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THERMOELASTIC RESPONSE OF A LONG TUBE SUBJECTED TO PERIODIC HEATING

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Abstract. Thermoelastic behavior of a long tube subjected to periodic heating is investigated analytically. The temperature distribution in the tube under a periodic boundary condition is obtained by the solution of the heat conduction equation and the application of Duhamel's theorem afterwards. Based on plane strain condition, the thermoelastic solution of the tube is derived. This article outlines the complete solution procedure and presents some important results of relevance to engineering.

1. introduction

Cylindrical structures subjected to thermal loads are widely used in engineering applications. When the temperature gradient is high enough, stresses build up in the structure and, this results in the deformation of the body. A detailed understanding of these stresses is an important issue of engineering design to predict the failures and improve the reliability of the mechanical structures [17, 3, 4].

Elastic and plastic analysis of long cylinders and tubes under thermal or both mechanical and thermal loads have been extensively studied in the last decade. Orcan and Eraslan examined a uniform heat generating tube with temperature dependent material properties [14]. This study was then extended to the transient solution of the thermoelastic-plastic deformation of a heat generating tube by Eraslan and Orcan [8]. In a later work the authors also investigated the effect of various parameters on the critical heat transfer coefficient for the heat generating tube with convective boundary condition [9]. Eraslan and Argeso developed a computational model to estimate the stresses in plane strain axisymmetric structures in polar coordinates [6] and later extended this study to include the cylinders and tubes with temperature dependent physical properties [7, 1]. However, in spite of the importance and relevance of periodic heat loads and generation, very little work

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has been done on the analysis of stress and strain distributions in such structures. Jahanian extended the study of thick walled cylinder subjected to rapid heating and cooling to a cyclic thermal and mechanical loads [11, 10]. Mack and Plöchl presented a semi-analytical solution to the stress distribution in a rotating shrink fit with solid shaft subjected to a temperature cycle [12]. Segall developed a close-form axisymmetric solution to a thick-walled tube under an arbitrary thermal loading [16]. Radu et al. developed a set of analytical solutions for sinusoidal transient temperature field and the associated elastic thermal stress distributions in a hollow circular cylinder [15]. Arslan et al. studied the elastic-plastic stress distributions in a rotating solid shaft with stress free surfaces subject to a temperature cycle [2].

In the present work, the thermoelastic response of a long tube subjected to periodic heating on the inner surface is studied by analytical means. In this regard, the present investigation may be considered as the extension of the elastic part of the solution of Arslan et al. [2]. Though, the tube geometry adds considerable mathematical difficulties into the solution.

The heating of the tube is realized by the linearly increasing temperature from its inner surface. When the inner surface reaches a certain temperature, it is kept at that temperature for some time, and then lowered with the same rate to the initial temperature. The outer surface of the tube is assumed to be isolated. The temperature distribution in the tube is obtained by the application of Duhamel's theorem. The thermoelastic behavior of the tube is modeled by considering free inner and radially constrained outer surfaces. The generalized plane strain assumption is adopted in the formulation. The results of this solution are compared to those of a numerical solution based on a combination of collocation and shooting methods.

2. Formulation of the Problem

2.1. **Temperature Distribution.** A long tube of inner and outer radii a and b, respectively, is taken into account. The unsteady heat conduction equation in cylindrical coordinates reads

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = \frac{1}{\kappa}\frac{\partial T}{\partial t} \quad \text{in } \; a < r < b, \;\; t > 0, \tag{2.1}$$

where T represents the temperature in the tube, r the radial coordinate, κ the thermal diffusivity, and t the time. As the tube is initially at a zero reference temperature and possesses heated inner and isolated outer surfaces the initial and boundary conditions take the forms

$$T(r,0) = 0 \quad \text{for } \; a < r < b, \tag{2.2}$$

$$T(a,t) = f(t), \tag{2.3}$$

$$k\frac{\partial T}{\partial r}(b,t) = 0 \quad \text{for } t > 0, \tag{2.4}$$

where $f(t)$ represents time dependent surface temperature, and k the thermal conductivity. The inner surface temperature $f(t)$ is given by the following equations.

