is infeasible, hence STOP. Otherwise, proceed to Step 6.

3.2.4 Step 6. Select a potential Mobile-TP: Let N_{M_i} be the set of potential Mobile-TPs of cluster i. If set N_{M_i} is empty this means that a VRP for all potential Mobile-TPs of that cluster is already solved and nothing is remained to be processed, hence proceed to Step 7. Otherwise, select a Mobile-TP, j, delete it from N_{M_i} and go to Step 8.

3.2.5 Step 7. Check infeasibility: If no VRP has a feasible solution this means that the demands of the units of this cluster cannot be satisfied in the given problem setting according to the specified constraints, hence STOP. Otherwise, if at least one feasible solution exists for the VRP of a Mobile-TP then this means that we processed all potential Mobile-TPs of this cluster and no transfer point remains to be solved a VRP for. We are then ready to process a new cluster, hence go to Step 3.

3.2.6 Step 8. Solve VRP: In this step, we solve a VRP including the combat units of cluster i and Mobile-TP j using the vehicle set V_{M_i}. We also use the following valid inequalities which we find useful. Valid inequalities (V1) maintain that if a vehicle drop (take) an ammo type to (from) a node then that node must be on that vehicle's route.

$$k_{ivp} \leq \sum_{j \in N_C} x_{ijv} ; \forall i \in N_M , v \in V_M , p \in P \tag{V1a}$$

$$k_{ivp} \leq \sum_{\substack{j \in N_{MC} \\ j \neq i}} x_{jiv} ; \forall i \in N_C , v \in V_M , p \in P \tag{V1b}$$

3.2.7 Step 9. Update VRP cost: There are two differences between the objective values of the VRP in the heuristic and that of the original model. The first difference is the fixed opening costs of Mobile-TPs. Original model includes these costs but heuristic model does not, since there is only one transfer point in each VRP of Step 8 and this transfer point is already considered open. The second difference is the vehicle acquisition costs. Heuristic model considers changed costs of the vehicles which are modified in Step 2. Hence to get the real total cost and to compare it with the cost of the original formulation, we need to correct these differences.

To do so, we add the fixed cost of Mobile-TP j to the VRP cost and add the difference between the actual and modified cost of each used ammo truck to the VRP cost. Go to Step 6.

3.3 Phase 3. Location routing problem (LRP)

In this phase, we decide which Mobile-TPs and Fixed-TPs to open and how to distribute ammo from Fixed-TPs to Mobile-TPs by using commercial trucks.

3.3.1 Step 10. Solve LRP: We incorporate all previous VRPs into the LRP objective function as a simple cost parameter. To do so, we add the fixed cost of opened Mobile-TPs, the acquisition cost of the ammo trucks that are used by opened Mobile-TPs and the distribution cost of ammo to combat units from opened Mobile-TPs to the LRP objective.

We also use the following valid inequalities for solving the LRP to help reduce the solution time. Valid inequalities (V1c) require that if a vehicle drops (take) an ammo type to (from) a node then that node must be on that vehicle's route. Valid inequalities (V2) set the lower bound for the number of transfer points to be opened.

$$k_{ivp} \leq \sum_{\substack{j \in N_{FM} \\ j \neq i}} x_{ivi}; \forall i \in N_M, v \in V_F, p \in P \qquad \text{(V1c)}$$

$$\sum_{i \in N_F} y_i \geq \left\lceil \frac{\sum_{p \in P} \sum_{i \in N_C} Q_{ip}}{\max_{p \in P, i \in N_F} \{CD_{ip}\}} \right\rceil \qquad \text{(V2)}$$

4. LRP FIRST-VRP SECOND HEURISTIC

``LRP first-VRP second" is a three phase heuristic method. Phase 1 is the clustering part that partitions the combat units into clusters. Phase 2 is the location and routing part that decides the locations of the transfer points to open and the routes of commercial trucks distributing ammo from Fixed-TPs to Mobile-TPs. Phase 3 is the routing part that finds the routes of ammo trucks distributing ammo from Mobile-TPs to units.

4.1 Phase 1. Clustering

In this phase, we group all combat units into clusters such that each cluster includes at least one potential Mobile-TP.

4.1.1 Step 1. Form the clusters: This step is the same as Step 1 of the first heuristic method.

4.2 Phase 2. Location routing problem (LRP)

In this phase we solve a LRP to decide (1) which Fixed-TP and Mobile-TP (one for each cluster) to open and (2) routes of the commercial trucks among open transfer points.

4.2.1 Step 2. Solve LRP: In this step, we solve an LRP model similar to that of the first heuristic. The only difference between the LRP models of the first and the second heuristics are in the objective functions. In the second method we do not solve any VRP for any Mobile-TP before solving LRP. Hence, we do not have any knowledge about the distribution system beyond Mobile-TPs. The only knowledge we have is the demand of the combat units. Thus, in the LRP model we open one Mobile-TP for each cluster and we try to send each open Mobile-TP an amount equal to the total demand of combat units that belong to the same cluster. We consider fixed opening costs of transfer points, commercial truck acquisition costs and distribution among transfer points.

4.3 Phase 3. Vehicle routing problem (VRP)

In this phase we solve a VRP for each open Mobile-TP.

4.3.1 Step 3. Modify ammo truck costs: This step is the same as Step 2 of the first heuristic method.

4.3.2 Step 4. Select a cluster: If set K is empty this means that all clusters have been processed already and we are done, hence STOP. Otherwise, select a cluster, i, delete it from K and proceed to Step 5.

