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SOME NEW MIXED SOFT SETS

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Abstract. In this paper we introduce the concepts of (τ_1, τ_2) - R - closed, (τ_1, τ_2) - A_R-, (τ_1, τ_2) - αAN_1-, (τ_1, τ_2) - αAN_2-, (τ_1, τ_2) - αNA_1-, (τ_1, τ_2) - αNA_2-, (τ_1, τ_2) - αNA_3-, (τ_1, τ_2) - αNA_4- and (τ_1, τ_2) - αNA_5- soft sets in the two soft topological spaces and show the relationships between defined new mixed soft sets. Also we investigate some properties of these soft sets. Moreover, we define the notions of mixed R -, mixed A_R-, mixed αAN_1-, mixed αAN_2-, mixed αNA_1-, mixed αNA_2-, mixed αNA_3-, mixed αNA_4- and mixed αNA_5- soft continuity. Finally, we obtain decomposition of mixed A_R - soft continuity, mixed α - soft continuity and τ_1 - soft continuity using two soft topological space.

1. Introduction

Some set theories such as theory of fuzzy sets [12], intuitionistic fuzzy sets [2], vague sets [3], rough sets [9] etc. can be deal with unclear concepts. But these theories are not sufficient to solve encountered difficulties. There are some vague problems in medical science, social science, economics etc. What is the reason of these problems? It is possible the inadequacy of the parametrization tool of the theories. In 1999, Molodtsov [8] introduced the concept of soft set theory as a general mathematical tool for coping with these problems. In 2001, Maji et al. [6] defined the notion of fuzzy soft set and [7] intuitionistic fuzzy soft set. Again in 2003, Maji et al. [5] introduced the theoretical notions of the soft set theory and studied some properties of these notions. In 2009, Ali et al. [1] investigated several operations on soft set theory and defined some new notions such as the restricted intersection etc. In 2011, Naz et al. [10] introduced some concepts such as soft topological space, soft interior, soft closure etc. Also in 2011, Hussain et al. [4] studied some properties of soft topological spaces. In 2012, Zorlutuna et al. [13] introduced the image and inverse image of soft set under a function and soft continuity in the soft topological spaces. Also they obtained decompositions of some forms of soft continuity.

In this paper we introduce the concepts of (τ_1, τ_2) - R - closed, (τ_1, τ_2) -

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A_R-, (τ_1, τ_2) - αAN_1-, (τ_1, τ_2) - αAN_2-, (τ_1, τ_2) - αNA_1-, (τ_1, τ_2) - αNA_2-, (τ_1, τ_2) - αNA_3-, (τ_1, τ_2) - αNA_4- and (τ_1, τ_2) - αNA_5- soft sets in the two soft topological spaces and show the relationships between defined new mixed soft sets. Also we investigate some properties of these soft sets. Additionally, we define the notions of mixed R -, mixed A_R-, mixed αAN_1-, mixed αAN_2-, mixed αNA_1-, mixed αNA_2-, mixed αNA_3-, mixed αNA_4- and mixed αNA_5- soft continuity. Consequently, we obtain decomposition of mixed A_R - soft continuity, mixed α - soft continuity and τ_1 - soft continuity using two soft topological space.

2. PRELIMINARIES

In this section we recall some known definitions and theorems.

Definition 2.1. [8] A pair (F, A), where F is mapping from A to $P(X)$, is called a soft set over X. The family of all soft sets on X denoted by $SS(X)_E$.

Definition 2.2. [5] Let (F, A) and (G, B) be two soft sets over a common universe X. Then (F, A) is said to be a soft subset of (G, B) if $A \subseteq B$ and $F(e) \subseteq G(e)$, for all $e \in A$. This relation is denoted by $(F, A)\widetilde{\subseteq}(G, B)$.
(F, A) is said to be soft equal to (G, B) if $(F, A)\widetilde{\subseteq}(G, B)$ and $(G, B)\widetilde{\subseteq}(F, A)$. This relation is denoted by $(F, A) = (G, B)$.

Definition 2.3. [1]. The complement of a soft set (F,A) is defined as $(F, A)^c = (F^c, A)$, where $F^c(e) = (F(e))^c = X - F(e)$, for all $e \in A$.

Definition 2.4. [10] The difference of two soft sets (F,A) and (G,A) is defined by $(F, A) - (G, A) = (F - G, A)$, where $(F - G)(e) = F(e) - G(e)$, for all $e \in A$.

Definition 2.5. [5] The union of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cup B$ and $H(e) = F(e)$ if $e \in A - B$ or $H(e) = G(e)$ if $e \in B - A$ or $H(e) = F(e) \cup G(e)$ if $e \in A \cap B$ for all $e \in C$.

Definition 2.6. [5] The intersection of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.

Definition 2.7. [5] Let (F, A) be a soft set over X. Then (F, A) is said to be a null soft set if $F(e) = \emptyset$, for all $e \in A$. This denoted by $\widetilde{\emptyset}$.

Definition 2.8. [5] Let (F, A) be a soft set over X. Then (F, A) is said to be an absolute soft set if $F(e) = X$, for all $e \in A$. This denoted by $\widetilde{X}$.

Definition 2.9. [10] Let τ be the collection of soft sets over X. Then τ is said to be a soft topology on X if

(1) $\widetilde{\emptyset}$, $\widetilde{X} \in \tau$;

(2) the intersection of any two soft sets in τ belongs to τ;

(3) the union of any number of soft sets in τ belongs to τ.

The triple (X, τ, E) is called a soft topological space over X. The members of τ are said to be $\tau-$ soft open sets or soft open sets in X. A soft set over X is said to be soft closed in X if its complement belongs to τ. The set of all soft open sets over X denoted by $OS(X, \tau, E)$ or $OS(X)$ and the set of all soft closed sets denoted by $CS(X, \tau, E)$ or $CS(X)$.

Definition 2.10. Let (X, τ, E) be a soft topological space and (F, E) be a soft set over X. Then

(1) [13] the soft interior of (F, E) is the soft set
$int(F, E) = \widetilde{\cup}\{(G, E) : (G, E)$ is soft open and $(G, E)\widetilde{\subseteq}(F, E)\}$;
(2) [10] the soft closure of (F, E) is the soft set
$cl(F, E) = \widetilde{\cap}\{(H, E) : (H, E)$ is soft closed and $(F, E)\widetilde{\subseteq}(H, E)\}$.

Theorem 2.1. [4] *Let (X, τ, E) be a soft topological space over X, (F, E) and (G, E) are soft sets over X. Then*

(1) $cl(\widetilde{\emptyset}) = \widetilde{\emptyset}$ *and* $cl(\widetilde{X}) = \widetilde{X}$.

(2) $(F, E)\widetilde{\subseteq}cl(F, E)$.

