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Examination of Stability Analysis of Sakarya and Turkey Scale Alcohol Use Model

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Abstract

This paper is devoted to studying the mathematical model of the alcohol-consuming population. For this purpose, the formulation of the model including the alcohol-consuming population is presented; then the balance points related to non-alcohol use and positive alcohol use are discussed. Hurwitz theorem is used to find the local stability of the model, and Lyapunov function theory is used to investigate the global stability. The same mathematical model with alcohol use is considered for Sakarya and Turkey, individual numerical results are presented, and stability analyzes are examined. Finally, using the numerical data, a simulation is made in Matlab with the Runge-Kutta fourth-order method.

Keywords: alcohol use model, differential equation system, stability analysis, equilibrium model of the mathematical model

1. INTRODUCTION

The mathematical model is a system model, and the terms are conjecture. Mathematical models are used in nature, fundamentals, engineering, and social sciences. Mathematical biology is an interdisciplinary branch in which mathematics is applied to the branch of biology, medicine, and biotechnology. This field is seen as the fastest growing and productive, and enlightening branch of mathematics in today's world. In mathematical biology, Daniel Bernoulli used mathematical tools to describe the impact of smallpox on the human population in the 18th century [1]. Likewise, we use mathematical biology to understand diseases and develop new perspectives for treatment. On an individual scale, it is possible to predict how the patient's immune system might respond to a virus and thus determine the course of treatment. On a larger scale, new methods can be developed to control epidemic diseases such as Ebola. In 1909, Brownlee took responsibility for the development of mathematical biology [1]. In 1912, He presented basic laws for epidemic spreading [2]. Details of epidemic studies were discussed by Kermark and McKendrick in 1927 [3]. Then they discussed different models of many diseases [4-17]. After all these researches, people's health deteriorated, and they became ill due to harmful habits such as alcohol, one of the social habits that spread rapidly to the world like a contagious disease.

Alcohol use causes significant health problems in society. Alcohol use is the main cause of some fatal diseases. Alcohol consumption negatively affects the social and economic lives of societies. The most important reason for the occurrence of

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violence and unconscious movements in society is alcohol. Some studies have shown that alcohol use starts at a young age. It is seen that the use as an infectious disease spreads in society. In this study, we created the mathematical model of alcohol use considering the SIR model of epidemic diseases [18-24].

We divided the population who use alcohol in society into five classes as follows:

X(t): Susceptible population,

$H_1(t)$: Alcohol drinkers class,

H_2(t): Irregular alcohol drinkers class,

Y(t): Regular alcohol drinkers class,

Z(t): Alcoholic quitters class.

Here 't' denotes time change. We formalized the alcohol use model according to the assumptions that we obtained. We applied the stability analysis of the model that we created. We examined the local stability of this model using the Hurwitz theorem and discussed its global stability with the help of the Lyapunov function.

2. MATHEMATICAL MODEL

2.1. Formation of the Model

We divided the community of the alcohol users class into five classes as X(t), $H_1(t)$, $H_2(t)$,Y(t), Z(t). Moreover, let λ, β_1, β_2, ω, γ, μ, α, ρ, d, respectively, denote the recruitment rate (birth ve migration), rate of conversion of sensitive populations to alcohol drinkers, rate at which alcohol drinkers become irregular alcohol drinkers, rate of which irregular alcohol drinkers become regular alcohol drinkers, quitting rate, natural death rate, relapse rate, the death rate of alcohol drinkers class due to alcohol use, death due to alcohol drinkers related diseases, then our model has been constructed as follows:

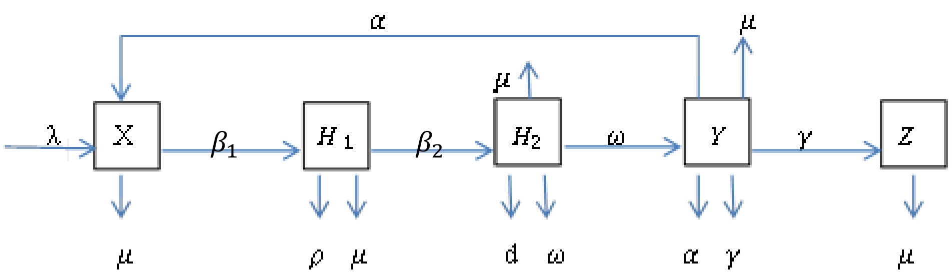


Figure 1 Schematic Diagram of Alcohol Use Pattern

Thus the differential equation system depending on our model has been given by

$$\frac{dX}{dt} = \lambda - \beta_1 X H_1 - \mu X + \alpha Y,$$

$$\frac{dH_1}{dt} = \beta_1 X H_1 - \beta_2 H_1 H_2 - (\rho + \mu) H_1,$$

$$\frac{dH_2}{dt} = \beta_2 H_1 H_2 - (d + \omega + \mu) H_2,$$

$$\frac{dY}{dt} = \omega H_2 - (\alpha + \gamma + \mu) Y,$$

$$\frac{dZ}{dt} = \gamma Y - \mu Z. \qquad (1)$$

In this system, we take the first four equations. We do not take Z(t) because it does not affect the generality. Our new model is converted to the following system:

$$\frac{dX}{dt} = \lambda - \beta_1 X H_1 - \mu X + \alpha Y,$$

$$\frac{dH_1}{dt} = \beta_1 X H_1 - \beta_2 H_1 H_2 - (\rho + \mu) H_1,$$

$$\frac{dH_2}{dt} = \beta_2 H_1 H_2 - (d + \omega + \mu) H_2,$$

$$\frac{dY}{dt} = \omega H_2 - (\alpha + \gamma + \mu) Y. \quad (2)$$

3. EQUILIBRIUM POINTS OF THE SYSTEM

3.1. No Alcohol Equilibrium Points

Let $H_1 = H_2 = Y = 0$, the balance point is obtained as $E_0 = (\frac{\lambda}{\mu}, 0,0,0)$. The Jacobian of the system (2) is found as

$$J=\begin{bmatrix} -\beta_1 H_1 - \mu & -\beta_1 X & 0 & \alpha \\ \beta_1 H_1 & \beta_1 X - \beta_2 H_2 - \rho + \mu & -\beta_2 H_1 & 0 \\ 0 & \beta_2 H_2 & \beta_2 H_1 - (d+\omega+\mu) & 0 \\ 0 & 0 & \omega & -(\alpha+\gamma+\mu) \end{bmatrix}.$$

So by considering the balance point in the last Jacobian matrix, we get

$$J(E_0) = \begin{bmatrix} -\mu & -\frac{\beta_1\lambda}{\mu} & 0 & \alpha \\ 0 & \frac{\beta_1\lambda}{\mu} - (\rho+\mu) & 0 & 0 \\ 0 & 0 & -(d+\omega+\mu) & 0 \\ 0 & 0 & \omega & -(\alpha+\gamma+\mu) \end{bmatrix}.$$

Concerning the Hurwitz theorem, we find the matrices of F and V as

$$F = \begin{bmatrix} \beta_1 \frac{\lambda}{\mu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and

$$V=\begin{bmatrix} (\rho+\mu) & 0 & 0 \\ 0 & (d+\omega+\mu) & 0 \\ 0 & -\omega & (\alpha+\gamma+\mu) \end{bmatrix}.$$

The dominant eigenvalue of $F.V^{-1}$ is $\frac{\beta_1\lambda}{\mu(\rho+\mu)}$,

So, $R_0 = \frac{\beta_1\lambda}{\mu(\rho+\mu)}$ is the value of the required reproductive number defined in [12].

3.2. Alcohol Present Equilibrium Points

Theorem 3.1. If $R_0 > 1$, then the drinking alcohol has a positive equilibrium point.

Proof. Let $R_0 > 1$. Then to determine the equilibrium E^*, we assume the left side of the system (2) is equal to zero.

