Mathematical Sciences And Applications E-Notes
Volume 1 No. 2 pp. 11–19 (2013) ©MSAEN

CARTAN-TYPE CRITERIONS FOR CONSTANCY OF ALMOST HERMITIAN MANIFOLDS

HAKAN METE TAŞTAN

(Communicated by Bayram ŞAHİN)

Abstract. We studied the axiom of anti-invariant 2-spheres and the axiom of co-holomorphic $(2n+1)$-spheres. We proved that a nearly Kählerian manifold satisfying the axiom of anti-invariant 2-spheres is a space of constant holomorphic sectional curvature. We also showed that an almost Hermitian manifold M of dimension $2m \geq 6$ satisfying the axiom of co-holomorphic $(2n+1)$-spheres for some n, where $(1 \leq n \leq m-1)$, the manifold M has pointwise constant type α if and only if M has pointwise constant anti-holomorphic sectional curvature α.

1. Introduction

E. Cartan [1] introduced the axiom of n-planes as: A Riemannian manifold M of dimension $m \geq 3$ is said to satisfy the axiom of n-planes, where n is a fixed integer $2 \leq n \leq m-1$, if for each point $p \in M$ and each n-dimensional subspace σ of the tangent space $T_p(M)$, there exists an n-dimensional totally geodesic submanifold N such that $p \in N$ and $T_p(N) = \sigma$. He also gave a criterion for constancy of sectional curvature for any Riemannian manifold of dimension $m \geq 3$ in the following theorem.

Theorem 1.1. *A Riemannian manifold of dimension $m \geq 3$ with the axiom of n-planes is a real space form.*

D.S. Leung and K. Nomizu [16] introduced *the axiom of n-spheres* by using totally umbilical submanifold N with parallel mean curvature vector field instead of totally geodesic submanifold N in the axiom of n-planes. They proved a generalization of Theorem 1.1.

Later on, Cartan's idea was applied to almost Hermitian manifolds in various studies. Kählerian manifolds were studied in [2, 5, 9, 14, 17, 22, 24]. The articles [20] and [21] discussed nearly Kählerian (almost Tachibana) manifolds. The results concerning larger classes of almost Hermitian manifolds can be found in [12, 13,

Date: Received: February 27, 2013; Revised: May 22, 2013; Accepted: June 3, 2013.
2000 *Mathematics Subject Classification.* 53B35, 53B25, 53C40.
Key words and phrases. Almost Hermitian manifold, Sectional curvature, Holomorphic sectional curvature, Constant type, Axiom of spheres.

18, 19, 20]. Here, we shall call the criterions used in all of the above papers as *Cartan-type criterions.*

2. Preliminaries

2.1. **Some classes of almost Hermitian manifolds.** Let M be an almost Hermitian manifold with an almost complex structure J in its tangent bundle and a Riemannian metric g such that $g(JX, JY) = g(X, Y)$ for all $X, Y \in \chi(M)$, where $\chi(M)$ is the Lie algebra of C^∞ vector fields on M. Let ∇ be the Riemannian connection on M. The Riemannian curvature tensor R associated with ∇ is defined by $R(X, Y) = \nabla_{[X,Y]} - [\nabla_X, \nabla_Y]$. We denote $g(R(X, Y)Z, U)$ by $R(X, Y, Z, U)$. Curvature identities are of fundamental importance for understanding the geometry of almost Hermitian manifolds. The following curvature identities are used in various studies e.g.([7, 8]):

(1) $R(X, Y, Z, U) = R(X, Y, JZ, JU)$,
(2) $R(X, Y, Z, U) = R(JX, JY, Z, U) + R(JX, Y, JZ, U)$
$+R(JX, Y, Z, JU)$,
(3) $R(X, Y, Z, U) = R(JX, JY, JZ, JU)$.