$$f(t) = \begin{cases} (T_m/t_t)t & \text{for} \quad 0 < t \le t_t \\ T_m & \text{for} \quad t_t < t \le t_t + t_c \\ T_m - (T_m/t_t)(t - t_t - t_c) & \text{for} \quad t_t + t_c < t \le 2t_t + t_c \\ 0 & \text{for} \quad t > 2t_t + t_c \end{cases} \tag{2.5}$$

Furthermore, the shape of $f(t)$ is depicted in Fig. 1, in which the meanings of the symbols T_m, t_t, and t_c can be deduced.

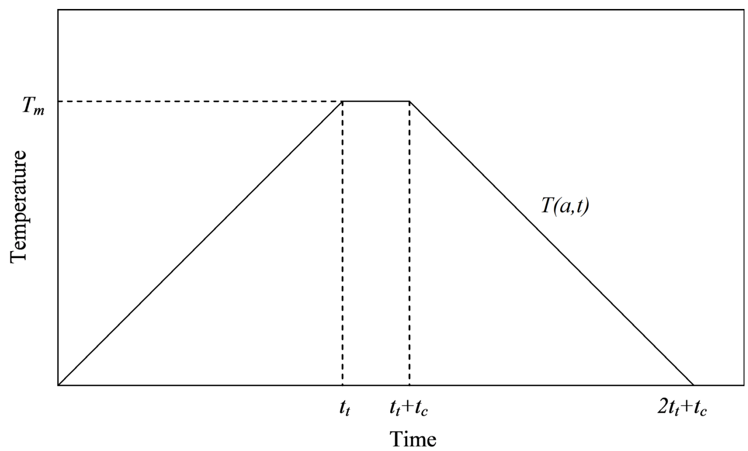

FIGURE 1. Periodic boundary condition applied to the inner surface of the tube.

In terms of the dimensionless variables: temperature: $\theta = T/T_m$, radial coordinate: $\overline{r} = r/b$, time: $\tau = \kappa t/b^2$, and inner radius $\overline{a} = a/b$, the heat conduction problem is rewritten as

$$\frac{\partial^2 \theta}{\partial r^2} + \frac{1}{r}\frac{\partial \theta}{\partial r} = \frac{\partial \theta}{\partial \tau} \quad \text{in} \;\; a < r < 1, \;\; \tau > 0, \tag{2.6}$$

$$\theta(r, 0) = 0 \quad \text{for} \;\; a < r < 1, \tag{2.7}$$

$$\theta(a, \tau) = f(\tau) = \tau/\tau_t, \tag{2.8}$$

$$\frac{\partial \theta}{\partial r}(1, \tau) = 0 \quad \text{for} \; \tau > 0, \tag{2.9}$$

where the overbars for r and a are not used for simplicity.

In order to treat periodic boundary condition Duhamel's theorem should be used. For this purpose an auxiliary problem is selected first. Here, it is chosen as

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{\partial \Phi}{\partial r} = \frac{\partial \Phi}{\partial \tau}, \tag{2.10}$$

with the following initial and boundary conditions

$$\Phi(r,0)=0 \qquad \text{for} \quad a<r<1, \tag{2.11}$$

$$\Phi(a,\tau)=1, \tag{2.12}$$

$$\frac{\partial \Phi}{\partial r}(1,\tau)=0 \qquad \text{for } \tau>0. \tag{2.13}$$

The solution of this auxiliary problem is given by Carslaw and Jaeger [5] as :

$$\Phi(r,\tau)=1+\pi\sum_{n=1}^{\infty} e^{-\lambda_n^2\tau}\frac{C_0(r,\lambda_n)}{F(\lambda_n)}P(\lambda_n) \tag{2.14}$$

where

$$\begin{aligned} C_0(r,\lambda_n)&=J_0(r\lambda_n)Y_0(a\lambda_n)-Y_0(r\lambda_n)J_0(a\lambda_n),\\ F(\lambda_n)&=J_0(a\lambda_n)^2-J_1(\lambda_n)^2,\\ P(\lambda_n)&=J_1(\lambda_n)^2, \end{aligned} \tag{2.15}$$