The third equation of the system (2) implies that

$$H_1^* = \frac{(d+\omega+\mu)}{\beta_2}.$$

From the second equation of the system (2), we have

$$X^* = \frac{\beta_2 H_2^* + (\rho+\mu)}{\beta_1}.$$

Also, the fourth equation implies that

$$Y^* = \frac{\omega H_2^*}{(\alpha+\gamma+\mu)}.$$

Similarly, the first equation reveals that

$$H_2^* = \frac{(\alpha+\gamma+\mu)(\rho+\mu)[\beta_2\mu(R_0-1) - \beta_1(d+\omega+\mu)]}{(\gamma+\mu)(\beta_1\beta_2\omega) + (\alpha+\gamma+\mu)(\beta_1\beta_2(d+\mu) + \beta_2^2\mu)}.$$

We have $\beta_2\mu(R_0 - 1) > \beta_1(d+\omega+\mu))$ since $R_0 > 1$. Thus, H_2^* is positive. So, the required positive equilibrium point $E^*$is found as

$$E^*(X^*, H_1^*, H_2^*, Y^*) = \left(\frac{\beta_2 H_2^* + (\rho+\mu)}{\beta_1}, \frac{(d+\omega+\mu)}{\beta_2}, \frac{\omega H_2^*}{(\alpha+\gamma+\mu)}, \frac{(\alpha+\gamma+\mu)\,(\rho+\mu)[\beta_2\mu\,(R_0-1)-\beta_1(d+\omega+\mu)]}{(\boldsymbol{\gamma}+\boldsymbol{\mu})(\boldsymbol{\beta_1\beta_2\omega})+(\alpha+\gamma+\mu)\left(\beta_1\beta_2(d+\mu)+\beta_2^2\mu\right)}\right).$$

4. STABILITY OF THE MODEL

4.1. Local Stability

Theorem 4.1. If $R_0 < 1$, then the system (2) is locally stable, and if $R_0 > 1$, then system (2) is unstable.

Proof. For the local stability at E_0, the Jacobian matrix of the system (2) is given by

$$J(E_0) = \begin{bmatrix} -\mu & -\frac{\beta_1\lambda}{\mu} & 0 & \alpha \\ 0 & \frac{\beta_1\lambda}{\mu} - (\rho+\mu) & 0 & 0 \\ 0 & 0 & -(d+\omega+\mu) & 0 \\ 0 & 0 & \omega & -(\alpha+\gamma+\mu) \end{bmatrix}.$$

The eigenvalues λ_1, λ_2, λ_3 and λ_4 of $J(E_0)$ are obtained as

$\lambda_1 = -\mu < 0,$

$\lambda_2 = (\rho+\mu)\,(R_0 - 1) < 0,$

$\lambda_3 = -(d+\omega+\mu) < 0,$

$\lambda_4 = -(\alpha+\gamma+\mu) < 0.$

Assuming $R_0 < 1$ implies that that $\lambda_2 < 0$. On the other hand if $R_0 = 1$ then $\lambda_2 = 0$. Finally, if $R_0 > 1$, then $\lambda_2 > 0$.

Theorem 4.2. If $R_0 > \frac{\beta_2\lambda}{(d+\omega+\mu)(\rho+\mu)}$, then the system (2) is locally stable at E^*, otherwise unstable.

Proof. For the local stability at E^*, the Jacobian matrix of the system (2) becomes

$$J(E^*) = \begin{bmatrix} -\beta_1 H_1^* - \mu & -\beta_1 X^* & 0 & \alpha \\ \beta_1 H_1^* & \beta_1 X^* - \beta_2 H_2^* - \rho + \mu & -\beta_2 H_1^* & 0 \\ 0 & \beta_2 H_2^* & \beta_2 H_1^* - (d+\omega+\mu) & 0 \\ 0 & 0 & \omega & -(\alpha+\gamma+\mu) \end{bmatrix}.$$