Let AH_i denote the subclass of the class AH of almost Hermitian manifolds satisfying the curvature identity (i)., $i = 1, 2, 3$. We know that

$$AH_1 \subset AH_2 \subset AH_3 \subset AH,$$

from [7]. Some authors call AH_1-manifold as a *para-Kählerian manifold* and call AH_3-manifold as an *RK-manifold* ([19]). An almost Hermitian manifold M is called *Kählerian* if $\nabla_X J = 0$ for all $X \in \chi(M)$ and *nearly Kählerian (almost Tachibana)* if $(\nabla_X J)X = 0$ for all $X \in \chi(M)$. It is well-known that a Kählerian manifold is AH_1-manifold and a nearly Kählerian manifold (non para-Kählerian) manifold is AH_2-manifold, see ([8, 19]).

A two-dimensional linear subspace of a tangent space $T_p(M)$ is called a *plane section.* A plane section σ is said to be *holomorphic* (resp. *anti-holomorphic* or *totally real*) if $J\sigma = \sigma$ (resp. $J\sigma \perp \sigma$) ([2], [22]). The sectional curvature K of M which is determined by orthonormal vector fields X and Y is given by $K(X, Y) = R(X, Y, X, Y)$. The sectional curvature of M restricted to a holomorphic (resp. an anti-holomorphic) plane σ is called *holomorphic* (resp. *anti-holomorphic*) *sectional curvature.* If the holomorphic (resp. anti-holomorphic) sectional curvature at each point $p \in M$ does not depend on σ, then M is said to be *pointwise constant holomorphic* (resp. *pointwise constant anti-holomorphic) sectional curvature.* A connected Riemannian (resp. Kählerian) manifold of (global) constant sectional curvature (resp. of constant holomorphic sectional curvature) is called a *real space form* (resp. a *complex space form*) ([2, 20, 12, 23]). The following useful notion was defined by A. Gray in [6].

Definition 2.1. Let M be an almost Hermitian manifold. Then M is said to be of *constant type* at $p \in M$ provided that for all $X \in T_p(M)$, we have $\lambda(X, Y) = \lambda(X, Z)$ whenever the planes $span\{X, Y\}$ and $span\{X, Z\}$ are anti-holomorphic and $g(Y, Y) = g(Z, Z)$, where the function λ is defined by $\lambda(X, Y) = R(X, Y, X, Y) - R(X, Y, JX, JY)$. If this holds for all $p \in M$, then we say that M has *(pointwise) constant type.* Finally, if for $X, Y \in \chi(M)$ with $g(X, Y) = g(JX, Y) = 0$, the value

$\lambda(X,Y)$ is constant whenever $g(X,X) = g(Y,Y) = 1$, then we say that M has *global constant type.*

It follows that any AH_1-manifold has global vanishing constant type from Definition 2.1.

Let M be a $2m$-dimensional Kählerian manifold, for all $X,Y,Z,U \in T_p(M)$ and $p \in M$, the Bochner curvature tensor B [15] is defined by

$$\begin{aligned} B(X,Y,Z,U) = R(X,Y,Z,U) &- L(Y,Z)g(X,U) + L(Y,U)g(X,Z) \\ &- L(X,U)g(Y,Z) + L(X,Z)g(Y,U) - L(Y,JZ)g(X,JU) \\ &+ L(Y,JU)g(X,JZ) - L(X,JU)g(Y,JZ) + L(X,JZ)g(Y,JU) \\ &+ 2L(Z,JU)g(X,JY) + 2L(X,JY)g(Z,JU), \end{aligned}$$

where $L = \frac{\varrho}{2(m+2)} - \frac{\tau}{8(m+1)(m+2)}g$, ϱ is the *Ricci tensor* and τ is the *scalar curvature* of M. It is well-known that the Bochner curvature tensor is a complex analogue of the Weyl conformal curvature tensor [23] on a Riemannian manifold.