λ_n are the positive roots of the following eigenvalue equation,

$$\lambda\left[J_0(a\lambda)Y_1(\lambda)-Y_0(a\lambda)J_1(\lambda)\right]=0. \tag{2.16}$$

According to Duhamel's theorem, the solution of the original heat conduction problem is obtained by the integration of the solution of the auxiliary equation as

$$\theta(r,\tau)=\int_{\gamma=0}^{\tau} f(\gamma)\frac{\partial \Phi(r,\tau-\gamma)}{\partial \tau}d\gamma, \tag{2.17}$$

where

$$\Phi(r,\tau-\gamma)=1+\pi\sum_{n=1}^{\infty} e^{-\lambda_n^2(\tau-\gamma)}\frac{C_0(r,\lambda_n)}{F(\lambda_n)}P(\lambda_n), \tag{2.18}$$

$$\frac{\partial \Phi(r,\tau-\gamma)}{\partial \tau}=\pi\sum_{n=1}^{\infty}(-\lambda_n^2)e^{-\lambda_n^2(\tau-\gamma)}\frac{C_0(r,\lambda_n)}{F(\lambda_n)}P(\lambda_n), \tag{2.19}$$

and

$$f(\gamma)=\frac{\gamma}{\tau_t}. \tag{2.20}$$

Substituting Eqs. (2.19) and (2.20) into Eq.(2.17) and performing the integration, we obtain

$$\theta(r,\tau)=-\frac{\pi}{\tau_t}\tau\sum_{n=1}^{\infty}\frac{C_0(r,\lambda_n)}{F(\lambda_n)}P(\lambda_n)+\frac{\pi}{\tau_t}\sum_{n=1}^{\infty}(1-e^{-\lambda_n^2\tau})\frac{C_0(r,\lambda_n)}{\lambda_n^2F(\lambda_n)}P(\lambda_n). \tag{2.21}$$

Note that

$$\int_{\gamma=0}^{\tau}\gamma e^{\lambda^2\gamma}d\gamma=\frac{1}{\lambda^4}\left[1-e^{\lambda^2\tau}+\lambda^2\tau e^{\lambda^2\tau}\right]. \tag{2.22}$$

For $\tau=0$, the solution of the auxiliary problem Eq.(2.14) should be equal to the initial temperature $\Phi(r,0)=0$; thus

$$0 = 1 + \pi \sum_{n=1}^{\infty} \frac{C_0(r, \lambda_n)}{F(\lambda_n)} P(\lambda_n)$$

which gives the closed-form expression for the first series on the right hand side of Eq. (2.21) as

$$\pi \sum_{n=1}^{\infty} \frac{C_0(r, \lambda_n)}{F(\lambda_n)} P(\lambda_n) = -1 \tag{2.23}$$

Substituting Eq. (2.23) into Eq.(2.21) ,the final solution becomes

$$\theta(r, \tau) = \frac{\tau}{\tau_t} + \frac{\pi}{\tau_t} \sum_{n=1}^{\infty} (1 - e^{-\lambda_n^2 \tau}) \frac{C_0(r, \lambda_n)}{\lambda_n^2 F(\lambda_n)} P(\lambda_n). \tag{2.24}$$

2.2. **Thermoelastic Solution.** In this part, the stress and strain distributions in the tube are determined under thermal load. In addition to the nondimensional variables in the temperature distribution calculations, in this part, the dimensionless normal stress is $\overline{\sigma}_j = \sigma_j/\sigma_0$, normalized strain $\overline{\epsilon}_j = \epsilon_j G/\sigma_0$, dimensionless radial displacement $\overline{u}_j = u_j G/(\sigma_0 b)$, and dimensionless heat load parameter $q = \alpha T_m G/\sigma_0$, where σ_0, G, ν, and α denote the yield limit, the modulus of rigidity, the Poisson's ratio, and the thermal expansion coefficient, respectively. We do not use the overbars as before.