(3) (F, E) *is a closed set if and only if* $(F, E) = cl(F, E)$.

(4) $cl(cl(F, E)) = cl(F, E)$.

(5) $(F, E)\widetilde{\subseteq}(G, E)$ *implies* $cl(F, E)\widetilde{\subseteq}cl(G, E)$.

(6) $cl((F, E)\widetilde{\cup}(G, E)) = cl(F, E)\widetilde{\cup}cl(G, E)$.

(7) $cl((F, E)\widetilde{\cap}(G, E))\widetilde{\subseteq}cl(F, E)\widetilde{\cap}cl(G, E)$.

Theorem 2.2. [4] *Let (X, τ, E) be a soft topological space over X and (F, E) and (G, E) are soft sets over X. Then*

(1) $int\widetilde{\emptyset} = \widetilde{\emptyset}$ *and* $int\widetilde{X} = \widetilde{X}$.

(2) $int(F, E)\widetilde{\subseteq}(F, E)$.

(3) $int(int(F, E)) = int(F, E)$.

(4) (F, E) *is a soft open set if and only if* $int(F, E) = (F, E)$.

(5) $(F, E)\widetilde{\subseteq}(G, E)$ *implies* $int(F, E)\widetilde{\subseteq}int(G, E)$.

(6) $int(F, E)\widetilde{\cap}int(G, E) = int((F, E)\widetilde{\cap}(G, E))$.

(7) $int(F,E)\widetilde{\cup}int(G,E)\widetilde{\subseteq}int((F,E)\widetilde{\cup}(G,E))$.

Definition 2.11. [13] Let $SS(X)_A$ and $SS(Y)_B$ be families of soft sets, $u : X \to Y$ and $p : A \to B$ be mappings. Then the mapping $f_{pu} : SS(X)_A \to SS(Y)_B$ is defined as:

(1) Let $(F,A) \in SS(X)_A$. The image of (F,A) under f_{pu}, written as $f_{pu}(F,A) = (f_{pu}(F), p(A))$, is a soft set in $SS(Y)_B$ such that $f_{pu}(F)(y) = \bigcup_{x\in p^{-1}(y)\cap A} u(F(x))$ if $p^{-1}(y) \cap A \neq \emptyset$ and $f_{pu}(F)(y) = \emptyset$ if $p^{-1}(y) \cap A = \emptyset$ for all $y \in B$.

(2) Let $(G,B) \in SS(Y)_B$. The inverse image of (G,B) under f_{pu}, written as $f_{pu}^{-1}(G,B) = (f_{pu}^{-1}(G), p^{-1}(B))$, is a soft set in $SS(X)_A$ such that $f_{pu}^{-1}(G)(x) = u^{-1}(G(p(x)))$ if $p(x) \in B$ and $f_{pu}^{-1}(G)(x) = \emptyset$ if $p(x) \notin B$ for all $x \in A$.

Definition 2.12. [13] Let (X,τ,A) and $(Y,\tau*,B)$ be soft topological spaces and $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then The function f_{pu} is called soft continuous (soft - cts or τ - soft continuous) if $f_{pu}^{-1}(G,B) \in \tau$ for all $(G,B) \in \tau*$.

Theorem 2.3. [11] *Let (X,τ_1,E), (X,τ_2,E) be two soft topological space and $(F,E) \in SS(X)_E$. If (G,E) is a τ_1 - soft open set, then $(G,E)\ \widetilde{\cap}\ cl_2(F,E)\ \widetilde{\subseteq}\ cl_2((G,E)\widetilde{\cap}(F,E))$.*

3. (τ_1,τ_2) - R - CLOSED, (τ_1,τ_2) - A_R-, (τ_1,τ_2) - αAN_1-, (τ_1,τ_2) - αAN_2-, (τ_1,τ_2) - αNA_1-, (τ_1,τ_2) - αNA_2-, (τ_1,τ_2) - αNA_3-, (τ_1,τ_2) - αNA_4- AND (τ_1,τ_2) - αNA_5- SOFT SETS

In this section we define (τ_1,τ_2) - R - closed, (τ_1,τ_2) - A_R-, (τ_1,τ_2) - αAN_1-, (τ_1,τ_2) - αAN_2-, (τ_1,τ_2) - αNA_1-, (τ_1,τ_2) - αNA_2-, (τ_1,τ_2) - αNA_3-, (τ_1,τ_2) - αNA_4- and (τ_1,τ_2) - αNA_5- soft sets and investigate some properties of these soft sets. In this paper, we will write int_1, cl_1, int_2, cl_2 instead of int_{τ_1}, cl_{τ_1}, int_{τ_2}, cl_{τ_2}, respectively.

Definition 3.1. Let (X,τ_1,E), (X,τ_2,E) be two soft topological spaces and $(F,E) \in SS(X)_E$. Then (F,E) is called

(1) (τ_1,τ_2) - α - open soft set [11] if $(F,E)\widetilde{\subseteq}int_1(cl_2(int_1(F,E)))$,

(2) (τ_1,τ_2) - semi - open soft set [11] if $(F,E)\widetilde{\subseteq}cl_2(int_1(F,E))$,

(3) (τ_1,τ_2) - pre - open soft set [11] if $(F,E)\widetilde{\subseteq}int_1(cl_2(F,E))$,

(4) τ_2 - soft closed set [10] if $(F,E) = cl_2(F,E)$,

(5) (τ_1,τ_2) - t - soft set if $int_1(F,E) = int_1(cl_2(F,E))$,

(6) (τ_1,τ_2) - $t*$ - soft set if $int_1(F,E) = int_1(cl_2(int_1(F,E)))$.

Definition 3.2. Let (X,τ_1,E), (X,τ_2,E) be two soft topological spaces and $(F,E) \in SS(X)_E$. Then (F,E) is called

(1) a (τ_1, τ_2) - weakly soft locally - closed set (briefly, (τ_1, τ_2) - WLC - soft set) if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where (G, E) is τ_1 - soft open and (H, E) is τ_2 - soft closed,

(2) a (τ_1, τ_2) - B - soft set if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where (G, E) is τ_1 - soft open and (H, E) is (τ_1, τ_2) - t - soft set,

(3) a (τ_1, τ_2) - C - soft set if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where (G, E) is τ_1 - soft open and (H, E) is (τ_1, τ_2) - $t*$ - soft set.

The family of all (τ_1, τ_2) - α - open soft (resp. (τ_1, τ_2) - semi - open soft, (τ_1, τ_2) - pre - open soft, (τ_1, τ_2) - weakly soft locally - closed, (τ_1, τ_2) - B - soft and (τ_1, τ_2) - C - soft) sets in two soft topological spaces (X, τ_1, E), (X, τ_2, E) is denoted by $\alpha OS(\tau_1, \tau_2)$ (resp. $SOS(\tau_1, \tau_2)$, $POS(\tau_1, \tau_2)$, $WLCS(\tau_1, \tau_2)$, $BS(\tau_1, \tau_2)$ and $CS(\tau_1, \tau_2)$).

