

Satıhtaki Bir Denizaltının Yalpa Salınım Hareketinin Matematiksel Modeli

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ÖZET

Bir denizaltı için yalpa hareketi diğer gemi hareketlerinin arasında en kritik olanıdır. Özellikle satıh durumunda, aşırı yalpa hareketi denizaltı mürettebatı ve ekipmanı için tehlikeli olabilir. Bu sebeple, aşırı genlikli yalpa hareketlerine maruz kalınmaması için dizayn aşamasında yalpa hareketinin analiz edilmesi gereklidir. Hareket genliklerini elde etmek için yalpa hareketinin matematiksel modeli kullanılabilir. Yalpa hareketinin matematiksel modelinin oluşturulmasında en önemli kısım yalpa sönümünün tahminidir. Yalpa sönümü deneysel, sayısal ve ampirik olarak elde edilebilir. Deneysel yöntemler pahalı ve zaman alıcı, ampirik ifadeler ise her gemi tipi için uygun değildir. Bu sebeple, son zamanlarda bilgisayar teknolojisindeki gelişiminde etkisiyle daha hızlı ve ucuz bir yöntem olan sayısal metotlar sıklıkla kullanılmaktadır. Bu çalışmada, satıhtaki bir denizaltının serbest yalpa hareketinin matematiksel modeli elde edilmiştir. Yalpa sönüm terimi denizaltının serbest yalpa salınım hareketinin ticari bir Hesaplamalı Akışkanlar Dinamiği (HAD) kodu ile sayısal olarak modellenmesi ile elde edilmiştir. Atalet ve doğrultma terimleri ise denizaltı geometrisi kullanılarak ampirik olarak hesaplanmıştır. Matematiksel model elde edilmiştir ve 20 derece serbest bırakma açısındaki denizaltı yalpa salınım hareketi sıfır hızda ve 5 kn, 7.5 kn ve 10 kn ileri hızda hesaplanmıştır. Elde edilen sonuçlar sayısal analiz sonuçları ile karşılaştırılmış ve uyumlu sonuçlar gözlemlenmiştir.

Anahtar kelimeler: HAD, Matematiksel model, Yalpa sönümü, Yalpa Hareketi, Denizaltı

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Mathematical Model of Roll Decay Motion for a Surfaced Submarine

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ABSTRACT

Among all ship motions, the most critical one is the roll motion for a submarine. The excessive roll motion can be harmful to both ship's crew and equipment, especially in surfaced condition. For this reason, it is important to predict the roll motion during the design stage to overcome excessive roll amplitudes. A mathematical model of the roll motion can be used to predict the motion responses of a submarine. Estimation of the roll damping is the most critical part to develop the mathematical model for a better prediction of the roll motion. The roll damping can be obtained experimentally, numerically and empirically. Experimental methods are expensive and time-consuming and empirical methods cannot be applied for all ship types. For this reason, numerical methods are frequently used due to the development of computer technologies in recent years, being economical and fast. In this study, the mathematical model of roll decay motion for a submarine at surfaced condition was obtained. The roll damping term was obtained numerically by carrying out roll decay simulations using a commercial Computational Fluid Dynamic (CFD) code. The inertia and restoring terms were obtained empirically by using the submarine geometry. The mathematical model was obtained and the roll decay motion of the surfaced submarine was calculated with a 20 degrees initial roll angle at zero speed and 5 kn, 7.5 kn and 10 kn forward speeds. The results were compared with numerical results and a good agreement was observed.

Keywords: CFD, Mathematical Model, Roll Motion, Roll Damping, Submarine

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1. Introduction

Submarines are generally used for search, rescue, touristic and military purposes. Contrary to general opinion, submarines spend a significant time of their life cruising at the sea surface (Vogels, 2015). During surface cruising, submarines are highly affected by adverse weather conditions, and they are exposed to high roll angles. The excessive roll motion affects the equipment and personnel, therefore, predicting the roll motion of submarines during the design stage is necessary for safe operation. The prediction of roll motion can be achieved by carrying out physical model experiments or numerical simulations via CFD. The strip theory can be also used for the roll motion prediction, however not taking into account the viscosity will cause incorrect prediction.

For a more accurate roll motion prediction, the roll damping should be determined correctly. The roll damping can be calculated by carrying out roll decay tests or forced oscillation tests by using both experiments or numerical simulations. Besides, a semi-empirical formula developed by Ikeda can also be used for roll damping estimation. In the formula, Ikeda et al. (1978), (see also Himeno (1981)), divided roll damping into five components: friction, wave, eddy, naked hull lift and bilge keel damping and developed a formula for each component. However, the experiments take a long time, cost a lot of money and empirical methods ignore nonlinearities. On the other hand, numerical simulations with CFD is a cost-effective and faster method compared to experiments. It also captures the effects of viscosity, effective creation of vortices in the boundary layer, vortex shedding and turbulence which were usually not taken into account in the empirical methods. These advantages and the recent developments in CFD technology lead researchers to use numerical simulations for estimation of roll damping.

There are a lot of papers in the literature related to the prediction of ship roll motion by using experimental or numerical techniques. However, the roll motion of a surfaced submarine has not been studied by many researchers. There are a few papers in the literature that investigates the roll motion of a surfaced submarine. In one of these papers, Hedberg (2006) investigated how to simulate the roll motion of an Australian Collins Class submarine. The roll characteristics of the surfaced submarine, through both linear and nonlinear approach was studied. The effect of different sea states, adding appendages and varying the transverse metacentric height were examined. Thornhill and Hermanski (2008) carried out a set of CFD simulations and physical experiments on a 2D surfaced submarine section to determine the effects on roll motion of a closed versus free flooding casing. Letter (2009) investigated the roll motion experienced by a surfaced submarine in beam seas using CFD techniques and physical model scale experiments.

In this study, a mathematical model for the roll decay motion of a surfaced submarine was developed. The roll damping of the surfaced submarine was calculated by carrying out numerical roll decay simulations. After obtaining the roll damping coefficients, the other hydrodynamic and hydrostatic coefficients were obtained by using submarine geometry. Finally, the mathematical model was obtained and the roll decay motion of surfaced submarine was calculated at zero speed and various forward speeds. The results were compared to numerical simulations and a good agreement was found.

