



Original Research / Orijinal Araştırma

Predicting screening/classification products via the pseudorandom number selection routine

Eleme ve sınıflandırma ürünlerinin sözde rastgele sayı üretme rutiniyle tahmini

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A B S T R A C T

Screening/classification is performed for the separation of particles by their sizes. There are empirical, phenomenological, and numerical models for predicting the size distributions of screening/classification products. This paper introduces a new algorithm for the same purpose, which partially mimics phenomenological and numerical models. The algorithm iteratively selects the monosize fractions with pre-defined probabilities, then carries particle masses from the selected fractions either to the oversize or undersize product. The applicability of the algorithm was validated against the product size distributions from some industrial-scale screening/classification equipment - namely rake classifier, sieve bend (0.212 mm), vibrating screen (20 mm), and hydrocyclone - which are provided in the literature. The results show that the algorithm is predictive if each particle has a selection probability proportional to the mass of its monosize fraction and some power of its diameter. Results also suggest that vibrating screens can provide the sharpest size separation.

Keywords: Screening, Classification, Pseudorandom number generation, Particle selection, Algorithm

Introduction

Screening/classification operations include the separation of minerals or other particulate materials based on their sizes. Comprehensive reviews on the fundamentals of both operations are provided in the relevant literature (Gupta and Yan, 2016; Mular, 2009; Wills and Finch, 2016): The separation in screening is achieved by carrying the particles to screen apertures, which are either retained over or pass through the apertures. Meanwhile, the particles are classified under a moving fluid such that they are separated by their velocities in the fluid. The choice for screening/classification generally depends on the size distribution of the feed material: The latter is preferred over the former if the feed particles are finer, which may cause blinding at screen apertures. However, some successful attempts have been made to replace hydrocyclones with high-frequency vibration screens (Dündar, 2020; Frausto et al., 2021).

Three main approaches, namely empirical (Austin, et al., 1984; Coelho and Medronho, 1992; King, 2012; Mular, 2009; Nageswararao et al., 2004; Napier-Munn and Lynch, 1992; Wills

and Finch, 2016), phenomenological (Elskamp and Kruggel-Emden, 2015; Heiskanen, 1996; King, 2012; Muñoz et al., 2017; Nageswararao et al., 2004; Napier-Munn and Lynch, 1992; Wills and Finch, 2016), and numerical (Elskamp and Kruggel-Emden, 2015; Heiskanen, 1996; Khoshdast et al., 2017; Mangadoddy et al., 2020; Narasimha et al., 2007; Wills and Finch, 2016) models, are adopted for predicting the coarse and fine product size distributions of screening/ classification process. Empirical models are the mathematical functions - e.g., partition curves (Gupta and Yan, 2016; Svarovsky and Svarovsky, 1992) - to predict the process outputs although they cannot describe the separation process. Phenomenological models - e.g., first-order screening kinetics (Elskamp and Kruggel-Emden, 2015) and particle velocity equations in hydrocyclone (Heiskanen, 1996) - are the semi-empirical functions that are based on the fundamental aspects of separation. Numerical models use the iterative computation routines - e.g., (i) Discrete Element Model (Davoodi et al., 2019; Dong and Yu, 2012; Elskamp and Kruggel-Emden, 2015; Kruggel-Emden and Elskamp, 2014; Zhao et al., 2016) for vibrating screens/ sieve bends, (ii) Computational Fluid Dynamics (Khoshdast et al.,

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2017; Mangadoddy et al., 2020; Narasimha et al., 2007) and their coupling (Mangadoddy et al., 2020; Tang et al., 2018) for hydrocyclones/hydraulic classifiers - to predict the motion of particles in the separation vessels.

