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Research Article

Investigation of Electromagnetic Wave Propagation Frequencies in Two-Dimensional Photonic Crystals with Finite Differences Method

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Abstract

In this study, frequencies of electromagnetic wave propagation in two-dimensional photonic crystal structures are investigated. For this purpose, the electromagnetic wave propagation equation obtained by using Maxwell equations is solved with the finite differences method. The fundamental frequencies of electromagnetic wave propagation have been obtained with a low error rate of 3.10%.

Keywords: Electromagnetic wave propagation, Photonic crystals, Maxwell equations, Finite differences method.

İki Boyutlu Fotonik Kristallerde Elektromanyetik Dalga Yayılımı Frekanslarının Sonlu Farklar Yöntemiyle İncelenmesi

Öz

Bu çalışmada, iki boyutlu fotonik kristal yapılarda elektromanyetik dalga yayılımı frekansları incelenmektedir. Bu amaçla, Maxwell denklemleri kullanılarak elde edilen elektromanyetik dalga yayılımı denklemi sonlu farklar yöntemi ile çözülmektedir. Elektromanyetik dalga yayılımı temel frekansları %3,10 gibi düşük bir hata oranıyla elde edilmiştir.

Anahtar Kelimeler: Elektromanyetik dalga yayılımı, Fotonik kristaller, Maxwell denklemleri, Sonlu farklar yöntemi.

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1. Introduction

In recent years, developments in the fields of optics and telecommunications have gradually increased the need for structures such as photonic crystals and metamaterials (Pendry, 2000; Shalaev ve ark., 2005; Slyusar, 2009; Fathallah et al., 2014; Soltani et al., 2017). Structures such as photonic crystals or metamaterials are produced by positioning various structures with different or same material properties as periodic, non-periodic, sequential or layered. Each part of these structures with different material properties has a different effect on the electromagnetic wave propagation occurring in the structures. Dielectric constants (ε) and magnetic permeability constants (μ) of the structures are among the material properties whose effects on the electromagnetic wave propagation are examined. The frequencies of electromagnetic wave propagation in structures such as photonic crystals and metamaterials are determined as a result of the individual examination of the electromagnetic wave propagation behavior of each part of the structures. There are many studies in the literature that examine electromagnetic wave propagation frequencies (Busch et al., 2007; Itin 2010; Yamashita, 2011; Martellesio et al., 2015; Inampudi et al., 2016; Gu and He, 2020; Basmaci, 2020; Huang, 2021). It is possible to absorb electromagnetic waves in structures whose each layer having different material properties. (Sreekanthe et al., 2012; Watts et al., 2012; Basmacı and Filiz, 2021; Kumar et al., 2021). Furthermore, there are many studies on the production and experimental investigation of structures that allow the absorption of electromagnetic waves are also included in the literature (Micheli et al., 2015; Takayama et al., 2018; Faruque et al., 2020; Mabhouti et al., 2021).

Maxwell equations are used to theoretically examine the frequencies of electromagnetic wave propagation in two-dimensional structures, each point of which has different material properties. The partial differential equation of electromagnetic wave propagation obtained using Maxwell equations can be solved by various numerical methods. There are many studies in the literature in which the solution is performed using the finite difference method (Liu & Zhong, 2013; Chapra & Canale, 2015; Jia et al., 2018). In these studies, the finite difference method is preferred for the solution since the structures whose electromagnetic wave propagation behavior is examined are two-dimensional and each point of the structures has different material properties. Moreover, the frequencies of electromagnetic wave propagation in optical structures are examined using the finite difference method by drilling small circular holes or additions to the structures (Alipour-Banaei et al., 2017; Dideban et al., 2017; Xia et al., 2017).

In this study, frequencies of the electromagnetic wave propagation in two-dimensional plates are examined theoretically. These two plates differ from each other in terms of the distribution of material property parameters (ε, μ). The nodes in the center of the first plate have larger material property parameters than the nodes in their walls, while the nodes in the center of the second plate have smaller material property parameters than the nodes in their walls. In order to obtain electromagnetic wave propagation frequencies, the finite differences method is used.