2. Roll motion

The 1-DOF (degrees of freedom) linear roll motion equation is shown in Equation 1. In the equation I_{44} , A_{44} , B_{44} , C_{44} refer to the mass inertia coefficient, added mass inertia coefficient, damping coefficient,

restoring coefficient, respectively. $\phi_4(t)$ is roll angle, $\dot{\phi}_4(t)$ is roll angular velocity and $\ddot{\phi}_4(t)$ is roll angular acceleration. Finally, M_{44} refers to the roll exciting moment in the roll motion equation.

$$(I_{44} + A_{44})\ddot{\phi}_4(t) + B_{44}\dot{\phi}_4(t) + C_{44}\phi_4(t) = M_{44} \quad (1)$$

For solving the equation, the hydrostatic and hydrodynamic term should be obtained. Calculation of mass inertia coefficient, I_{44} , is estimated with the help of the formula given in Equation 2.

$$I_{44} = k_4^2 M \quad (2)$$

$$k_4 = \frac{\sqrt{B^2 + D^2}}{2\sqrt{3}} \quad (3)$$

k_4 is the radius of gyration, M is the mass of the submarine, B is the width of the submarine and D is the depth of the submarine.

Added mass inertia coefficient for the submarine can be obtained by using the roll period formula. The roll period formula is shown in Equation 4.

$$T = 2\pi \sqrt{\frac{I_{44} + A_{44}}{C_{44}}} \quad (4)$$

T is the natural roll period, I_{44} is the mass inertia coefficient, A_{44} is added mass inertia coefficient and C_{44} is restoring coefficient.

The hydrostatic restoring moment is a moment caused by the geometry of the submarine. It is a moment caused by the effect of static buoyancy. The restoring term can be obtained by using Equation 5.

$$C_{44} = \rho g \nabla GM_0 \quad (5)$$

The damping coefficient can be calculated by using the roll decay test. To take nonlinearities into account, the roll damping is divided into linear and nonlinear term as it is shown in Equation 6. This roll damping model is named as a linear-plus-cubic model where B_1 is the linear damping term and B_3 is the nonlinear term.

The roll decay motion equation with nonlinear damping term is shown in Equation 6. The submarine is brought to a certain angle and released to free for the roll decay test, therefore, there is no excitation force. This means that the right part of the equation is zero.

$$(I_{44} + A_{44})\ddot{\phi} + B_1 \dot{\phi} + B_3 \dot{\phi}^3 + C_{44}\phi = 0 \quad (6)$$

After the submarine model is initially inclined and then released, the initial roll angle ϕ_0 and the other peaks of roll angles ϕ_j, ϕ_{j+1} etc. at each cycle are obtained from roll decay time history as shown in Figure 1. Using the measured roll amplitudes, the mean roll angles (ϕ_m) and roll decrements ($\Delta\phi$) are calculated. ϕ_m and $\Delta\phi$ are calculated as shown in Equation 7 and 8, respectively.

$$\phi_m = \frac{|\phi_j| + |\phi_{j+1}|}{2} \quad (7)$$

$$\Delta\phi = |\phi_j| - |\phi_{j+1}| \quad (8)$$

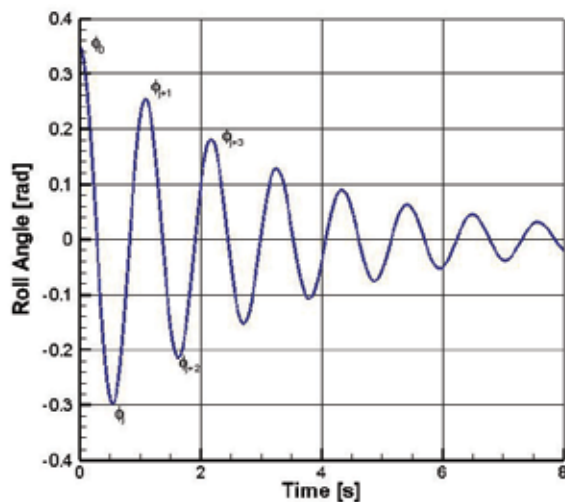


Figure 1. Typical roll decay time history

Thus, the roll extinction curve is obtained by plotting the roll decrements against the mean roll angles as it is shown in Figure 2 and the equation of the curve is obtained by fitting a cubic polynomial to the data as it is shown in Equation 9. Here, a and c are the extinction coefficients.

$$\Delta\phi = a\phi_m + c\phi_m^3 \quad (9)$$

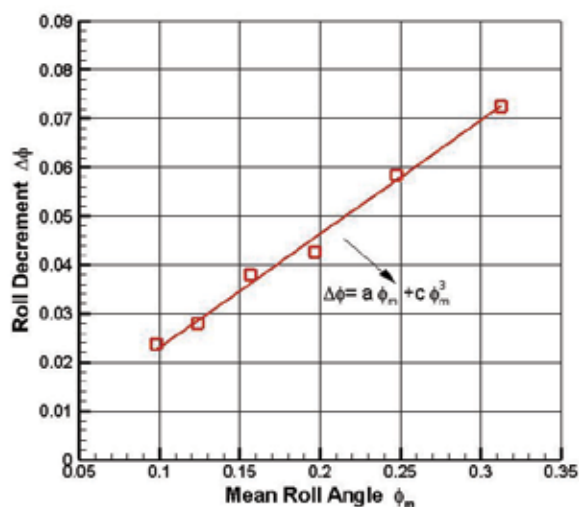


Figure 2. Roll extinction curve

In Equation 6, when both sides are divided by $(I_{44}+A_{44})$, the following equation is obtained.

$$\ddot{\phi}_4 + 2\alpha\dot{\phi}_4 + \gamma\phi_4^3 + \omega_n^2\phi_4 = 0 \quad (10)$$

In this equation, 2α and γ , c , ω_n show damping, stiffness and natural frequency respectively.

$$2\alpha = \frac{B_1}{(I_{44} + A_{44})} \quad (11)$$

$$\gamma = \frac{B_3}{(I_{44} + A_{44})} \quad (12)$$

$$c = \frac{C_{44}}{(I_{44} + A_{44})} \quad (13)$$

$$\omega_n = \sqrt{\frac{C_{44}}{(I_{44} + A_{44})}} \quad (14)$$

An equivalent damping coefficient term shown in Equation 15 was used by Himeno (1981) to find the nonlinear roll damping coefficients.

$$B_{eq} = B_1\dot{\phi} + \frac{3}{4}\omega_n^2\dot{\phi}^3 B_3 \quad (15)$$

Himeno proposed the relation between ϕ_m and $\Delta\phi$ as in Equation 16 (Himeno, 1981).