This paper presents a computational algorithm to predict the size distributions of oversize (coarse) and undersize (fine) products of different screening/classification operations. The algorithm implements a pseudorandom number generator into a particle selection routine, which iteratively distributes particles to the coarse or fine product. The proposed algorithm mimics (i) the phenomenological models by taking account of the size-mass balance, and (ii) the numerical models by iterative carriage of particles to the coarse or fine product. However, the algorithm cannot predict the percentage of water recovery from feed to undersize (or oversize) at wet classification. The applicability of the algorithm was validated against the coarse and fine product size distributions of some industrial-scale separations (Austin et al., 1984; Olson and Turner, 2002) that were performed with the rake classifier, sieve bend (0.212 mm), vibrating screen (20 mm), and hydrocyclone.

1. Experimental methodology

Figure 1 demonstrates the flowsheet for the simulation of

screening/classification. The algorithm started by selecting a size fraction from the initial feed mass, using the pseudorandom selection routine through the pre-defined selection probabilities. The mass of the mean particle of the selected size fraction was removed from the feed and further moved to the same size fraction of the undersize product. Then, the masses and the size distributions of the remaining feed and undersize product were calculated. The mean particle masses were (i) successively selected from the feed size fractions and (ii) moved to corresponding undersize fractions until the simulated 80 % passing size (d_{80}) of the remaining feed exceeded the experimental d_{80} of oversize. The remaining feed mass was then assigned to the oversize product. Finally, the simulated masses and size distributions of the oversize and undersize products were calculated. The simulated data were compared with the corresponding data of experimental products. During the successive particle selection stage, the masses of particles were calculated assuming that they were spherical. The mean particle size of each size fraction, except the finest unbounded size fraction (pan), was taken as the geometric mean of its lower and upper screen sizes. Meanwhile, the mean particle size of the pan was taken as the average between the aperture size of the finest screen and zero.