The equation of motion in the radial direction

$$(r\sigma_r)' - \sigma_\theta = 0, \tag{2.25}$$

the strain-displacement relations or geometric relations

$$\epsilon_r = u', \ \epsilon_\theta = \frac{u}{r}, \tag{2.26}$$

Generalized Hooke's law for cylindrical coordinates

$$\epsilon_r = \frac{1}{2(1+\nu)} [\sigma_r - \nu(\sigma_\theta + \sigma_z)] + q\theta(r, \tau), \tag{2.27}$$

$$\epsilon_\theta = \frac{1}{2(1+\nu)} [\sigma_\theta - \nu(\sigma_r + \sigma_z)] + q\theta(r, \tau), \tag{2.28}$$

$$\epsilon_z = \frac{1}{2(1+\nu)} [\sigma_z - \nu(\sigma_r + \sigma_\theta)] + q\theta(r, \tau). \tag{2.29}$$

are the basic equations of the system. Solving Eq.(2.29) for the axial stress based on generalized plane strain condition $\epsilon_z = \epsilon_0$ we obtain

$$\sigma_z = \nu(\sigma_r + \sigma_\theta) + 2(1+\nu) [\epsilon_0 - q\theta(r, \tau)] . \tag{2.30}$$

By using geometric relations, and Hooke's law, elastic stresses are written in terms of radial displacement as

$$\sigma_r = \frac{2}{r(1-2\nu)}[\nu u + (1-\nu)ru' + (\epsilon_0\nu - (1+\nu))q\theta(r,\tau)], \tag{2.31}$$

$$\sigma_\theta = \frac{2}{r(1-2\nu)}[(1-\nu)u + \nu ru' + (\epsilon_0\nu - (1+\nu))q\theta(r,\tau)]. \tag{2.32}$$

Substituting the stresses into the equation of motion Eq.(2.25), the nonhomogeneous Cauchy-Euler equation is obtained

$$r^2\frac{d^2u}{dr^2} + r\frac{du}{dr} - u = \frac{r^2q(1+\nu)}{1-\nu}\theta'(r,\tau). \tag{2.33}$$

Homogeneous solution of Eq.(2.33) is

$$u_h(r) = \frac{C_1}{r} + C_2 r, \tag{2.34}$$

where C_1 , C_2 are arbitrary integration constants. Particular solution is obtained by the method of variation of parameters as

$$u_p(r) = Y_1(r)U_1(r) + Y_2(r)U_2(r) \tag{2.35}$$

The fundamental solutions Y_1 and Y_2 are $Y_1(r) = 1/r$, $Y_2(r) = r$ and,

$$U_1(r) = -\int \frac{Y_2(r)F(r)}{W(r)}dr \text{ and, } \quad U_2(r) = \int \frac{Y_1(r)F(r)}{W(r)}dr, \tag{2.36}$$

where the Wronskian is $W(r) = 2/r$, and $F(r)$ is the non-homogeneous part of the Eq.(2.33)

$$F(r) = \frac{q(1+\nu)}{1-\nu}\theta'(r,\tau). \tag{2.37}$$

Hence

$$U_1(r) = -\frac{q(1+\nu)}{2(1-\nu)}\int r^2\theta'(r,\tau)dr\ , \quad U_2(r) = \frac{q(1+\nu)}{2(1-\nu)}\theta(r,\tau). \tag{2.38}$$

Therefrom, the particular solution is obtained as

$$u_p(r) = \frac{q(1+\nu)}{2(1-\nu)}\theta(r,\tau) - \frac{q(1+\nu)}{2r(1-\nu)}\int r^2\theta'(r,\tau)dr. \tag{2.39}$$