Finally, we can use the following expression for the roll decrement as a function of the mean roll amplitude (Himeno, 1981) to obtain the linear and nonlinear roll damping coefficients.

$$\Delta\phi = \frac{\pi}{2} 2\alpha\phi_m + \frac{3\pi}{8}\omega_n\gamma\phi_m^3 \quad (16)$$

Using the extinction coefficients a and c , the regression coefficients α and γ are calculated. Finally, the linear damping coefficient, B_1 and nonlinear damping coefficient B_3 are calculated using the α and γ regression coefficients as follows.

$$\alpha = \frac{a\omega_n}{\pi} \quad (17)$$

$$\gamma = \frac{8c}{3\pi\omega_n} \quad (18)$$

$$B_1 = 2\alpha(I_{44} + A_{44}) \quad (19)$$

$$B_3 = \gamma(I_{44} + A_{44}) \quad (20)$$

The calculated roll damping coefficients for 0 kn, 5 kn, 7.5 kn and 10 kn speeds are shown in Table 3.

3. Submarine model

In this study, a submarine model was used for the investigation of the roll decay motion. The subject submarine is a swimmer delivery vehicle (SDV) known as CE4F which was designed for the Turkish Navy's elite special forces. The submarine is currently used for some research and development

projects at the Yildiz Technical University. Instead of rudders in +-configuration in the original model, rudders in x-configuration is applied in this study. Besides, the model without stabilizers were used for the calculations. The main particulars of the selected submarine are listed in Table 1. The 2D geometry including the dimensions of the full scale model is shown in Figure 3 and the 3D geometry in Figure 4.

Table 1. Main particulars of the submarine

Main Dimensions	Model Scale	Full Scale
Length	0.928 m	9.28 m
Breadth	0.14 m	1.4 m
Depth	0.14 m	1.4 m
Draft	0.12 m	1.2 m
Displacement	10.10 kg	10.10 ton
KG	0.0556 m	0.556 m
GM	0.0146 m	0.146 m
I_{44}	0.0329 kgm^2	3.29 tonm^2
A_{44}	0.2358 * I_{44} kgm^2	0.2358 * I_{44} tonm^2
C_{44}	1.4827 kgm^2/s^2	14,46583 tonm^2/s^2

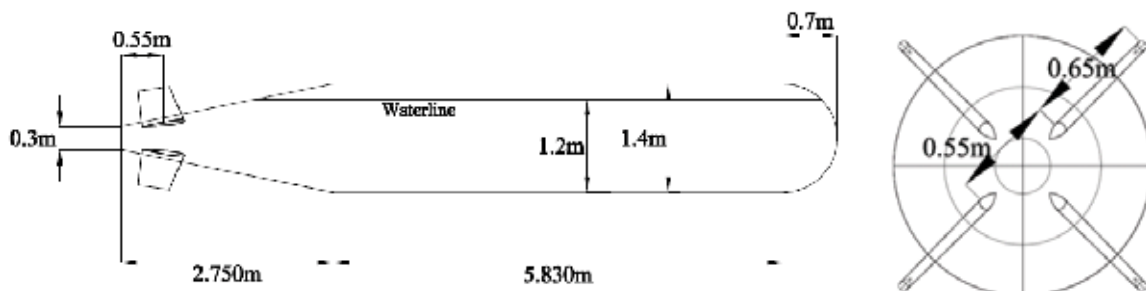


Figure 3. 2D view of the submarine (full scale)

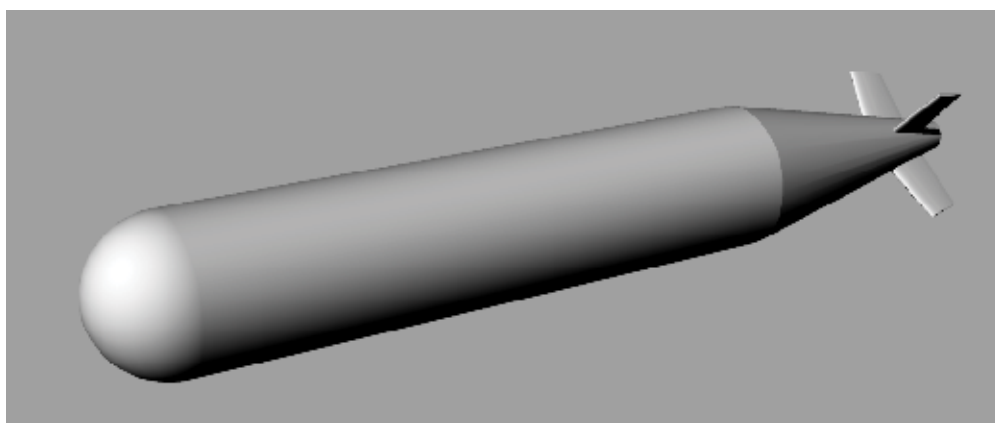


Figure 4. 3D view of the submarine

4. Numerical model

In this study, the roll motion of a submarine in surface condition was modelled. The submarine was subjected to a two-phase flow which was resolved by a numerical modelling technique. In order to solve the viscous flow around the submarine numerically, a commercial CFD software was used that discretizes the RANS(Reynolds Averaged Navier Stokes) equations using the finite volume method and solves them iteratively.

The basic equations used to model the flow around a body in a viscous fluid are mass and momentum conservation equations. The flow around the submarine was considered incompressible, turbulent and time-dependent. The Semi-Implicit Method for Pressure-Linked Equations (SIMPLE) was used to resolve the pressure and velocity field. A suitable $k-\epsilon$ turbulence model was chosen to decrease the CPU-time compared to $k-\omega$ model, which requires higher CPU time (Tezdogan et al., 2015; Querard et al., 2008). It is also known that the effect of turbulence is small when calculating the roll motion of the hulls with round corners (Yu, 2008). The volume of fluid (VOF) method was used to solve the multiphase flows with the free surface at the air-water interface. Figure 5 shows the free surface and the separation of air and water phases.