Under the assumption, we get

$$J(E^*) = \begin{bmatrix} -\beta_1 H_1^* - \mu & -\beta_1 X^* & 0 & \alpha \\ \beta_1 H_1^* & 0 & -\beta_2 H_1^* & 0 \\ 0 & \beta_2 H_2^* & 0 & 0 \\ 0 & 0 & \omega & -(\alpha+\gamma+\mu) \end{bmatrix},$$

$$= \begin{bmatrix} -\beta_1 H_1^* - \mu & -\beta_1 X^* & 0 & \alpha \\ -\mu & -\beta_1 X^* & -\beta_2 H_1^* & \alpha \\ \beta_1 H_1^* & \beta_2 H_2^* & -\beta_2 H_1^* & 0 \\ 0 & 0 & \omega & -(\alpha+\gamma+\mu) \end{bmatrix},$$

$$= \begin{bmatrix} -\beta_1 H_1^* - \mu & -\beta_1 X^* & 0 & \alpha \\ 0 & -\beta_1 X^* + \frac{\mu\beta_2{}' H_2^*}{\beta_1{}' H_1^*} & -\beta_2 H_1^* - \frac{\mu\beta_2 H_1^*}{\beta_1 H_1^*} & \alpha \\ \beta_1 H_1^* & \beta_2 H_2^* & -\beta_2 H_1^* & 0 \\ 0 & 0 & \omega & -(\alpha+\gamma+\mu) \end{bmatrix},$$

$$J(E^*) = \begin{bmatrix} -\beta_1 H_1^* - \mu & -\beta_1 X^* & \frac{\omega\alpha}{(\alpha+\gamma+\mu)} & 0 \\ 0 & -\beta_1 X^* + \frac{\mu\beta_2 H_2^*}{\beta_1 H_1^*} & -\beta_2 H_1^* - \frac{\mu\beta_2 H_1^*}{\beta_1 H_1^*} + \frac{\omega\alpha}{(\alpha+\gamma+\mu)} & 0 \\ \beta_1 H_1^* & \beta_2 H_2^* & -\beta_2 H_1^* & 0 \\ 0 & 0 & \omega & -(\alpha+\gamma+\mu) \end{bmatrix}.$$

For the sake of simplicity, this matrix can be written as

$$J(E^*) = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$$

where

$$A = \begin{pmatrix} -\beta_1 H_1^* - \mu & -\beta_1 X^* \\ 0 & -\beta_1 X^* + \frac{\mu\beta_2 H_2^*}{\beta_1 H_1^*} \end{pmatrix},$$

$$B = \begin{pmatrix} \frac{\omega\alpha}{(\alpha+\gamma+\mu)} & 0 \\ -\beta_2 H_1^* - \frac{\mu\beta_2 H_1^*}{\beta_1 H_1^*} + \frac{\omega\alpha}{(\alpha+\gamma+\mu)} & 0 \end{pmatrix},$$

$$C = \begin{pmatrix} \beta_1 H_1^* & \beta_2 H_2^* \\ 0 & 0 \end{pmatrix},$$

$$D = \begin{pmatrix} -\beta_2 H_1^* & 0 \\ \omega & -(\alpha+\gamma+\mu) \end{pmatrix}.$$

Since the eigenvalues of $J(E^*)$ depend on the eigenvalues of A and D, the eigenvalues of A are given as follows:

$$\lambda_1 = -\beta_1 H_1^* - \mu < 0,$$

$$\lambda_2 = -(\rho+\mu) + \frac{\mu\beta_2 H_2^*}{\beta_1 H_1^* \lambda}\left(\lambda - H_1^*(\rho+\mu)R_0\right).$$

If $R_0 > \frac{\beta_2\lambda}{(d+\omega+\mu)(\rho+\mu)}$, then $\lambda_2 < 0$.

Moreover, the eigenvalues of D are

$$\lambda_3 = -\beta_2 H_1^* < 0,$$

$$\lambda_4 = -(\alpha+\gamma+\mu) < 0.$$

This completes the proof.

4.2. Global Stability

Theorem 4.3. If $R_0 < 1$, then the system (2) is globally stable.