2. Theoretical Analysis

Fundamental frequencies of the electromagnetic wave propagation in a two-dimensional plate having larger material parameters defined as $\left(\frac{1}{\mu\varepsilon}\right)$ in its center, and fundamental frequencies of electromagnetic wave propagation in a two-dimensional plate having smaller material parameters in its center are examined theoretically. For this purpose, the solution of the electromagnetic wave propagation equation of the two-dimensional plates obtained by using Maxwell equations is investigated by using the finite differences method. Figure 1a illustrates a plate with greater material parameters in its center compared to its walls, while Figure 1b depicts a plate with smaller material parameters in its center compared to its walls.

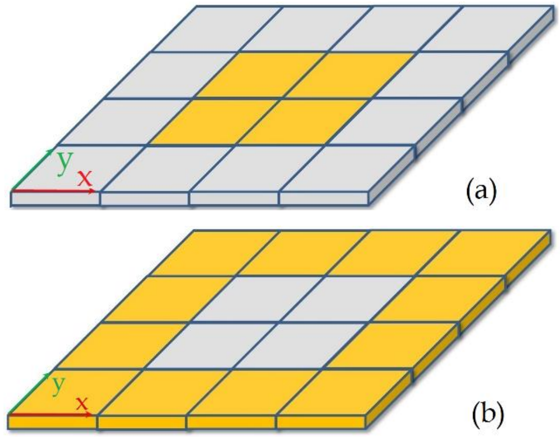

Figure 1. (a) A plate with greater material parameters in its center compared to its walls (S1 plate), (b) A plate with smaller material parameters in its center compared to its walls (S2 plate)

Maxwell equations used in forming the electromagnetic wave propagation equation related to the plates are as follows (Pozar, 2012):

$$\nabla \times \vec{E} = -if\mu\vec{H} \tag{1a}$$

$$\nabla \times \vec{H} = -if\varepsilon\vec{E} \tag{1b}$$

where, μ is the magnetic permeability constant, ε is the dielectric constant, f is the electromagnetic wave frequencies, $\bar{E}$ is the electric field, $i = \sqrt{-1}$, and $\bar{H}$ is the magnetic field.

Solving both Equation (1a) and Equation (1b) together, the electromagnetic wave propagation equation is obtained as follows:

$$\frac{\partial^2 H_x}{\partial x^2} + \frac{\partial^2 H_y}{\partial y^2} - \mu\varepsilon\frac{\partial^2 H}{\partial t^2} = 0 \tag{2}$$

By applying the displacement function given as $H(x, y, t) = w(x, y)\, e^{-ift}$ to Equation (2), the solution of the electromagnetic wave propagation equation is obtained as follows:

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \mu\varepsilon f^2 w = 0 \tag{3}$$

where, w represents the amplitude of the travelling electromagnetic wave. By creating material parameters separately for the x and y axes in Equation (3), Equation (4) is obtained as follows:

$$A(x)\frac{\partial^2 w}{\partial x^2}+B(y)\frac{\partial^2 w}{\partial y^2}+f^2 w=0 \quad (4)$$

where, *A(x)* and *B(y)* represents the material parameter of the two-dimensional plate along the *x*-axis and the material parameter of the two-dimensional plate along the *y*-axis, respectively. In this study, unlike other studies in the literature the material parameters of the plate are taken into account considering that the plate is two-dimensional.

In order to solve Equation (4) with the central finite differences method, it should be arranged as follows:

$$(x)\left[\frac{-w_{m-2,n}+16w_{m-1,n}-30w_{m,n}+16w_{m+1,n}-w_{m+2,n}}{12\Delta x^2}\right]+$$
$$B(y)\left[\frac{-w_{m,n-2}+16w_{m,n-1}-30w_{m,n}+16w_{m,n+1}-w_{m,n+2}}{12\Delta y^2}\right]+f^2 w_{m,n}=0 \quad (5)$$

The error rate of the derivative expressions in Equation (5) is of the order of $O(h^4)$ [17]. This equation is solved by defining the each node' material parameters of the two-dimensional plate separately. This approach makes it possible to determine the electromagnetic wave propagation frequencies of plates, each node having different material parameters. The two-dimensional plate given as the nodes *(m, n)* and the step lengths *(Δx, Δy)* between the nodes, has a total of 4 nodes as shown in Figure 2.