The flow within the boundary layer has to be solved correctly for a better calculation of the boundary layer dynamics. Therefore, y^+ values on the hull surface should remain within the limits for the $k-\epsilon$ turbulence model. In this study, the y^+ values on the submarine surface were around 45 and that is considered to be suitable since it remains between the recommended ranges 30-300 for the selected turbulence model (CD-Adapco, 2014).

The roll motion of the submarine was defined by using the dynamic fluid body interaction (DFBI) module of the numerical software with the moving mesh technique. The submarine was set free to only roll motion.

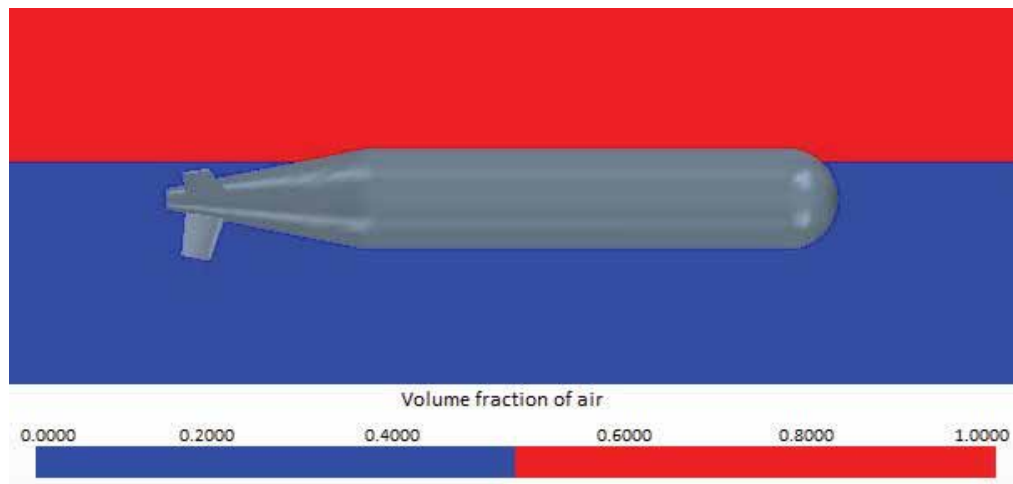


Figure 5. Volume fraction field of the numerical model

4.1. Mesh structure and boundary conditions

It is very important to determine the computational volume. In this study, no parametric study was conducted to determine the calculation volume, similar studies previously conducted in the literature and recommendations published by ITTC in 2011 were taken into account (Kahramanoglu et al., 2020; ITTC, 2011).

The main dimensions of the computational volume were given in Figure 6. The computational volume extends 3 L front of and 8 L behind, 3 L to the side, 3 L the under and 3 L the top of the boundaries of the overset region.

The left and right side of the computational volume was defined as velocity inlet and pressure outlet, respectively in Figure 7. The other surfaces were also defined as velocity inlet. The submarine surface was selected as a wall. The size of the domain is large enough, therefore no reflection was observed

from the boundaries. In addition, a velocity value equal to the ship velocity was defined only in the minus x-direction.

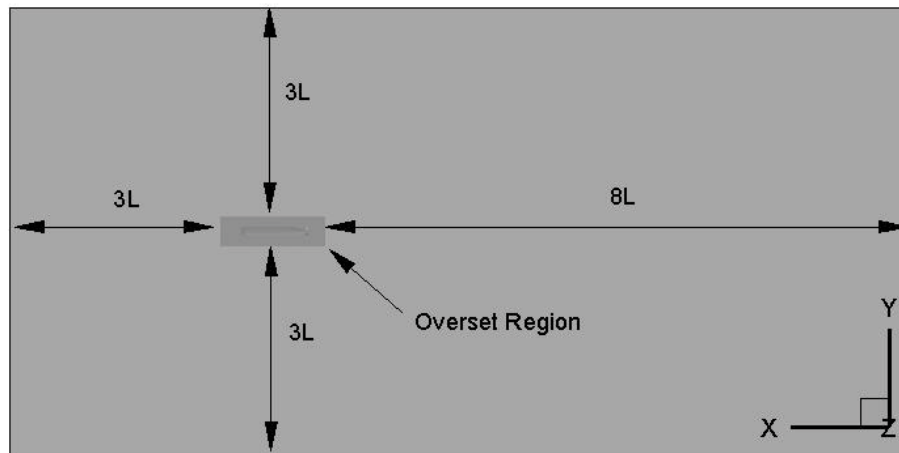


Figure 6. Size of the computational volume

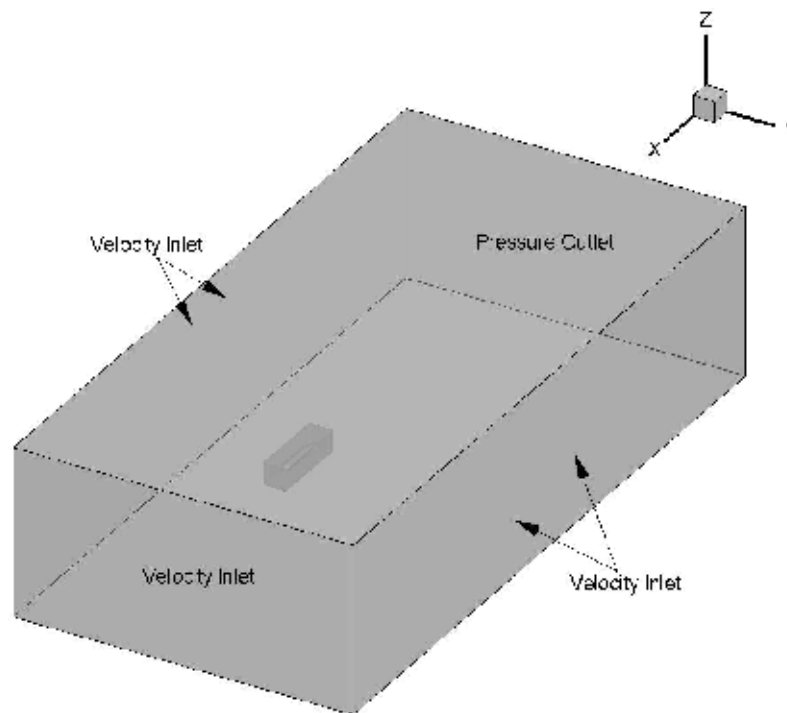


Figure 7. Boundary conditions of the computational volume

The computational volume was decomposed by the finite volume method using hexahedral mesh. Overset mesh technique was used which was shown to be able to calculate the horizontal and vertical motion of the ship with high accuracy in previous studies (Cakici et al., 2017; Cakici et al., 2018; Yildiz et al., 2017; Yildiz et al., 2019). Due to the mesh motion synchronized with the body, the overset mesh has great capability compared to rigid and deforming mesh techniques (Cakici et al., 2017). The overset mesh technique requires two different regions; the background and overset regions are shown in Figure 6. The overset mesh region was defined as a small rectangle surrounding the submarine. The grid system around the hull in the overset region was allowed to move freely. Three overlap zones were defined between the overset and background region. The data is transferred between the