Definition 3.3. Let (X, τ_1, E), (X, τ_2, E) be two soft topological spaces and $(F, E) \in SS(X)_E$. Then (F, E) is called a (τ_1, τ_2) - R - closed soft if $(F, E) = cl_2(int_1(F, E))$.

Definition 3.4. Let (X, τ_1, E), (X, τ_2, E) be two soft topological spaces and $(F, E) \in SS(X)_E$. Then (F, E) is called a (τ_1, τ_2) - A_R - soft set if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where (G, E) is τ_1 - soft open and (H, E) is a (τ_1, τ_2) - R - closed soft.

We denote the family of all (τ_1, τ_2) - A_R - ((τ_1, τ_2) - R - closed) soft sets by $A_RS(\tau_1, \tau_2)$ ($RS(\tau_1, \tau_2)$).

Definition 3.5. Let (X, τ_1, E), (X, τ_2, E) be two soft topological spaces and $(F, E) \in SS(X)_E$. Then (F, E) is called

(1) a (τ_1, τ_2) - αAN_1 - soft set if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where $(G, E) \in \alpha OS(\tau_1, \tau_2)$ and $cl_2(int_1(H, E)) = \widetilde{X}$,

(2) a (τ_1, τ_2) - αAN_2 - soft set if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where $(G, E) \in \alpha OS(\tau_1, \tau_2)$ and $cl_2(H, E) = \widetilde{X}$,

(3) a (τ_1, τ_2) - αNA_1 - soft set if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where $(G, E) \in \alpha OS(\tau_1, \tau_2)$ and $cl_2(int_1(H, E)) \widetilde{\subseteq} (H, E)$,

(4) a (τ_1, τ_2) - αNA_2 - soft set if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where $(G, E) \in \alpha OS(\tau_1, \tau_2)$ and $cl_2(int_1(H, E)) = (H, E)$,

(5) a (τ_1, τ_2) - αNA_3 - soft set if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where $(G, E) \in \alpha OS(\tau_1, \tau_2)$ and $int_1(cl_2(H, E)) = int_1(H, E)$,

(6) a (τ_1, τ_2) - αNA_4 - soft set if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where $(G, E) \in \alpha OS(\tau_1, \tau_2)$ and $int_1(cl_2(int_1(H, E))) \widetilde{\subseteq} (H, E)$,

(7) a (τ_1, τ_2) - αNA_5 - soft set if $(F, E) = (G, E)\widetilde{\cap}(H, E)$, where $(G, E) \in \alpha OS(\tau_1, \tau_2)$ and $cl_2(H, E) = (H, E)$.

We denote the family of all (τ_1, τ_2) - αAN_1- (resp. (τ_1, τ_2) - αAN_2-, (τ_1, τ_2) - αNA_1-, (τ_1, τ_2) - αNA_2-, (τ_1, τ_2) - αNA_3-, (τ_1, τ_2) - αNA_4- and (τ_1, τ_2) -

αNA_5-) sets in the soft topological spaces (X, τ_1, E), (X, τ_2, E) by $\alpha AN_1S(\tau_1, \tau_2)$ (resp. $\alpha AN_2S(\tau_1, \tau_2)$, $\alpha NA_1S(\tau_1, \tau_2)$,
$\alpha NA_2S(\tau_1, \tau_2)$, $\alpha NA_3S(\tau_1, \tau_2)$, $\alpha NA_4S(\tau_1, \tau_2)$ and $\alpha NA_5S(\tau_1, \tau_2)$).

Proposition 3.1. *Every (τ_1, τ_2) - αAN_1 - soft set is (τ_1, τ_2) - αAN_2 - soft set.*

Proof. It is obvious from Definition 3.5. □

Remark 3.1. The converse of Proposition 3.6 need not be true as shown in the following example.

Example 3.1. Let $X = \{a, b, c\}$, $E = \{e\}$, $\tau_1 = \{\widetilde{\emptyset}, \widetilde{X}, (F_1, E)\}$ and $\tau_2 = \{\widetilde{\emptyset}, \widetilde{X}, (F_2, E)\}$, where (F_1, E), (F_2, E) are soft sets over X defined as follows:
$(F_1, E) = \{(e, \{c\})\}$,
$(F_2, E) = \{(e, \{a, b\})\}$.
The soft set $(G, E) = \{(e, \{a, c\})\}$ is a (τ_1, τ_2) - αAN_2 - soft set, but it is not a (τ_1, τ_2) - αAN_1 - soft set.

Proposition 3.2. *Let (X, τ_1, E), (X, τ_2, E) be two soft topological spaces and $(F, E) \in SS(X)_E$. Then the following hold:*

1. *If (F, E) is a (τ_1, τ_2) - αNA_2 - soft set, then (F, E) is a (τ_1, τ_2) - αNA_5 - soft set.*
2. *If (F, E) is a (τ_1, τ_2) - αNA_5 - soft set, then (F, E) is a (τ_1, τ_2) - αNA_3 - soft set.*
3. *If (F, E) is a (τ_1, τ_2) - αNA_3 - soft set, then (F, E) is a (τ_1, τ_2) - αNA_4 - soft set.*

Proof. (1) Let $(F, E) = (G, E)\widetilde{\cap}(H, E) \in \alpha NA_2S(\tau_1, \tau_2)$, where $(G, E) \in \alpha OS(\tau_1, \tau_2)$ and $cl_2(int_1(H, E)) = (H, E)$. Since $cl_2(int_1(H, E)) = cl_2(H, E) = (H, E)$, we obtain $(F, E) \in \alpha NA_5S(\tau_1, \tau_2)$.
(2) It is obvious from Definition 3.5.
(3) It is clear from Definition 3.5. □

Remark 3.2. The converses of Proposition 3.9 need not be true as shown in the following examples.

Example 3.2. Let $X = \{a, b\}$, $E = \{e_1, e_2\}$, $\tau_1 = \{\widetilde{\emptyset}, \widetilde{X}, (F, E)\}$ and $\tau_2 = \{\widetilde{\emptyset}, \widetilde{X}\}$, where (F, E) is a soft set over X defined as follows:
$(F, E) = \{(e_1, \{a\}), (e_2, \{b\})\}$.
The soft set $(G, E) = \{(e_1, \{b\}), (e_2, \{a\})\}$ is a (τ_1, τ_2) - αNA_5 - soft set, but it is not a (τ_1, τ_2) - αNA_2 - soft set.