2.2. **Submanifolds of a Riemannian manifold.** Let N be a submanifold of a Riemannian manifold M with a Riemannian metric g. Then Gauss and Weingarten formulas are respectively given by $\nabla_X Y = \hat{\nabla}_X Y + h(X,Y)$ and $\nabla_X \xi = -A_\xi X + \nabla^\perp_X \xi$ for all $X, Y \in \chi(N)$ and $\xi \in \chi^\perp(N)$. Here $\nabla, \hat{\nabla}$, and $\nabla^\perp$ are respectively the Riemannian, induced Riemannian, and induced normal connection in M, N, and the normal bundle $\chi^\perp(N)$ of N, and h is the second fundamental form related to shape operator A corresponding to the normal vector field ξ by $g(h(X,Y),\xi) = g(A_\xi X, Y)$. A submanifold N is said to be *totally geodesic* if its second fundamental form identically vanishes: $h = 0$, or equivalently $A_\xi = 0$. We say that N is *totally umbilical* submanifold in M if for all $X, Y \in \chi(N)$, we have

$$h(X,Y) = g(X,Y)\eta \ , \tag{2.1}$$

where $\eta \in \chi^\perp(N)$ is the mean curvature vector field of N in M. A vector field $\xi \in \chi^\perp(N)$ is said to be *parallel* if $\nabla^\perp_X \xi = 0$ for each $X \in \chi(N)$. The Codazzi equation is given by

$$(R(X,Y)Z)^\perp = (\nabla_X h)(Y,Z) - (\nabla_Y h)(X,Z) \ , \tag{2.2}$$

for all $X,Y,Z \in \chi(N)$, where $^\perp$ denotes the normal component and the covariant derivative of h denoted by $\nabla_X h$ is defined by

$$(\nabla_X h)(Y,Z) = \nabla^\perp_X (h(Y,Z)) - h(\hat{\nabla}_X Y, Z) - h(Y, \hat{\nabla}_X Z) \ , \tag{2.3}$$

for all $X,Y,Z \in \chi(N)$ ([2, 5, 9, 22]).

2.3. **Anti-invariant submanifolds of an almost Hermitian manifold.** Let M be a $2m$-dimensional almost Hermitian manifold endowed with an almost complex structure J and a Hermitian metric g. An n-dimensional Riemannian manifold N isometrically immersed in M is called an *anti-invariant submanifold* of M (or *totally real submanifold* of M) if $JT_p(N) \subset T_p(N)^\perp$ for each point p of N. Then we have $m \geq n$ ([23]).

3. The axiom of anti-invariant 2-spheres

S. Yamaguchi and M. Kon [22] introduced the axiom of anti-invariant 2-spheres as: An almost Hermitian manifold M is said to satisfy the axiom of anti-invariant 2-spheres, if for each point $p \in M$ and each anti-holomorphic 2-plane σ of the tangent space $T_p(M)$, there exists a 2-dimensional totally umbilical anti-invariant submanifold N such that $p \in N$ and $T_p(N) = \sigma$. They proved in Theorem 1([22]) that a Kählerian manifold with the axiom of anti-invariant 2-spheres is a complex space form. Here, we give a generalization of Theorem 1([22]). From now on, we shall assume that all manifolds are connected throughout this study.

We shall need the following Lemma for the proof of the main Theorem 3.1 below.

Lemma 3.1. *([11]) Let N be an anti-invariant submanifold of a nearly Kählerian manifold M. Then*

$$A_{JZ}X = A_{JX}Z \tag{3.1}$$

holds for any two vectors X and Z tangent to N, where A is the shape operator of N.

Let N be given as in Lemma 3.1, then

$$g(A_{JZ}X, Y) = g(A_{JX}Z, Y) = g(A_{JY}X, Z) \ , \tag{3.2}$$

for any vectors X, Y and Z tangent to N, from (3.1). This equation is equivalent to

$$g(h(X, Y), JZ) = g(h(Z, Y), JX) = g(h(X, Z), JY) \ , \tag{3.3}$$

where h is the second fundamental form of N.