overset and the background via the overset boundaries. Based on numerical experience, a much higher resolution is needed near the hull to capture the boundary layer. A hexahedral fine grid in the near wall boundary layer was applied and the cell sizes were gradually increased starting from the boundary layer to the outer boundaries. Finally, the total mesh number is equal to 1,396,366. The total number of cells used in the background region is 1,225,142 since it is 171,224 in the overset region. Figure 8-11 shows the generated mesh around the submarine.

ITTC recommends the time step not greater than at least %1 of each period for the release test. With this recommendation, the time step of the analysis was chosen as 0,005 seconds (ITTC, 2011).

The numerical simulations were performed with an 18 cores workstation that has 128 GB RAM capacity. Each simulation run required 12 hours to simulate 7 seconds roll decay motion.

More details about the numerical simulations and sensitivity analysis for the grid and time step can be found in Cansiz, 2020.

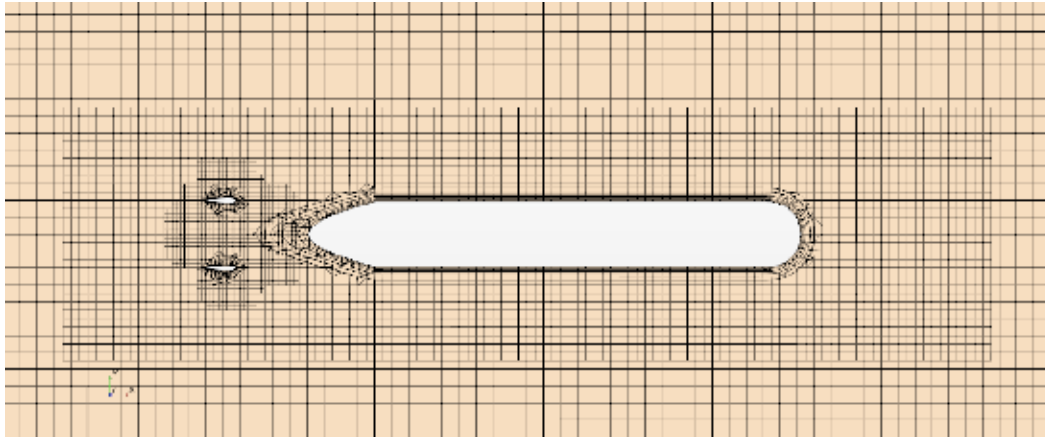


Figure 8. The mesh structure around the submarine (profile view)

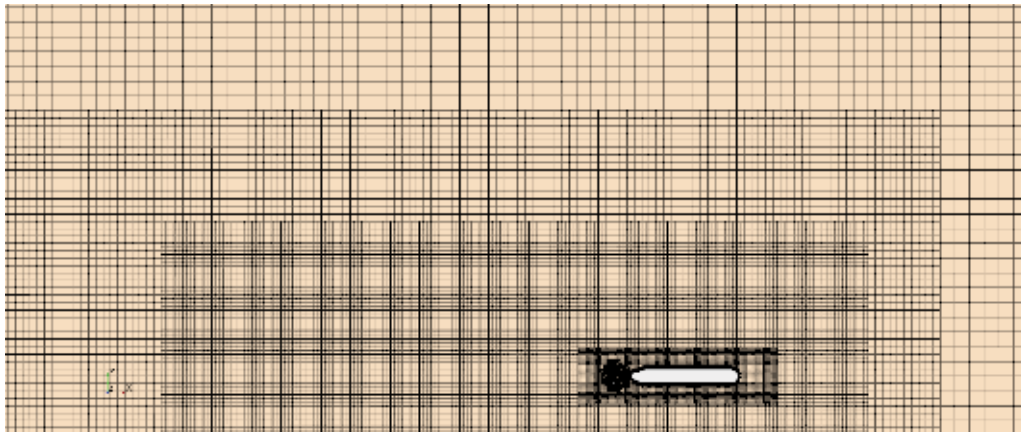


Figure 9. The overlap zones of the mesh structure

5. Numerical calculation for roll damping via roll decay curve

The roll decay time histories obtained with CFD is shown in Figure 12. The roll decay simulations were carried out at zero speed and 5 kn, 7.5 kn and 10 kn forward speeds. The roll extinction curve for zero speed condition is shown in Figure 13. The fitted cubic polynomial is also shown in Figure 13. The other roll extinction curves with varying forward speeds are given in Appendix A. The obtained extinction coefficients (a and c values) to find the linear and nonlinear damping coefficients is shown in Table 2.

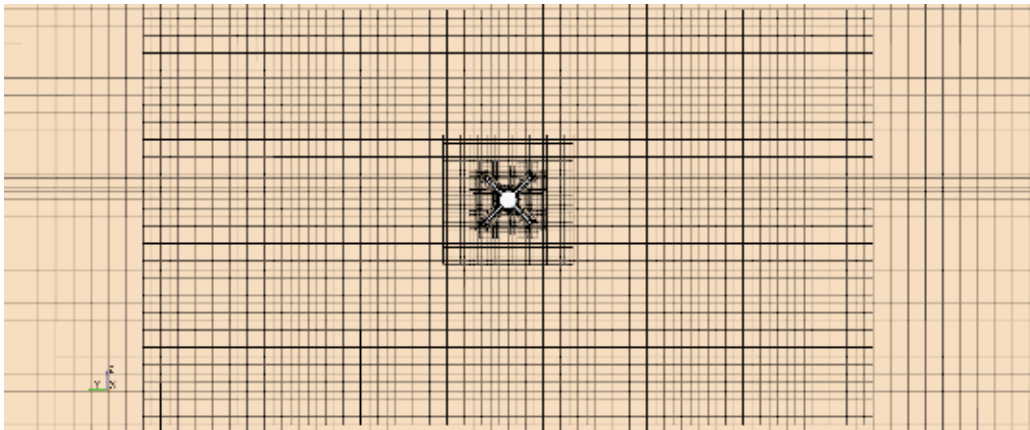


Figure 10. The mesh structure around the submarine (back view)