As

$$\int r^2\theta'(r,\tau)dr = r^2\theta(r,\tau) - 2\int r\theta(r,\tau)dr,$$

by integrations by parts, the final form of the particular solution becomes

$$u_p(r) = \frac{q(1+\nu)}{r(1-\nu)}\int_a^r \xi\theta(\xi,\tau)d\xi. \tag{2.40}$$

By summing up the homogeneous and particular solutions, the general solution to the radial displacement takes the form

$$u = \frac{C_1}{r} + C_2 r + \frac{q(1+\nu)}{r(1-\nu)}\int_a^r \xi\theta(\xi,\tau)d\xi. \tag{2.41}$$

The radial and circumferential stresses are then expressed as

$$\sigma_r = -\frac{2C_1}{r^2} + \frac{2}{1-2\nu}(C_2+\nu\epsilon_0) - \frac{2q(1+\nu)}{(1-\nu)r^2}\int_a^r \xi\theta(\xi,\tau)d\xi \tag{2.42}$$

$$\sigma_\theta = \frac{2C_1}{r^2} + \frac{2}{1-2\nu}(C_2+\nu\epsilon_0) - \frac{2q(1+\nu)}{(1-\nu)r^2}\left(r^2\theta(r,\tau) - \int_a^r \xi\theta(\xi,\tau)d\xi\right). \tag{2.43}$$

If we substitute σ_r and σ_θ into σ_z ,Eq.(2.30), the axial stress is also derived in the form

$$\sigma_z = \frac{2}{1-2\nu}\left(2\nu C_2 + (1-\nu)\epsilon_0\right) - \frac{2q(1+\nu)}{1-\nu}\theta(r,\tau). \tag{2.44}$$

To complete the solution the unknowns: C_1, C_2, and ϵ_0 are to be calculated. Free inner and radially constrained outer surfaces imply the conditions

$$\sigma_r = 0\ , \quad \text{at } r = a, \tag{2.45}$$

$$u = 0\ , \quad \text{at } r = 1. \tag{2.46}$$

Since the tube is allowed to expand freely in the axial direction we have the additional condition

$$F_z = 2\pi\int_a^1 r\sigma_z dr = 0, \tag{2.47}$$

in which F_z is the axial force. The third condition is then obtained from $F_z = 0$ as

$$-\frac{2q(1+\nu)}{(1-\nu)}\int_a^1 r\theta(r,\tau)dr - \frac{\left(a^2-1\right)}{1-2\nu}\left[(1-\nu)\epsilon_0 + 2\nu C_2\right] = 0.$$

The unknowns are obtained by means of these three conditions as

$$C_1 = \frac{a^2(1+\nu)(a^2(1+\nu)-1+\nu)}{(1-a^2)(1-\nu)\left[1-\nu+a^2(1+\nu)\right]}q\int_a^1 r\theta(r,\tau)dr, \tag{2.48}$$

$$C_2 = \frac{(1+\nu)(a^2(1-3\nu)-1+\nu)}{(1-a^2)(1-\nu)\left[1-\nu+a^2(1+\nu)\right]}q\int_a^1 r\theta(r,\tau)dr, \tag{2.49}$$

$$\epsilon_0 = \frac{2(1+\nu)(1+a^2)}{(1-a^2)\left[1-\nu+a^2(1+\nu)\right]}q\int_a^1 r\theta(r,\tau)dr. \tag{2.50}$$

