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İleri ve Geri Saçılma İçin Difüzyon Uzunluklarının P_N ve Modifiye U_N Metodu ile Hesaplanması

Ahmet BÜLBÜL[1]

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Özet

Nötron kaynağının olmadığı dilim geometride difüzyon uzunluğu hesaplamaları için Modifiye U_1 ve P_1 yaklaşımları kullanıldı. Yaklaşımlar ileri-geri saçılmalı tek hızlı nötron transport denklemine uygulandı. Analitik difüzyon uzunluğu denklemleri elde edildi ve farklı çarpışma parametreleri (c) için modifiye U_1 ve P_1 yaklaşımından elde edilen sayısal sonuçlar birbiri arasında kıyaslandı.

Anahtar kelimeler: U_1 yaklaşımı, Transport Denklemi, Difüzyon Uzunluğu, İleri- Geri Saçılma.

Calculation of Diffusion Lengths using P_N and Modified U_N Method for Forward and Backward Scattering

Abstract

Modified U_1 and P_1 approximations are used for the calculation of diffusion lengths in a slab geometry without neutron source case. The approximations are applied to one-speed neutron transport equation with forward and backward scattering. Analytical diffusion length equations are obtained and the numerical results obtained from modified U_1 and P_1 approximations for different collision parameters (c) are compared with each other.

Keywords: *U_1 Approximation, Transport Equation, Diffusion length, Forward-backward scattering*

[1]Assoc. Prof. Dr. Ahmet BÜLBÜL, Osmaniye Korkut Ata University, Electric and Energy Department, ahmetbulbul@osmaniye.edu.tr

1. Introduction

In nuclear reactor theory, neutron population has a major significance for the best designing a reactor. Neutrons maintain fission chain reactions in a nuclear power reactor, and they which move in complicated paths interact with all materials throughout the system. Since the neutrons repeated nuclear collisions and interact with reactor materials, designing a reactor became a hardcase. To overcome this problem, the scientist should predict neutron behaviors and its components in the system absolutely.

Neutron transport equation is used for the description of neutron behaviors in the system and also it is not easy to find an accurate solution for the transport problems. It can be said that diffusion approximation is a good solution for the prediction of neutron distributions in the system. This approximation is generally used for the prediction of the properties of nuclear reactors, for example, neutron transport and energy spectrum (Stacey, 2007), and mostly used for the first estimation of reactor properties such as size, buckling, and material composition. Neutron diffusion theory which is the simplest and most widely used method by scientist to describe the neutron distributions mathematically (Stacey). Diffusion theory was used to account for chemical diffusions such as gas molecules and neutral particles, also diffusion equation relating the current to the gradient of the neutron flux is based on Fick’s law (Lamarsh and Baratta, 2001).

Neutron transport problems have studied using various methods by many scientists. Among methods, the spherical harmonics method (P_N) is mostly used in lots of studies (Yildiz, 1998; Gulecyuz and Senyigit, 2018; Case,1967).The scientist also used different approximations to solve neutron transport problems, i.e. criticality, diffusion lengths, the efficiency of reflection coefficients and they studied Chebyshev polynomials in their studies (Anli, Yasa, Güngör and Öztürk, H, 2006; Öztürk, Bülbül, and Kara, 2006; Öztürk, Anli and Güngör, 2007; Bülbül, Ulutas, and Anli, 2008).

In the present study, Chebyshev polynomial approximation is used for the solution of diffusion equation. Recently, the scientist gives quite coherent results with the widely used P_N approximation using Chebyshev polynomials in literature. In these studies, Ozturk et al. (2007) calculated critical slab thicknesses for one-speed neutrons using different reflection coefficients and compared their results P_N method. Bulbul et al. (2008) applied the second kind of Chebyshev polynomials $U_N$to neutron transport equation in slab geometry including isotropic scattering and reflective boundary conditions and presented convenient results for the critical thicknesses. Also modified U_N approximation is applied to neutron transport equation and critical thickness results are obtained in slab geometry (Bülbül and Anli, 2009). In the present study, firstly neutron angular flux is expanded in terms of Chebyshev polynomials and then flux moment equations are obtained. Using Fick’s Law, diffusion equation is given and diffusion lengths are calculated for forward-backward scattering in slab geometry The results obtained from the polynomial approximations are presented in the tables for comparison and the exact values of diffusion lengths are given from Bell and Glasstone (1972).