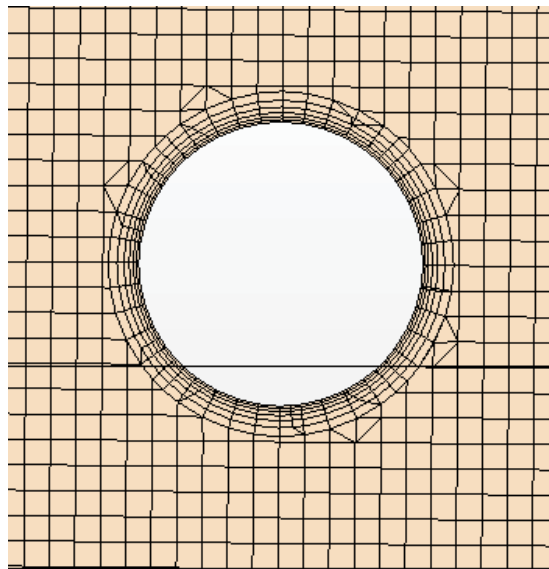


Figure 11. The prism layer around the boundary layer of the submarine (front view)

Table 2. The extinction coefficients for 0 kn, 5 kn, 7.5 kn and 10 kn

Velocity [kn]	a	c
0	0.1820	-0.21600
5	0.3095	-0.47890
7.5	0.3265	-0.63480
10	0.2310	0.01173

Table 3. The roll damping coefficients for 0 kn, 5 kn, 7.5 kn and 10 kn

Velocity [kn]	B_1 [tm^2/s^2]	B_3 [tm^2/s^2]
0	0.889883503	-0.397022612
5	1.513290901	-0.880250597
7.5	0.050334075	-0.003678283
10	0.035611551	6.79683E-05

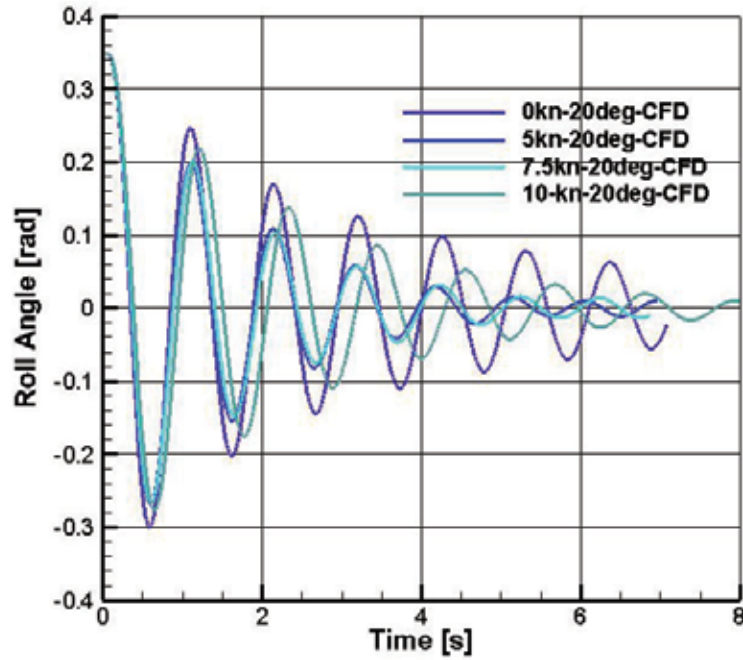


Figure 12. Roll decay time histories for all speeds

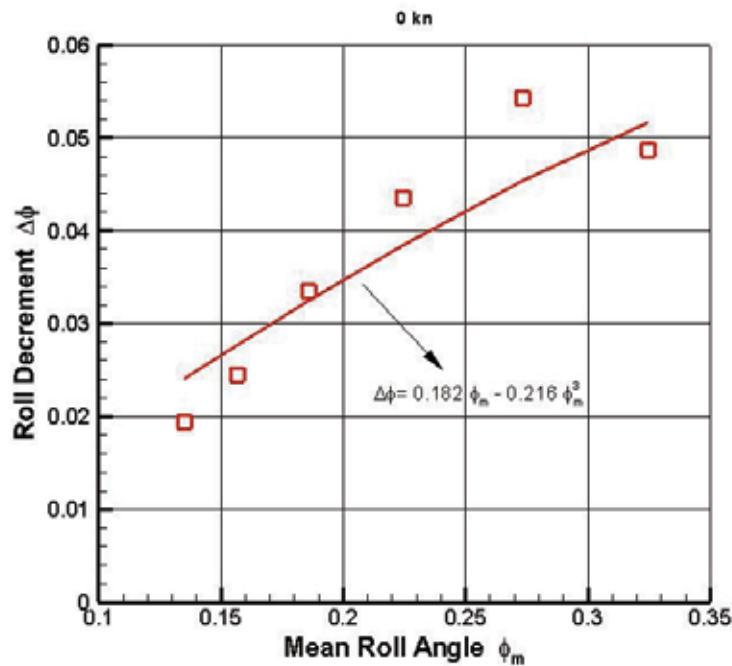


Figure 13. Roll extinction curve for 0kn

The natural roll periods were calculated by using the roll decay time histories for all speeds. The natural roll period was obtained as 1.055 s by averaging all the natural roll periods. Finally, A_{44} was calculated by using the natural roll period.

6. Mathematical model of roll decay motion

After calculating all the coefficients in the roll decay motion equation, the mathematical model for the roll decay motion of the surfaced submarine was obtained. The mathematical model was used to

obtain the roll decay time histories at zero speed and various forward speeds with an initial release angle. The mathematical model was solved using the Euler method and the results were compared with CFD results. The compared results are given in Figure 14-17. As it can be seen from the figures, there is a good agreement between the roll decay time histories.