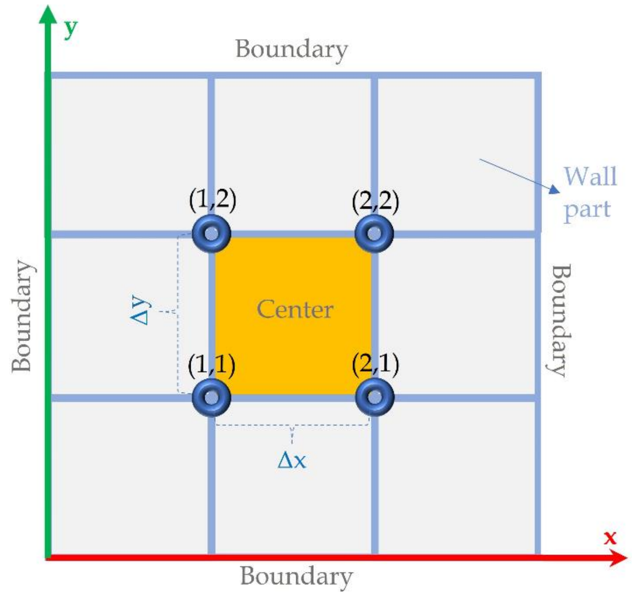

Figure 2. The nodes of the two-dimensional plate

In order to determine the electromagnetic wave propagation frequencies of the plate, it is necessary to determine the boundary conditions. The electromagnetic interaction is not observed in the parts and boundaries of the plate except for the nodes locating near the center. The nodes of the plate having electromagnetic interaction between them are *(1, 1)*, *(2, 1)*, *(1, 2)*, and *(m, n)*, respectively. These nodes are used to determine the frequencies of the electromagnetic wave propagation in plates by solving Equation (5).

K(f) is an eigenvector, and it is arranged due to all the specified nodes given as *(1, 1)*, *(2, 1)*, *(1, 2)*, and *(m, n)*, respectively to obtain the following equation:

$$K(f)=\left|\begin{bmatrix} a(f) & d & 0 \\ d & b(f) & \ddots \\ 0 & \ddots & \ddots \end{bmatrix}\right|_{mxn} \quad (6)$$

where, *a(f)* and *b(f)* are frequency-dependent terms, and *d* is the non-frequency-dependent term. By equating the determinant of the matrix given in Equation (6) to zero, the *f* frequencies related to the electromagnetic wave propagation are obtained.

In this study, the frequencies of electromagnetic wave propagation in two plates with different material parameters are investigated. The first of these plates, the S1 plate, has greater material parameters in its center compared to its other parts. The second of these plates, the S2 plate, has greater material parameters on its walls compared to its other parts. Moreover, both plates include 9 nodes.

3. Results and Discussion

Mode 1 (fundamental) frequencies regarding the electromagnetic wave propagation in the S1 plate are given in Table 1a. The values in the table have been calculated for five different cases obtained by increasing the material parameters of the plate from 1 to 5 in the center of the plate. Mode 1 frequencies of electromagnetic wave propagation in the S2 plate are given in Table 1b. The values in the table have been calculated for five different cases created by increasing the material parameters on the walls of the plate from 1 to 5. The values in Table 1 have been obtained by the central finite differences method with the error rate having order of $O(h^4)$. In addition, the Mode 1 frequencies in the table have been obtained with an error rate of 3.10%. This error rate shows that the accuracy of the Mode 1 frequencies is sufficient for the use of the central finite differences method. It should be noted that Mode 1 frequencies increase by increasing the material parameters.

Table 1. Fundamental electromagnetic wave propagation frequencies for (a) the S1 plate, (b) the S2 plate

(a)	
Ratio of Material Parameters of S1 plate (In the center /At the walls)	**Mode 1 frequencies (rad/s)**
1/1	*4.580*
2/1	*4.871*
3/1	*4.961*
4/1	*5.004*
5/1	*5.030*
(b)	
Ratio of Material Parameters of S2 plate (In the center /At the walls)	**Mode 1 frequencies (rad/s)**
1/1	*4.580*
1/2	*5.662*
1/3	*6.126*
1/4	*6.365*
1/5	*6.506*

Mode 1 frequencies of electromagnetic wave propagation are affected more by the increase in material parameters of the center of the plate than by the increase in its walls. The fact that the material parameters of the plate are at the center or walls of the plate has an important effect on Mod 1 frequencies, at least as well as the material parameters. As shown in Figure 3 and Figure 4, electromagnetic field distribution maps have been obtained with ANSYS Lumerical program for Mode 1 frequencies by increasing the material parameters of S1 and S2 plates from 1 to 5.