Example 3.3. Let $X = \{a, b, c\}$, $E = \{e\}$, $\tau_1 = \{\widetilde{\emptyset}, \widetilde{X}, (F_1, E)\}$ and $\tau_2 = \{\widetilde{\emptyset}, \widetilde{X}, (F_2, E)\}$, where (F_1, E), (F_2, E) are soft sets over X defined as follows:
$(F_1, E) = \{(e, \{a\})\}$,
$(F_2, E) = \{(e, \{b\})\}$,
The soft set $(G, E) = \{(e, \{a\})\}$ is a (τ_1, τ_2) - αNA_3 - soft set, but it is not a (τ_1, τ_2) - αNA_5 - soft set.

Example 3.4. Let $X = \{a, b, c, d\}$, $E = \{e\}$, $\tau_1 = \{\widetilde{\emptyset}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ and $\tau_2 = \{\widetilde{\emptyset}, \widetilde{X}, (F_1, E)\}$, where (F_1, E), (F_2, E), (F_3, E) are soft sets over X defined as follows:
$(F_1, E) = \{(e, \{a, b\})\}$,
$(F_2, E) = \{(e, \{d\})\}$,
$(F_3, E) = \{(e, \{a, b, d\})\}$.
The soft set $(G, E) = \{(e, \{b, c, d\})\}$ is a (τ_1, τ_2) - αNA_4 - soft set, but it is not a (τ_1, τ_2) - αNA_3 - soft set.

Proposition 3.3. *Let (X, τ_1, E), (X, τ_2, E) be two soft topological spaces and $(F, E)) \in SS(X)_E$. Then the following hold:*

1. *If (F, E) is a (τ_1, τ_2) - αNA_5 - soft set, then (F, E) is a (τ_1, τ_2) - αNA_1 - soft set.*
2. *If (F, E) is a (τ_1, τ_2) - αNA_1 - soft set, then (F, E) is a (τ_1, τ_2) - αNA_4 - soft set.*

Proof. This proof is obvious from Definition 3.5. □

Remark 3.3. The converses of Proposition 3.14 need not be true as shown in the following examples.

Example 3.5. Let $X = \{a, b, c\}$, $E = \{e\}$, $\tau_1 = \{\widetilde{\emptyset}, \widetilde{X}, (F, E)\}$ and $\tau_2 = \{\widetilde{\emptyset}, \widetilde{X}\}$, where (F, E) is a soft set over X defined as follows:
$(F, E) = \{(e, \{b, c\})\}$.
The soft set $(G, E) = \{(e, \{a, c\})\}$ is a (τ_1, τ_2) - αNA_1 - soft set, but it is not a (τ_1, τ_2) - αNA_5 - soft set.

Example 3.6. Let $X = \{a, b, c\}$, $E = \{e\}$, $\tau_1 = \{\widetilde{\emptyset}, \widetilde{X}, (F_1, E), (F_2, E), (F_3, E)\}$ and $\tau_2 = \{\widetilde{\emptyset}, \widetilde{X}, (F_2, E)\}$, where (F_1, E), (F_2, E), (F_3, E) are soft sets over X defined as follows:
$(F_1, E) = \{(e, \{a\})\}$,
$(F_2, E) = \{(e, \{b\})\}$,
$(F_3, E) = \{(e, \{a, b\})\}$.
The soft set $(G, E) = \{(e, \{a\})\}$ is a (τ_1, τ_2) - αNA_4 - soft set, but it is not a (τ_1, τ_2) - αNA_1 - soft set.

Definition 3.6. Let (X, τ_1, E), (X, τ_2, E) be two soft topological spaces and $(F, E) \in SS(X)_E$. Then

1. (F, E) is said to be τ_2 - soft - dense if $cl_2(F, E) = \widetilde{X}$.
2. τ_1 is said to be τ_2 - submaximal if each τ_2 - soft - dense (F, E) is a τ_1 - soft open set.

Definition 3.7. [11] Let (X, τ_1, E), (X, τ_2, E) be two soft topological spaces. τ_1 is said to be τ_2 - $ESDC$ if the τ_2 - soft closure of every τ_1 - soft open set is τ_1 - soft open.

Theorem 3.1. *Let (X, τ_1, E), (X, τ_2, E) be two soft topological spaces. If τ_1 is a τ_2 - $ESDC$, then $\alpha NA_4S(\tau_1, \tau_2) = \alpha NA_1S(\tau_1, \tau_2)$.*

Proof. Let $(F,E) = (G,E)\widetilde{\cap}(H,E) \in \alpha NA_4S(\tau_1,\tau_2)$, where $(G,E) \in \alpha OS(\tau_1,\tau_2)$ and $int_1(cl_2(int_1(H,E))) \widetilde{\subseteq} (H,E)$. Since the soft topology τ_1 is τ_2 - $ESDC$, we obtain $int_1(cl_2(int_1(H,E))) = cl_2(int_1(H,E))$ and so $cl_2(int_1(H,E)) \widetilde{\subseteq} (H,E)$. As a consequence, we obtain $\alpha NA_4S(\tau_1,\tau_2) \widetilde{\subseteq} \alpha NA_1S(\tau_1,\tau_2)$ and $\alpha NA_4S(\tau_1,\tau_2) = \alpha NA_1S(\tau_1,\tau_2)$ by Proposition 3.14. □

Proposition 3.4. *Let (X,τ_1,E), (X,τ_2,E) be two soft topological spaces and $(F,E) \in SS(X)_E$. Then the following hold:*

(1) *If (F,E) is a (τ_1,τ_2) - A_R - soft set, then (F,E) is a (τ_1,τ_2) - WLC - soft.*

(2) *If (F,E) is a (τ_1,τ_2) - WLC - soft, then (F,E) is a (τ_1,τ_2) - B - soft set.*

(3) *If (F,E) is a (τ_1,τ_2) - B - soft set, then (F,E) is a (τ_1,τ_2) - C - soft set.*

Proof. (1) Let $(F,E) = (G,E)\widetilde{\cap}(H,E) \in A_RS(\tau_1,\tau_2)$, where (G,E) is τ_1 - soft open and $cl_2(int_1(H,E)) = (H,E)$. Then we obtain

$$cl_2(cl_2(int_1(H,E))) = cl_2(H,E) = cl_2(int_1(H,E))$$

and so $(H,E) = cl_2(H,E)$. As a consequence, (F,E) is (τ_1,τ_2) - WLC - soft.
(2) It is obvious.
(3) It is clear. □

Proposition 3.5. *Let (X,τ_1,E), (X,τ_2,E) be two soft topological spaces and $(F,E) \in SS(X)_E$. Then the following hold:*

(1) *If (F,E) is a (τ_1,τ_2) - A_R - soft set, then (F,E) is a (τ_1,τ_2) - αNA_2 - soft set.*

(2) *If (F,E) is a (τ_1,τ_2) - WLC - soft, then (F,E) is a (τ_1,τ_2) - αNA_5 - soft set.*