Proof. First, we construct the Lyapunov function L as

$$L = \ln\frac{X}{X_0} + \ln\frac{H_1}{H_{1_0}} + H_2 + Y. \quad (4)$$

Differentiating Eq. (4) with respect to time, we get

$$L' = \frac{\lambda}{X} - \beta_1 H_1 + \frac{\alpha Y}{X} - \mu + \beta_1 X - \beta_2 H_2 - (\rho+\mu) + \beta_1 H_1 H_2 - (d+\omega+\mu)H_2 + \omega H_2 - (\alpha+\gamma+\mu)Y.$$

Using the values E_0 in the above equation, we find

$$L' = \mu - \mu + \frac{\beta_1\lambda}{\mu} - (\rho+\mu).$$

By successive calculation, we obtain that

$$L' = R_0(\rho+\mu) - (\rho+\mu),$$

$$L' = (\rho+\mu)(R_0 - 1).$$

Therefore, if $R_0 < 1$, then $L' < 0$, which implies that the system (2) is globally stable.

5. NUMERICAL RESULTS

Table 1 Parameters of the model

Parameter	Definition	Rates in Turkey	Rates in Sakarya
λ	Recruitment rate (birth ve migration)	0,0022	0,0018
β_1	Rate of conversion of sensitive populations to alcohol drinkers	0,003	0,003
β_2	Rate at which alcohol drinkers become irregular alcohol drinkers	0,004	0,004
ω	Rate of which irregular alcohol drinkers become regular alcohol drinkers	0,002	0,002
γ	Quitting rate	0,086	0,084
μ	Naturel death rate	0,005	0,006
α	Relapse rate	0,002	0,002
ρ	The death rate of alcohol drinkers class due to alcohol use	0,003	0,003
d	Death due to alcohol drinkers related diseases	0,003	0,003

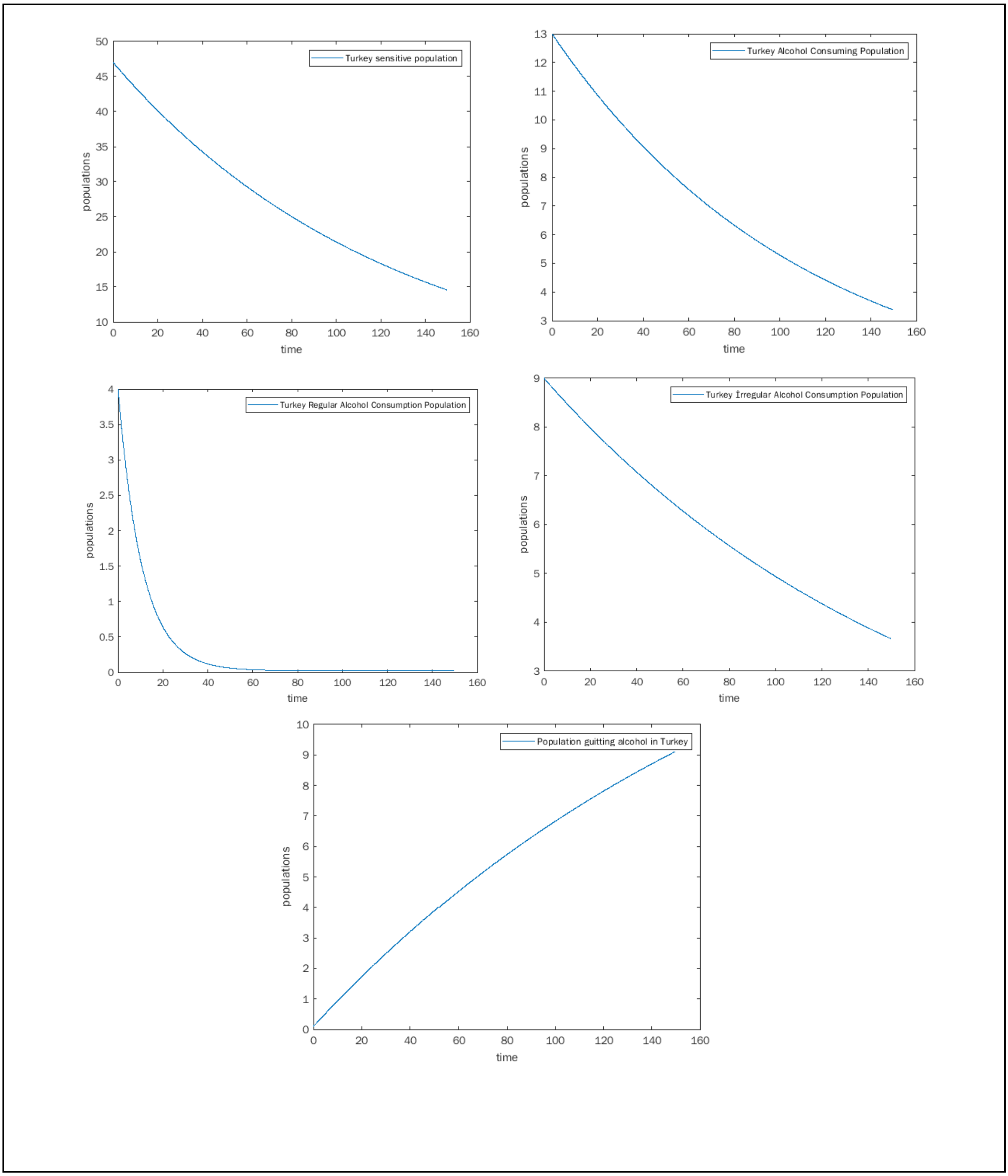
Table 2 The classes of the population who use alcohol