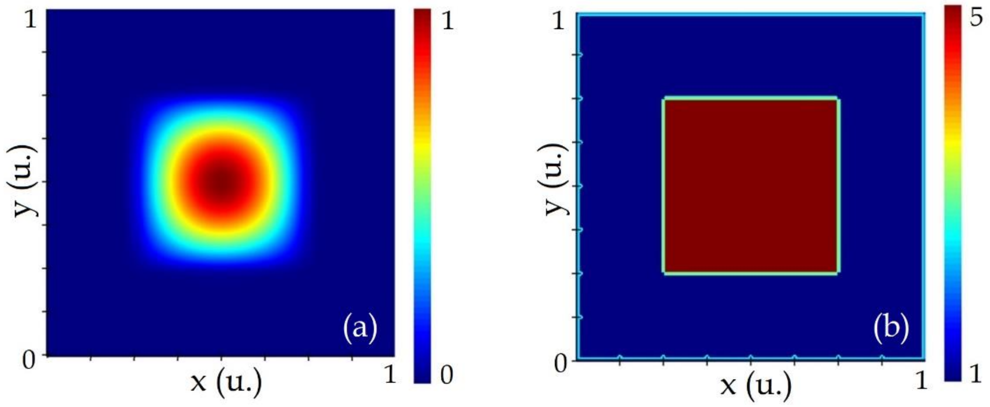

Figure 3. (a) Electromagnetic field distribution map of the S1 plate, (b) the material parameter variation of the S1 plate

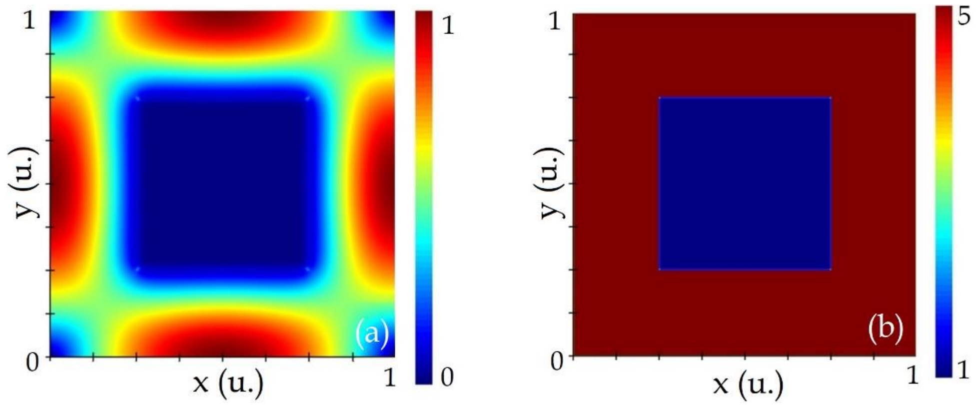

Figure 4. (a) Electromagnetic field distribution map of the S2 plate, (b) the material parameter variation of the S2 plate

4. Conclusions

In this study, the fundamental frequencies of electromagnetic wave propagation in a two-dimensional plate with larger material parameters in its center compared to its other parts, and the fundamental frequencies of electromagnetic wave propagation in another two-dimensional plate with smaller material parameters in its center compared to its other parts are obtained. For this purpose, the electromagnetic wave propagation equation obtained by Maxwell equations is solved by using the finite differences method. As a result, fundamental frequencies of electromagnetic wave propagation have been obtained with an error rate of 3.10%. The error rate of 3.10% has shown that the finite difference method is suitable for the solution. It has been concluded that the electromagnetic wave propagation frequencies are affected by the material parameters of the plates in terms of the parts they belong to. In other words, fundamental frequencies of electromagnetic wave propagation are affected more by the increase in material parameters of the center of the plate than by the increase in material parameters of the walls. This result enables the detection of the parts having different material parameters of the two-dimensional plates whose electromagnetic wave propagation frequencies are known. It is also possible that the analyzes conducted within the scope of this study can be applied to nanoplates, nanotubes, and phononic structures in future research.

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