(3) *If (F,E) is a (τ_1,τ_2) - B - soft set, then (F,E) is a (τ_1,τ_2) - αNA_3 - soft set.*

(4) *If (F,E) is a (τ_1,τ_2) - C - soft set, then (F,E) is a (τ_1,τ_2) - αNA_4 - soft set.*

Proof. It is obvious since every τ_1 - soft open set is (τ_1,τ_2) - α - open soft set. □

Theorem 3.2. *Let (X,τ_1,E), (X,τ_2,E) be two soft topological spaces. Let τ_1 be τ_2 - submaximal and τ_2 - $ESDC$. Then the following hold:*

(1) $A_RS(\tau_1,\tau_2) = \alpha NA_2S(\tau_1,\tau_2)$.
(2) $WLCS(\tau_1,\tau_2) = \alpha NA_5S(\tau_1,\tau_2)$.
(3) $BS(\tau_1,\tau_2) = \alpha NA_3S(\tau_1,\tau_2)$.
(4) $CS(\tau_1,\tau_2) = \alpha NA_4S(\tau_1,\tau_2)$.

Proof. It is clear since $\tau_1 = \alpha OS(\tau_1,\tau_2)$ if τ_1 be τ_2 - submaximal and τ_2 - $ESDC$. □

Proposition 3.6. [11] *Let (X,τ_1,E), (X,τ_2,E) be two soft topological spaces.*

(1) *If $(F,E) \in SOS(\tau_1,\tau_2)$ and $(G,E) \in \alpha OS(\tau_1,\tau_2)$, then $(F,E)\widetilde{\cap}(G,E) \in SOS(\tau_1,\tau_2)$.*

(2) *If* $(F,E) \in POS(\tau_1,\tau_2)$ *and* $(G,E) \in \alpha OS(\tau_1,\tau_2)$*, then* $(F,E)\widetilde{\cap}(G,E) \in POS(\tau_1,\tau_2)$.

Proposition 3.7. *Every* (τ_1,τ_2) *-* αNA_2 *- soft set is* (τ_1,τ_2) *- semi - open soft set.*

Proof. Let $(F,E) = (G,E)\widetilde{\cap}(H,E) \in \alpha NA_2S(\tau_1,\tau_2)$, where $(F,E) \in \alpha OS(\tau_1,\tau_2)$ and $cl_2(int_1(H,E)) = (H,E)$. Thus (H,E) is (τ_1,τ_2) - semi - open soft set. We obtain that (F,E) is (τ_1,τ_2) - semi - open soft set using Proposition 3.24. □

Theorem 3.3. *Let* $(F,E) \in SS(X)_E$*.* (F,E) *is a* (τ_1,τ_2) *-* A_R *- soft set if and only if* (F,E) *is a* (τ_1,τ_2) *- semi - open soft set and a* (τ_1,τ_2) *-* WLC *- soft.*

Proof. Let $(F,E) = (G,E)\widetilde{\cap}(H,E) \in A_RS(\tau_1,\tau_2)$, where (G,E) is τ_1 - soft open and $cl_2(int_1(H,E)) = (H,E)$. Then we obtain

$$cl_2(cl_2(int_1(H,E))) = cl_2(int_1(H,E)) = cl_2(H,E) = (H,E).$$

Also since $int_1(F,E) = (G,E)\,\widetilde{\cap}\,int_1(H,E)$, we have $(F,E) = (G,E)\widetilde{\cap}\,cl_2(int_1(H,E))$ $= cl_2((G,E)\widetilde{\cap}\,int_1(H,E))$ by Theorem 2.15. Hence $(F,E) \widetilde{\subseteq} cl_2(int_1(F,E))$. As a consequence, (F,E) is (τ_1,τ_2) - semi - open soft set.

Conversely, let (F,E) be (τ_1,τ_2) - semi - open soft set and (τ_1,τ_2) - WLC - soft. Then $(F,E) = (G,E)\widetilde{\cap}(H,E)$, where (G,E) is τ_1 - soft open and $cl_2(H,E) = (H,E)$. Since (F,E) is (τ_1,τ_2) - semi - open soft, we have $(F,E) \widetilde{\subseteq} cl_2(int_1(F,E)) \widetilde{\subseteq} cl_2(F,E)$ and so $cl_2(F,E) \widetilde{\subseteq} cl_2(int_1(F,E)) \widetilde{\subseteq} cl_2(F,E)$. Thus we obtain $cl_2(F,E) = cl_2(int_1(F,E))$. Also since $(F,E) = (G,E)\widetilde{\cap}(H,E)$ and $(F,E) \widetilde{\subseteq} (G,E)\widetilde{\cup}cl_2(F,E)$, we obtain $(F,E) \widetilde{\subseteq} (G,E)\widetilde{\cap}cl_2(F,E) \widetilde{\subseteq} (G,E)\widetilde{\cap}\ cl_2((G,E)\widetilde{\cap}(H,E)) \widetilde{\subseteq} (G,E)\widetilde{\cap}$ $[cl_2(G,E)\widetilde{\cap}\,cl_2(H,E)] = [(G,E)\widetilde{\cap}cl_2(G,E)]\,\widetilde{\cap}cl_2(H,E) = (G,E)\widetilde{\cap}cl_2(H,E) = (G,E)\widetilde{\cap}$ $(H,E) = (F,E)$. As a consequence, $(F,E) = (G,E)\widetilde{\cap}cl_2(F,E) = (G,E)\widetilde{\cap}\,cl_2(int_1(F,E))$ and so (F,E) is a (τ_1,τ_2) - A_R - soft set. □

Theorem 3.4. *Let* (X,τ_1,E)*,* (X,τ_2,E) *be two soft topological spaces. Then* $A_RS(\tau_1,\tau_2) = \alpha NA_2S(\tau_1,\tau_2) \cap WLCS(\tau_1,\tau_2)$.