Classes	Explanations	Turkey data	Sakarya data
$X(t)$	Sensitive Alcohol Drinkers	47.236.000 (%78)	420.550 (%65)
$H_1(t)$	Alcohol Consumers	13.324.075 (%22)	226.450 (%35)
$H_2(t)$	Irregular Alcohol Consumers	9.145.435 (%15.1)	163.044 (%25,2)
Y(t)	Regular Alcohol Consumers	4.178.640 (%6,9)	63.406 (%9,8)
Z(t)	Those Who Quit Alcohol	111.914 (%8,4)	19.021 (%8,4)

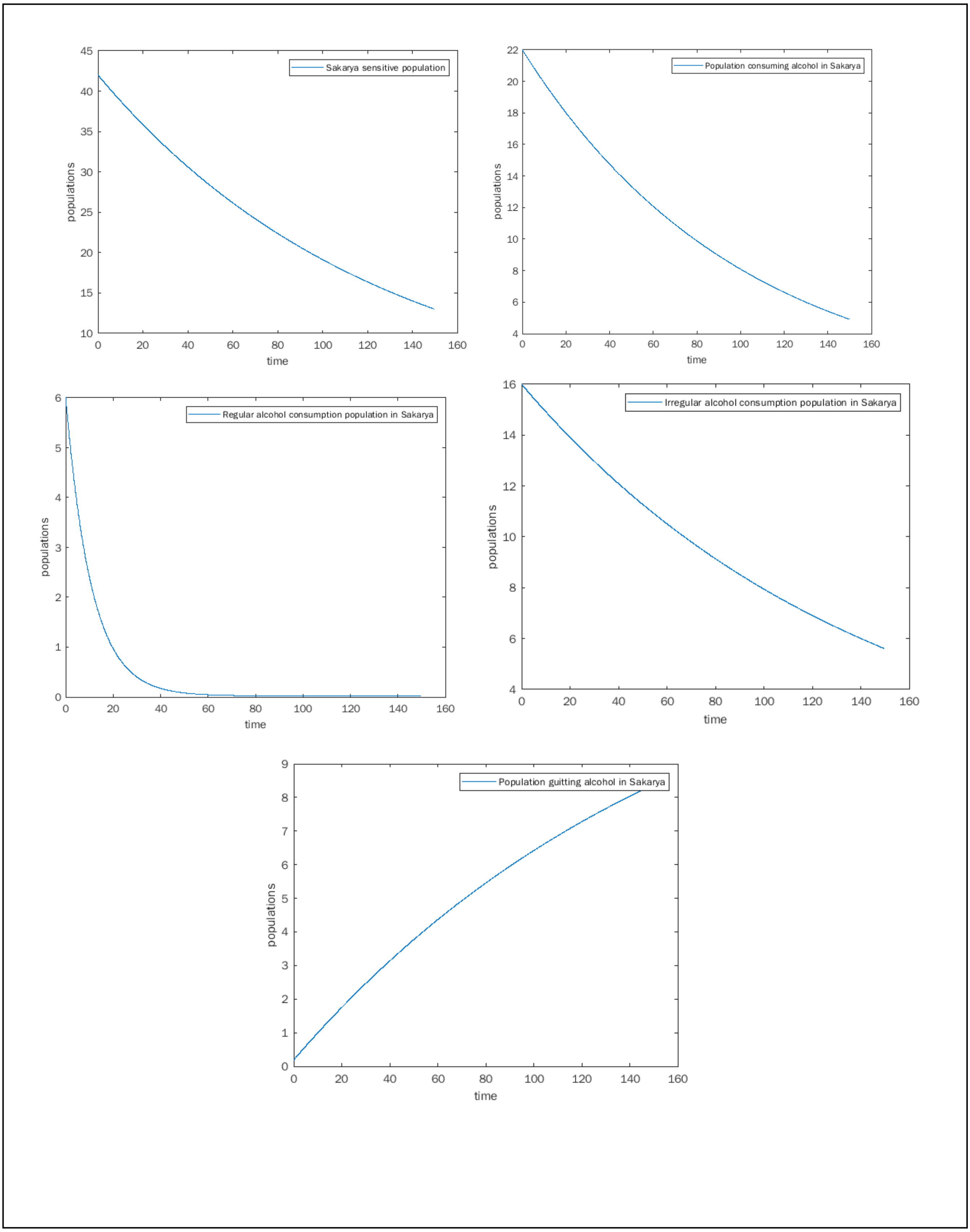
Turkey's population in 2019 is approximately 83 million and over 18 years old is 60 million 560 thousand 75.