4.3.3 Step 5. Update ammo truck set: This step is the same as Step 4 of the first heuristic method.

4.3.4 Step 6. Check infeasibility: If set V_{M_i} is empty this means that there is no unused ammo truck left for that cluster to dispatch and the problem is infeasible, hence STOP. Otherwise, proceed to Step 7.

4.3.5 Step 7. Select the open Mobile-TP: There exists exactly one open Mobile-TP for cluster i in the LRP solution and let Mobile-TP j^* be this one. Select this Mobile-TP, j^*, delete it from N_{M_i} and proceed to Step 8.

4.3.6 Step 8. Solve VRP: Solve exactly the same VRP model of the first heuristic model for cluster i and Mobile-TP j^* using vehicle set V_{M_i} to determine the routes of ammo trucks distributing ammo to combat units.

4.3.7 Step 9. Check VRP solution: If VRP does not have a feasible solution this means that the demands of the units of this cluster cannot be satisfied in the given problem setting according to the specified constraints from Mobile-TP j^*. Then we need to check a new Mobile-TP, hence proceed to Step 10. Otherwise, if VRP has a feasible solution with Mobile-TP j^* then go to Step 4.

4.3.8 Step 10. Check infeasibility: If no Mobile-TP is left to be solved a VRP for, this means that the demands of the units of this cluster cannot be satisfied in the given problem setting according to the specified constraints from any Mobile-TP of this cluster. In other words the problem is infeasible, hence STOP. Otherwise, select and delete a Mobile-TP j from N_{M_i} and go to Step 8.

5. COMPUTATIONAL EXPERIMENTS

This section compares the performances of the two heuristic approaches by using the same nine problem instances of Toyoglu et al. (2010).

The running procedure of "VRP first-LRP second" heuristic is as follows:

- Run eight VRPs either to completion or at most 60 seconds,
- Run a single LRP until it reaches the objective value that is obtained by the original model in 24 hours,
- Total run time is the sum of the run times of all eight VRPs and that of the single LRP.

The running procedure of "LRP first-VRP second" heuristic is as follows:

- Run a single LRP either to completion or at most 3600 seconds,
- Run four VRPs (one for each open MTP) either to completion or at most 60 seconds,
- Total run time is the sum of the run times of all four VRPs and that of the single LRP.

We use GAMS/Cplex 9.1 as the solver and GAMS 22.0 as the modeling language. Table 2 compares both the run times (seconds) and the first objective function values of the exact solution methodology (branch and bound) with those of the two heuristics. On the average, the original model reaches an objective value of 1445.85 in approximately 19 hours (66675 seconds), whereas heuristic 1 reaches an objective value of 1439.28 in approximately 16 minutes (930 seconds). From another point of view, heuristic 1 provides a better solution with 0.5% less cost within 99% less time. Heuristic 2 cannot find a solution in three problem instances within the allowed run time.

Table 3 compares both the run times (seconds) and the second objective function values of the original model with those of the two heuristics. On the average, the original model reaches an objective value of 1061.51 in approximately 18 hours (65901 seconds), whereas heuristic 1 reaches an objective value of 1058.45 in approximately 13 minutes (799 seconds) and heuristic 2 reaches an objective value of 1058.42 in approximately 64 minutes (3835 seconds). From another point of view, heuristic 1 provides a better solution with 0.3% less cost within 99% less time and heuristic 2 provides a better solution with 0.3% less cost within 94% less time.

Table 2. Comparison with the first objective

1st Objective	Branch and Bound		VRP first – LRP second		LRP first – VRP second	
	run time	cost	run time	cost	run time	cost
PI A	79236	1301.84	288	1301.84	157	1301.84
PI B	78035	1301.84	727	1301.84	3129	1301.84
PI C	7005	1688.26	3780	1686.17	-	-
PI D	83157	1304.80	365	1301.84	197	1301.84
PI E	85911	1314.80	346	1305.83	375	1301.85
PI F	82383	1686.17	682	1686.17	-	-
PI G	86198	1342.75	518	1304.79	467	1304.62
PI H	84541	1386.06	530	1378.83	1152	1306.13
PI I	13605	1686.17	1132	1686.17	-	-
AVERAGE	**66675**	**1445.85**	**930**	**1439.28**	**-**	**-**

Table 3. Comparison with the second objective

2nd Objective	Branch and Bound		VRP first – LRP second		LRP first – VRP second	
	run time	cost	run time	cost	run time	cost
PI A	52175	922.03	493	921.60	3814	921.64
PI B	81250	921.89	1512	921.60	3837	921.64
PI C	41800	1332.01	1355	1331.94	3840	1332.01
PI D	77941	922.02	576	921.94	3840	921.64
PI E	83123	922.53	615	921.99	3827	921.77
PI F	30340	1331.72	694	1331.67	3840	1331.72
PI G	80375	922.05	661	921.85	3840	921.85
PI H	82937	947.32	500	921.75	3840	921.75
PI I	63165	1332.01	783	1331.67	3840	1331.72
AVERAGE	**65901**	**1061.51**	**799**	**1058.45**	**3835**	**1058.42**

6. CONCLUSION

With the first objective, on the average VRP first-LRP second heuristic reaches a better objective function value than that of the original formulation within 99% less time (930 seconds). There exist three instances that LRP first-VRP second heuristic cannot find a solution within the allowed time.

With the second objective, on the average both heuristics are better than the original formulation. However, VRP first-LRP second heuristic attains a similar objective function value to that of the LRP first-VRP second heuristic in 799 seconds, whereas the run time of latter is 3835 seconds

These computational results assert that the VRP first-LRP second heuristic outperforms the LRP first-VRP second heuristic. Since this heuristic offers a better computational efficiency than the branch and bound method does, we believe that it can help solve real world problems in a reasonable amount of time.

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