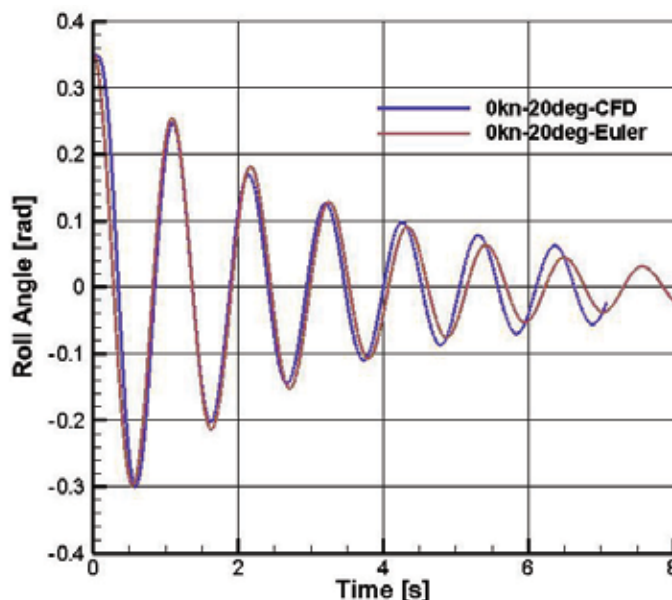


Figure 14. Comparison between CFD and Euler code for 0kn

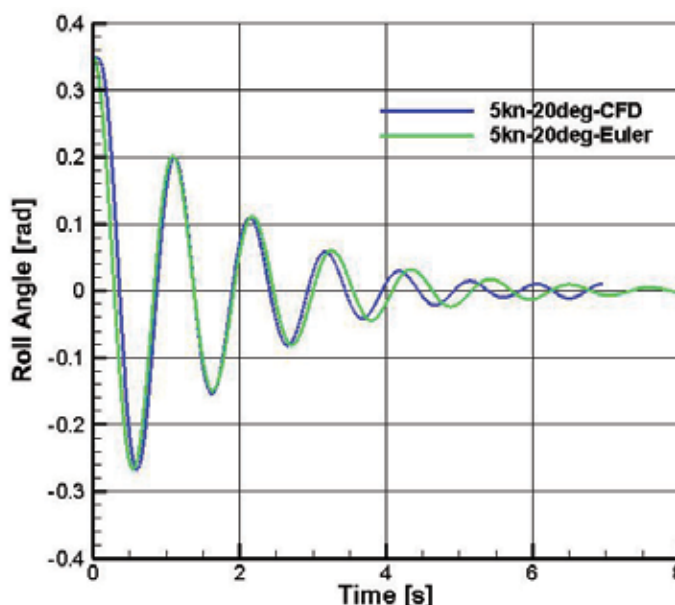


Figure 15. Comparison between CFD and Euler code for 5kn

The roll decay results of the mathematical model for 0 kn, 5 kn and 7.5 kn, especially until the third cycle, were the same as CFD results. After the third cycle, the results started to show discrepancies related to change in roll period at CFD results. The reason why the roll period changes after some time is still an unsolved problem for CFD simulations. The same problem has been observed when comparing the numerical and experimental results by other researchers (Gokce and Kinaci, 2018). Unlike the roll period, the peaks of the roll decay motion were found compatible for all speeds. The results of the mathematical model and CFD simulations showed discrepancies starting from the first

cycle. There is a phase difference observed between the results. The reason for this situation will be investigated in future work.

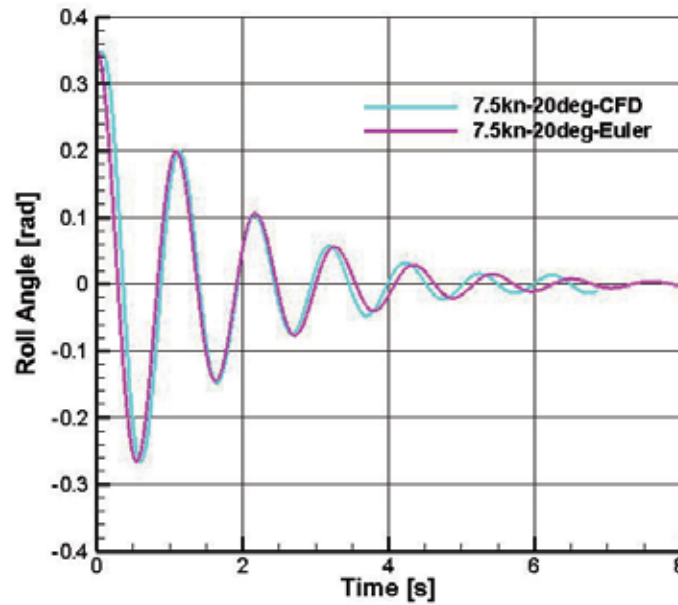


Figure 16. Comparison between CFD and Euler code 7.5kn

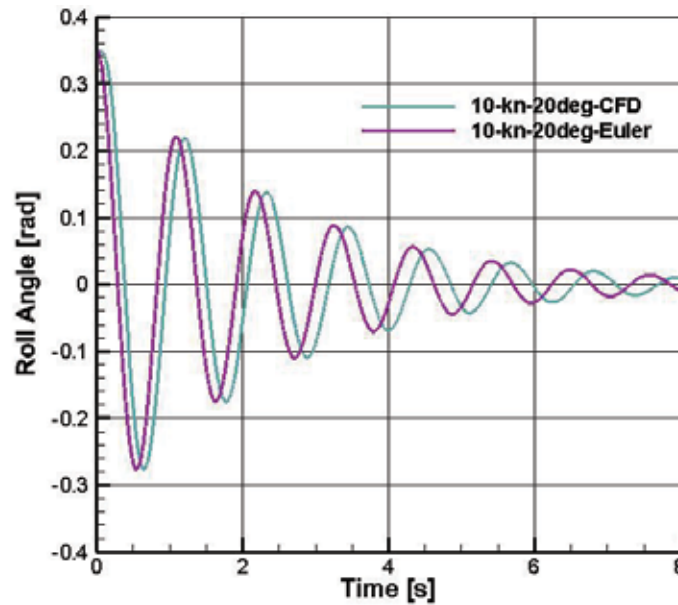


Figure 17. Comparison between CFD and Euler code 10kn

7. Conclusion

Submarines can stay extended periods on the sea surface related to operational requirements. Unlike surface ships, they can be more vulnerable to roll motions due to hull design. Extreme roll motion is dangerous for the submarine's crew and equipment. Therefore, the roll motion of submarines under surfaced condition should be investigated during the design phase. In this study, the roll decay motion of a surfaced submarine was investigated and the following outcomes were found.