2. Theory

The neutron transport equation for one-speed neutrons with forward and backward scattering and no sources are taken into account and is given in plane geometry as,

$$\mu\frac{\partial\psi(x,\mu)}{\partial x}+\sigma_T(1-\alpha c)\psi(x,\mu)=\frac{c\sigma_T}{2}(1-\alpha-\beta)\int_{-1}^{1}\psi(x,\mu')d\mu'+\beta c\sigma_T\psi(x,-\mu) \quad (1)$$

In Equation (1), $\psi(x,\mu)$ is the neutron angular flux at position x traveling in direction μ, cosine of the angle between the neutron velocity vector and the positive x axis, c is the number of secondary neutrons per collision and σ_T is the total macroscopic cross-section (Davison,1958).Forward and backward parameters are also given α, β which are the scattering probabilities in a collision.

In the present approximation, in order to obtain analytical equation of diffusion length the angular flux is described as a combination of the second kind Chebyshev polynomials and the scalar flux depend on x.

In order to solve Eq.(1), the neutronangular flux is described in terms of modified U_N as

$$\psi(x,\mu) = \frac{2}{\pi}\sum_{n=0}^{N} \Phi_n(x)\, U_n(\mu), \quad -1 \leq \mu \leq 1 \tag{2}$$

Firstly, the neutron angular flux $\psi(x,\mu)$ given in Eq. (2) is inserted into neutron transport equation given in Eq. (1). Then obtained equation is multiplied by second kind Chebyshev polynomial, $U_n(\mu)$, and integrated over $\mu \in [-1,1]$In resulting equation, if one used the orthogonality properties and the recurrence relations of the second kind of Chebyshev polynomials given below, neutron flux moment equations are obtained.

$$\int_{-1}^{1} U_n(\mu) U_m(\mu)\sqrt{1-\mu^2}\, d\mu = \begin{cases} \pi/2, & n = m \\ 0, & n \neq m \end{cases} \tag{3}$$

$$2\mu U_n(\mu) = U_{n+1}(\mu) + U_{n-1}(\mu), \quad -1 \leq \mu \leq 1 \tag{4}$$

One can obtain the modified U_N flux moment equation for n = 0 and n = 1 respectively as

$$\frac{d\Phi_1(x)}{dx} + 2\sigma_T[1 - c(\alpha+\beta)]\Phi_0(x) = 2c\sigma_T(1-\alpha-\beta)\sum_{n=0}^{N}\frac{\Phi_{2n}(x)}{2n+1} \tag{5}$$

$$\frac{d\Phi_2(x)}{dx} + \frac{d\Phi_0(x)}{dx} + 2\sigma_T[1 - c(\alpha-\beta)]\Phi_1(x) = 0 \tag{6}$$

Using Eqs. (5) and (6) which are modified U_1 flux moment equations of the present method diffusion length term is obtained. If one uses the condition for n = 1 and by setting , then resulting equation is inserted into Eq.(5), thesecond order-homogeneous differential equation is obtained. For the present study, a familiar equation known as Fick's law is obtained by taking $d\Phi_2/dx = 0$ in Eq. (6) as,

$$\Phi_1(x) = -\frac{1}{2\sigma_T[1-c(\alpha-\beta)]}\frac{d\Phi_0(x)}{dx}\,. \tag{7}$$

Eq. (7) is the description of the current and if Eq. (7) is inserted into Eq.(5), one could obtain the diffusion equation in the present method as,

$$\frac{d^2\Phi_0(x)}{dx^2} - 4{\sigma_T}^2(1-c)[1-c(\alpha-\beta)]\Phi_0(x) = 0 \tag{8}$$

Eq.(8) is the diffusion equation and it depends on c, α, β and σ_T. Eq.(8) gives the diffusion length (L) for modified U_1 approximation as,

$$L = \frac{1}{2\sigma_T\sqrt{(1-c)[1-c(\alpha-\beta)]}} \tag{9}$$

By following the same procedure for the P_N approximation, the diffusion equation which depends on c, α, β and σ_T can be obtained for the P_1 method as,

$$\frac{d^2\Phi_0(x)}{dx^2} - 3\sigma_T{}^2(1-c)[1-c(\alpha-\beta)]\Phi_0(x) = 0 \tag{10}$$

and from Eq.(10) the diffusion length is given as,

$$L = \frac{1}{\sigma_T\sqrt{3(1-c)[1-c(\alpha-\beta)]}} \tag{11}$$

These results are derived for one-speed neutrons with forward and backward scattering in slab geometry. The analytical equation of separation of variables (exact) method for diffusion length is not solved in this study because the results of the method for different collision values can be found in literature (Bell and Glasstone,1972) and the equation for the isotropic scattering is given as,