Proof. We have $\alpha NA_2S(\tau_1,\tau_2) \subseteq SOS(\tau_1,\tau_2)$ by Proposition 3.25 and $A_RS(\tau_1,\tau_2)$ $= SOS(\tau_1,\tau_2) \cap WLCS(\tau_1,\tau_2)$ by Theorem 3.26. Then $\alpha NA_2S(\tau_1,\tau_2) \cap WLCS(\tau_1,\tau_2)$ $\subseteq SOS(\tau_1,\tau_2) \cap WLCS(\tau_1,\tau_2) = A_RS(\tau_1,\tau_2)$. Hence, we obtain $\alpha NA_2S(\tau_1,\tau_2)$ $\cap\, WLCS(\tau_1,\tau_2) \subseteq A_RS(\tau_1,\tau_2)$. We have $A_RS(\tau_1,\tau_2) \subseteq \alpha NA_2S(\tau_1,\tau_2)$ by Proposition 3.22. Since $A_RS(\tau_1,\tau_2) = SOS(\tau_1,\tau_2) \cap WLCS(\tau_1,\tau_2)$, $A_RS(\tau_1,\tau_2) \subseteq WLCS(\tau_1,\tau_2)$ and so $A_RS(\tau_1,\tau_2) \subseteq \alpha NA_2S(\tau_1,\tau_2) \cap WLCS(\tau_1,\tau_2)$. As a consequence, we obtain $A_RS(\tau_1,\tau_2) = \alpha NA_2S(\tau_1,\tau_2) \cap WLCS(\tau_1,\tau_2)$. □

Proposition 3.8. *Every* (τ_1,τ_2) *-* αNA_2 *- soft set is* (τ_1,τ_2) *- pre - open soft set.*

Proof. Let $(F,E) = (G,E)\,\widetilde{\cap}\,(H,E) \in \alpha NA_2S(\tau_1,\tau_2)$, where $(G,E) \in \alpha OS(\tau_1,\tau_2)$ and $cl_2(H,E) = \widetilde{X}$. Since $int_1(cl_2(H,E)) = \widetilde{X}$, then $(H,E) \widetilde{\subseteq} int_1(cl_2(H,E))$ and so (H,E) is (τ_1,τ_2) - pre - open soft. By Proposition 3.24, we obtain that (F,E) is (τ_1,τ_2) - pre - open soft. □

Theorem 3.5. *For two soft topological spaces* (X,τ_1,E) *and* (X,τ_2,E)*,* $\alpha OS(\tau_1,\tau_2)$ $= POS(\tau_1,\tau_2) \cap \alpha NA_3S(\tau_1,\tau_2)$.

Proof. Let (F,E) be a (τ_1,τ_2) - α - open soft set. Put $(G,E) = (F,E)$ and $(H,E) = \widetilde{X}$. Then $(F,E) = (G,E)\widetilde{\cap}\,(H,E)$, where $(G,E) \in \alpha OS(\tau_1,\tau_2)$ and $int_1(cl_2(H,E))$ $= int_1(H,E)$. Since (F,E) is (τ_1,τ_2) - pre - open soft, $\alpha OS(\tau_1,\tau_2) \subseteq POS(\tau_1,\tau_2)$ $\cap\, \alpha NA_3S(\tau_1,\tau_2)$.

Let (F,E) be (τ_1,τ_2) - pre - open soft set and (τ_1,τ_2) - αNA_3 - soft set. Since (F,E) is a (τ_1,τ_2) - pre - open soft set, we have $(F,E) \widetilde{\subseteq} int_1(cl_2(F,E))$. Also we have $(F,E) = (G,E) \widetilde{\subseteq} (H,E)$, where (G,E) is (τ_1,τ_2) - α - open soft set and $int_1(cl_2(H,E)) = int_1(H,E)$ since (F,E) is a (τ_1,τ_2) - αNA_3 - soft set. Then (F,E) $\widetilde{\subseteq} int_1(cl_2(F,E)) = int_1(cl_2((G,E)\widetilde{\cap}(H,E))) \widetilde{\subseteq} int_1(cl_2(int_1(G,E)) \widetilde{\cap} cl_2(H,E)) =$ $int_1(cl_2(\ (int_1(G,E))) \widetilde{\cap} int_1(cl_2(H,E)) = int_1(cl_2(int_1(G,E))) \widetilde{\cap} int_1(H,E)$ and so $(F,E) \widetilde{\subseteq} int_1[(cl_2(int_1(G,E))) \widetilde{\cap} int_1(H,E)] \widetilde{\subseteq} int_1(cl_2[int_1\ (G,E) \widetilde{\cap} int_1(H,E)]$ $= int_1(cl_2(int_1((G,E) \widetilde{\cap}\ (H,E)))) = int_1(cl_2(int_1\ (F,E)))$. As a consequence, $(F,E) \in \alpha OS(\tau_1,\tau_2)$. □

Lemma 3.1. [11] *Let (X,τ_1,E), (X,τ_2,E) be two soft topological spaces and $(F,E) \in SS(X)_E$. (F,E) is (τ_1,τ_2) - α - open soft if and only if it is (τ_1,τ_2) - semi - open soft and (τ_1,τ_2) - pre - open soft.*

Theorem 3.6. *For two soft topological spaces (X,τ_1,E) and (X,τ_2,E), the following hold:*

(1) $\alpha OS(\tau_1,\tau_2) = POS(\tau_1,\tau_2) \cap \alpha NA_5S(\tau_1,\tau_2)$.
(2) $\alpha OS(\tau_1,\tau_2) = \alpha AN_2S(\tau_1,\tau_2) \cap \alpha NA_2S(\tau_1,\tau_2)$.

Proof. (1) Since $\alpha OS(\tau_1,\tau_2) = POS(\tau_1,\tau_2) \cap SOS(\tau_1,\tau_2)$ by Lemma 3.30, then $\alpha OS(\tau_1,\tau_2) \subseteq POS(\tau_1,\tau_2)$. Also since $\alpha OS(\tau_1,\tau_2) \subseteq \alpha NA_5\ S(\tau_1,\tau_2)$, we obtain that $\alpha OS(\tau_1,\tau_2) \subseteq POS(\tau_1,\tau_2) \cap \alpha NA_5S(\tau_1,\tau_2)$. We have $\alpha OS(\tau_1,\tau_2) =$ $POS(\tau_1,\tau_2) \cap \alpha NA_3S(\tau_1,\tau_2)$ by Theorem 3.29. Also since $\alpha NA_5S(\tau_1,\tau_2) \subseteq \alpha NA_3$ $S(\tau_1,\tau_2)$, we obtain that $POS(\tau_1,\tau_2) \cap \alpha NA_5S(\tau_1,\tau_2) \subseteq POS(\tau_1,\tau_2) \cap \alpha NA_3$ $S(\tau_1,\tau_2) = \alpha OS(\tau_1,\tau_2)$.
(2) We have $\alpha AN_2S(\tau_1,\tau_2) \subseteq POS(\tau_1,\tau_2)$ by Proposition 3.28 and $\alpha NA_2S(\tau_1,\tau_2)$ $\subseteq SOS(\tau_1,\tau_2)$ by Proposition 3.25. Since $\alpha OS(\tau_1,\tau_2) = POS(\tau_1,\tau_2) \cap SOS(\tau_1,\tau_2)$, then $\alpha AN_2S(\tau_1,\tau_2) \cap \alpha NA_2S(\tau_1,\tau_2) \subseteq POS(\tau_1,\tau_2) \cap SOS(\tau_1,\tau_2) = \alpha OS(\tau_1,\tau_2)$. Also since $\alpha OS(\tau_1,\tau_2) \subseteq \alpha AN_2S(\tau_1,\tau_2) \cap \alpha NA_2S(\tau_1,\tau_2)$, we obtain that αO $S(\tau_1,\tau_2) = \alpha AN_2\ S(\tau_1,\tau_2) \cap \alpha NA_2S(\tau_1,\tau_2)$. □