The temperature integral in these equations is determined as

$$\begin{aligned}\int_a^r \xi\theta(\xi,\tau)d\xi &= \frac{\tau}{\tau_t}r + \frac{\pi}{\tau_t}\sum_{n=1}^{\infty}(1-e^{-\lambda_n^2\tau})\frac{P(\lambda_n)}{\lambda_n^2F(\lambda_n)}\int_a^r \xi C_0(\xi,\lambda_n)\\ &= \frac{\tau}{\tau_t}\left\{\frac{(r^2-a^2)}{2}\right\}\\ &+ \frac{\pi}{\tau_t}\sum_{n=1}^{\infty}(1-e^{-\lambda_n^2\tau})\frac{P(\lambda_n)}{\lambda_n^3F(\lambda_n)}\left\{Y_0(a\lambda_n)\left[rJ_1(r\lambda_n)-aJ_1(a\lambda_n)\right]\right.\\ &\left.-J_0(a\lambda_n)\left[rY_1(r\lambda_n)-aY_1(a\lambda_n)\right]\right\}.\end{aligned} \tag{2.51}$$

3. Numerical Results of the Solutions

The inner radius of the tube is taken as $a = 0.5$. In addition, the temperature distribution calculations are carried out by using the following parameters: $\tau_t = 1.2$, $\tau_c = 0.4$. The period is completed when $\tau = 2.8$. The heat conduction equation is also solved numerically by using a semi-analytical collocation method [13]. Fig. 2 shows the temperature distribution time history in the tube corresponding to important heating and cooling paths. In this figure solid lines belong to the analytical solution and dots to numerical solution. As seen in Fig. 2, analytical and numerical results are in perfect agreement.

The thermoelastic response of the tube corresponding to the temperature distribution depicted in Fig. 2 is then calculated. In these calculations, the following numerical values of the parameters are used: Poisson ratio $\nu = 0.287$, and the heat load parameter $q = 0.15$. Fig. 3 and 4 show, respectively, the evolution of nondimensional radial and tangential stress distributions during the periodic heating. The variation of the radial displacement is depicted in Fig. 5. The stresses and displacement increase with the increasing temperature until $\tau = 1.2$, and start to decrease when the cooling starts at $\tau = 1.6$. In these figures, as before, solid lines show the analytical results, while dots the numerical results. Numerical solution of the mechanical equations are performed by using the shooting method. To integrate the initial value system Runge-Kutta-Fehlberg fourth-fifth order integration method is used. Fig. 6 presents the evolution of the axial strain for different values of inner radius during the temperature cycle. The axial strain slightly changes and increases with the decreasing thickness of the tube. It is noted that the axial strain follows the similar trend as the plot of the periodic heating boundary condition.

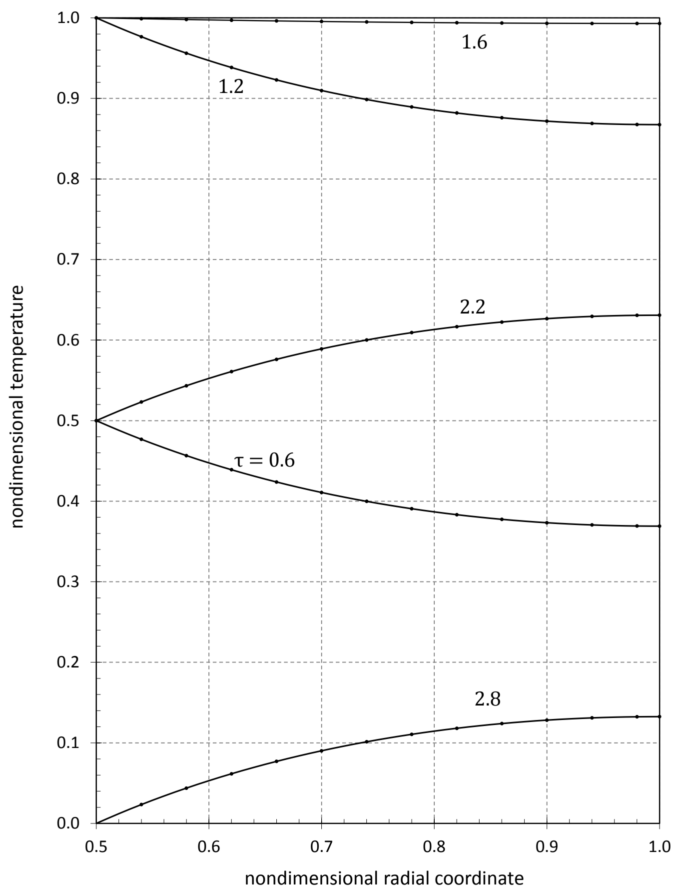

FIGURE 2. Temperature distribution in the tube at different time steps. Solid line denotes the analytical solution, while dotes indicate the numerical. The parameters are used: $\tau_t = 1.2$ and $\tau_c = 0.4$.