$$1 = c\,L\ \tanh^{-1}\frac{1}{L}. \tag{12}$$

3. Numerical Results and Discussion

Diffusion length equations and results are obtained for one-speed neutrons with forward-backward scattering using polynomial approximations in the present study. In section Theory, firstly, the conservation equation is described for neutrons in a reactor according to chosen geometry and scattering parameters and to solve the problemthe neutron angular flux is expanded in terms of the second kind of Chebyshev polynomials. The first and second terms of the series expansion of the angular flux are taken (n = 0 and n = 1) to obtain the flux moment equations, i.e. modified U_1 approximation. In this study, both equations are adequate to solve the problem for the first order approximation. Here, Fick's Law has an important description mentioned in Section Introduction and using the law, analytic expressions are given Eqs. (9) and (11) for both polynomial approximations. If one analyzes both equations, it can be seen that diffusion lengths depend on collision parameter c, forward and backward scattering parameters α and β and the total macroscopic cross-section is assumed to be its normalized value, $\sigma_T = 1$ cm^{-1}.

Diffusion lengths are calculated for the values of collision parameter c ranging from 0 (weakly absorbing medium) to 1 (highly scattering medium) and for the values of scattering parameters. In the case of isotropic scattering (α = 0.0 and β = 0.0), forward scattering (α = 0.3 and β = 0.0) and backward scattering (α = 0.0 and β = 0.3) are tabulated in Table1-3. The exact results are exist only for the isotropic scattering and one could get them from literature (Bell and Glasstone ,1972).

In Table 1, the results obtained from modified U_1 approximation, P_1 approximation, and from the exact method are given for the comparison, and can be seen the differences of values of all methods. In Table 2-3, the results are given for the forward and backward scattering respectively. All the results show that modified U_N approximation is convenient to solve the neutron transport equation. It can be said that the present method is an alternative technique from the similarity of the modified U_1 results with P_1 results for the diffusion length application.

Table 1. Diffusion lengths L obtained from modified U_1 and P_1 approximation for (α = 0.0 and β = 0.0) and comparison with literature values, (cm).

c	U_1(Eq.(9))Present Method	P_1(Eq.(11))	*Exact*(Eq.(12))
0.99	5.000000000	5.773502690	5.796729451
0.97	2.886751345	3.333333333	3.374031386
0.95	2.236067978	2.581988898	2.635148834
0.90	1.581138830	1.825741858	1.903204856
0.85	1.290994449	1.490711985	1.588558625
0.77	1.042572070	1.203858531	1.331415486
0.75	1.000000000	1.154700538	1.289463525
0.65	0.8451542545	0.9759000726	1.146703049
0.50	0.7071067810	0.8164965812	1.044382034
0.00	0.5000000000	0.5773502690	1.000000000

In Tables 1-3, if the values of c decrease, the diffusion length decreases. On the other hand, one can see that the results of diffusion approximation for the forward scattering are higher than the isotropic and backward scattering, respectively. Diffusion approximation is a good method and gives convenient results for the transport problem near the c =1. Consequently, it can be said that Chebyshev polynomial approximation can be applied to other problems especially particle transfer problems in science and engineering.

Table 2. Diffusion lengths L obtained from modified U_1 approximation and P_1 approximation for α = 0.3 and β = 0.0 (cm).

c	U_1(Eq.(9))Present Method	P_1(Eq.(11))
0.99	5.963378040	6.885915834
0.97	3.428358736	3.958727680
0.95	2.644429427	3.053524083
0.90	1.850583026	2.136869216
0.85	1.495706009	1.727092534
0.77	1.188893401	1.372815850
0.75	1.135923668	1.311651672
0.65	0.9419721050	1.087695697
0.50	0.7669649890	0.8856148856
0.00	0.5000000000	0.5773502690

Table 3. Diffusion lengths L obtained from modified U_1 approximation and P_1 approximation for α = 0.0 and β = 0.3 (cm).

c	U_1(Eq.(9))Present Method	P_1(Eq.(11))
0.99	4.390358822	5.069549695
0.97	2.540658279	2.933699482
0.95	1.972574608	2.277732962
0.90	1.403033834	1.620083922
0.85	1.152398042	1.330674640
0.77	0.9396736740	1.085041697
0.75	0.9035079030	1.043281062
0.65	0.7731291165	0.8927326075
0.50	0.6593804735	0.7613869874
0.00	0.5000000000	0.5773502690

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