Proposition 3.9. *Let (X,τ_1,E), (X,τ_2,E) be two soft topological spaces. For $(F,E) \in SS(X)_E$, the following equivalent:*

(1) *(F,E) is a τ_1 - soft open set;*
(2) *(F,E) is a (τ_1,τ_2) - α - open soft set and (τ_1,τ_2) - A_R - soft set;*
(3) *(F,E) is a (τ_1,τ_2) - pre - open soft set and (τ_1,τ_2) - A_R - soft set.*

Proof. (1) ⇒ (2). Let (F,E) be a τ_1 - soft open set. Then (F,E) is a (τ_1,τ_2) - α - open soft set. Also $(F,E) = (F,E) \widetilde{\cap} \widetilde{X}$, where (F,E) is a τ_1 - soft open and $\widetilde{X}$ is a (τ_1,τ_2) - R - closed soft. As a consequence, (F,E) is a (τ_1,τ_2) - A_R - soft set.
(2) ⇒ (3). It is clear from every (τ_1,τ_2) - α - open soft set is (τ_1,τ_2) - pre - open soft.
(3) ⇒ (1). Let (F,E) be (τ_1,τ_2) - pre - open soft and (τ_1,τ_2) - A_R - soft set. We have $(F,E) \widetilde{\subseteq} int_1(cl_2(F,E))$ and $(F,E) = (G,E) \widetilde{\cap} (H,E)$, where (G,E) is τ_1 - soft open and $cl_2(int_1(H,E)) = (H,E)$. Hence we have $(F,E) \widetilde{\subseteq} int_1(cl_2(F,E))$ $= int_1(cl_2((G,E)\widetilde{\cap}(H,E))) \widetilde{\subseteq} int_1(cl_2(G,E)\widetilde{\cap}cl_2(H,E))$. Every (τ_1,τ_2) - R -closed soft set is soft closed, (H,E) is τ_2 - soft closed and $cl_2(H,E) = (H,E)$. Thus we have $int_1(cl_2(G,E) \widetilde{\cap}\ (H,E)) = int_1(cl_2(G,E)) \widetilde{\cap}\ int_1(H,E)$ and $(G,E) \widetilde{\cap}$ $(H,E) \widetilde{\subseteq} (G,E) \widetilde{\cap} int_1(cl_2(G,E)) \widetilde{\cap} int_1(H,E) = int_1((G,E) \widetilde{\cap} cl_2(G,E) \widetilde{\cap} (H,E))$

$= int_1((G,E) \widetilde{\cap} (H,E))$. As a consequence, we obtain that $(G,E) \widetilde{\cap} (H,E) \widetilde{\subseteq} int_1((G,E) \widetilde{\cap} (H,E))$ and (F,E) is τ_1 - soft open. □

Theorem 3.7. *$\tau_1 = \alpha AN_2S(\tau_1,\tau_2) \cap \alpha NA_2S(\tau_1,\tau_2) \cap WLCS(\tau_1,\tau_2)$ for two soft topological spaces (X,τ_1,E), (X,τ_2,E).*

Proof. We have $\tau_1 = \alpha OS(\tau_1,\tau_2) \cap A_RS(\tau_1,\tau_2)$ by Proposition 3.32 and $A_RS(\tau_1,\tau_2) = \alpha NA_2S(\tau_1,\tau_2) \cap WLCS(\tau_1,\tau_2)$ by Theorem 3.27. Also we have $\alpha OS(\tau_1,\tau_2) = \alpha AN_2S(\tau_1,\tau_2) \cap \alpha NA_2S(\tau_1,\tau_2)$ by Theorem 3.31. As a consequence, we obtain $\tau_1 = \alpha AN_2S(\tau_1,\tau_2) \cap \alpha NA_2S(\tau_1,\tau_2) \cap WLCS(\tau_1,\tau_2)$. □

$$\begin{array}{ccccccc}
A_RS(\tau_1,\tau_2) & \longrightarrow & WLCS(\tau_1,\tau_2) & \longrightarrow & BS(\tau_1,\tau_2) & \longrightarrow & CS(\tau_1,\tau_2) \\
\downarrow & & \downarrow & & \downarrow & & \searrow \\
\alpha NA_2S(\tau_1,\tau_2) & \longrightarrow & \alpha NA_5S(\tau_1,\tau_2) & \longrightarrow & \alpha NA_3S(\tau_1,\tau_2) & \longrightarrow & \alpha NA_4S(\tau_1,\tau_2)
\end{array}$$

Diagram 1. The relationships between the mixed soft sets defined above.

4. Decompositions of some soft continuities

In this section we define some new mixed soft continuities and obtain some decompositions.

Definition 4.1. Let (X,τ_1,A), (X,τ_2,A) and $(Y,\tau*,B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then the function is said to be mixed α - soft continuous (resp. mixed semi - soft continuous, mixed pre - soft continuous, mixed WLC - soft continuous, mixed B - soft continuous, mixed C - soft continuous) if $f_{pu}^{-1}(G,B) \in \alpha OS(\tau_1,\tau_2)$ (resp. $SOS(\tau_1,\tau_2)$, $POS(\tau_1,\tau_2)$, $WLCS(\tau_1,\tau_2)$, $BS(\tau_1,\tau_2)$, $CS(\tau_1,\tau_2)$) for all $(G,B) \in OS(Y)$.

Definition 4.2. Let (X,τ_1,A), (X,τ_2,A) and $(Y,\tau*,B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then the function is said to be mixed R - soft continuous (mixed A_R - soft continuous) if $f_{pu}^{-1}(G,B) \in RS(\tau_1,\tau_2)$ $(A_RS(\tau_1,\tau_2))$ for all $(G,B) \in OS(Y)$.

Definition 4.3. Let (X,τ_1,A), (X,τ_2,A) and $(Y,\tau*,B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. Then the function is said to be mixed αAN_1 - soft continuous (resp. mixed αAN_2 - soft continuous, mixed αNA_1 - soft continuous, mixed αNA_2 - soft continuous, mixed αNA_3 - soft continuous, mixed αNA_4 - soft continuous, mixed αNA_5 - soft continuous) if $f_{pu}^{-1}(G,B) \in \alpha AN_1S(\tau_1,\tau_2)$ (resp. $\alpha AN_2S(\tau_1,\tau_2)$, $\alpha NA_1S(\tau_1,\tau_2)$, $\alpha NA_2S(\tau_1,\tau_2)$, $\alpha NA_3S(\tau_1,\tau_2)$, $\alpha NA_4S(\tau_1,\tau_2)$, $\alpha NA_5S(\tau_1,\tau_2)$) for all $(G,B) \in OS(Y)$.