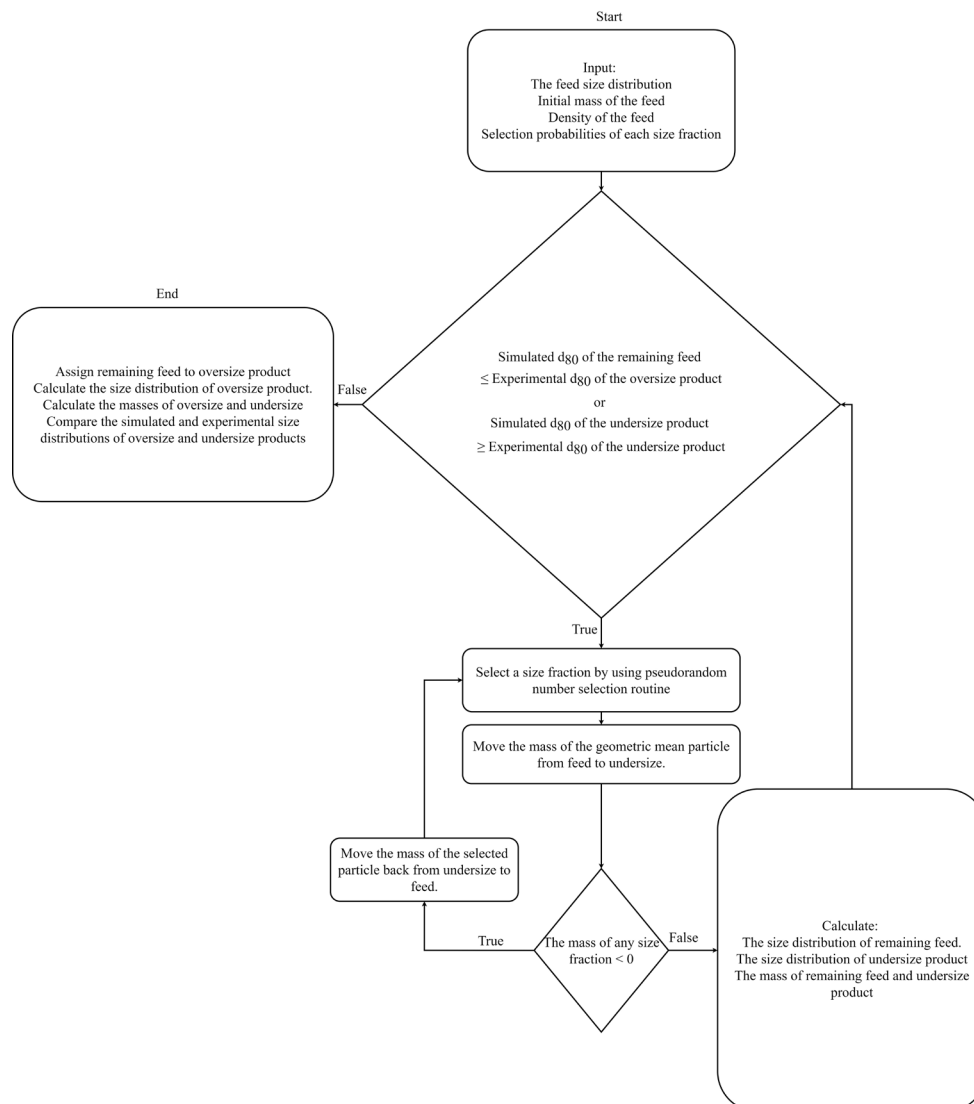


Figure 1. The flowsheet for the simulation of screening/classification by selecting and carrying feed particles to undersize product

Each particle selection event was simulated by using the MATLAB's built-in DATASAMPLE function. The routine used the MATLAB's built-in DATASAMPLE function to generate the integer index of a size fraction for particle selection (Figure 1). The function used the Mersenne Twister algorithm (Matsumoto and Nishimura, 1998) coupled with a binary search tree algorithm (Wong and Easton, 1980) for the weighted sampling of the data. The size fractions were selected with a probability (p_i) being proportional to their mass (m_i) and a power (n) of its geometric mean diameter (x_i):

$$p_i = \frac{m_i x_i^n}{\sum_{i=1}^z (m_i x_i^n)} \quad (1)$$

where i=1 is the top size fraction, and z is the sink size fraction of the feed.

The algorithm was tested against the coarse and fine product size distributions of some industrial-scale separations achieved at (i) a rake classifier, (ii) a sieve bend (0.212 mm), (iii) a vibrating screen (20 mm), and (iv) a hydrocyclone. The experimental feed and product (oversize, undersize) size distributions at (i)-(ii)-(iii) were taken from Austin et al. (1984). Meanwhile, the respective experimental size distributions at (iv) were taken from Olson and Turner (2002). However, it was not explicitly stated whether the experimental size distribution errors were reduced by performing mass balancing around (i)-(iv). For the purpose of simulation, the feeds for (i) and (iii) were assumed as quartz (density = 2.75 g/cm³) while the feeds for (ii) and (iv) were coal (1.3 g/cm³) and iron ore (3 g/cm³), respectively. The top size fractions that had been unbounded in the experimental feed size distributions were discarded from the simulation and evaluation. The total mass of oversize product after an experimental screening/classification process was estimated by averaging mass balances on the experimental size distribution curves:

$$\text{Oversize or Coarse Mass (\% Feed Mass)} = \frac{\sum_j 100 \frac{U_j - F_j}{U_j - O_j}, U_j \neq 0_j}{q} \quad (2)$$

where q is the total number of screens around which the mass balances are taken. F_j, U_j, and O_j are the cumulative masses % of feed, undersize, and oversize, respectively, which are passing through the screen j.