- 1) The roll damping of the surfaced submarine was calculated by carrying out roll decay tests via CFD at zero speed and different forwards speeds. The roll damping increased with the effect

- of increased speed until a critical speed which is 7.5 kn for the subject submarine. After critical speed, the roll damping started to decrease.
- 2) After calculating all the coefficients in the roll decay motion equation, the mathematical model was obtained and it was solved using the Euler method to get the roll decay time histories at different speeds.
 - 3) The results were compatible with the CFD results, especially until the third cycle. The peaks of the roll angle were captured well with the mathematical model, however, the roll period showed some discrepancies. The roll period changed in CFD simulations after the third cycle which was constant in the mathematical model.

The obtained mathematical model results showed compatible results with the numerical results, therefore, the roll decay time histories and roll damping coefficients at various speeds can be calculated by solving the obtained mathematical model.

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APPENDIX A

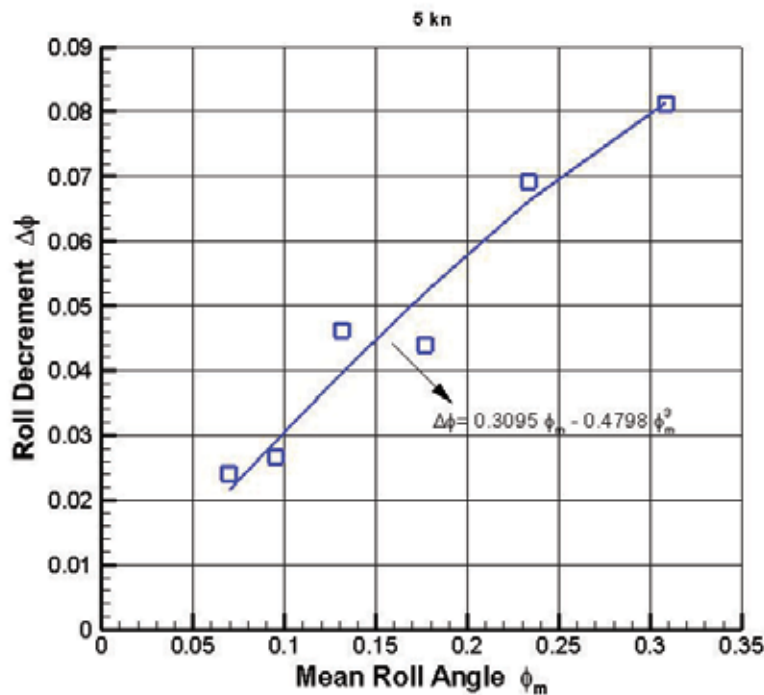


Figure 18. Roll extinction curve for 5kn

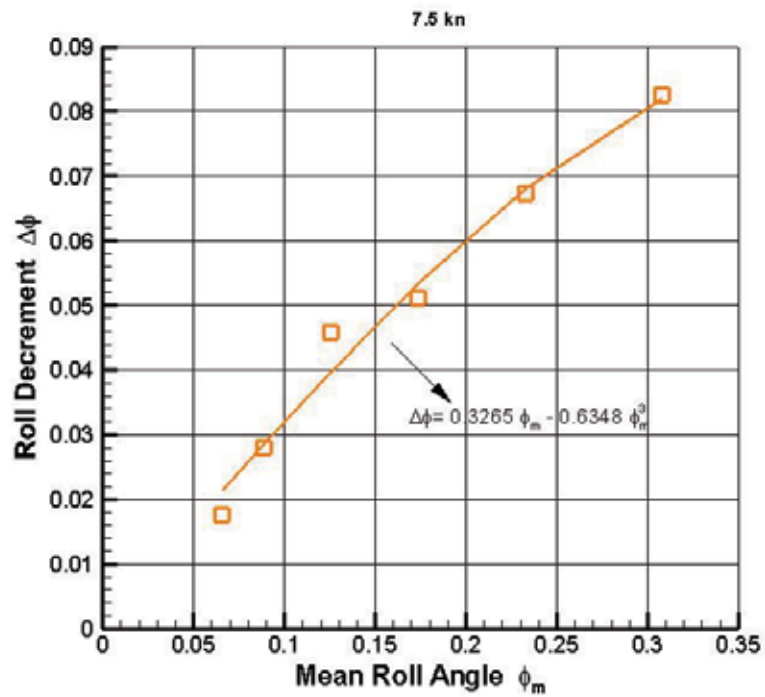


Figure 19. Roll extinction curve for 7.5kn

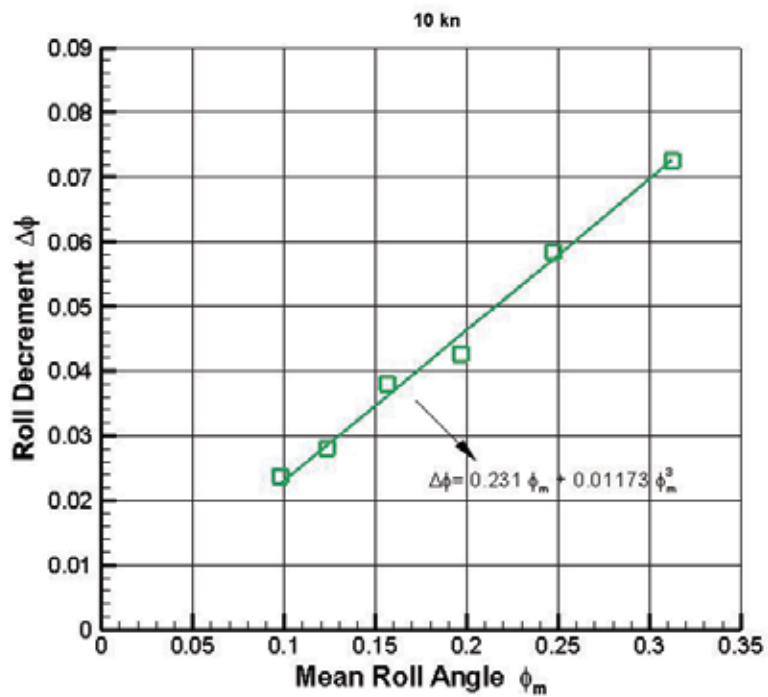


Figure 20. Roll extinction curve for 10kn