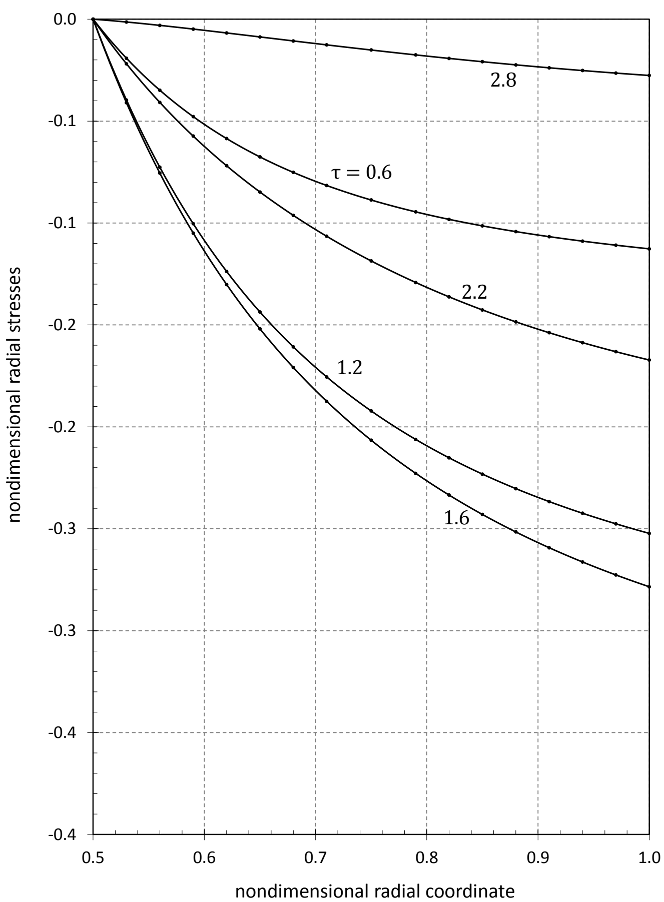

FIGURE 3. Nondimensional radial stress. Solid lines denote the analytical solution, while dotes indicate the numerical results. $q = 0.15$.