Theorem 3.1. *Let M be a nearly Kählerian manifold of dimension $2m \geq 6$. If M satisfies the axiom of anti-invariant 2-spheres, then M is a space of constant holomorphic sectional curvature.*

Proof. Let p be any point of M. Let X and Y be any orthonormal vectors of $T_p(M)$ spanning an anti-holomorphic 2-plane σ. By the axiom of anti-invariant 2-spheres, there exists a two-dimensional totally umbilical anti-invariant submanifold N such that $p \in N$ and $T_p(N) = \sigma$. Then, we have

$$(R(X, Y)Y)^{\perp} = \nabla^{\perp}_X \eta \ , \tag{3.4}$$

with the help of (2.1) and (2.3) from (2.2), where η is the mean curvature vector field of N in M. We find

$$R(X, Y, Y, JX) = g(\nabla^{\perp}_X \eta, JX) \ , \tag{3.5}$$

from (3.4), since JX is normal to N. If we put $Y = Z$ into (3.3), we obtain

$$g(\eta, JX) = 0 \ , \tag{3.6}$$

by using (2.1). Similarly, we also have $g(\eta, JY) = g(\eta, JZ) = 0$, for any vectors $X, Y,$ and Z tangent to N. If we differentiate the equation (3.6) with respect to X, then we have

$$0 = X[g(\eta, JX)] = g(\nabla^{\perp}_X \eta, JX) + g(\eta, \nabla^{\perp}_X JX) \ . \tag{3.7}$$

Upon straightforward calculation, we see that $g(\eta, \nabla^{\perp}_X JX) = 0$. Thus, we obtain

$$g(\nabla^{\perp}_X \eta, JX) = 0 \ , \tag{3.8}$$

from (3.7). We get

$$R(X,Y,Y,JX) = 0 \tag{3.9}$$

by combining (3.8) with (3.5). For all orthonormal vectors $X, Y \in T_p(M)$ with $g(X, JY) = 0$, we derive

$$R(JY,Y,Y,JX) = 0, \tag{3.10}$$

by replacing X by $\frac{1}{\sqrt{2}}(X+JY)$ in (3.9). In this case, it follows that M has pointwise constant holomorphic sectional curvature, using (3.10) and Lemma 1([12]). We conclude that M is a space of constant holomorphic sectional curvature by Theorem 6([10]). □

4. The axiom of co-holomorphic $(2n+1)$-spheres

Let M be a $2m$-dimensional almost Hermitian manifold. L. Vanhecke [20] defined a co-holomorphic $(2n+1)$-plane as a $(2n+1)$-plane containing a holomorphic $2n$-plane for the manifold M. It is not difficult to see that a co-holomorphic $(2n+1)$-plane contains an anti-holomorphic $(n+1)$-plane and that $1 \leq n \leq m-1$. He also gave the axiom of co-holomorphic $(2n+1)$-spheres as: A $2m$-dimensional almost Hermitian manifold M which is said to satisfy the axiom of co-holomorphic $(2n+1)$-spheres, if for each point $p \in M$ and each co-holomorphic $(2n+1)$-plane σ of the tangent space $T_p(M)$, there exists a $(2n+1)$-dimensional totally umbilical submanifold N such that $p \in N$ and $T_p(N) = \sigma$. He studied this axiom for AH_3-manifolds and obtained several results.

Now, we study this axiom for larger classes of almost Hermitian manifolds.

Lemma 4.1. *Let M be an almost Hermitian manifold of dimension $2m \geq 4$. If M satisfies the axiom of co-holomorphic $(2n+1)$-spheres, then we have*

$$\lambda(X,Y) = K(X,Y)\ , \tag{4.1}$$

for all orthonormal vectors $X, Y \in T_p(M)$ with $g(X, JY) = 0$, where $\lambda(X,Y) = R(X,Y,X,Y) - R(X,Y,JX,JY)$ and K denotes anti-holomorphic sectional curvature.

Proof. Let p be an arbitrary point of M. Let X and Y be any orthonormal vectors in $T_p(M)$ with $g(X, JY) = 0$, that is, they span an anti-holomorphic plane. Consider the co-holomorphic $(2n+1)$-plane σ containing X, JX, and Y such that JY is normal to σ. By the axiom of co-holomorphic $(2n+1)$-spheres, there exists a $(2n+1)$-dimensional totally umbilical submanifold N such that $p \in N$ and $T_p(N) = \sigma$. Then, we have

$$(R(X,Y)JX)^{\perp} = 0\ . \tag{4.2}$$

with the help of (2.1) and (2.3), from (2.2). We get

$$R(X,Y,JX,JY) = 0\ , \tag{4.3}$$

from (4.2), since JY is normal to N. Thus, our assertion follows from Definition 2.1 and (4.3). □

Now, we are ready to prove our second main result.