2. Results and discussion

Table 1 shows the simulated and experimental masses of oversize (coarse) and undersize (fine) products of the different screening/classification equipment. The results show that simulated masses are comparable to experimental masses, indicating the algorithm's success in predicting the separation products. Meanwhile, Figure 2 demonstrates the experimental and simulated size distributions of oversize and undersize products of the rake classifier (Figure 2a), the sieve bend (Figure 2b), the vibrating screen (Figure 2c), and the hydrocyclone (Figure 2d). The experimental feed size distributions are also shown with symbols in the figure. Figure 2a-d shows that the simulated size distributions of oversize and undersize products shift downwards and upwards from their corresponding feed size distributions, respectively. In other words, the simulated oversize is qualitatively coarser than its feed, yet the simulated fine product is finer. This suffices to prove that the algorithm can make logical predictions on the separation products. Figure 2 also shows that all simulated size distributions of undersize products agree quite well with the respective experimental size distributions. However, the simulated size distributions of coarse products may deviate from the respective experimental curves at the fine size scale. Such deviations are visible between the simulated and experimental curves of the coarse products of rake classifier (Figure 2a), sieve bend (Figure 2b), and vibrating screen (Figure 2c). A possible reason for these deviations may be the misclassification of fines to the coarse product in the screening/classification operations, which may not be reflected in the particle selection routine (Section 2). Some operational factors that may cause the fines' misclassification can be defined as (i) insufficient drainage and passage of fine particles to the screen aperture (Dong et al., 2013), and (ii) bypassing of fines with water flow in classifiers (Kelly, 1991).

Table 1. The simulated and experimental masses of the oversize products of the screening/classification equipment

Equipment	Oversize Mass (% of the Feed)	
	Simulated	Experimental
Rake Classifier	55.49	62.16
Sieve Bend	50.55	55.45
Vibrating Screen	32.90	32.89
Hydrocyclone	66.06	59.97