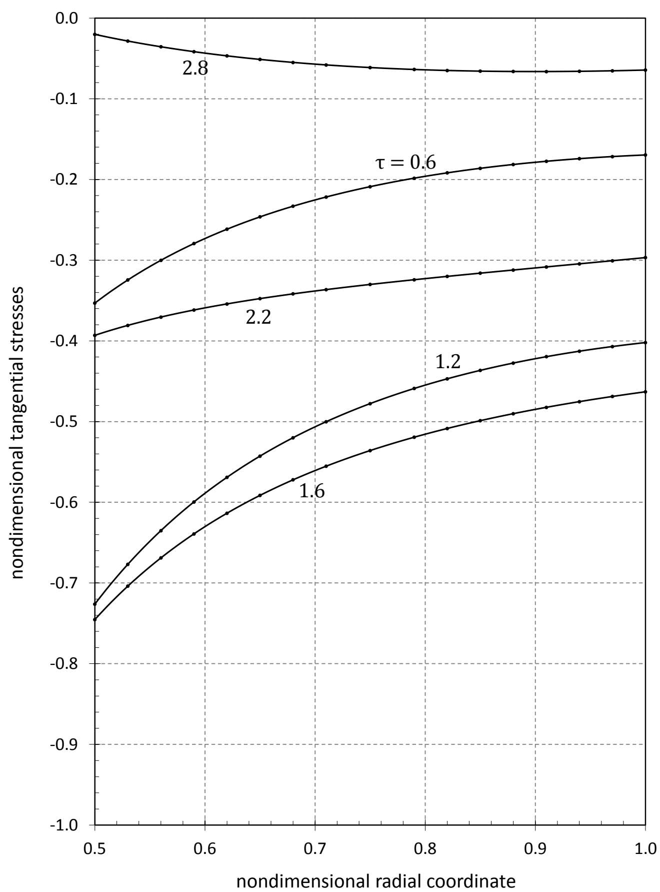

FIGURE 4. Nondimensional tangential stress. Solid lines denote the analytical solution, while dotes indicate the numerical results. $q = 0.15$.

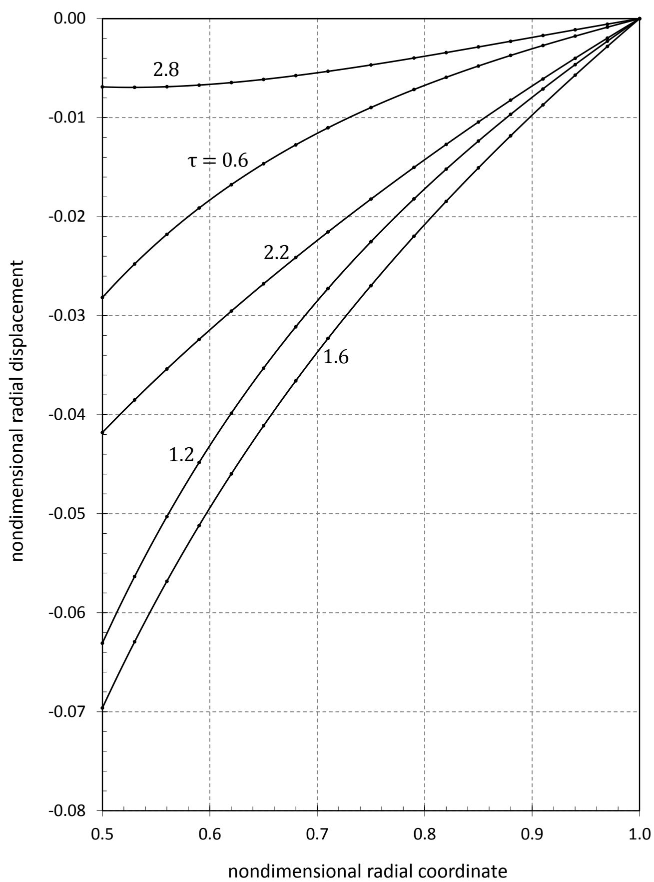

FIGURE 5. Nondimensional radial displacement. Solid lines denote the analytical solution, while dotes indicate the numerical results. $q = 0.15$.

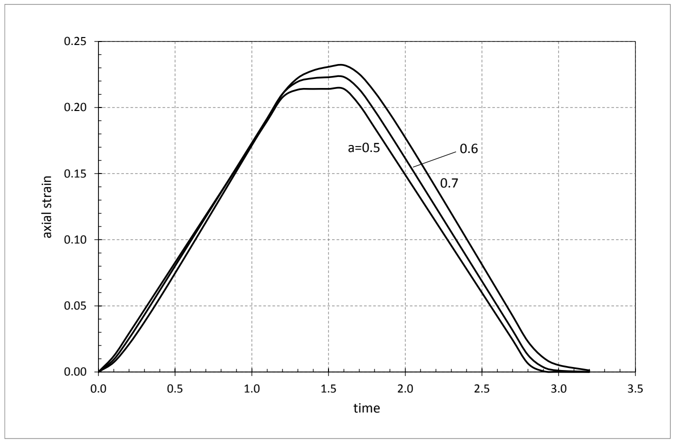

FIGURE 6. Evolution of the axial strain for different values of the inner radius during periodic heating. $q = 0.15$.

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