Theorem 4.1. *Let M be an almost Hermitian manifold of dimension $2m \geq 6$. If M satisfies the axiom of co-holomorphic $(2n+1)$-spheres for some n, then M has pointwise constant type α if and only if M has pointwise constant anti-holomorphic sectional curvature α.*

Proof. Let M be an almost Hermitian manifold of dimension $2m \geq 6$ satisfying the axiom of co-holomorphic $(2n+1)$-spheres for some n. If M has pointwise constant type, that is, M has constant type at p, for all $p \in M$. Then, for all $X, Y, Z \in T_p(M)$ whenever the planes $span\{X, Y\}$ and $span\{X, Z\}$ are anti-holomorphic and $g(Y, Y) = g(Z, Z)$, we have

$$\lambda(X, Y) = \lambda(X, Z) \ . \tag{4.4}$$

Here, we can assume that $g(Y, Y) = g(Z, Z) = 1$. Thus, for all orthonormal vectors $X, Y, Z \in T_p(M)$ with $g(X, JY) = g(X, JZ) = 0$, we get

$$K(X, Y) = K(X, Z) \ , \tag{4.5}$$

from Lemma 4.1.
On the other hand, we can choose a unit vector U in $(span\{X, JX\})^{\perp} \cap (span\{Z, JZ\})^{\perp}$, since the dimension of M is greater than 6. Then, we have

$$K(X, U) = K(X, Z) \ , \tag{4.6}$$

from (4.5). This implies that the sectional curvature is same for all anti-holomorphic sections which contain any given vector X. Hence, we write

$$K(X, Y) = K(Y, Z) = K(Z, U) \ . \tag{4.7}$$

Therefore, we find

$$K(X, Y) = K(Z, U) \ , \tag{4.8}$$

for all $X, Y, Z, U \in T_p(M)$ whenever the planes $span\{X, Y\}$ and $span\{Z, U\}$ are anti-holomorphic. It follows that the sectional curvature is same for all anti-holomorphic sections at $p \in M$. Namely, M has pointwise constant anti-holomorphic sectional curvature.
Conversely, let M be of pointwise constant anti-holomorphic sectional curvature and let p be any point of M. Then for all orthonormal vectors $X, Y, Z \in T_p(M)$ with $g(X, JY) = g(X, JZ) = 0$, ($span\{X, Y\}$ and $span\{X, Z\}$ are anti-holomorphic planes and $g(X, X) = g(Y, Y) = g(Z, Z) = 1$), we have

$$K(X, Y) = K(X, Z) \ . \tag{4.9}$$

By Lemma 4.1, we get

$$\lambda(X, Y) = \lambda(X, Z) \ , \tag{4.10}$$

for all orthonormal vectors $X, Y, Z \in T_p(M)$ whenever the planes $span\{X, Y\}$ and $span\{X, Z\}$ are anti-holomorphic. It is not difficult to see that (4.10) also holds in the case $g(Y, Y) = g(Z, Z) \neq 1$. It follows that M has constant type at p. Additionally, if the constant value of $\lambda(X, Y)$ equals α, then the pointwise constant anti-holomorphic sectional curvature K must be α, because of Lemma 4.1. □

We remark that above technical method was used also in Theorem 3.4([18]).

Next, we give some applications of Theorem 4.1.