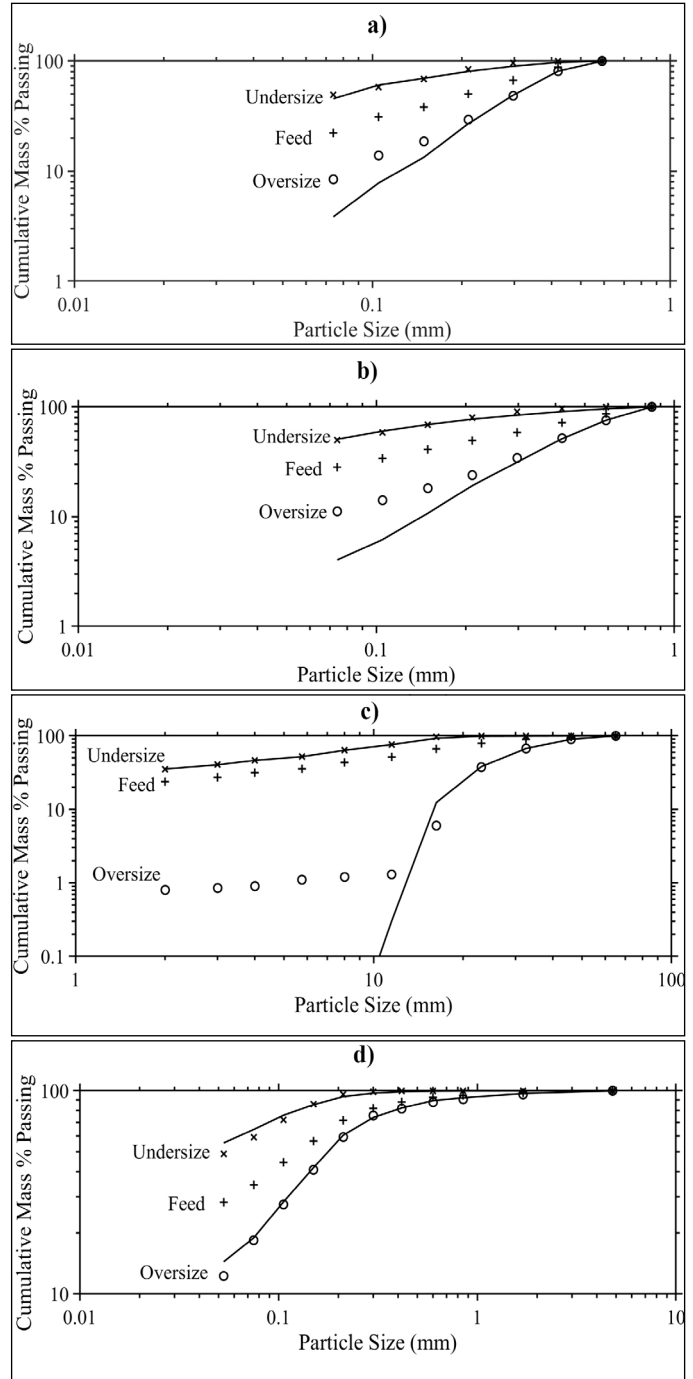


Figure 2. The experimental (symbols) and simulated (lines) size distributions of oversize and undersize products of (a) rake classifier, (b) sieve bend, (c) vibrating screen, and (d) hydrocyclone. The experimental feed size distributions are also included in the figure. The experimental size distributions in (a)-(c) and (d) were taken from Austin et al. (1984), and Olson and Turner (2002), respectively.

Table 2 shows the ‘n’ values used to generate the probabilities (Equation 1) for the selection of particles to undersize products at different screening/classification equipment. The table shows that the ‘n’ values required to simulate the products at rake classifier, sieve bend and hydrocyclone are similar to each other. However, the respective ‘n’ value for the vibrating screen are visibly the lowest. Using lower ‘n’ values must provide more chance for relatively finer particles to get selected into the undersize product, as indicated by Equation 1. Therefore, this result strongly suggests that the vibrating screens can outperform sharp separation of fine particles from the coarse ones. Some additional evidences are also available in the literature showing that the vibrating screens yield sharper tromp curves as compared to hydrocyclones (Dündar, 2020; Wills and Finch, 2016).

Table 2. The ‘n’ values used to generate the probabilities (Equation 1) for particle selection to undersize products of different screening/classification equipment

Equipment	“n” value (Equation 1)
Rake Classifier	-4.5
Sieve Bend	-4.25
Vibrating Screen	-6.5
Hydrocyclone	-4.2

The default form of the algorithm is to select and carry particles from feed to undersize product till the simulated d_{80} of the remaining feed is larger than the experimental d_{80} of oversize product (Figure 1). Also, the algorithm can be revised in a way that the feed particles were selected and carried to the oversize product (Figure 3). In this case, the algorithm is executed till the simulated d_{80} of the remaining feed is lower than the experimental d_{80} of the undersize product. The applicability of this revised algorithm was also tested against the experimental product size distributions of the rake classifier, the sieve bend, the vibrating screen, and the hydrocyclone (Section 2). Figure 4 shows that the simulated and experimental product size distributions are in good agreement when the selection probability of feed particles to oversize product is calculated by the p_i formulation (Equation 1). Table 3 also shows the simulated masses of oversize (coarse), which are comparable to the experimental masses (Table 1).

Table 3. The simulated masses of the oversize products of the screening/classification equipment

Equipment	Simulated Oversize Mass (% of the Feed)
Rake Classifier	62.70
Sieve Bend	57.49
Vibrating Screen	33.18
Hydrocyclone	67.63