Theorem 4.1. *Let (X,τ_1,A), (X,τ_2,A) and $(Y,\tau*,B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. If a function f_{pu} is mixed αAN_1 - soft continuous (resp. mixed αNA_2 - soft continuous, mixed αNA_5 - soft continuous, mixed αNA_3 - soft continuous), then f_{pu} is mixed αAN_2 - soft continuous (resp. mixed αNA_5 - soft continuous, mixed αNA_3 - soft continuous, mixed αNA_4 - soft continuous).*

Proof. It is obvious from Proposition 3.6 and 3.9. □

Theorem 4.2. *Let* (X, τ_1, A)*,* (X, τ_2, A) *and* $(Y, \tau*, B)$ *be soft topological spaces. Let* $u : X \to Y$ *and* $p : A \to B$ *be mappings. Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a function. If a function* f_{pu} *is mixed* αNA_5 *- soft continuous (mixed* αNA_1 *- soft continuous), then* f_{pu} *is mixed* αNA_1 *- soft continuous (mixed* αNA_4 *- soft continuous).*

Proof. The proof is clear from Proposition 3.14. □

Theorem 4.3. *Let* (X, τ_1, A)*,* (X, τ_2, A) *and* $(Y, \tau*, B)$ *be soft topological spaces. Let* $u : X \to Y$ *and* $p : A \to B$ *be mappings. Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a function. If a function* f_{pu} *is mixed* A_R *- soft continuous (resp. mixed* WLC *- soft continuous, mixed* B *- soft continuous), then* f_{pu} *is mixed* WLC *- soft continuous (resp. mixed* B *- soft continuous, mixed* C *- soft continuous).*

Proof. This is evident from Proposition 3.21. □

Theorem 4.4. *Let* (X, τ_1, A)*,* (X, τ_2, A) *and* $(Y, \tau*, B)$ *be soft topological spaces. Let* $u : X \to Y$ *and* $p : A \to B$ *be mappings. Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a function. If a function* f_{pu} *is mixed* A_R *- soft continuous (resp. mixed* WLC *- soft continuous, mixed* B *- soft continuous, mixed* C *- soft continuous), then* f_{pu} *is mixed* αNA_2 *- soft continuous (resp. mixed* αNA_5 *- soft continuous, mixed* αNA_3 *- soft continuous, mixed* αNA_4 *- soft continuous).*

Proof. It is a straight consequence of Proposition 3.22. □

Theorem 4.5. *Let* (X, τ_1, A)*,* (X, τ_2, A) *and* $(Y, \tau*, B)$ *be soft topological spaces. Let* $u : X \to Y$ *and* $p : A \to B$ *be mappings. Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a function. If a function* f_{pu} *is mixed* αNA_2 *- soft continuous, then* f_{pu} *is mixed semi - soft continuous.*

Proof. The proof is apparent from Proposition 3.25. □

Theorem 4.6. *Let* (X, τ_1, A)*,* (X, τ_2, A) *and* $(Y, \tau*, B)$ *be soft topological spaces. Let* $u : X \to Y$ *and* $p : A \to B$ *be mappings. Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a function. A function* f_{pu} *is mixed* A_R *- soft continuous if and only if it is both mixed semi - soft continuous and mixed* WLC *- soft continuous.*

Proof. This is a direct consequence of Theorem 3.26. □

Theorem 4.7. *Let* (X, τ_1, A)*,* (X, τ_2, A) *and* $(Y, \tau*, B)$ *be soft topological spaces. Let* $u : X \to Y$ *and* $p : A \to B$ *be mappings. Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a function. A function* f_{pu} *is mixed* A_R *- soft continuous if and only if it is both mixed* αNA_2 *- soft continuous and mixed* WLC *- soft continuous.*

Proof. This is an immediate consequence of Theorem 3.27. □

Theorem 4.8. *Let* (X, τ_1, A)*,* (X, τ_2, A) *and* $(Y, \tau*, B)$ *be soft topological spaces. Let* $u : X \to Y$ *and* $p : A \to B$ *be mappings. Let* $f_{pu} : SS(X)_A \to SS(Y)_B$ *be a function. If a function* f_{pu} *is mixed* αAN_2 *- soft continuous, then* f_{pu} *is mixed pre - soft continuous.*

Proof. It is clear from Proposition 3.28. □

Theorem 4.9. *Let (X, τ_1, A), (X, τ_2, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. A function f_{pu} is mixed α - soft continuous if and only if it is both mixed pre - soft continuous and mixed αNA_3 - soft continuous.*

Proof. It is an evident consequence of Theorem 3.29. □

Theorem 4.10. *Let (X, τ_1, A), (X, τ_2, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. The following properties are equivalent:*

(1) *f_{pu} is mixed pre - soft continuous and mixed αNA_5 - soft continuous;*
(2) *f_{pu} is mixed α - soft continuous;*
(3) *f_{pu} is mixed αAN_2 - soft continuous and mixed αNA_2 - soft continuous.*

Proof. This is clear from Theorem 3.31. □

Theorem 4.11. *Let (X, τ_1, A), (X, τ_2, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. The following properties are equivalent:*

(1) *f_{pu} is τ_1 - soft continuous;*
(2) *f_{pu} is mixed α - soft continuous and mixed A_R - soft continuous;*
(3) *f_{pu} is mixed pre - soft continuous and mixed A_R - soft continuous.*

Proof. It follows immediately from Proposition 3.32. □

Theorem 4.12. *Let (X, τ_1, A), (X, τ_2, A) and $(Y, \tau*, B)$ be soft topological spaces. Let $u : X \to Y$ and $p : A \to B$ be mappings. Let $f_{pu} : SS(X)_A \to SS(Y)_B$ be a function. A function f_{pu} is τ_1 - soft continuous if and only if it is mixed αAN_2 - soft continuous, mixed αNA_2 - soft continuous and mixed WLC - soft continuous.*

Proof. Obvious from Theorem 3.33. □

5. Conclusion

The concepts of (τ_1, τ_2) - R - closed, (τ_1, τ_2) - A_R-, (τ_1, τ_2) - αAN_1-, (τ_1, τ_2) - αAN_2-, (τ_1, τ_2) - αNA_1-, (τ_1, τ_2) - αNA_2-, (τ_1, τ_2) - αNA_3-, (τ_1, τ_2) - αNA_4- and (τ_1, τ_2) - αNA_5- soft sets have been introduced and the notions of some soft continuities have been defined. Also, some properties of these new mixed soft sets and these mixed continuities has been investigated. Consequently, decompositions of some soft continuities have been obtained. These concepts may be used other topological spaces and can be defined in different forms.

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