Theorem 4.2. *Let M be a $2m$-dimensional almost Hermitian manifold with pointwise constant type α and $m \geq 3$. If M satisfies the axiom of co-holomorphic $(2n+1)$-spheres for some n, then*
i) *M is a space of constant curvature α and M has global constant type α,*
ii) *M is an AH_2-manifold.*

Proof. Let p be any point of M. Let X and Y be any orthonormal vectors in $T_p(M)$ with $g(X, JY) = 0$. Consider the co-holomorphic $(2n+1)$-plane σ containing X, JX, and JY such that Y is normal to σ. By the axiom of co-holomorphic $(2n + 1)$-spheres, there exists a $(2n + 1)$-dimensional totally umbilical submanifold N such that $p \in N$ and $T_p(N) = \sigma$. Then, we have

$$(R(X, JX)JX)^{\perp} = 0 \ . \tag{4.11}$$

with the help of (2.1) and (2.3), from (2.2). Hence, we get

$$R(X, JX, JX, Y) = 0 \ , \tag{4.12}$$

for all orthonormal vectors $X, Y \in T_p(M)$ with $g(X, JY) = 0$, since Y is normal to N. It follows that M is an AH_3-manifold with pointwise constant holomorphic sectional curvature using (4.12) and Lemma 1([12]) together with Lemma 3([12]). On the other hand, we have that M has constant anti-holomorphic sectional curvature α at p, from Theorem 4.1. In this case, we obtain that the constant holomorphic sectional curvature H of M is α at p from Theorem 5([19]). By using Theorem 4([19])in Theorem 2([19]), we obtain

$$K(X, Y) = \alpha \ , \tag{4.13}$$

for all orthonormal vectors $X, Y \in T_p(M)$, where $K(X, Y) = R(X, Y, X, Y)$ is sectional curvature. It is not difficult to see that (4.13) is also true for all $X, Y \in T_p(M)$. By the well-known Schur's theorem([23]) it follows that M is a space of constant curvature α and M has global constant type.
Now, we prove the part **ii)**. From the part **i)**, automatically, both holomorphic and anti-holomorphic sectional curvature equal to α. In which case, it follows from Theorem 3([4]) that M is an AH_2-manifold. □

Remark 4.1. Theorem 4.2 without part **ii)** was also obtained by O.T. Kassabov in ([13]) with different approach. It is a generalization of Theorem 1([20]) concerning AH_3-manifolds.

By Theorem 4.1, Theorem 4.2 and Theorem 5([19]), we have the following result which is a generalization of Corollary 1([20]) concerning AH_3-manifolds.

Corollary 4.1. *Let M be a $2m$-dimensional almost Hermitian manifold with vanishing constant type and $m \geq 3$. If M satisfies the axiom of co-holomorphic $(2n+1)$-spheres for some n, then M is a flat AH_2-manifold.*

We end this paper by giving a result related to the Bochner curvature tensor of a Kählerian manifold satisfying the axiom of co-holomorphic $(2n + 1)$-spheres.

Theorem 4.3. *Let M be a Kählerian manifold of dimension $2m \geq 6$. If M satisfies the axiom of co-holomorphic $(2n + 1)$-spheres for some n, then M has a vanishing Bochner curvature tensor.*

Proof. Let p be any point of M. Let X, Y, and Z be any unit vectors of $T_p(M)$, which span an anti-holomorphic 3-plane, that is, $g(X,Y) = g(X,Z) = g(Y,Z) = 0$, and $g(X,JY) = g(X,JZ) = g(Y,JZ) = 0$. Consider the co-holomorphic $(2n+1)$-plane σ containing X, JX, and Y such that Z is normal to σ. By the axiom of co-holomorphic $(2n+1)$-spheres, there exists a $(2n+1)$-dimensional totally umbilical submanifold N such that $p \in N$ and $T_p(N) = \sigma$. Then, we have

$$(R(X,JX)Y)^{\perp} = 0 \ , \tag{4.14}$$

and

$$(R(X,Y)JX)^{\perp} = 0 \ . \tag{4.15}$$

with the help of (2.1) and (2.3) from (2.2). For all unit vectors $X, Y, Z \in T_p(M)$, which span an anti-holomorphic 3-plane, we respectively get

$$R(X,JX,Y,Z) = 0 \tag{4.16}$$

and

$$R(X,Y,JX,Z) = 0 \ , \tag{4.17}$$

from (4.14) and (4.15), since Z is normal to N. Thus, our assertion follows from (4.16), (4.17) and Lemma([14]). □

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Department of Mathematics, İstanbul University, Vezneciler, 34134, İstanbul, TURKEY
E-mail address: hakmete@istanbul.edu.tr