The population of Sakarya in 2019 is approximately 1 million 45 thousand and over the age of 18 is 647 thousand [25-28]

Graphics 1. MatLab simulations of the Turkey alcohol use model (system 1), that are plotted according to the approximate parameters given in Table 1 and the numerical data presented in Table 2. They are scaled to 1/1.000.000.



Graphics 2 MatLab simulations of the Sakarya alcohol use model (system 1), that are plotted according to the approximate parameters given in Table 1 and the numerical data presented in Table 2. They are scaled to 1/1.000.000.



6. CONCLUSION AND DISCUSSION

The ratio R_0 of a mathematical model gives us information about the increase or decrease of an incident. R_0 consists of the parameters of the model. In our model, if R_0 =1, the prevalence of alcohol use is stable. It never disappears. If R_0>1, the prevalence of alcohol use increases, and if R_0 < 1, the prevalence of alcohol use decreases over time.

6.1. Non-Alcohol Equilibrium Points

Since R_0= 0.165 < 1 for Turkey, the balance point is found as $(E_0) =$ (0,44 ; 0; 0; 0). This means that the social spread of alcohol use is decreasing. Alcohol use will decrease over time.

Similarly, since R_0= 0.1 < 1 for Turkey, the balance point is found as $(E_0) =$ (0,3 ; 0; 0; 0). This can be interpreted that the social spread of alcohol use is decreasing. Alcohol use will decrease over time.

6.2. Alcohol Use Equilibrium Points

If R_0> 1, the equilibrium point is positive. Since we found R_0< 1 for Turkey, alcohol consumption does not have a positive equilibrium point. Since we found R_0< 1 for Sakarya, alcohol consumption does not have a positive equilibrium point, too.

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The Declaration of Conflict of Interest/ Common Interest

No conflict of interest or common interest has been declared by the authors.

Authors' Contribution

The first author contributed 70%, and the second author contributed 30%.

The Declaration of Ethics Committee Approval

This study does not require ethics committee approval or any special permission.

The Declaration of Research and Publication Ethics

I declare that I have complied with the scientific, ethical, and attribution rules of SAUJS in all processes of the article and have not made any falsifications on the data collected. I accept that I am not responsible for ethical violations that may be encountered, and this study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

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