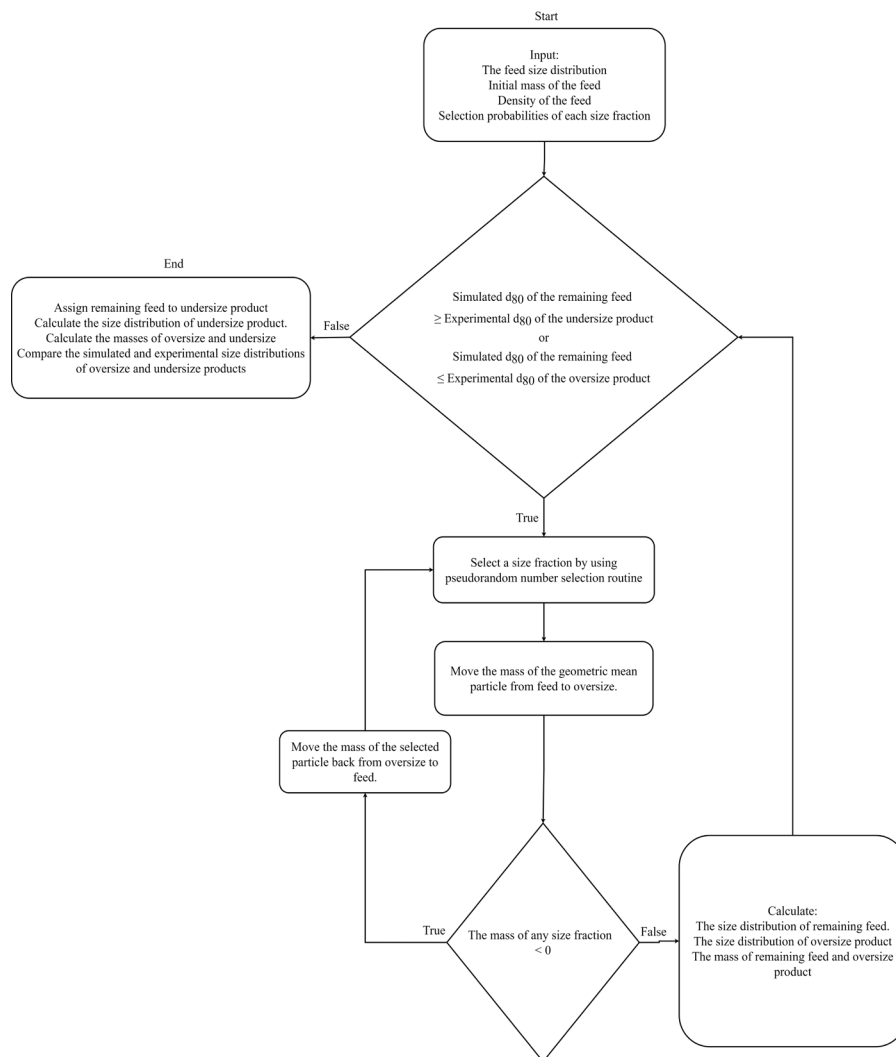


Figure 3. The flowsheet for the simulation of screening/classification by selecting and carrying feed particles to oversize product

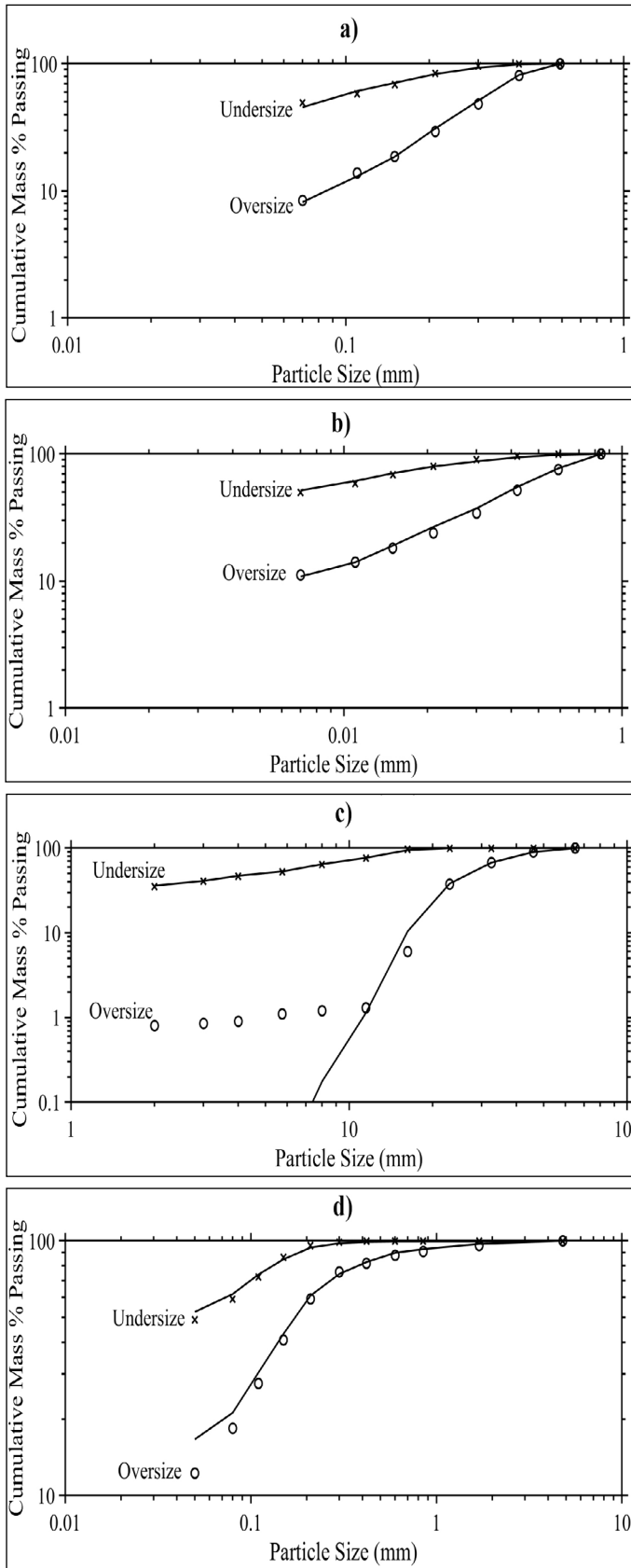


Figure 4. The experimental (symbols) and simulated (lines) size distributions of coarse and fine products of (a) the rake classifier, (b) the sieve bend, (c) the vibrating screen, and (d) the hydrocyclone. The simulated size distributions were produced by selecting and carrying feed particles to oversize products. The experimental size distributions in (a)-(c) and (d) were taken from Austin et al. (1984), and Olson and Turner (2002), respectively.

Table 4 shows the 'n' values used to generate the selection probabilities (p_s) of particles for the oversize products. These values are quite higher than the respective 'n' values used to select particles for the undersize products (Table 2). Therefore, that coarse and fine particles in the feed have more chance to be carried to the oversize and undersize products, respectively. More importantly, if there is a large difference between the p_s , congruently the 'n' values, for oversize and undersize selection, a sharp size separation can be achieved. Referring to the 'n' exponents in Table 2 and Table 4, the greatest differences in the p_s are possible at vibrating screen, suggesting that it yields the sharpest size separation. When a particle is to be separated by a vibrating screen either to oversize or undersize fraction, its separation only depends on the particle dimensions and the aperture size of the screen. Meanwhile, the separation of a particle to oversize or undersize fraction is based on its motion (trajectory) in a classifier. However, this trajectory is not only affected by the particle size but also particle density (Wills and Finch, 2016). Therefore, it is very likely that vibrating screen should provide better size separation than the classifiers, as supported by sharper partition curves in the former (Dündar, 2020).

Table 4. The "n" values used to generate the (Equation 1) for the particle selection to oversize products of different screening/classification equipment

Equipment	"n" value (Equation 1)
Rake Classifier	-1.8
Sieve Bend	-2
Vibrating Screen	+2
Hydrocyclone	-2.1

Conclusions

An algorithm is presented to predict the size distributions of oversize and undersize products of screening/classification operations. The algorithm mimics (i) the phenomenological models by taking account of the size-mass balance, and (ii) the numerical models by iterative carriage of particles to coarse or fine products. The algorithm iteratively (i) selects a monosize fraction, (ii) takes a particle from the size fraction, and (iii) carries the particle either to oversize or undersize product. The algorithm can predict the size distributions of screening/classification products if each particle has a selection probability proportional to the mass of its monosize fraction and some power of its diameter. Results also suggest that vibrating screens can provide the sharpest size separation.

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